Elastic NC neutrino-nucleon scattering

and strangeness of the nucleon

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Looking for strangeness with ν scattering

The measurement of NC neutrino cross sections

$$\nu_\mu(\overline{\nu}_\mu) + N \longrightarrow \nu_\mu(\overline{\nu}_\mu) + N \tag{1}$$

is very important tool for the determination of the matrix elements of the strange current:

$$\langle p, s|\bar{S}\gamma^\alpha\gamma^5 S|p, s\rangle = 2Ms^\alpha g_A^s$$

$S, \bar{S}$ strange quark fields

$|p, s\rangle$ proton (momentum, spin) state vector.

CC processes also considered:

$$\begin{aligned} \nu_\mu + n &\longrightarrow \mu^- + p\,, \\ \overline{\nu}_\mu + p &\longrightarrow \mu^+ + n\,. \end{aligned} \tag{2}$$

One nucleon matrix element of axial quark current:

$$\langle p, s|\bar{q}\gamma^\alpha\gamma^5 q|p, s\rangle = 2Ms^\alpha g_A^q \qquad (q = u, d, s)$$

constants g_A^u, g_A^d, g_A^s determined from:

- QCD sum rule (polarized structure function, from DIS)

$$\Gamma_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$$

  where

$$\Delta q = \int_0^1 \sum_{r=\pm 1} r\left[q^{(r)}(x) + \overline{q}^{(r)}(x)\right] dx$$

  is difference of the total numbers of quarks and antiquarks in the nucleon with helicity equal and opposite to the helicity of the nucleon.

  $\Rightarrow$ Δq is the contribution of the q-quarks and $\bar{q}$-antiquarks to the spin of the proton.

- relation $g_A = g_A^u - g_A^d$
  with $g_A = 1.2573 \pm 0.0028$ from neutron decay
- relation $3F - D = g_A^u + g_A^d - 2g_A^s$
  F, D from semileptonic decay of hyperons.

Determination of various g_A^q subject to several assumptions.

$\Rightarrow$ Need independent (possibly model-independent) determination of this quantity.

Basic ingredients

Standard Lagrangian of interaction of leptons and quarks with vector bosons

- Neutral Current Lagrangian

$$\mathcal{L}_I^{NC} = -\frac{g}{2\cos\theta_W} j_\alpha^{NC} Z^\alpha \,,$$

where θ_W is Weinberg angle, $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}$, j_α^{NC} is the neutral current

$$\begin{aligned} j_\alpha^{NC} &= 2j_\alpha^3 - 2\sin^2\theta_W j_\alpha^{em} \\ &= \sum_{q=u,c,t} \overline{q}\gamma_\alpha \frac{(1-\gamma_5)}{2} q - \sum_{q=d,s,b} \overline{q}\gamma_\alpha \frac{(1-\gamma_5)}{2} q + \\ &+ \sum_{\ell=e,\mu,\tau} \overline{\nu}_\ell \gamma_\alpha \frac{(1-\gamma_5)}{2} \nu_\ell - \sum_{\ell=e,\mu,\tau} \overline{\ell}\gamma_\alpha \frac{(1-\gamma_5)}{2} \ell + -2\sin^2\theta_W j_\alpha^{em} \,. \end{aligned}$$

In the quark sector:

$$j_{\alpha}^{NC;q} = v_{\alpha}^{3} - a_{\alpha}^{3} - \frac{1}{2}\left(v_{\alpha}^{s} - a_{\alpha}^{s}\right) - 2\sin^2\theta_W j_{\alpha}^{em}\,.$$

with

$$v_{\alpha}^{3} = \overline{u}\gamma_{\alpha}\frac{1}{2}u - \overline{d}\gamma_{\alpha}\frac{1}{2}d \equiv \overline{N}\gamma_{\alpha}\frac{1}{2}\tau_3 N,$$

$$a_{\alpha}^{3} = \overline{u}\gamma_{\alpha}\gamma_5\frac{1}{2}u - \overline{d}\gamma_{\alpha}\gamma_5\frac{1}{2}d \equiv \overline{N}\gamma_{\alpha}\gamma_5\frac{1}{2}\tau_3 N,$$

Here $N = (u\ d)$ (doublet of isotopic SU(2) group)

$$v_{\alpha}^{i} = \overline{N}\gamma_{\alpha}\frac{1}{2}\tau^{i}N, \qquad \text{isovector vector current}$$

$$a_{\alpha}^{i} = \overline{N}\gamma_{\alpha}\gamma_5\frac{1}{2}\tau^{i}N \qquad \text{isovector axial}-\text{vector current}$$

Instead v_α^s and a_α^s are isoscalars:

$$v_\alpha^s = \overline{s}\gamma_\alpha s, \quad a_\alpha^s = \overline{s}\gamma_\alpha\gamma_5 s$$

- Electromagnetic interaction Lagrangian

$$\mathcal{L}_I^{em} = -e j_\alpha^{em} A^\alpha \, ,$$

with

$$j_\alpha^{em} = \sum_{\ell=e,\mu,\tau} (-1)\overline{\ell}\gamma_\alpha \ell + \sum_{q=u,d,\ldots} e_q \overline{q}\gamma_\alpha q$$

NB

$$j_\alpha^{em;q} = v_\alpha^3 + v_\alpha^0 \qquad v_\alpha^0 = \frac{1}{6}\overline{N}\gamma_\alpha N - \frac{1}{3}\overline{s}\gamma_\alpha s$$

- Charged current Lagrangian

$$\mathcal{L}_I^{CC} = -\frac{g}{2\sqrt{2}}\, j_\alpha^{CC} W^\alpha + \text{h.c.}$$

where j_α^{CC} expressed in terms of left-handed fields is

$$j_\alpha^{CC} = 2 \sum_{\ell=e,\mu,\tau} \overline{\nu}_{\ell L} \gamma_\alpha \ell_L + 2 \left[\overline{u}_L \gamma_\alpha d_L^{\text{mix}} + \overline{c}_L \gamma_\alpha s_L^{\text{mix}} + \overline{t}_L \gamma_\alpha b_L^{\text{mix}} \right]$$

and the quark fields are related to fields with definite masses by usual CKM mixing matrix:

$$d_L^{\text{mix}} = \sum_{q=d,s,b} V_{uq} q_L, \quad s_L^{\text{mix}} = \sum_{q=d,s,b} V_{cq} q_L, \quad b_L^{\text{mix}} = \sum_{q=d,s,b} V_{tq} q_L,$$

If only light quarks are considered:

$$j_\alpha^{CC;q} = 2V_{ud}(j_\alpha^1 + i j_\alpha^2) = 2V_{ud}\overline{u}\gamma_\alpha(1-\gamma_5)d$$

Neutrino-nucleon elastic (NC) and inelastic (CC) scattering (A)

Consider the NC processes

$$\nu_\mu(\overline{\nu}_\mu) + N \longrightarrow \nu_\mu(\overline{\nu}_\mu) + N$$

The amplitude are given by

$$\langle f|S|i\rangle = \mp\frac{G_F}{\sqrt{2}}\overline{u}(k')\gamma^\alpha\left(1\mp\gamma_5\right)u(k)\langle p'|J_\alpha^{NC}|p\rangle(2\pi)^8\delta^{(4)}(p'-p-q)$$

k and k': momenta of the initial and final neutrino (antineutrino)
p and p': momenta of the initial and final nucleon, $q = k - k'$ and

$$J_\alpha^{NC} = V_\alpha^{NC} - A_\alpha^{NC}$$

States and current operators are in Heisenberg representation.
Explicitly:

$$\langle p'|J_\alpha^{NC}(0)|p\rangle = \langle p'|\left(V_\alpha^3 - A_\alpha^3\right)|p\rangle - \frac{1}{2}\langle p'|\left(V_\alpha^s - A_\alpha^s\right)|p\rangle + -2\sin^2\theta_W\langle p'|J_\alpha^{em}|p\rangle$$

Isotopic invariance of strong interactions implies:

$$ {}_p\langle p'|V^3_\alpha|p\rangle_p = -{}_n\langle p'|V^3_\alpha|p\rangle_n\,, \qquad {}_p\langle p'|V^0_\alpha|p\rangle_p = +{}_n\langle p'|V^0_\alpha|p\rangle_n $$

Moreover, being:

$$ {}_{p(n)}\langle p'|J^{em}_\alpha|p\rangle_{p(n)} = {}_{p(n)}\langle p'|V^3_\alpha|p\rangle_{p(n)} + {}_{p(n)}\langle p'|V^0_\alpha|p\rangle_{p(n)} $$

the matrix elements of vector component of NC current are:

$$ {}_p\langle p'|V^3_\alpha|p\rangle_p = \frac{1}{2}\left[{}_p\langle p'|J^{em}_\alpha|p\rangle_p - {}_n\langle p'|J^{em}_\alpha|p\rangle_n\right] = -{}_n\langle p'|V^3_\alpha|p\rangle_n $$

Remember the general form:

$$ \langle p'|J^{em}_\alpha|p\rangle = \overline{u}(p')\left[\gamma_\alpha F_1(Q^2) + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2(Q^2)\right]u(p) $$

with $F_1(0) = e_N, \quad F_2(0) = \kappa_N$ (anomalous magnetic moment).

Similarly for the CC processes

$$\begin{aligned} \nu_\mu + n &\longrightarrow \mu^- + p \\ \overline{\nu}_\mu + p &\longrightarrow \mu^+ + n \end{aligned}$$

the amplitudes are given by

$$\begin{aligned} \langle f|S|i\rangle &= -i\frac{G_F}{\sqrt{2}}\overline{u}(k')\gamma^\alpha(1-\gamma_5)u(k)_p\langle p'|J_\alpha^{CC}|p\rangle_n\,(2\pi)^4\delta^{(4)}(p'-p-q) \\ \langle f|S|i\rangle &= -i\frac{G_F}{\sqrt{2}}\overline{u}(k')\gamma^\alpha(1+\gamma_5)u(k)_n\langle p'|J_\alpha^{CC\,\dagger}|p\rangle_p\,(2\pi)^4\delta^{(4)}(p'-p-q) \end{aligned}$$

where

$$j_\alpha^{CC} = V_{ud}\overline{N}\gamma_\alpha(1-\gamma_5)\frac{1}{2}(\tau_1 + i\tau 2)N \equiv v_\alpha^{1+i2} - a_\alpha^{1+i2}$$

NB: charge symmetry of strong interactions entails

$$_p\langle p'|V_\alpha^{1+i2}|p\rangle_n = {}_n\langle p'|V_\alpha^{1-i2}|p\rangle_p = {}_p\langle p'|J_\alpha^{em}|p\rangle_p - {}_n\langle p'|J_\alpha^{em}|p\rangle_n\,.$$

Moreover:

$$ {}_p\langle p'|A_\alpha^{1+i2}|p\rangle_n = {}_n\langle p'|A_\alpha^{1-i2}|p\rangle_p = {}_p\langle p|A_\alpha^{1+i2}|p'\rangle_n^* $$

