Phenomenology of neutrino oscillations

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INTRODUCTION

In 1957 violation of parity in the weak interaction was discovered

Soon after this discovery Landau, Lee and Yang and Salam proposed the theory of the two-component neutrino

The neutrino field $\nu(x)$ satisfies the Dirac equation

$$(i\,\gamma^{\alpha}\ \partial_{\alpha} - m)\ \nu(x) = 0$$

The field $\nu(x)$ can be presented in the form

$$\nu(x) = \nu_L(x) + \nu_R(x)$$

$$\nu_{L,R}(x) = \frac{1 \mp \gamma_5}{2}\nu(x)$$

left-handed and right-handed components of the field $\nu(x)$

$$i\,\gamma^{\alpha}\,\partial_{\alpha}\,\nu_L(x) - m\;\;\nu_R(x) = 0$$

$$i\,\gamma^{\alpha}\,\partial_{\alpha}\,\nu_R(x) - m\;\;\nu_L(x) = 0$$

Landau, Lee and Young and Salam assumed that neutrino mass m is equal to zero

For massless neutrino two decoupled equations

$$i\,\gamma^{\alpha}\,\partial_{\alpha}\nu_{L,R}(x) = 0$$

Two-component theory: m=0 and neutrino field is ν_L(or ν_R)

Major consequence: helicity of neutrino is equal to -1 and helicity of antineutrino is equal to +1 (assuming ν_L; for ν_R opposite)

For the state of massless neutrino with momentum p and helicity r ($\vec{k} = \frac{\vec{p}}{p}$)

$$\gamma p\;\; u^r(p) = 0 \quad \vec{\Sigma}\vec{p} = \gamma_5\gamma^0\vec{\gamma}\vec{k}\,u^r(p) = r u^r(p)$$

From Dirac equation $\vec{\gamma}\vec{k}\;\; u^r(p) = \gamma^0\;\; u^r(p)$

$$\gamma_5\;\; u^r(p) = r\;\; u^r(p)$$

$$\gamma_5\;\; u^r(-p) = -r\;\; u^r(-p)$$

For the left-handed neutrino field

$$\nu_L(x) = \tfrac{1-\gamma_5}{2}\nu_L(x) =$$
$$\int N_p \left(u^{-1}(p)c_{-1}(p)\, e^{-ipx} + u^1(-p)d_1^\dagger(p)\, e^{ipx}\right)\, d^3p$$

Neutrino helicity was measured in spectacular Goldhaber et al experiment

$$\begin{array}{lll} e^- + \mathrm{Gd} \to \nu_\mathrm{e} + & \mathrm{Sm}^* & \\ & \downarrow & \\ & \mathrm{Sm} & + \gamma \end{array}$$

It was found $h = -1 \pm 0.3$. Neutrino is a particle with negative helicity

The two-component neutrino theory was the first step in the creation of the universal $V-A$ theory of Feynman and Gell-Mann, Marshak and Sudarshan (1958) and later of the Standard Model.

V-A theory: In the weak interaction Hamiltonian fields of all particles enters in the form of left-handed components (as neutrino field)

$$\mathcal{H}_I^{\beta} = \frac{G_F}{\sqrt{2}} 4\, \bar{p}_L \gamma_\alpha n_L \;\; \bar{e}_L \gamma^\alpha \nu_L + h.c.$$

Standard Model

$$\psi^q_{1L} = \begin{pmatrix} u'_L \\ d'_L \end{pmatrix}, \quad \psi^l_{1L} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, \ldots$$

All particles which take part in the weak interaction together with neutrino have mass. For neutrino mass was only upper bound. At the time when V-A theory and the Standard Model were proposed $m_\nu \lesssim 100$ eV.

During many years there was a general belief that neutrinos are massless particles

The first idea about massive neutrinos and neutrino oscillations was put forward by B. Pontecorvo in 1958 soon after the two-component neutrino theory was established

Possibility of nonzero neutrino masses became popular after the GUT models appeared (end of the seventies)

NEUTRINO INTERACTION

It was established by numerous experiments (β-decay of neutron and nuclei, μ-decay, π and K-decays, neutrino processes $\nu_\mu N \to \mu^- X$ etc, $Z \to \nu\bar{\nu}$ and $W^+ \to l^+ \nu_l$ etc etc) that

Fundamental neutrino interactions

Charged current interaction

$$\mathcal{L}_I^{CC} = -\frac{g}{2\sqrt{2}} j_\alpha^{CC} W^\alpha + \text{h.c.}$$

$$j_\alpha^{CC} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha l_L$$

Neutral current interaction

$$\mathcal{L}_I^{NC} = -\frac{g}{2\cos\theta_W} j_\alpha^{NC} Z^\alpha.$$

$$j_\alpha^{NC} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_\alpha \nu_{lL}$$

NUMBER OF FLAVOR NEUTRINOS ν_l

For the width of the decay $Z \to \nu\bar{\nu}$ (invisible width) we have

$$\Gamma_Z = n_\nu \; \Gamma^{SM}_{Z \to \nu_l \nu_l}$$

From the data of the LEP experiments

$$n_\nu = 2.984 \pm 0.008$$

Three flavor neutrinos ν_e, ν_μ, ν_τ exist in nature

All three neutrinos were observed in neutrino experiments

In the modern theories the sources of masses and mixing of leptons, quarks and neutrinos are mass terms

NEUTRINO MASS TERMS

The mass term of charged leptons

$$\mathcal{L}(x) = -\sum_{l',l} \overline{l'}_L(x)\, M_{l'l}\, l'_R(x) + \text{h.c.}$$

Lorenz-invariant product of left-handed and right-handed components; M is 3×3 matrix.

After the diagonalization of the matrix M

The standard mass term

$$\mathcal{L}(x) = -\sum_{l=e,\mu,\tau} m_l\, \overline{l}(x)\, l(x)$$

$l(x)$ is the field of lepton l ($l = e, \mu, \tau$) with mass m_l. Fields $l'(x)$ and $l(x)$ are connected by an unitary transformation

This mass term is called Dirac mass term: $l(x)$ is the Dirac field of particles l^- and antiparticles l^+

Fundamental difference between neutrinos and other fermions

Electric charges of neutrinos are equal to zero

Neutrinos can be Dirac particles (neutrino and antineutrino are different) or Majorana particles (neutrino and antineutrino are identical)

In the Dirac case exists conserved lepton number which distinguish neutrino and antineutrino. In the Majorana case there is no conserved lepton numbers

Not only masses and mixing but also nature of neutrino is determined by the neutrino mass term

Dirac neutrino mass term

Dirac neutrino mass term can be generated by the Standard Higgs mechanism if we assume that right-handed singlets ν_{lR} enter into Lagrangian. Can be other mechanisms

