VARIATIONAL ADAPTIVE METHODS FOR OBJECT / BACKGROUND IDENTIFICATION IN PHYSICAL DATA.

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Problem formalization.

Let us consider the simplest representation of the data $f(i)$:

$$f(i) = f_0(i) + b(i), \quad i = 1, 2, 3, ..., m. \quad (1)$$

as linear combination of an interesting component $f_0(i)$ and that of background $b(i)$.
$f_0 \in F_0$, but $b(i) \in B, \quad B \cap F_0 = \emptyset$.
Our goal is to get $b_e(i)$ - an estimate of $b(i)$, i.e. a function which belongs to the class B, but not to F_0.

We define formally our solution $b_e(i)$ as an envelope from below, because it touches only minima of the peak-like functions in the sum $s(i)$. Qualitatively, $b_e(i)$ roughly contours $s(i)$ at the points of its minima and is more or less smooth and slowly varying between these minima.

Mathematically, we should build a metric ρ, which provides for the closeness of $b_e(i)$ to $b(i)$, and ignoring of high values of peak-like functions, where $b_e(i)$ should be smooth enough. Then $b_e(i)$ is a solution of the following variational problem:

minimize the expression $\rho(b_e(i), s(i))$ under certain conditions imposed on $b_e(i)$.

As such a metric the sum of the weighted quadratic deviations and of squared first differences of $b_e(i)$ can be taken. So formally we can write our problem so: minimize

$$\sum_{i=2}^{m-1} b_e'(i)^2 + \beta \cdot q \cdot \sum_{i=1}^{m} w(i) \cdot (b_e(i) - s(i))^2 \quad (2)$$

under the boundary conditions
$b_e(1) = s(1), \quad b_e(m) = s(m),$
and an inequality

$$b_e(i) \leq s(i) \text{ for all } i. \quad (3)$$

$b_e'(i)$ is the first difference (discrete analog of the first derivative).
The parameter β regulates degrees of the influence of the 1st or 2nd members in (2): more smooth or more close to $s(i)$;
q is the normalization coefficient, $q = 1/\sum w(i)$.

The weights $w(i)$, providing for the above-numbered qualities of the solution in the general case may fail to exist. Therefore, we use an iterational process, the steps of which include:

1. building the weights providing the closeness of the solution mainly to the low-valued ordinates of $s(i)$ - for getting an initial estimate of $b_e(i)$;
2. building the weights providing the maximum distance of the solution from the high-valued ordinates of $s(i)$ - for getting the final estimate of $b_e(i)$.

Now we have to define the sequential values of $w(i)$ in order to make the solution to touch only the minima of $s(i)$ and be a smooth function between these minima.

The first step is obtaining the initial estimate of $b_e(i)$; let it be $b_0(i)$. The weights here should be inversely proportional either to some degree of $s(i)$ or $s''(i)$.

At the next steps the weights should be inversely proportional to some degree of the $||s(i) - b_n i||$, where b_n is the current estimate of $b_e(i)$, and n is the number of the iteration.

The convergence of the iteration process is rather stable if the region where the spectrum is low-valued is not strongly overlapped by the region of high-valued spectrum ordinates, and the number of iterations is not large.

Method for solution.

Euler's equation for finding minimum of (2) is

$$b_e(i+1) - 2b_e(i) + b_e(i-1) - \beta w_n(i) b_e(i) =$$
$$-\beta w_n(i) s(i), \quad i = 1, ..., m-1,$$

at $b_e(0) = s(0), \quad b_e(m) = s(m)$ and
$w_n(i) = w(i)q.$

This equation can be solved by the "chasing" method

$$b_e(i) = c(i) b_e(i+1) + h(i), \;\; c(0) = 0, \;\; h(0) = s(0), i =$$

If we put $b_e(i)$ into the equation, we get

$$b_e(i+1) - 2b_e(i) + c(i-1) b_e(i) + h(i-1) - \beta w_n(i) b_e(i) =$$

From this we get the formula for coefficients c and h

$$c(i) = 1/(2 + \beta w_n(i) - c(i-1)),$$
$$h(i) = c(i)(\beta w_n(i) s(i) + h(i-1)),$$

using which, we find solutions at each iteration in succession.

The projection $p(i)$ of the solution on the inequalities (3) is made simply $p(i) = b_e(i)$, if (3) holds, and $p(i) = s(i)$ otherwise;

This $p(i)$ is the solution of our problem.