One nucleon matrix elements of the currents

- vector and axial NC:

$$ {}_{p(n)}\langle p'|V_\alpha^{NC}|p\rangle_{p(n)} = \overline{u}(p')\left[\gamma_\alpha F_1^{NC;p(n)}(Q^2) + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^{NC;p(n)}(Q^2)\right]u(p) $$

$$ {}_{p(n)}\langle p'|A_\alpha^{NC}|p\rangle_{p(n)} = \overline{u}(p')\gamma_\alpha\gamma_5 G_A^{NC;p(n)}u(p) + \overline{u}(p')\frac{1}{2M}G_P(Q^2)Q_\alpha\gamma_5 u(p) $$

where the NC form factors are given by

$$ F_{1,2}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{F^p_{1,2}(Q^2) - F^n_{1,2}(Q^2)\right\} - 2\sin^2\theta_W F_{1,2}^{p(n)}(Q^2) - \frac{1}{2}F^s_{1,2}(Q^2 $$

$$ G_A^{NC;p(n)}(Q^2) = \pm\frac{1}{2}G_A(Q^2) - \frac{1}{2}G_A^s(Q^2) $$

Equivalently, NC Sachs form factors are used ($G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$):

$$G_E{}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{G_E^p(Q^2) - G_E^n(Q^2)\right\} - 2\sin^2\theta_W G_E^{p(n)}(Q^2) - \frac{1}{2}G_E^s(Q^2$$

$$G_M{}^{NC;p(n)}(Q^2) = \pm\frac{1}{2}\left\{G_M^p(Q^2) - G_M^n(Q^2)\right\} - 2\sin^2\theta_W G_M^{p(n)}(Q^2) - \frac{1}{2}G_M^s(Q$$

- vector and axial CC:

$$_p\langle p'|V_\alpha^{1+i2}|p\rangle_n = \overline{u}(p')\left[\gamma_\alpha F_1^{CC}(Q^2) + \frac{i}{2M}\sigma_{\alpha\beta}q^\beta F_2^{CC}(Q^2)\right]u(p)$$

(or with Sach's F.F.)

$$_p\langle p'|A_\alpha^{1+i2}|p\rangle_n = \overline{u}(p')\left[\gamma_\alpha\gamma_5 G_A(Q^2) + \frac{1}{2M}q_\alpha\gamma_5 G_P^{CC}(Q^2)\right]u(p)$$

where usually $G_P^{CC}(Q^2)$ can be neglected.

Interesting Observables

Consider ν–proton elastic cross sections or ν–nucleus elastic and inelastic cross sections

NC over CC ratio (considered at Fermilab):

$$R_{NC/CC}(Q^2) = \frac{\left(d\sigma/dQ^2\right)_{\nu}^{NC}}{\left(d\sigma/dQ^2\right)_{\nu}^{CC}}$$

Proton to neutron ratio (in quasielastic processes with emission of one nucleon)

$$R^{\nu}_{p/n}(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)_{(\nu,p)}^{NC}}{\left(\frac{d\sigma}{dQ^2}\right)_{(\nu,n)}^{NC}}$$

Neutrino-antineutrino Asymmetry:

$$\mathcal{A}(Q^2) = \frac{\left(\dfrac{d\sigma}{dQ^2}\right)_{\nu}^{NC} - \left(\dfrac{d\sigma}{dQ^2}\right)_{\overline{\nu}}^{NC}}{\left(\dfrac{d\sigma}{dQ^2}\right)_{\nu}^{CC} - \left(\dfrac{d\sigma}{dQ^2}\right)_{\overline{\nu}}^{CC}}$$

Neutrino-nucleon elastic (NC) and inelastic (CC) scattering (B)

NC Differential cross sections:

$$\left(\frac{d\sigma}{dQ^2}\right)^{NC}_{\nu(\overline{\nu})} = \frac{G_F^2}{2\pi}\left[\frac{1}{2}y^2(G_M^{NC})^2 + \left(1 - y - \frac{M}{2E}y\right)\frac{(G_E^{NC})^2 + \frac{E}{2M}y(G_M^{NC})^2}{1 + \frac{E}{2M}y}\right.$$

$$\left. + \left(\frac{1}{2}y^2 + 1 - y + \frac{M}{2E}y\right)(G_A^{NC})^2 \pm 2y\left(1 - \frac{1}{2}y\right)G_M^{NC}G_A^{NC}\right].$$

with

$$y = \frac{p\cdot q}{p\cdot k} = \frac{Q^2}{2p\cdot k}$$

E is the energy of neutrino (antineutrino) in the laboratory system.

CC Differential cross sections:

$$\left(\frac{d\sigma}{dQ^2}\right)^{CC}_{\nu(\overline{\nu})} = \frac{G_F^2}{2\pi}\left[\frac{1}{2}y^2(G_M^{CC})^2 + \left(1 - y - \frac{M}{2E}y\right)\frac{(G_E^{CC})^2 + \frac{E}{2M}y(G_M^{CC})^2}{1 + \frac{E}{2M}y} +\right.$$

$$\left. + \left(\frac{1}{2}y^2 + 1 - y + \frac{M}{2E}y\right)(G_A)^2 \pm 2y\left(1 - \frac{1}{2}y\right)G_M^{CC}G_A\right].$$

Normally employed flux averaged neutrino cross sections:

$$\langle\frac{d\sigma}{dQ^2}\rangle^{NC}_{\nu(\overline{\nu})} = \frac{\int dE_{\nu(\overline{\nu})}\left(d\sigma/dQ^2\right)^{NC}_{\nu(\overline{\nu})}\Phi_{\nu(\overline{\nu})}\left(E_{\nu(\overline{\nu})}\right)}{\int dE_{\nu(\overline{\nu})}\Phi_{\nu(\overline{\nu})}\left(E_{\nu(\overline{\nu})}\right)}.$$

The $\nu - \bar{\nu}$ asymmetry

The neutrino-antineutrino asymmetry in $\nu(\bar{\nu})$-nucleon elastic scattering reads:

$$\mathcal{A}_{p(n)} = \frac{1}{4}\left(\pm 1 - \frac{G_A^s}{G_A}\right)\left(\pm 1 - 2\sin^2\theta_W \frac{{G_M}^{p(n)}}{G_M^3} - \frac{1}{2}\frac{G_M^s}{G_M^3}\right) .$$

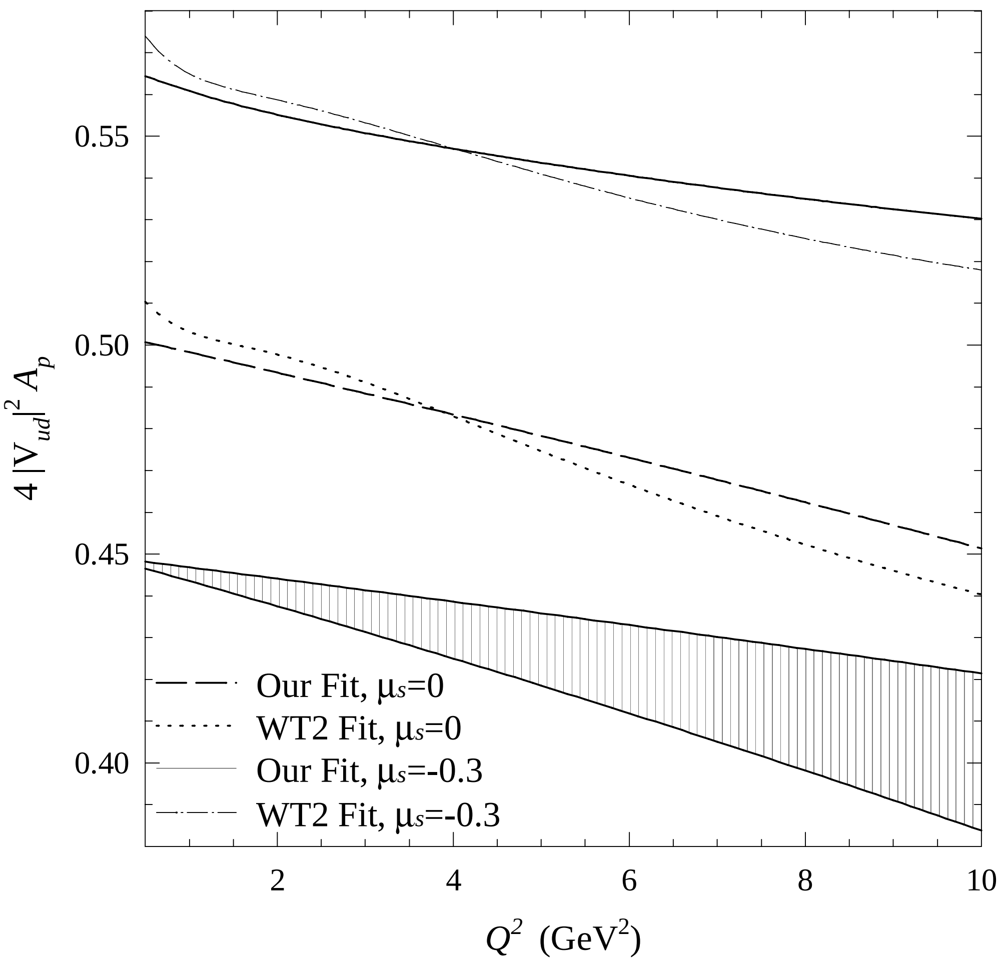
Thus, in the asymmetry $\mathcal{A}$ the strange axial and vector form factors enter in the form of ratios, G_A^s/G_A and G_M^s/G_M^3.