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l',l} \bar{\nu}_{l'L}\, M^{\mathrm{D}}_{l'l}\, \nu_{lR} + \mathrm{h.c.}.$$

M^{D} is a 3×3 complex matrix

Diagonalization

We have

$$M^{\mathrm{D}} = U\ m\ V^{\dagger}$$

$$VV^{\dagger} = 1\ UU^{\dagger} = 1$$

Proof

Consider any complex matrix M

$M\,M^\dagger$ is hermitian matrix with positive eigenvalues

Can be diagonalized by unitary transformation

$$M\,M^\dagger = U\,m^2\,U^\dagger \quad m^2_{ik} = m^2_i\,\delta_{ik} \quad UU^\dagger = 1$$

The matrix M can be presented in the form

$$M = U\,m\,V^\dagger$$

$$V^\dagger = m^{-1}\,U^\dagger\,M$$

Proof that V is unitary matrix

$$V = M^\dagger\,U\,m^{-1}$$

$$V^\dagger\,V = m^{-1}\,U^\dagger\,U\,m^2\,U^\dagger\,U\,m^{-1} = 1$$

After the diagonalization of the matrix M^{D}

$$\mathcal{L}^{\mathsf{D}} = -\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i$$

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\ \nu_{iL} \quad (l = e, \mu, \tau)$$

I. $\nu_i(x)$ is the field of neutrino with mass m_i

II. Flavor field ν_{lL} is mixture (unitary combination) of the left-handed components of the fields of neutrinos with definite masses

U is called 3×3 PMNS mixing matrix

The total Lagrangian is invariant under the following global gauge transformations

$$\nu_{lL}(x) \to e^{i\Lambda}\nu_{lL}(x), \quad \nu_{lR}(x) \to e^{i\Lambda}\nu_{lR}(x)$$

$$l_L(x) \to e^{i\Lambda} l_L(x), \quad l_R(x) \to e^{i\Lambda} l_R(x)$$

Λ is an arbitrary constant phase

The lepton charge L, the same for all charged leptons and all neutrinos is conserved

$$L = L_e + L_\mu + L_\tau$$

Neutrinoless double β-decay $A, Z \to A, Z+2 + e^- + e^-$ and other similar processes are forbidden.

Majorana mass term

If neutrino masses are equal to zero neutrino fields ν_{lL} (two-component theory)

Vice versa: if neutrino fields ν_{lL} does this means that neutrino masses are equal to zero?

Yes, if lepton numbers are conserved

If lepton number is not conserved neutrinos can have Majorana masses (Gribov and Pontecorvo)

Any mass term is a sum of Lorenz-invariant products of left-handed and right-handed fields

Let us consider charge-conjugated fields

$$(\nu_{lL})^c = C\,\bar{\nu}_{lL}^T, \quad (\nu_{lR})^c = C\,\bar{\nu}_{lR}^T$$

C is the matrix of the charge conjugation

$$C\,\gamma_\alpha^T\,C^{-1} = -\gamma_\alpha, \quad C^T = -C\ .$$

Proof: The left-handed and right-handed components are determined by

$$\gamma_5\,\nu_{lL} = -\nu_{lL}, \quad \gamma_5\,\nu_{lR} = \nu_{lR}$$

By hermitian conjugation and multiplication by γ^0 from the right we find

$$\bar{\nu}_{lL}\,\gamma_5 = \bar{\nu}_{lL}, \quad \bar{\nu}_{lR}\,\gamma_5 = -\bar{\nu}_{lR}$$

From here by transposition and multiplication by C from left

$$\gamma_5\ (\nu_{lL})^c = (\nu_{lL})^c, \quad \gamma_5\ (\nu_{lR})^c = -(\nu_{lR})^c$$

We take into account that $C\,\gamma_5^T\,C^{-1} = \gamma_5^T$. Thus, $(\nu_{lL})^c$ and $(\nu_{lR})^c$ are right-handed and left-handed fields

Majorana mass term

$$\mathcal{L}^{\mathsf{M}} = -\frac{1}{2} \sum_{l',l=e,\mu,\tau} \bar{\nu}_{l'L}\, M^{\mathsf{M}}_{l'l}\, (\nu_{lL})^{c} + \mathrm{h.c.}$$

M^{M} is symmetrical matrix

$$\sum_{l',l} \bar{\nu}_{l'L}\, M^{\mathsf{M}}_{l'l} C \bar{\nu}^{T}_{lL} = -\sum_{l',l} \bar{\nu}_{lL}\, M^{\mathsf{M}}_{l'l} C^{T} \bar{\nu}^{T}_{l'L}$$

$C^{T} = -C$ and $l \leftrightarrows l'$ we find

$$M^{\mathsf{M}} = (M^{\mathsf{M}})^{T}$$

For symmetrical matrix

$$M^{\mathsf{M}} = U\, m\, U^{T}$$

U is an unitary matrix and

$m_{ik} = m_i\, \delta_{ik}, \;\; m_i > 0.$

From these relations

$$\mathcal{L}^{\mathsf{M}} = -\frac{1}{2} \sum_{i=1}^{3} m_i\; \bar{\nu}_i\; \nu_i$$

$$\nu_i = \sum_l U_{li}^* \nu_{lL} + \sum_l (U_{li}^* \nu_{lL})^c$$

1.ν_i is the field on neutrinos with mass m_i

2. the field ν_i satisfies Majorana condition

$$\nu_i = \nu_i^c = C \bar{\nu}_i^T$$

General spin 1/2 field

$$\nu = \int N_p \left(c_r(p)\; u^r(p)\, e^{-i\,p\,x} + d_r^\dagger(p)\, u^r(-p)\, e^{i\,p\,x} \right)\, d^3p$$

From Majorana condition

$$c_r(p) = d_r(p)$$

particle $\equiv$ antiparticle

Mixing relation

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\, \nu_{iL}$$

No global gauge invariance and conserved lepton number which allows to distinguish neutrinos and antineutrinos.

The most general mass term, in which left-handed flavor fields ν_{lL} and right-handed fields ν_{lR} enter is the sum of the left-handed Majorana, Dirac and right-handed Majorana mass terms

It is called

Dirac and Majorana mass term

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \sum_{l',l} \bar{\nu}'_{l'L} \, (M_L^{\mathrm{M}})_{l'l} (\nu'_{lL})^c - \sum_{l'l} \bar{\nu}'_{l'L} \, (M^{\mathrm{D}})_{l',l} \, \nu'_{lR} - \frac{1}{2} \sum_{l',l} \overline{(\nu'_{l'R})^c} \, (M_R^{\mathrm{M}})_{l,'l} \nu'_{lR} + \mathrm{h.c.}$$

After the diagonalization

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2} \sum_{i=1}^{6} m_i \, \bar{\nu}_i \, \nu_i$$

ν_i is the field of the Majorana neutrinos

Six Majorana neutrinos with definite masses

Neutrino mixing in the case of D+M mass term

$$\nu_{lL}(x) = \sum_{i=1}^{6} U_{li}\,\nu_{iL}(x), \quad (\nu_{lR}(x))^c = \sum_{i=1}^{6} U_{\bar{l}i}\,\nu_{iL}(x)$$

U is 6×6 matrix

If all masses are light possible transitions $\nu_l \to \nu_{l'}$ and $\nu_l \to \bar{\nu}_{l'L}$

$\bar{\nu}_{l'L}$ (quantum of right-handed field $\nu_{lR}(x)$) sterile state (do not interact via SM interaction)

The minimal number of the light neutrinos is equal to the number of the flavor neutrinos (three). If there are more than three light neutrinos, sterile neutrinos must exist

General mixing

$$\nu_{lL} = \sum_{i=1}^{3+n_s} U_{li}\,\nu_{iL}, \quad \nu_{sL} = \sum_{i=1}^{3+n_s} U_{li}\,\nu_{iL}$$

$l = e, \mu, \tau$, $s = s_1, s_2, ...s_{n_s}$, n_s is the number of the sterile neutrinos, U is $(3+n_s)\times(3+n_s)$ unitary mixing matrix.

The simplest case of two neutrino fields

$$\mathcal{L}^{\mathrm{D+M}} = -\frac{1}{2}\,\bar{n}_L\,M^{\mathrm{D+M}}(n_L)^c + \mathrm{h.c.}. \qquad (1)$$

$$n_L = \binom{\nu_L}{(\nu_R)^c}\ .$$

$$M^{\mathrm{D+M}} = \begin{pmatrix} m_L & m_D \\ m_D & m_R\ . \end{pmatrix}$$

The matrix $M^{\mathrm{D+M}}$ can be easily diagonalized

$$M^{\mathrm{D+M}} = O\,m'\,O^T\ .$$

Eigenvalues

$$m'_{1,2} = \frac{1}{2}(m_R + m_L) \mp \frac{1}{2}\sqrt{(m_R - m_L)^2 + 4\,m_D^2}$$

$$O = \left(\begin{array}{cc} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta, \end{array} \right)$$

$$\tan 2\,\theta = \frac{2 m_D}{m_R - m_L}, \quad \cos 2\,\theta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4\, m_D^2}} \,.$$

The eigenvalues of the symmetrical matrix $M^{\mathrm{D+M}}$ can be positive and negative. Let us write down

$$m'_i = m_i\,\mu_i \ , \qquad (2)$$

where $m_i = |m'_i|$ and $\mu_i = \pm 1$.

$$M^{\mathrm{D+M}} = U\, m\, U^T \ ,$$

Here

$$U = O\,\sqrt{\mu}$$

U is the unitary matrix.

The fields ν_L and $(\nu_R)^c$ are connected with the left-handed components of the Majorana fields ν_{1L} and ν_{2L}

$$\begin{aligned} \nu_L &= \cos\theta\sqrt{\eta_1}\,\nu_{1L} + \sin\theta\sqrt{\eta_2}\,\nu_{2L} \\ (\nu_R)^c &= -\sin\theta\sqrt{\eta_1}\,\nu_{1L} + \cos\theta\sqrt{\eta_2}\,\nu_{2L} \end{aligned}$$

Some special cases

1. $m_D = 0$

$\theta = 0$, no mixing and

$$\nu_L + (\nu_L)^c = \nu_1, \quad \nu_R + (\nu_R)^c = \nu_2 \ ,$$

ν_1 and ν_2 are Majorana fields with masses $m_1 = m_L$ and $m_2 = m_R$. Mixing is due to nondiagonal term

2. $m_R = m_L = m > 0$

$$m'_{1,2} = m_{1,2} = m \mp m_D \tag{3}$$

$$\theta = \frac{\pi}{4}$$

$$\nu_L = \frac{1}{\sqrt{2}}(\nu_{1L}+\nu_{2L}), \quad (\nu_R)^c = \frac{1}{\sqrt{2}}(-\nu_{1L}+\nu_{2L}) \ .$$

If diagonal terms are equal, mixing is maximal

3. $m_L = 0, \quad m_R \gg m_D$

Eigenvalues

$$m'_1 = \frac{1}{2}m_R - \frac{1}{2}\sqrt{m_R^2 + 4\,m_D^2} \simeq -\frac{m_D^2}{m_R}\ ,$$
$$m'_2 = \frac{1}{2}m_R + \frac{1}{2}\sqrt{m_R^2 + 4\,m_D^2} \simeq m_R$$

The mixing angle

$$\tan 2\theta = \frac{2\,m_D}{m_R} \ll 1\ .$$

$$\theta \ll 1,\ \mu_1 = -1,\ \mu_2 = 1$$

$$\nu_L \simeq i\,\nu_{1L}, \quad (\nu_R)^c \simeq \nu_{2L}.$$

The masses of the Majorana particles are equal to

$$m_1 \simeq \frac{m_D^2}{m_R} \ll m_D, \quad m_2 \simeq m_R\ .$$

Famous see-saw relations. The most plausible mechanism of the generation of neutrino masses

OSCILLATIONS OF NEUTRINOS IN VACUUM

Neutrinos interact with other particles via the standard charged current

$$j_{\alpha}^{CC}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\, \gamma_{\alpha}\, l_L(x)$$

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x)$$

What is the state of neutrino produced in the decay

$$a \to b + l^{+} + \nu_l$$

together with lepton l^{+} $(l = e.\mu, \tau)$?