Taking into account only terms which linearly depend on the strange form factors:

$$\mathcal{A}_{p(n)} = \mathcal{A}_{p(n)}^0 \mp \frac{1}{8}\frac{G_M^s}{G_M^3} \mp \frac{G_A^s}{G_A}\mathcal{A}_{p(n)}^0$$

with

$$\mathcal{A}_{p(n)}^0 = \frac{1}{4}\left(1 \mp 2\sin^2\theta_W \frac{{G_M}^{p(n)}}{G_M^3}\right)$$



0.55
0.50
0.45
0.40
$4\,|V_{ud}|^2\,A_p$
2
4
6
8
10
Q^2 (GeV^2)
Our Fit, μ_s=0
WT2 Fit, μ_s=0
Our Fit, μ_s=-0.3
WT2 Fit, μ_s=-0.3

Form factor parameterization

Electromagnetic form-factors = dipole/Galster parameterization

$$G_A = 1.26\left(1+\frac{Q^2}{M_A^2}\right)^{-2} \quad ;$$

$$G_A^s = g_A^s\left(1+\frac{Q^2}{M_A^2}\right)^{-2} \quad \text{same } M_A \text{ as } G_A$$

$$G_M^s = \mu_s G_V^{\text{dipole}}(Q^2) \quad ; \; G_E^S = \rho_s \tau G_V^{\text{dipole}}(Q^2)$$

$$G_V^{\text{dipole}}(Q^2) = \left(1+\frac{Q^2}{M_V^2}\right)^{-2}$$

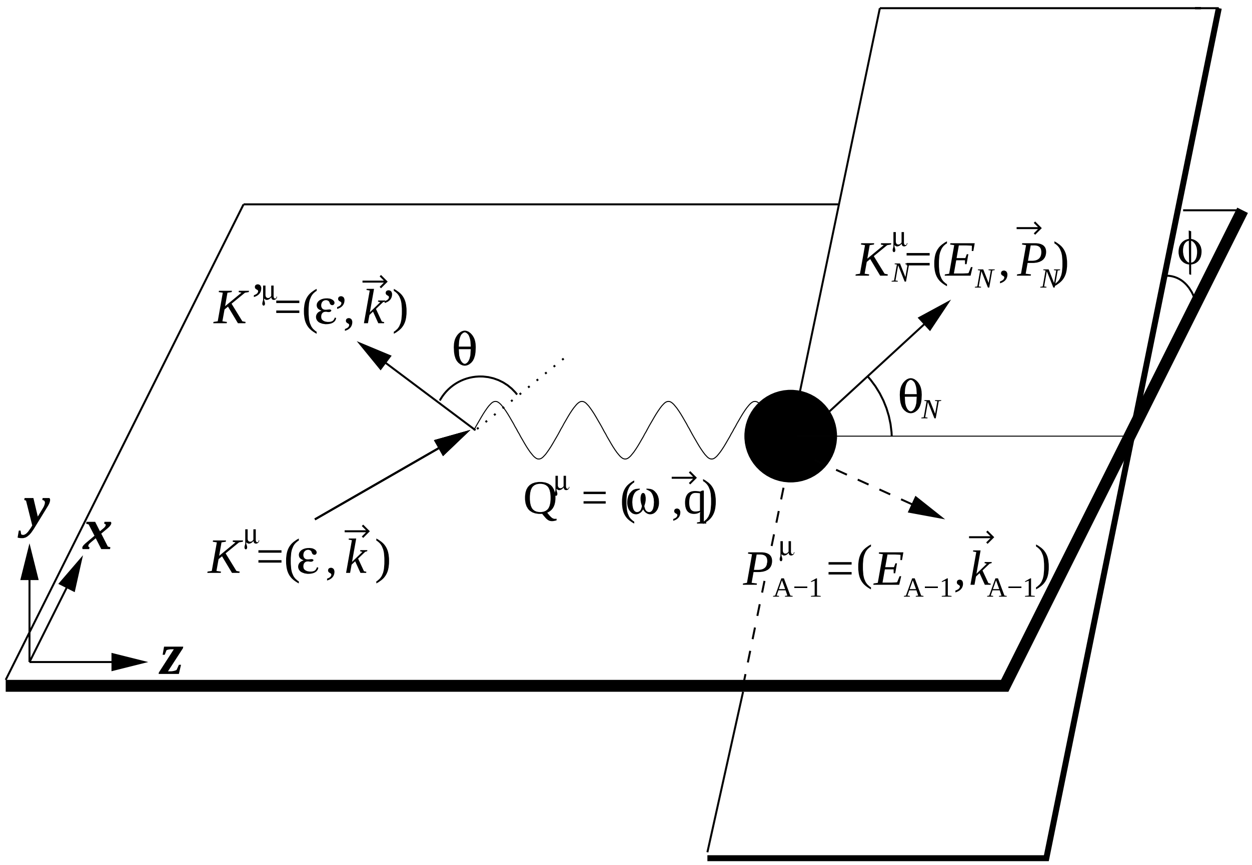
Neutrino nucleus scattering

Neutrino scattering realized both on free and bound nucleons: relevance in considering the effects of nuclear structure and dynamics.

Processes on a nucleus:

$$\nu_\mu(\overline{\nu}_\mu) + A \longrightarrow \nu_\mu(\overline{\nu}_\mu) + N + (A-1) \qquad \text{NC process}$$

$$\nu_\mu(\overline{\nu}_\mu) + A \longrightarrow \mu^-(\mu^+) + p(n) + (A-1) \qquad \text{CC process}$$



$K'^{\mu}=(\varepsilon', \vec{k}')$
θ
$K_N^{\mu}=(E_N, \vec{P}_N)$
ϕ
θ_N
$Q^{\mu} = (\omega, \vec{q})$
$K^{\mu}=(\varepsilon, \vec{k})$
$P_{A-1}^{\mu}=(E_{A-1}, \vec{k}_{A-1})$
y
x
z

CROSS SECTIONS

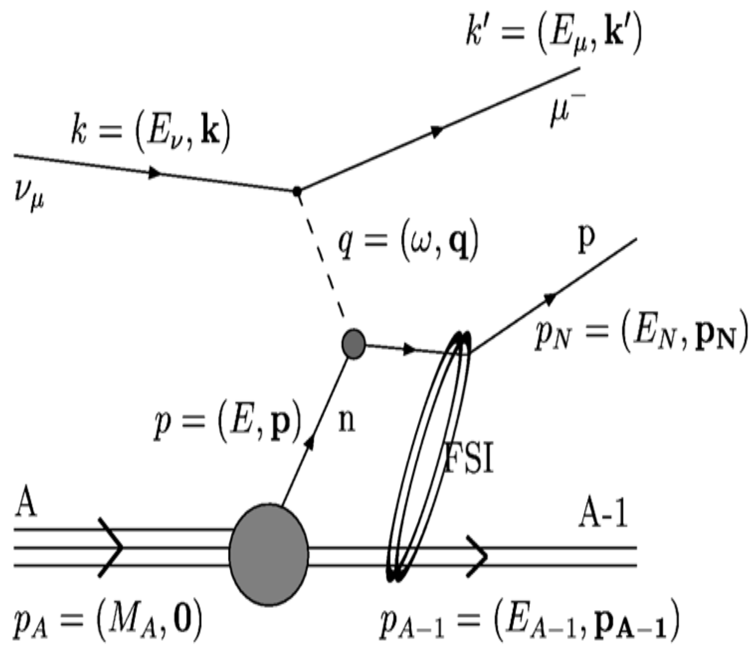
$$\frac{d\sigma}{d^3k'd^3p_N} = \frac{G_F^2}{2(2\pi)^5}\frac{2}{\epsilon\epsilon'}\eta_{\mu\nu}W^{\mu\nu}$$

$$\eta_{\mu\nu} \quad = \quad K_\mu K'_\nu - g_{\mu\mu}K\cdot K' + K'_\mu K_\nu \mp \epsilon_{\mu\nu\rho\sigma}K^\rho K'^\sigma$$

$$W^{\mu\nu} = \sum_{(A-1)} \langle A-1,\phi_N|\hat{J}^\mu(\mathbf{q})|A\rangle\langle A-1,\phi_N|\hat{J}^\nu(\mathbf{q})|A\rangle^*\delta(E_A+\omega-E_{A-1}-E_N)$$

Impulse Approximation

- neutrino interacts with only one nucleon in the target, which is then emitted, remaining (A-1) nucleons are spectators
- nuclear current sum of single nucleon currents
- target and residual nuclei described within and independent particle model



$$\langle A-1, \phi_N | \hat{J}^\mu | A \rangle \rightarrow \langle \phi_N | \hat{J}^\mu_{S.N.} | \psi_B \rangle = \int d\mathbf{r} e^{i\mathbf{q}\cdot\mathbf{r}} \overline{\phi}_N(\mathbf{r}) \hat{J}^\mu_{S.N.} \psi_B(\mathbf{r})$$

ϕ_N: outgoing nucleon wave functions

- plane wave Dirac spinor $\rightarrow$ PWIA
- distorted wave: ϕ_N is scattering solution of a Dirac-like equation

$$[i\vec{\alpha}\cdot\nabla - \beta(M+U_S) + E - U_V - U_C]\,\phi(\mathbf{r}) = 0\ ,$$

with scalar, and vector complex relativistic optical potentials (ROP) obtained from fits of elastic pA data (E.D. Cooper et al., PRC 47, 297 (1993)).

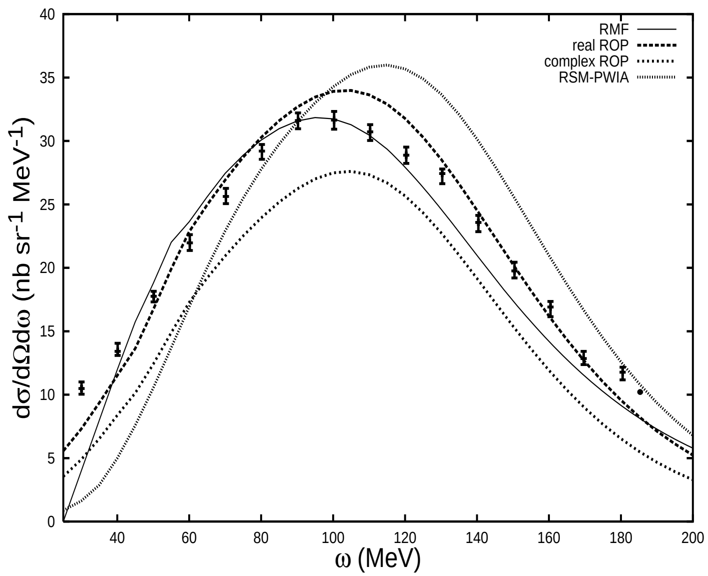
$$\phi_N(\vec{r}) = 4\pi\sqrt{\frac{E_N+M_N}{2E_N}}\sum_{\kappa\mu m} e^{-i\delta^*_\kappa} i^\ell \langle \ell m \frac{1}{2} s_N | j\mu\rangle \times Y_\ell^{m*}(\Omega_{k_N})\Psi^\mu_\kappa(\vec{r})\,,$$

Real part of potential: re-scattering

Imaginary part: absorption into unobserved channels

FSI in inclusive CC processes

- "real ROP": consider ϕ_N obtained by setting the imaginary part of the ROP to zero (flux conservation)
- relativistic mean field (RMF): consider ϕ_N solutions in the continuum of the same Dirac equation used to describe the initial bound nucleon.



Comparison of descriptions of FSI with electron scattering (e, e') data for a ^{12}C target and momentum transfer $q \simeq 400$ MeV/c.