The vector of the state of the final particles

$$|f\rangle = \sum_{i=1}^{3} |\nu_i\rangle \;\; |l^{+}\rangle \;\; |b\rangle \;\; \langle \nu_i\, l^{+}\, b\, |S|\, a\rangle$$

$|\nu_i\rangle$ is the vector of the state of neutrino with mass m_i and momentum $p_i = (E_i = \sqrt{p^2 + m_i^2},\ \vec{p})$

$\frac{m_i^2}{E^2} \leq 10^{-12}$ and neutrino masses m_i can be neglected in the matrix element

$$\langle \nu_i \, l^+ \, b \, |S| \, a \rangle \simeq U_{li}^* \; \langle \nu_l \, l^+ \, b \, |S| \, a \rangle_{SM}$$

SM matrix element does not depend on neutrino masses

$$|f\rangle = |\nu_l\rangle \;\; |l^+\rangle \;\; |b\rangle \;\; \langle \nu_l \, l^+ \, b \, |S| \, a \rangle_{SM}$$

Neutrino state

$$|\nu_l\rangle = \sum_i U_{li}^* \;\; |\nu_i\rangle \; .$$

These states are orthogonal and normalized

$$\langle \nu_{l'} | \nu_l \rangle = \sum_i^3 U_{l'i} \; U_{li}^* = \delta_{l'l}$$

States of flavor neutrinos ν_e, ν_μ, ν_τ are coherent superpositions of states of neutrinos with definite masses

Analogously, states of flavor antineutrinos $\bar{\nu}_l$, produced together with lepton l^-

$$|\bar{\nu}_l\rangle = \sum_{i=1}^3 U_{li} \;\; |\bar{\nu}_i\rangle$$

Evolution of the mixed flavor neutrino (antineutrino) states in vacuum

If initial state is $|\Psi(0)\rangle$ the state $t \geq 0$

$$|\Psi(t)\rangle = e^{-i\,H\,t}\,|\Psi(0)\rangle \ ,$$

H is the total Hamiltonian

For the mixed neutrino state

$$|\nu_l\rangle_t = e^{-i\,H\,t}\,|\nu_l\rangle = \sum_{i=1}^{3}\, e^{-i\,E_i\,t}\,U_{li}^{*}\ |\nu_i\rangle$$

During the time t different mass-components of the coherent neutrino state acquire *different phases*

Neutrinos are detected via the observation of weak processes

Developing the state $|\nu_l\rangle_t$ over the flavor states we have

$$|\nu_l\rangle_t = \sum_{l=e,\mu,\tau} \mathcal{A}(\nu_l \to \nu_{l'};\ t)|\nu_{l'}\rangle$$

The amplitude of the transition $\nu_l \to \nu_{l'}$ during the time t

$$\mathcal{A}(\nu_l \to \nu_{l'};\ t) = \sum_{i=1}^{3} U_{l'i}\, e^{-i\, E_i\, t}\, U_{li}^{*}$$

Has a simple meaning: the last factor U_{li}^{*} describes transition from the state $|\nu_l\rangle$ into the state with the definite mass $|\nu_i\rangle$, the factor $e^{-i\, E_i\, t}$ describes propagation in the state with the energy E_i and the first factor describes transition from the state $|\nu_i\rangle$ into the state $|\nu_{l'}\rangle$

The amplitude of the transition $\bar{\nu}_l \to \bar{\nu}_{l'}$ during the time t

$$\mathcal{A}(\bar{\nu}_l \to \bar{\nu}_{l'};\ t) = \sum_{i=1}^{3} U_{l'i}^{*}\, e^{-i\, E_i\, t}\, U_{li}$$

The probabilities of the transitions $\nu_l \to \nu_{l'}$ and $\bar{\nu}_l \to \bar{\nu}_{l'}$ are equal

$$\mathcal{P}(\nu_l \to \nu_{l'}) = |\sum_{i=1}^{3} U_{l'i}\, e^{-i\,E_i\,t}\, U_{li}^* \,|^2$$

$$\mathcal{P}(\bar{\nu}_l \to \bar{\nu}_{l'}) = |\sum_{i=1}^{3} U_{l'i}^*\, e^{-i\,E_i\,t}\, U_{li} \,|^2$$

Comparing these expressions we have

$$\mathrm{P}(\nu_\alpha \to \nu_{\alpha'}) = \mathrm{P}(\bar{\nu}_{\alpha'} \to \bar{\nu}_\alpha) \quad (CPT)$$

If the CP invariance in the lepton sector holds

$$U_{li}^* = U_{li}(\mathrm{Dirac}) \quad \mathrm{U}_{\mathrm{li}}^* = \mathrm{U}_{\mathrm{li}}\,\eta_{\mathrm{i}}(\mathrm{Majorana})$$

$\eta_i = \pm i$ is the CP-parity of the Majorana neutrino with the mass m_i

$$U_{l'i}\, U_{li}^* = U_{l'i}^*\, U_{li} \tag{4}$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \mathrm{P}(\bar{\nu}_l \to \bar{\nu}_{l'}) \quad (CP)$$

Transition probability

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \sum_{i,k} U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk} e^{-i\,(E_i - E_k)\,t}$$

$$\frac{m_i^2}{p^2} \ll 1 \qquad E_i \simeq p + \frac{m_i^2}{2p}$$

The phase difference

$$E_i - E_k \simeq \frac{\Delta m^2_{ki}}{2E}$$

$$\Delta m^2_{ki} = m_i^2 - m_k^2$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \sum_i |U_{l'i}|^2\, |U_{li}|^2$$

$$+2\,\mathrm{Re} \sum_{\mathrm{i>k}} \mathrm{U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk} e^{-i\,\frac{\Delta m^2_{ki}}{2E}\,L}}$$

L is distance between production and detection points

Convenient to present in another form. From the unitarity

$$\sum_i U_{l'i} U^*_{li} = \delta_{l'l}$$

From this relation

$$\sum_i |U_{l'i}|^2 \, |U_{li}|^2 = \delta_{l'l} - 2\,\mathrm{Re} \sum_{\mathrm{i>k}} \mathrm{U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk}}$$

Combining two expressions

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \delta_{l'l}$$

$$- \quad 2\,\mathrm{Re} \sum_{\mathrm{i>k}} \mathrm{U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk}} \left(1 - \mathrm{e}^{-\mathrm{i}\, \frac{\Delta m^2_{ki}}{2E} L}\right)$$

$$\mathrm{Re\, a\, b = Rea\, Reb - Ima\, Imb}$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \delta_{l'l}$$

$$- \quad 2 \sum_{i>k} \mathrm{Re} \left(\mathrm{U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk}}\right) \left(1 - \cos \frac{\Delta \mathrm{m}^2_{\mathrm{ki}}}{2\mathrm{E}} \mathrm{L}\right)$$

$$+ \quad 2 \sum_{i>k} \mathrm{Im} \left(\mathrm{U_{l'i}\, U^*_{l'k}\, U^*_{li}\, U_{lk}}\right) \sin \frac{\Delta \mathrm{m}^2_{\mathrm{ki}}}{2\mathrm{E}} \mathrm{L}$$

$$
\begin{aligned}
\mathrm{P}(\bar{\nu}_l \to \bar{\nu}_{l'}) &= \delta_{l'l} \\
&- 2 \sum_{i>k} \mathrm{Re}\,(\mathrm{U}_{l'i}\,\mathrm{U}^*_{l'k}\,\mathrm{U}^*_{li}\,\mathrm{U}_{lk})\,(1 - \cos\frac{\Delta \mathrm{m}^2_{ki}}{2\mathrm{E}}\,\mathrm{L}) \\
&- 2 \sum_{i>k} \mathrm{Im}\,(\mathrm{U}_{l'i}\,\mathrm{U}^*_{l'k}\,\mathrm{U}^*_{li}\,\mathrm{U}_{lk})\,\sin\frac{\Delta \mathrm{m}^2_{ki}}{2\mathrm{E}}\,\mathrm{L}
\end{aligned}
$$