RELATIVISTIC FERMI GAS (RFG)

In RFG the NC cross sections read:

$$\left(\frac{d^2\sigma}{dE_N d\Omega_N}\right)_{\nu(\overline{\nu})} = \frac{G_F^2}{(2\pi)^2}\frac{3\mathcal{N}}{4\pi p_F^3}\frac{M^2|\vec{p}_N|}{k_0}\int \frac{d^3k'}{k'_0}\frac{d^3p}{p_0}\times$$

$$\times\delta^{(3)}\left(\vec{k}-\vec{k}'+\vec{p}-\vec{p}_N\right)\delta\left(k_0-k'_0+p_0-E_N\right)\times$$

$$\times\theta(p_F-|\vec{p}\,|)\theta(|\vec{p}_N|-p_F)\left(L^{\alpha\beta}\mp L_5^{\alpha\beta}\right)\left(W_{\alpha\beta}^{NC}\right)_{s.n.} \quad ,$$

The single nucleon NC hadronic tensor being:

$$\left(W_{\alpha\beta}^{NC}\right)_{s.n.} = -\left[\tau\left(G_M^{NC}\right)^2+(1+\tau)\left(G_A^{NC}\right)^2\right]\left(g_{\alpha\beta}-\frac{q_\alpha q_\beta}{q^2}\right)+$$

$$+\left[\frac{\left(G_E^{NC}\right)^2+\tau\left(G_M^{NC}\right)^2}{1+\tau}+\left(G_A^{NC}\right)^2\right]\frac{X_\alpha X_\beta}{M^2}+$$

$$-\left(G_A^{NC}\right)^2\frac{q_\alpha q_\beta}{q^2}+\frac{i}{M^2}\epsilon_{\alpha\beta\mu\nu}p^\mu q^\nu G_A^{NC}G_M^{NC} \, ,$$

with

$$X_\alpha = p_\alpha - \frac{(p\cdot q)\, q_\alpha}{q^2}\,.$$

Explicitly:

$$\begin{aligned}
\left(\frac{d^2\sigma}{dE_N d\Omega_N}\right)_{\nu(\overline{\nu})} &= \frac{G_F^2}{(2\pi)^2}\frac{3\mathcal{N}}{4\pi p_F^3}\frac{|\vec{p}_N|}{k_0}\int \frac{d^3k'}{k_0'}\frac{d^3p}{p_0}\delta\left(k_0 - k_0' + p_0 - E_N\right) \\
&\times \quad \delta^{(3)}\left(\vec{k} - \vec{k}' + \vec{p} - \vec{p}_N\right)\theta(p_F - |\vec{p}\,|)\theta(|\vec{p}_N| - p_F) \\
&\times \quad \left\{V_M (G_M^{NC})^2 + V_{EM}\frac{(G_E^{NC})^2 + \tau(G_M^{NC})^2}{1+\tau} + \right. \\
&+ \quad \left. V_A (G_A^{NC})^2 \pm V_{AM} G_A^{NC} G_M^{NC}\right\}
\end{aligned}$$

where

$$V_M = 2M^2\tau\,(k\cdot k')$$

$$V_{EM} = 2\,(k\cdot p)\,(k'\cdot p) - M^2\,(k\cdot k')$$

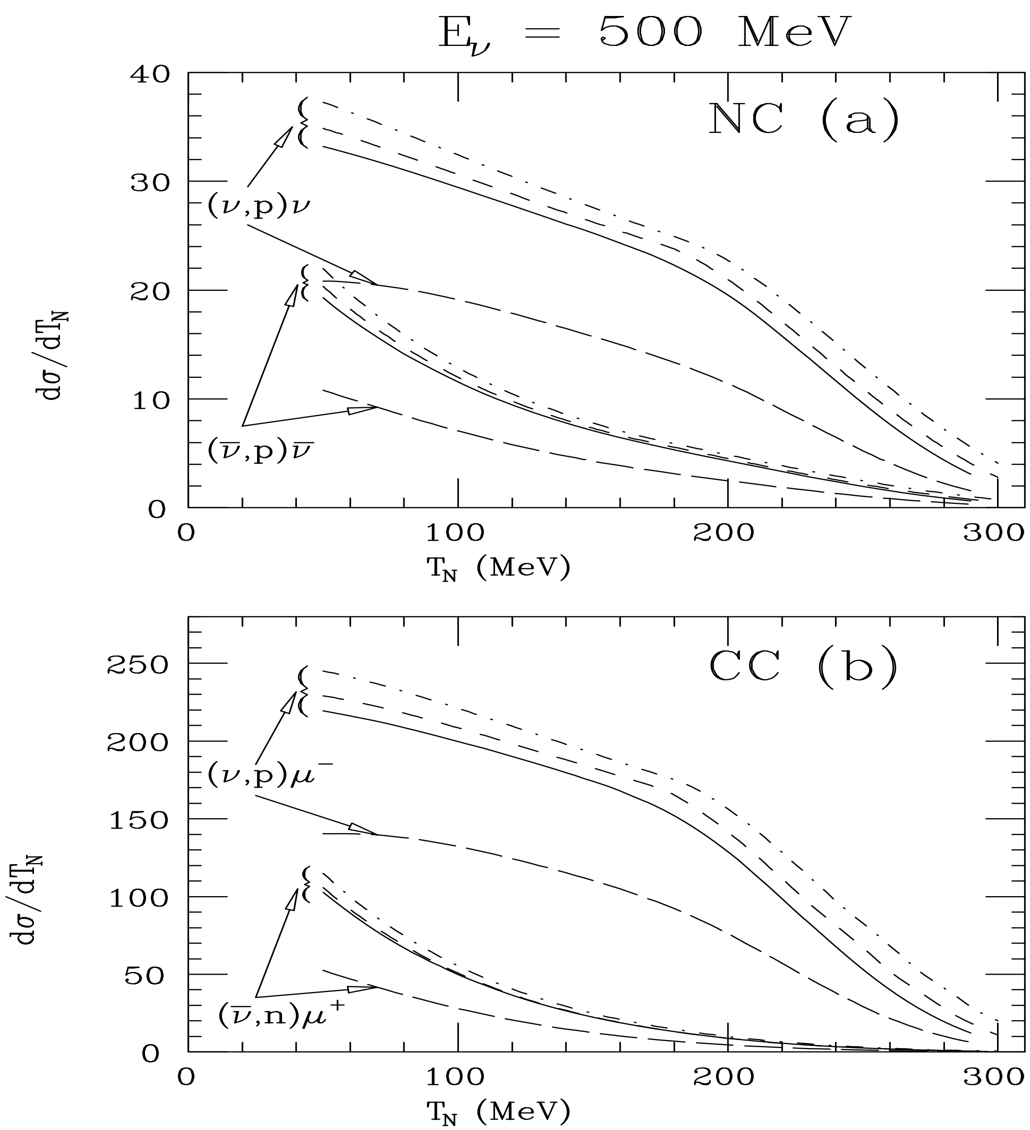
$$V_A = M^2\,(k\cdot k') + 2M^2\tau\,(k\cdot k') + 2\,(k\cdot p)\,(k'\cdot p)$$

$$V_{AM} = 2\,(k\cdot k')\,(k\cdot p + k'\cdot p)$$

Single differential cross sections then follow:

$$\left(\frac{d\sigma}{dT_N}\right)_{\nu(\overline{\nu})N} \equiv \left(\frac{d\sigma}{dE_N}\right)_{\nu(\overline{\nu})N} = \int d\Omega_N \left(\frac{d^2\sigma}{dE_N d\Omega_N}\right)_{\nu(\overline{\nu})N} ,$$

with T_N outgoing nucleon kinetic energy.

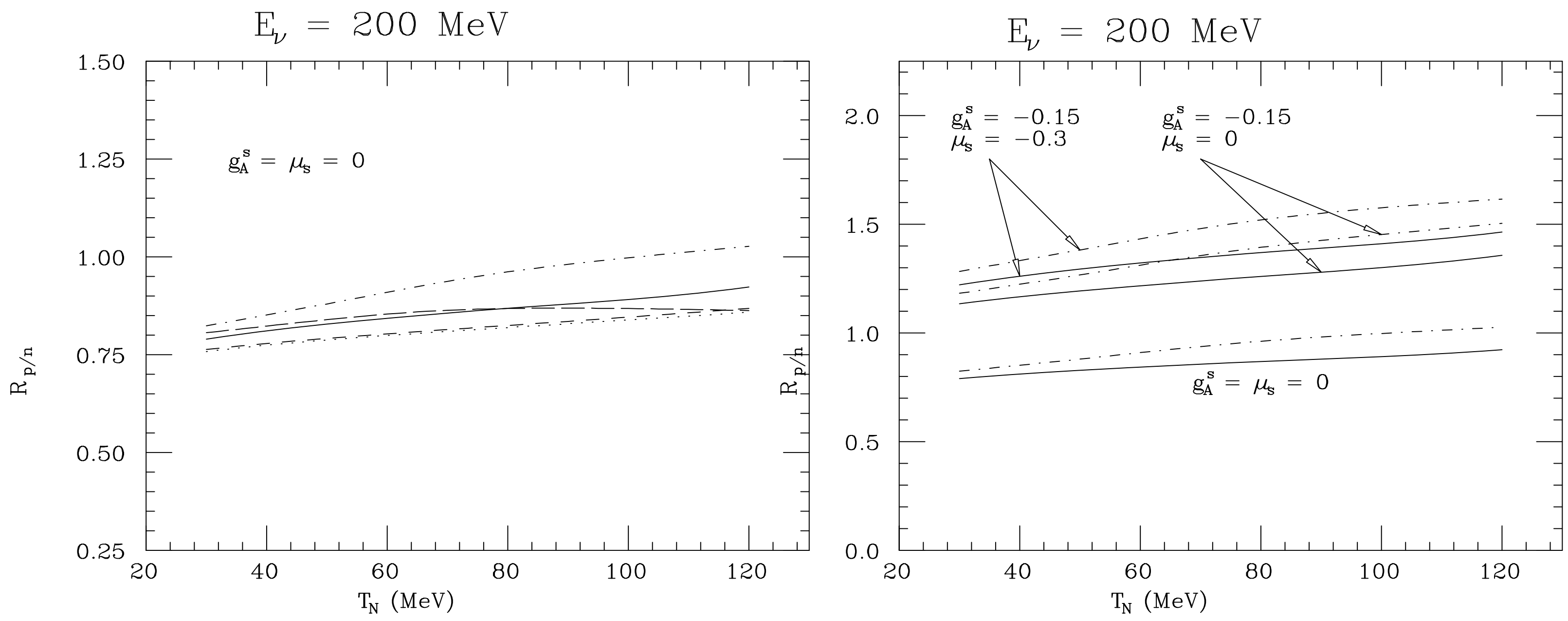


NC and CC $d\sigma/dT_N$, CARBON, 500 MeV

SOLID= RSM-PWIA,
DASHED=RFG with e_B,
DOT-DASHED=RFG,
LONG-DASHED=ROP

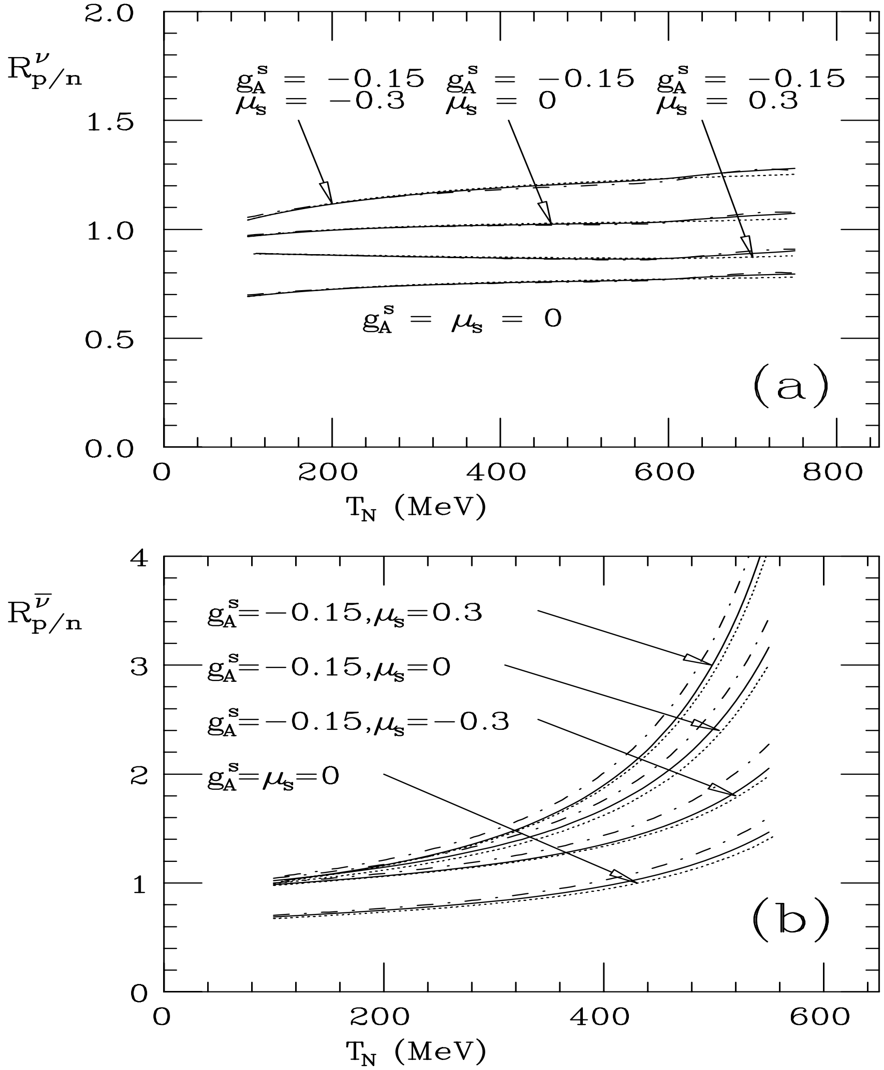
The ratio of proton to neutron yield

NC, p/n ratio, 200 MeV



SOLID= RSM-PWIA, DASHED (DOTTED) = RFG (with e_B), DOT-DASH=ROP,

LONG-DASH= ROP with $U_{Coulomb} = 0$ (proton)

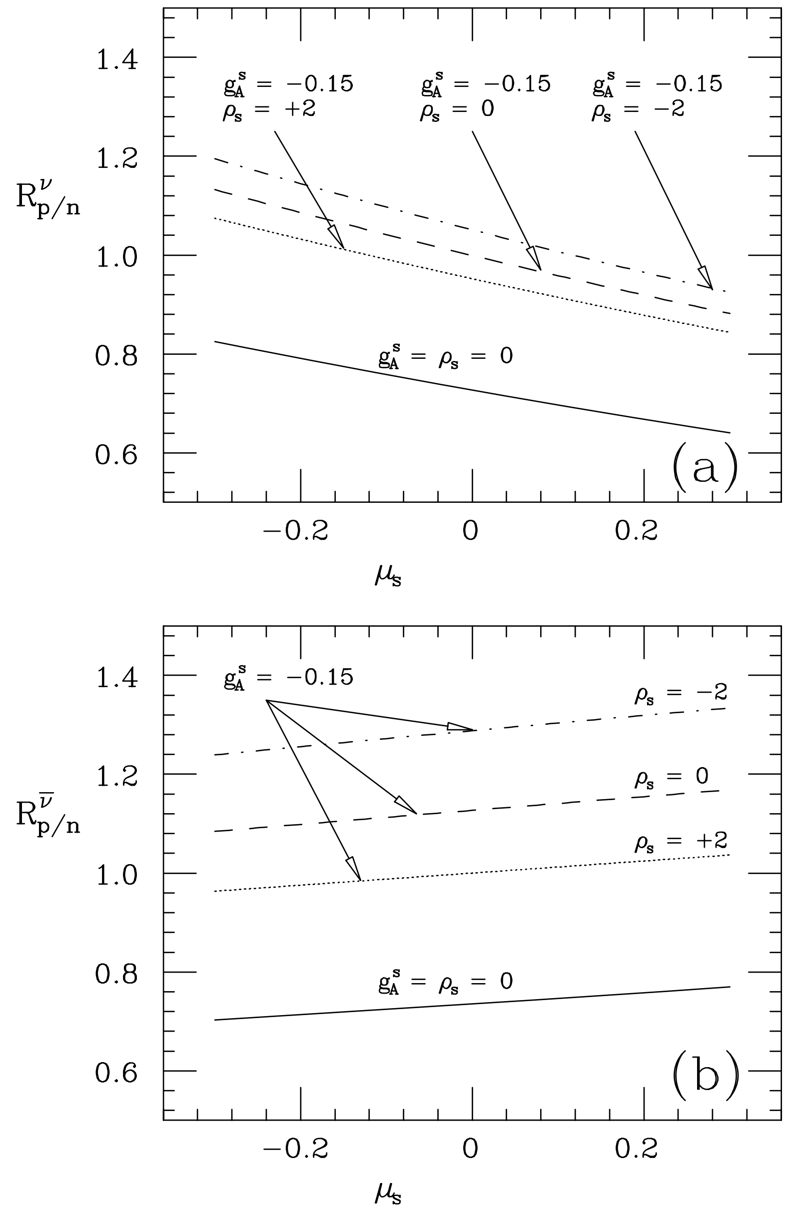


NC p/n ratio,
CARBON,
1 GeV

SOLID= RSM-PWIA,

DOTTED=RFG,

DOT-DASHED = ROP



$R^{\nu}_{p/n}$
$g_A^s = -0.15$
$\rho_s = +2$
$g_A^s = -0.15$
$\rho_s = 0$
$g_A^s = -0.15$
$\rho_s = -2$
$g_A^s = \rho_s = 0$
(a)
μ_s
$R^{\bar{\nu}}_{p/n}$
$g_A^s = -0.15$
$\rho_s = -2$
$\rho_s = 0$
$\rho_s = +2$
$g_A^s = \rho_s = 0$
(b)
μ_s

The BNL - 734 experiment

They measured:

$$
\begin{aligned}
R_\nu &= \frac{\langle\sigma\rangle_{(\nu p\to\nu p)}}{\langle\sigma\rangle_{(\nu n\to\mu^- p)}} = 0.153 \pm 0.007 \pm 0.017 \\
R_{\overline{\nu}} &= \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\overline{\nu} p\to\mu^+ n)}} = 0.218 \pm 0.012 \pm 0.023 \\
R &= \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\nu p\to\nu p)}} = 0.302 \pm 0.019 \pm 0.037 \ ,
\end{aligned}
$$

$\langle\sigma\rangle_{\nu(\bar{\nu})}$ is a total cross section integrated over the incident neutrino (antineutrino) energy and weighted by the $\nu(\bar{\nu})$ flux. The first error is statistical and the second is the systematic one.

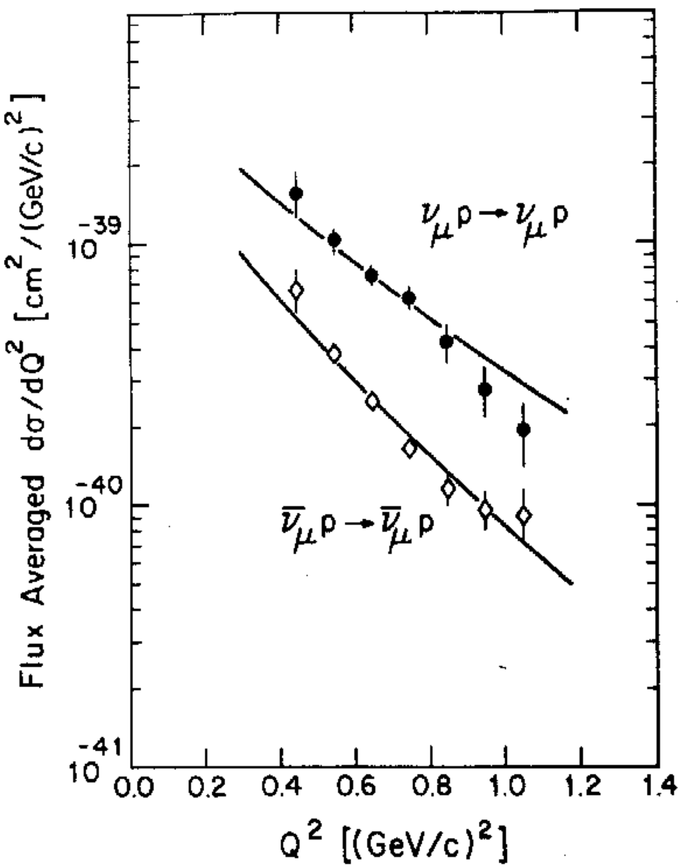


Fig. 2 – Flux averaged differential cross sections measured by Ahrens et al., PRD35 (1987). Solid curves are the best fit to the combined data.

NB Data are fitted with $M_A = 1.06$ GeV and $\sin^2_W = 0.220$. The same data are compatible (at 90% CL) with $-0.25 \leq G_A^s(0) \leq 0$ if the axial cutoff is constrained to $M_A = 1.032 \pm 0.036$ GeV.