Another convenient form (k is fixed)

$$
\mathrm{P}(\nu_l \to \nu_{l'}) = |\delta_{l'l} + \sum_{i \neq k} U_{l'i}\,(e^{-i\,\frac{\Delta m^2_{ki}}{2E}\,L} - 1)\,U^*_{li}|^2
$$

$$
\mathrm{P}(\bar{\nu}_l \to \bar{\nu}_{l'}) = |\delta_{l'l} + \sum_{i \neq k} U^*_{l'i}\,(e^{-i\,\frac{\Delta m^2_{ki}}{2E}\,L} - 1)\,U_{li}|^2
$$

Neutrino and antineutrino transition probabilities depend on the parameter $\frac{L}{E}$ and are determined by the elements of the neutrino mixing matrix and Δm^2_{ki}.

TWO NEUTRINO OSCILLATIONS

$$\nu_l \rightleftarrows \nu_{l'} \quad l, l' = e, \mu, \tau$$

$$m_1 < m_2,\ k = 1$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\delta_{l'l} + U_{l'2}\,(e^{-i\,\frac{\Delta m^2}{2E}\,L} - 1)\,U^*_{l2}|^2$$

$$\Delta m^2 = m_2^2 - m_1^2.$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = 2\,|U_{l'2}|^2\,|U_{l2}|^2\,(1 - \cos\frac{\Delta m^2}{2E}\,L) \quad (l' \neq l)\,.$$

2×2 real orthogonal matrix

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The standard two-neutrino probability

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \frac{1}{2}\,\sin^2 2\theta\,(1 - \cos\frac{\Delta m^2}{2E}\,L) \quad (l \neq l')\,.$$

$$\mathrm{P}(\nu_l \to \nu_l) = 1 - \mathrm{P}(\nu_l \to \nu_{l'})$$

$$= 1 - \frac{1}{2}\,\sin^2 2\theta\,(1 - \cos\frac{\Delta m^2}{2E}\,L)$$

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \frac{1}{2}\,\sin^2 2\theta\,(1 - \cos 2.54\,\frac{\Delta m^2}{E}\,L) \quad l \neq l'$$

Δm^2 in eV^2, E in MeV (GeV) L in m (km)

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \frac{1}{2}\,\sin^2 2\theta\,(1 - \cos 2\,\pi\,\frac{L}{L_{\mathrm{osc}}}) \quad (l \neq l')\,,$$

Oscillation length

$$L_{\mathrm{osc}} = 4\,\pi\,\frac{E}{\Delta m^2}$$

$$L_{\mathrm{osc}} \simeq 2.47\,\frac{E}{\Delta m^2}\,\mathrm{m}$$

Consider $P(\nu_\mu \to \nu_\tau)$ as a function of $\frac{L}{E}$.
Assume $\sin^2 2\theta = 1$. At the points

$$\frac{L}{E} = \frac{2\pi\,n}{2.54\,\Delta m^2} \quad \text{and} \quad \frac{\mathrm{L}}{\mathrm{E}} = \frac{2\pi\,(\mathrm{n} + \frac{1}{2})}{2.54\,\Delta \mathrm{m}^2} \quad (\mathrm{n} = 0, 1, 2, ...)$$

only ν_μ and ,correspondingly, ν_τ can be observed. At other values of $\frac{L}{E}$ both ν_μ and ν_τ can be found

NEUTRINO OSCILLATION IN THE LEADING APPROXIMATION

Consider neutrino oscillations in the minimal scheme of the three-neutrino mixing

PMNS mixing matrix is characterized by three angles θ_{12}, θ_{23} and θ_{13} and CP-phase δ

Three Euler rotations

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{12} = \cos\theta_{12}, \; s_{12} = \sin\theta_{12}, \ldots$$

Transition probabilities depend on six parameters: Δm^2_{12}, Δm^2_{23}, $\theta_{12}, \theta_{13}, \theta_{23}$ and δ and have rather complicated form

However, from the analysis of the neutrino oscillation data

$$\Delta m_{12}^2 \simeq 3 \cdot 10^{-2} \Delta m_{23}^2$$

We will consider first neutrino oscillations driven by Δm_{23}^2. For $k = 2$ we have

$$\mathrm{P}(\nu_l \to \nu_{l'}) = |\delta_{l'l} + \sum_{i \neq 2} U_{l'i} \, (e^{-i \frac{\Delta m_{2i}^2}{2E} L} - 1) \, U_{li}^*|^2$$

We are interested in such $\frac{L}{E}$ for which

$$\frac{\Delta m_{23}^2}{2E} \; L \gtrsim 1$$

Such condition is fulfilled in the atmospheric and long baseline accelerator (K2K, MINOS,...) and reactor (CHOOZ,...) experiments

$$\frac{\Delta m_{12}^2}{2E} \, L \ll 1$$

We will neglect Δm_{12}^2 in the transition probability

$$\mathrm{P}(\nu_l \to \nu_{l'}) \simeq |\delta_{l'l} + U_{l'3} \, (e^{-i \frac{\Delta m_{23}^2}{2E} L} - 1) \, U_{l3}^*|^2$$

The appearance probability

$$\mathrm{P}(\nu_l \to \nu_{l'}) = \frac{1}{2}\,\mathrm{A}_{l'l}\,(1 - \cos\Delta m^2_{23}\frac{L}{2E}) \quad (l' \neq l)\ ,$$

$$\mathrm{A}_{l'l} = 4|U_{l'3}|^2\ |U_{l3}|^2 = \mathrm{A}_{ll'}$$

is the oscillation amplitude

For the ν_l-survival probability we find

$$\mathrm{P}(\nu_l \to \nu_l) = 1 - \sum_{l' \neq l} \mathrm{P}(\nu_l \to \nu_{l'})$$

$$= 1 - \mathrm{B}_{ll}\,(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

$$\mathrm{B}_{ll} = \sum_{l' \neq l} \mathrm{A}_{l'l} = 4\ |U_{l3}|^2\ (1 - |U_{l3}|^2)$$

$|U_{e3}|^2 = \sin^2\theta_{13}$ The $\bar{\nu}_e$ survival probability we obtain

$$\mathrm{P}(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2}\ \sin^2 2\theta_{13}\,(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