In terms of these ratios, the "integrated" asymmetry reads:

$$\langle \mathcal{A}_p \rangle = \frac{R_\nu (1 - R)}{1 - R R_\nu / R_{\overline{\nu}}}$$

and from the experimental data we found

$$\langle \mathcal{A}_p \rangle = 0.136 \pm 0.008(\mathrm{stat}) \pm 0.019(\mathrm{syst})$$

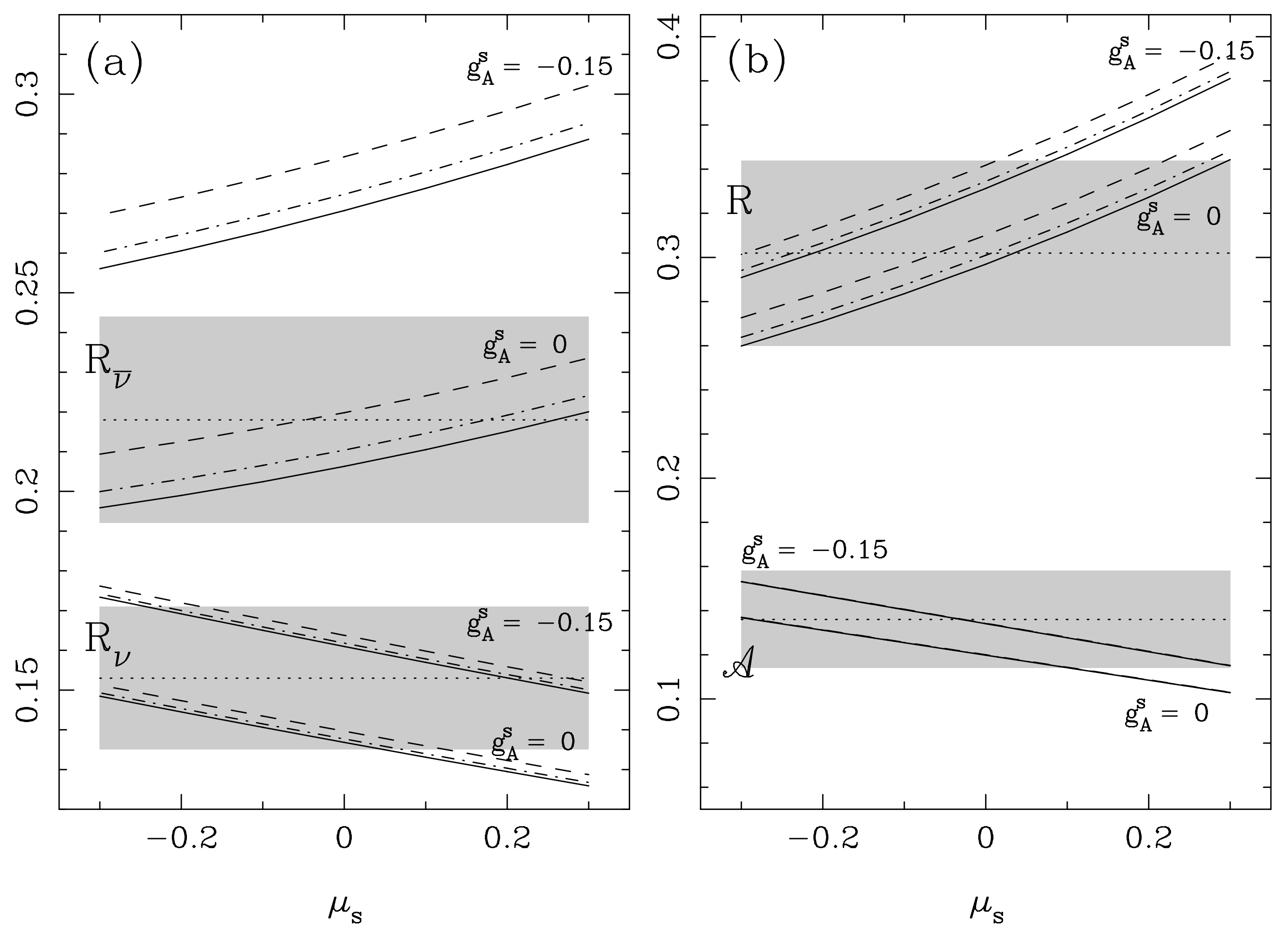


Fig. 3 – R_ν and $R_{\bar\nu}$, R and $\langle \mathcal{A}_p \rangle$, for $g_A^s = 0$ and $g_A^s = -0.15$.

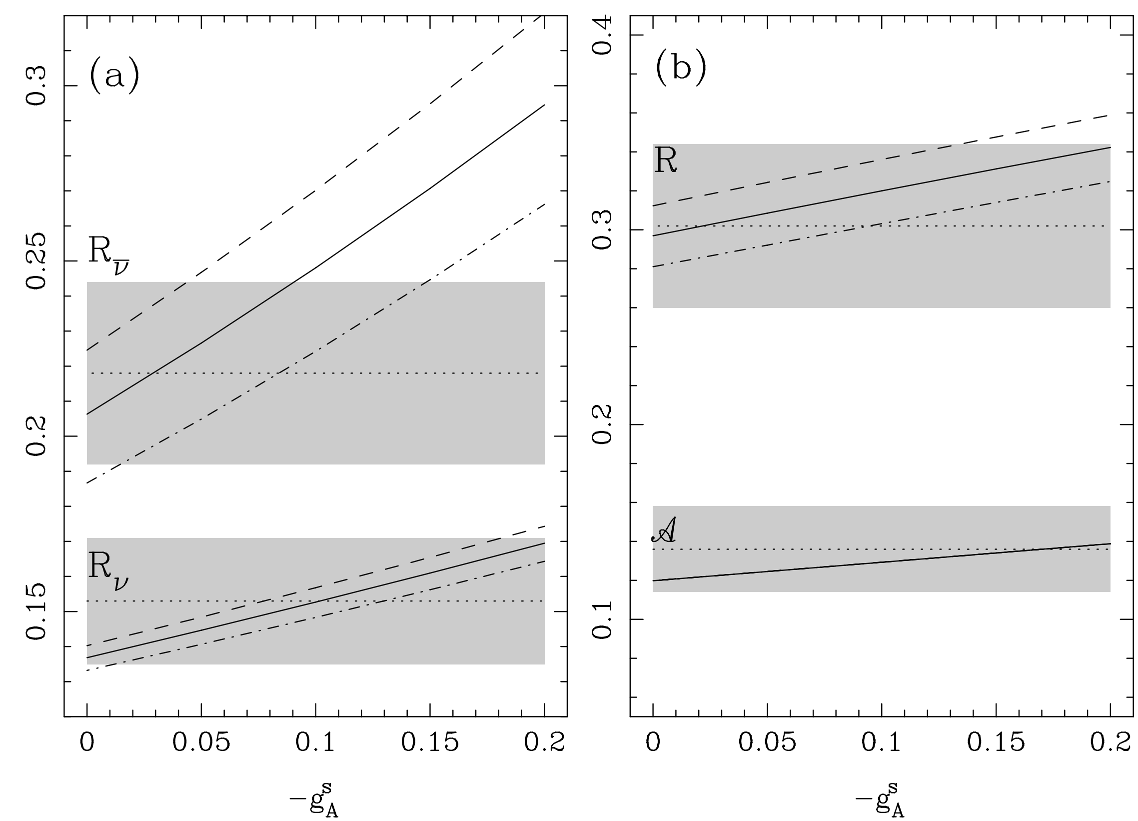


Fig. 4 – The ratios R_ν, $R_{\bar{\nu}}$, R and $\langle \mathcal{A}_p \rangle$ versus g_A^s: sensitivity to M_A. $M_A = 1.032$ GeV (solid), $M_A = 1.068$ GeV (dashed) and $M_A = 0.996$ GeV.

New model independent determination of ν-nucleus cross sections

Suggested by M.B. Barbaro, J.A. Caballero et al.: Phys. Rev. C71, 015501 (2003); Phys. Rev. C73, 035503 (2006); Phys. Rev. C75, 064617 (2007).

Inclusive NC neutrino-nucleus cross sections in the u-channel:

$$\frac{d\sigma}{d\Omega_N dp_N} \simeq \overline{\sigma}_{sn}^{(u)} F(\psi', q'),$$

where

$$F(\psi', q') \equiv \int_{\mathcal{D}_u} p dp \int \frac{d\mathcal{E}}{E} \Sigma \simeq F(\psi'),$$

CDMF (coherent density fluctuation model) scaling functions

$$f^{QE}(\psi') = \frac{1}{A}[Z f_p^{\mathrm{QE}}(\psi') + N f_n^{\mathrm{QE}}(\psi')].$$

$$f^{\rm QE}_{p(n),(1,2)}(\psi') \quad = \quad \int\limits_0^{\alpha_{p(n)}/(k_F^{p(n)}|\psi'|)} dR|F_{p(n)}(R)|^2 f^{p(n)}_{\rm RFG,(1,2)}(\psi'(R)),$$

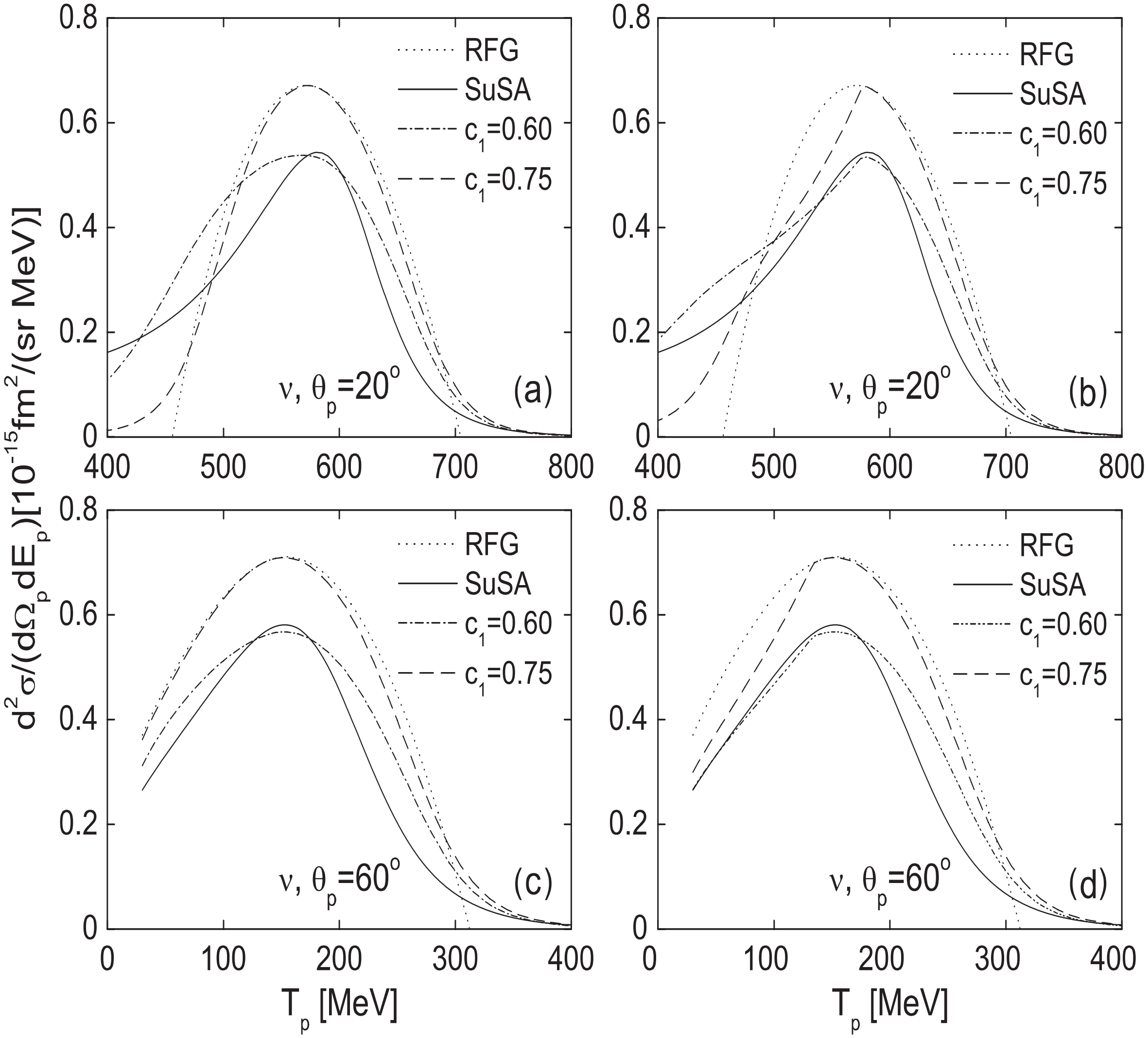
where

$$f^{p(n)}_{\rm RFG,1}(\psi'(R)) = c_1 \left[1 - \left(\frac{k_F^{p(n)} R|\psi'|}{\alpha_{p(n)}}\right)^2\right], \ \psi' \leq 0$$

and

$$f^{p(n)}_{\rm RFG,2}(\psi'(R)) = c_1 \exp\left[-\frac{k_F^{p(n)} R\psi'}{c_2\alpha_{p(n)}}\right], \ \psi' \geq 0.$$

• Parameterization of scaling functions fitted on (e, e') scattering data and employed in ν-scattering on the basis of super-scaling arguments.



RFG
SuSA
c1=0.60
c1=0.75
ν, θp=20o
(a)
(b)
ν, θp=60o
(c)
(d)
d2σ/(dΩp dEp)[10-15fm2/(sr MeV)]
Tp [MeV]

Future perspectives: the case at Fermilab

We have considered, for the expected low energy neutrino flux:

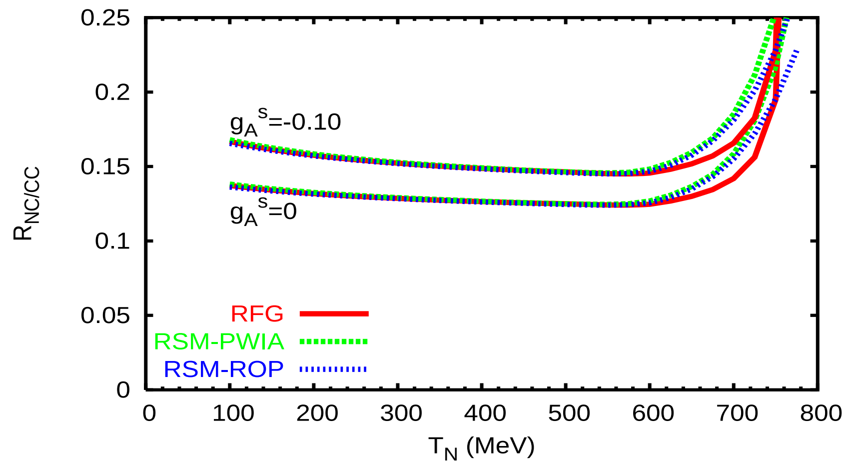
The ratio of NC and CC elastic νp scattering

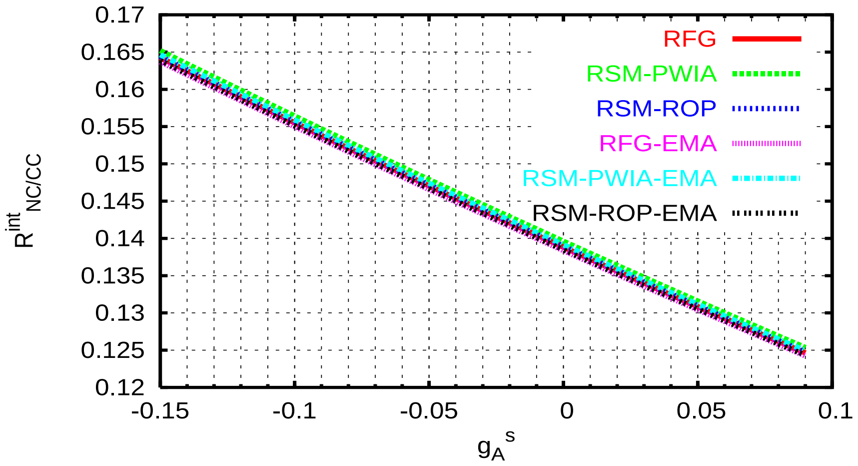
1. Sensitive to g_A^s, but not much affected by the cutoff mass of the axial form factors, assumed in the dipole form:

$$G_A(Q^2) = \frac{1.26}{(1 + Q^2/M_A^2)}, \; G_A^s(Q^2) = \frac{g_A^s}{(1 + Q^2/M_A^2)}$$

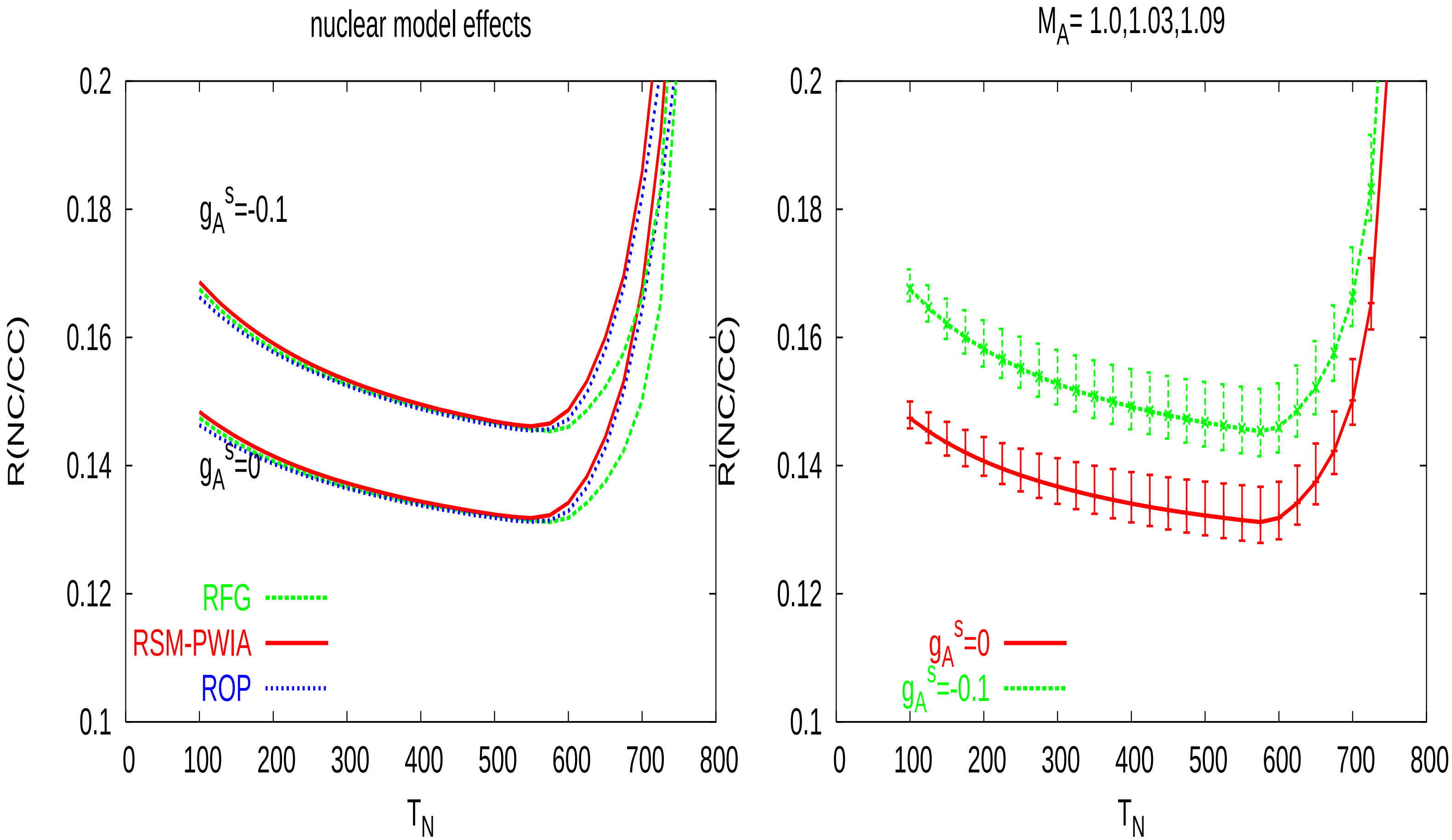
2. The e.m. form factors do not sensibly affect the ratio
3. Interference between axial and vector strange ff can mask effect of g_A^s.
4. The sensitivity to the flux is negligible, because of ratio
5. Nuclear effects are again negligible, because of ratio.

NC/CC ratio,1 GeV, effects of G_A^s

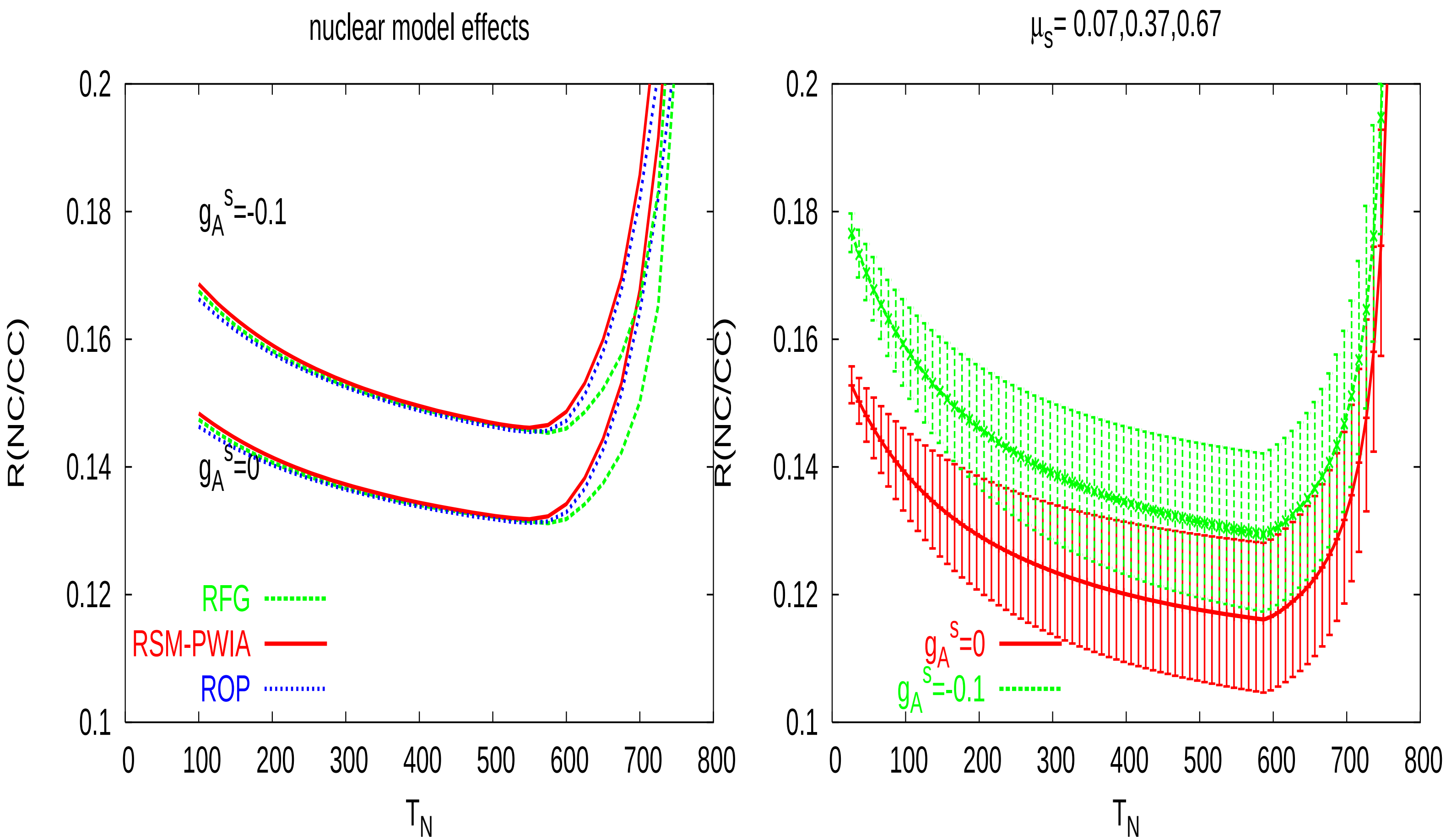




NC/CC ratio,1 GeV, effects of M_A



NC/CC ratio,1 GeV, effects of G_M^s



$$G_M^s = \mu_s G_D^V(Q^2) \qquad G_E^s = 0$$

Parity Violating Electron scattering

In electron-nucleon scattering both γ and Z_0 can be exchanged. Interference depends on electron polarization.