In the reactor CHOOZ experiment neutrino oscillations were not observed

$$\sin^2\theta_{13} \leq 5 \cdot 10^{-2}\ .$$

If we neglectΔm^2_{12} and $\sin^2\theta_{13}$ we have

$$A_{e\mu} = A_{\tau e} = 0, \quad B_{ee} = 0\ .$$

In the leading approximation neutrino oscillations driven by Δm^2_{23} are $\nu_\mu \rightleftarrows \nu_\tau$. For the probability of ν_μ to survive we obtain in this approximation

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{2}\ \sin^2 2\theta_{23}\,(1 - \cos\Delta m^2_{23}\frac{L}{2E})$$

Atmospheric and LBL accelerator neutrino oscillation data are perfectly described by this expression

Neutrino oscillations in experiments with $\frac{L}{E}$ which satisfy the inequality

$$\Delta m^2_{12}\,\frac{L}{2E} \geq 1, \quad \Delta m^2_{23}\,\frac{L}{2E} \gg 1$$

This condition is satisfied in the reactor KamLAND experiment

Effect of the large neutrino mass-squared difference is averaged.

$$\overline{\mathrm{P}}(\bar{\nu}_e \to \bar{\nu}_e) = |\sum_{i=1,2} |U_{ei}|^2 \, e^{i\,\frac{\Delta m^2_{i2}}{2E}\,L}|^2 + |U_{e3}|^4$$

The first term of this expression can be presented in the form

$$|\sum_{i=1,2} |U_{ei}|^2 \, e^{i\,\frac{\Delta m^2_{i2}}{2E}\,L}|^2 =$$

$$\sum_{i=1,2} |U_{ei}|^4 + 2\,|U_{e1}|^2\,|U_{e2}|^2 \,\cos\frac{\Delta m^2_{12}}{2E}\,L$$

Taking into account the unitarity of the mixing matrix

$$\sum_{i=1,2} |U_{ei}|^4 = (1 - |U_{e3}|^2)^2 - 2\,|U_{e1}|^2\,|U_{e2}|^2 \;.$$

$$\overline{\mathrm{P}}(\nu_e \to \nu_e) = \overline{\mathrm{P}}(\bar{\nu}_e \to \bar{\nu}_e) =$$
$$|U_{e3}|^4 + (1 - |U_{e3}|^2)^2 \,\mathrm{P}^{(12)}(\nu_e \to \nu_e)$$

$$\mathrm{P}^{(12)}(\nu_e \to \nu_e) = 1 - \frac{1}{2}\,\sin^2 2\,\theta_{12}\,(1 - \cos\frac{\Delta m^2_{12}}{2E}\,L)$$

General relation based on $\Delta m^2_{12} \ll \Delta m^2_{23}$.
Valid also in matter

In the leading approximation
$|U_{e3}|^2 = \sin^2\theta_{13} \to 0$

$$\mathrm{P}(\bar\nu_e \to \bar\nu_e) = 1 - \frac{1}{2}\sin^2 2\,\theta_{12}\,(1 - \cos\frac{\Delta m^2_{12}}{2E}\,L)\ .$$

Perfectly describes the data of the KamLAND reactor neutrino experiments in which neutrino oscillations driven by Δm^2_{12} were observed

EVIDENCE FOR NEUTRINO OSCILLATIONS

Super-Kamiokande atmospheric neutrino experiment

e and μ are detected in the large 50 ktonn water Cherenkov underground detector.

Large up-down asymmetry of muon events was observed

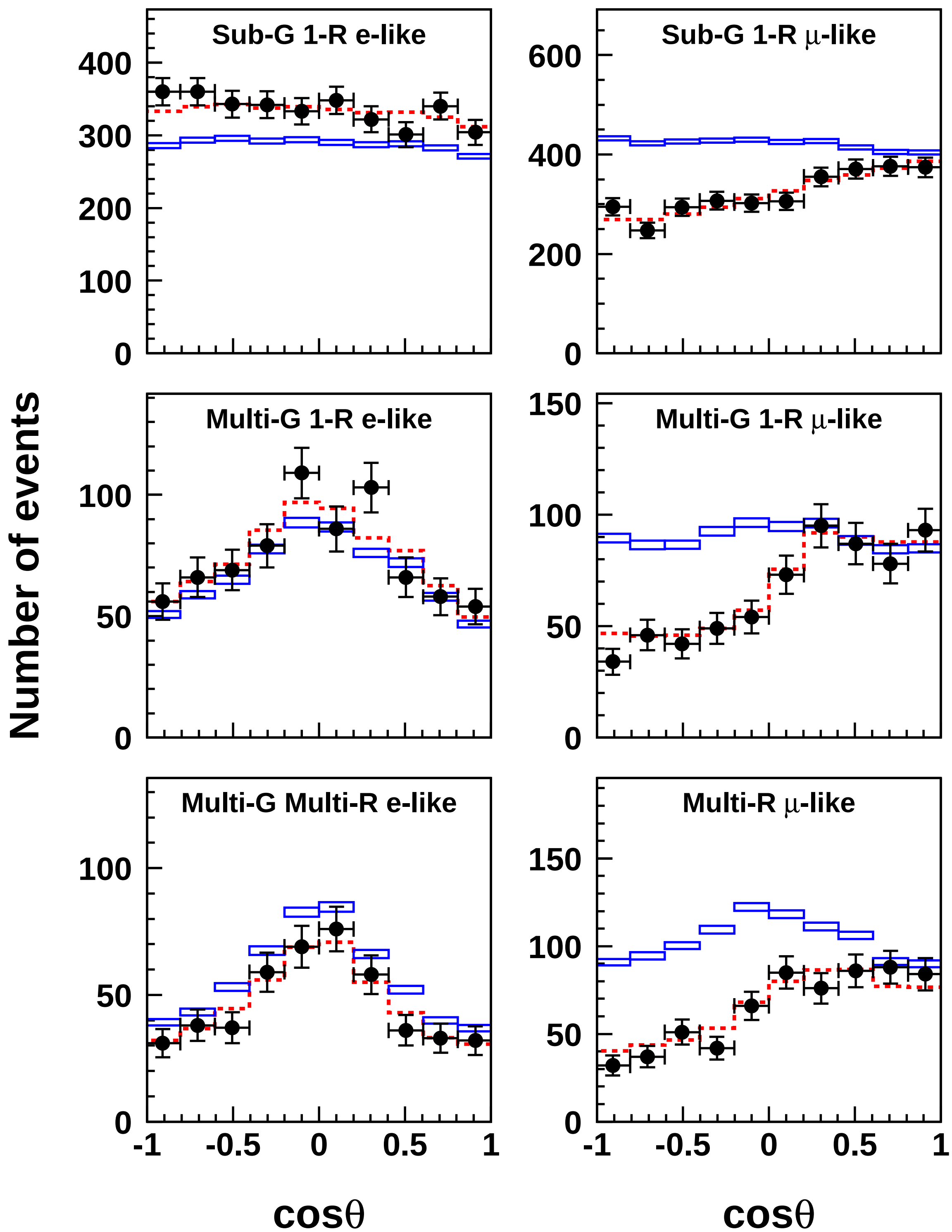
$$\left(\frac{U}{D}\right)_{\mu} = 0.551 \pm 0.035 \pm 0.004$$