Cross section for scattering of electrons with polarization λ on unpolarized nucleons:

$$\left(\frac{d\sigma}{d\Omega}\right)_\lambda = \left(\frac{d\sigma}{d\Omega}\right)_0 (1 + \lambda \mathcal{A})$$

$(d\sigma/d\Omega)_0$ cross section for scattering of unpolarized electrons

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \sigma_{Mott} \left\{ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tan^2\frac{\theta}{2} \tau G_M^2 \right\} .$$

$$\sigma_{Mott} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4\frac{\theta}{2}\left(1 + \frac{2E}{M}\sin^2\frac{\theta}{2}\right)}$$

The P–odd asymmetry is given by

$$\mathcal{A} = \frac{1}{\lambda} \frac{\left(\frac{d\sigma}{d\Omega}\right)_{\lambda} - \left(\frac{d\sigma}{d\Omega}\right)_{-\lambda}}{\left(\frac{d\sigma}{d\Omega}\right)_{\lambda} + \left(\frac{d\sigma}{d\Omega}\right)_{-\lambda}} \,.$$

Explicitly:

$$\mathcal{A} = -\mathcal{A}_0 \, \frac{\tau G_M G_M^{NC} + \varepsilon G_E G_E^{NC} + (1 - 4\sin^2\theta_W)\varepsilon' G_M G_A^{NC}}{\tau G_M^2 + \varepsilon G_E^2} \,,$$

where

$$\varepsilon = \frac{1}{1 + 2(1+\tau)\tan^2(\theta/2)} \,, \quad \varepsilon' = \sqrt{\tau(1+\tau)(1-\varepsilon^2)} \,.$$

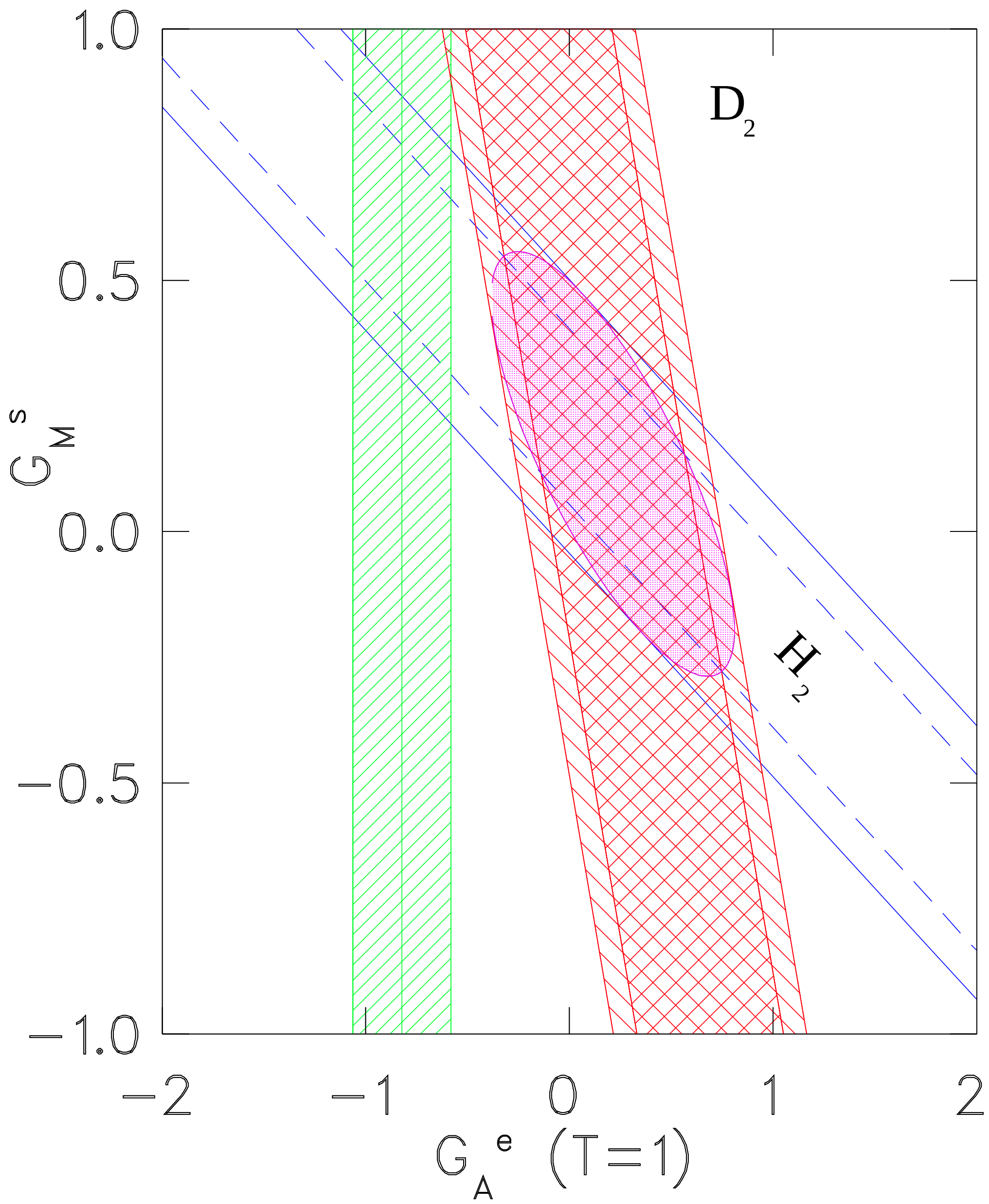
and

$$\mathcal{A}_0 \equiv \frac{G_F Q^2}{2\sqrt{2}\pi\alpha} = 1.798 \times 10^{-4} \frac{Q^2}{\mathrm{GeV}^2} \, .$$

NOTE:

- Axial neutral current suppressed by the smallness of electron vector coupling $g_V = -\frac{1}{2}\left(1 - 4\sin^2\theta_W\right)$
- Electric component suppressed at large angles and large momentum transfer
- Magnetic component enhanced at large Q^2 and scattering angles
- Nucleon axial coupling affected by radiative corrections.

Experiments done at BATES (SAMPLE) and TJNAF (HAPPEX) at various Q^2.



1.0
0.5
0.0
−0.5
−1.0
−2
−1
0
1
2
G_M^s
G_A^e (T=1)
D_2
H_2

Elastic scattering on $S = T = 0$ nuclei

Consider the processes

$$\nu\ (\overline{\nu}) + A \longrightarrow \nu\ (\overline{\nu}) + A$$

Axial current, A_α^{NC}, and isovector part of the vector NC, $V_\alpha^3(1 - 2\sin^2\theta_W)$, do not contribute. Cross sections are given by:

$$\frac{d\sigma_\nu}{dQ^2} = \frac{d\sigma_{\overline{\nu}}}{dQ^2} = \frac{G_F^2}{2\pi}\left(1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}\right)\left[F^{NC}(Q^2)\right]^2$$

with

$$F^{NC}(Q^2) = -2\sin^2\theta_W F(Q^2) - \frac{1}{2}F^s(Q^2)$$

The e.m. FF of the nucleus $F(Q^2)$ can be determined from elastic scattering of unpolarized electrons:

$$\frac{d\sigma_e}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left(1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}\right)\left[F(Q^2)\right]^2 .$$

Hence the strange form factor of the $S = T = 0$ nucleus can be obtained from measurable cross sections:

$$F^s(Q^2) = \pm 2F(Q^2) \left\{ \left(\frac{2\sqrt{2}\pi\alpha}{G_F Q^2} \right) \sqrt{\frac{(d\sigma_\nu/dQ^2)}{(d\sigma_e/dQ^2)}} \mp 2\sin^2\theta_W \right\}$$

or, equivalently:

$$F^s(Q^2) = \pm 2 \frac{1}{\sqrt{1 - \frac{p\cdot q}{M_A E} - \frac{Q^2}{4E^2}}} \times$$

$$\times \left\{ \sqrt{\frac{2\pi}{G_F^2} \frac{d\sigma_\nu}{dQ^2}} \mp 2\sin^2\theta_W \sqrt{\frac{Q^4}{4\pi\alpha^2} \frac{d\sigma_e}{dQ^2}} \right\}.$$

NOTE:

The observation of the process of the scattering of neutrino on nuclei requires the measurement of the small recoil energy of the final nucleus. It could be easier to detect the process of scattering of neutrinos and electrons on nuclei if the nucleus undergoes a transition to excited states, for example:

$$\begin{aligned} \nu + {}^4\mathrm{He} &\longrightarrow \nu + {}^4\mathrm{He}^* \\ e + {}^4\mathrm{He} &\longrightarrow e + {}^4\mathrm{He}^* \end{aligned}$$

where ${}^4\mathrm{He}^*$ is the excited state of ${}^4\mathrm{He}$ with with $S = 0$ and $T = 0$ and excitation energy of 20.1 MeV. This state can decay into p and radioactive ${}^3\mathrm{H}$.

Conclusions

- The experiments of ν-proton NC and CC scattering are highly interesting for the determination of $\Delta s \equiv g_A^s$.
- Problems of interference with strange vector form factors can be resolved by complementary experiments (PV electron scattering)
- If feasible, $\bar{\nu}$ scattering would offer relevant and complementary information and:
- would allow the determination of the neutrino asymmetry (a unique tool for unambiguous determination of Δs)

For reference, see:

W.M. Alberico, S.M. Bilenky and C. Maieron, Phys. Rep. 358 (2002) 227;
also: W.M. Alberico, et al., Z. Physik C 70 (1996) 463; Nucl. Phys. A623 (1997) 471; Phys. Lett. B438 (1998) 9; Nucl. Phys. A651 (1999) 277.