U is total number of the up-going muons (13000 -500 km). D is total number of the down-going muons (20 - 500 km). If there is no neutrino oscillations

$$N_l(\cos\theta) = N_l(-\cos\theta) \quad (l = e, \mu)$$

θ is zenith angle

Strong violation of this symmetry relation for muon events was observed

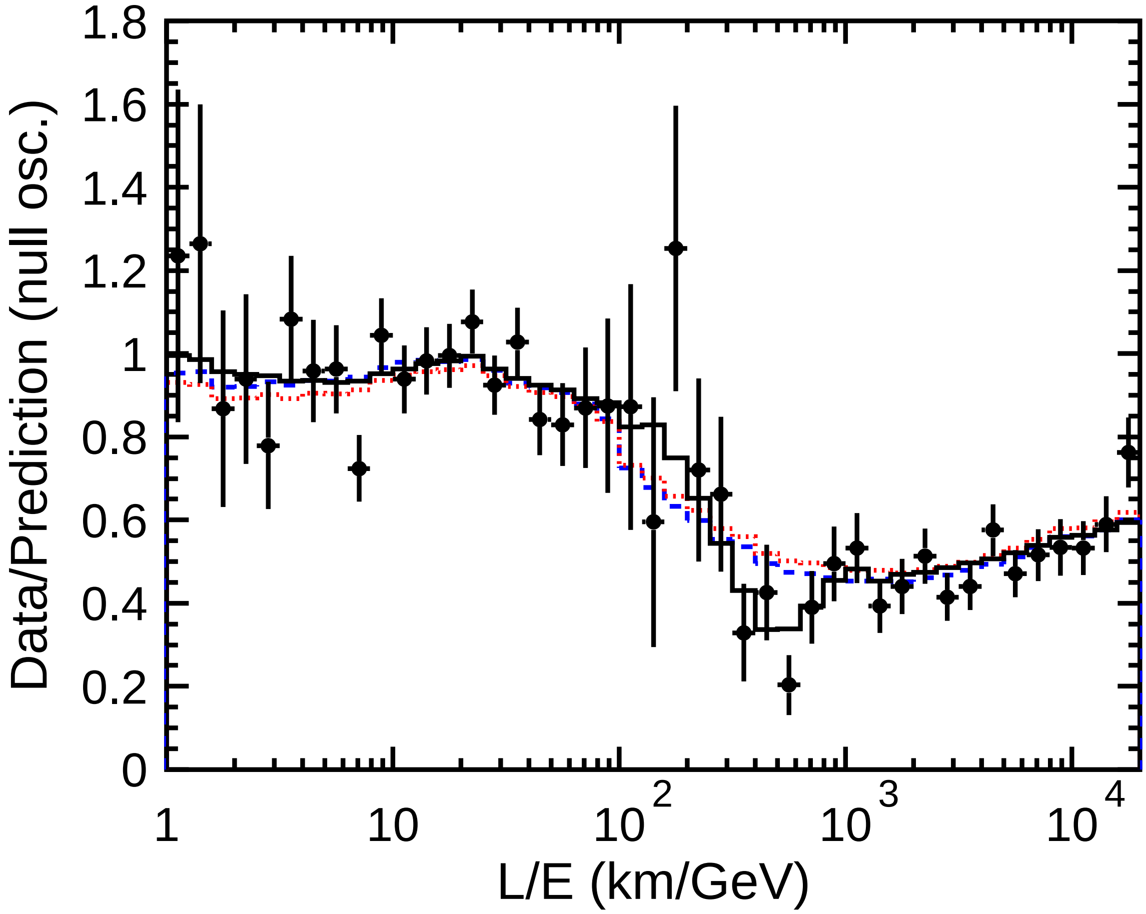


Number of events
Sub-G 1-R e-like
Sub-G 1-R μ-like
Multi-G 1-R e-like
Multi-G 1-R μ-like
Multi-G Multi-R e-like
Multi-R μ-like
cosθ
cosθ

In the SK experiment $\frac{L}{E}$ dependence of the ν_μ survival probability was measured

First minimum at 1.27 $\Delta m^2_{23}\frac{L}{E} = \pi/2$

First oscillation minimum



All SK data are described by two- neutrino oscillations

$$1.9 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.1 \cdot 10^{-3}\mathrm{eV}^2; \quad \sin^2 2\theta_{23} > 0.9. \ \ (90\%\ \mathrm{CL})$$

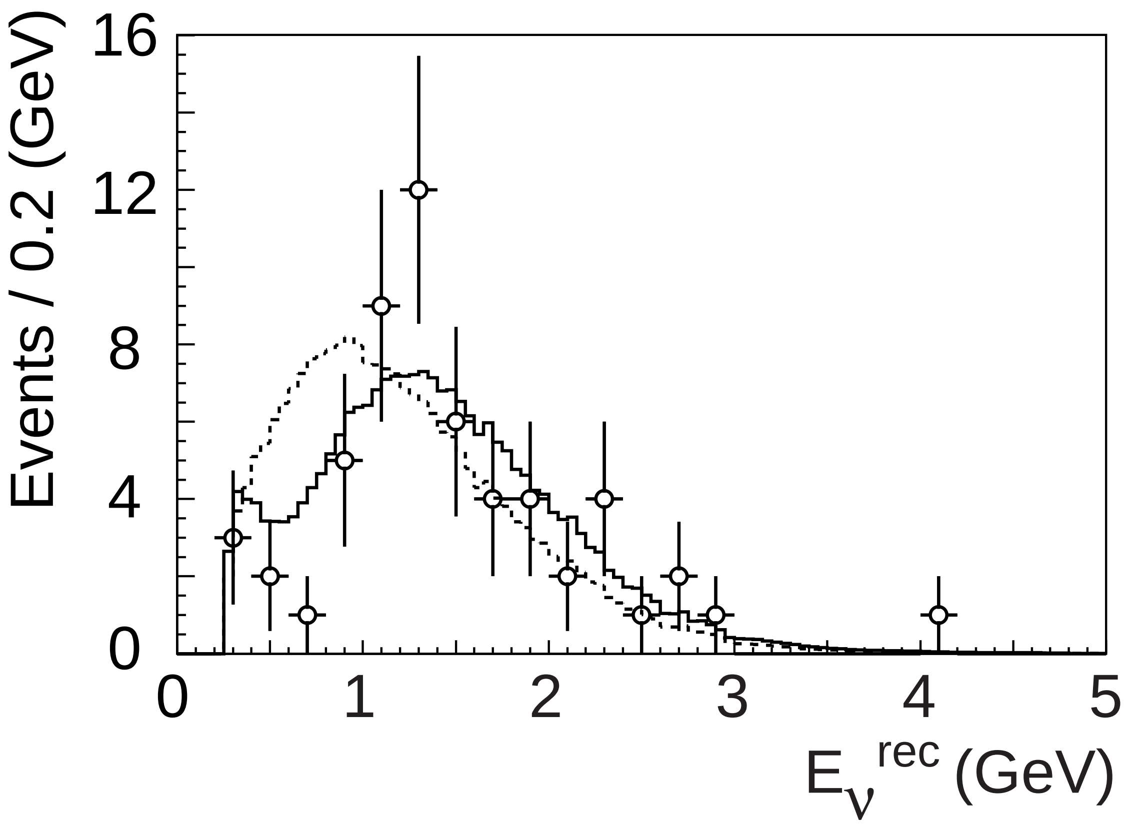
Best fit:

$$\Delta m^2_{23} = 2.5 \cdot 10^{-3}\mathrm{eV}^2; \quad \sin^2 2\theta_{23} = 1$$

$$(\chi^2/\mathrm{dof} = 839.7/755)$$

Super-Kamiokande atmospheric neutrino evidence for neutrino oscillations were confirmed by accelerator K2K and MINOS experiments

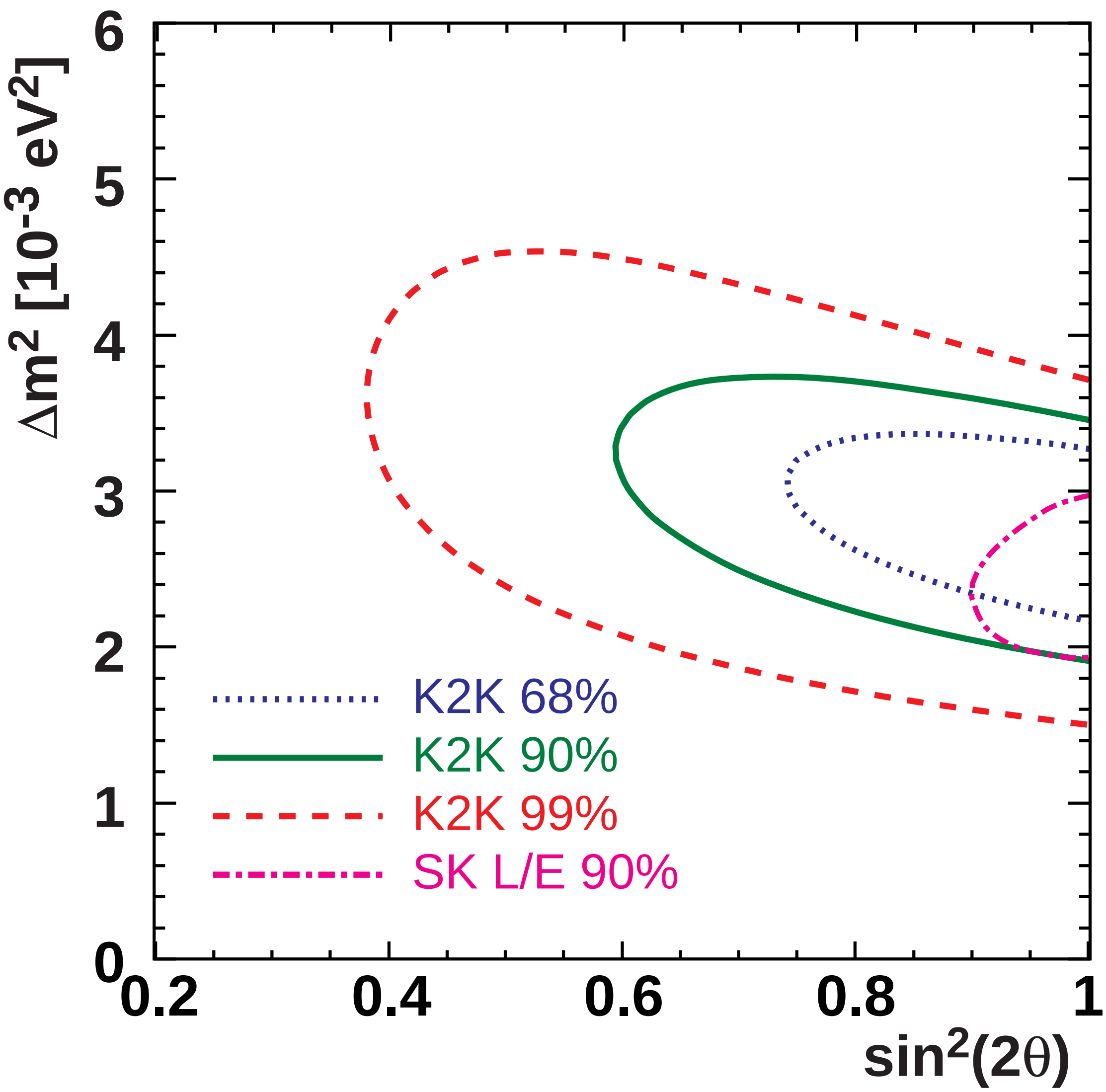
In K2K experiment (KEK-SK, 250 km)
spectrum distortion of ν_μ was observed



The K2K result is compatible with SK

The best fit values

$$\Delta m^2_{23} = 2.64 \cdot 10^{-3}\text{eV}^2; \quad \sin^2 2\theta_{23} = 1$$



In the MINOS accelerator experiment (Fermilab-Soudan, 735 km) the number of expected (without oscillations) $\nu_\mu + \bar{\nu}_\mu$ events with energy $\leq$ 30 MeV is 336 ± 14.4. The number of observed events is 215

$$2.31 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.43 \cdot 10^{-3}\mathrm{eV}^2$$
$$\sin^2 2\theta_{23} > 0.78. \quad (90\% \; CL)$$

Best fit:
$$\Delta m^2_{23} = 2.72 \cdot 10^{-3}\mathrm{eV}^2; \quad \sin^2 2\theta_{23} = 1$$

Latest preliminary results
$$\Delta m^2_{23} = (2.38^{+0.20}_{-0.16}) \cdot 10^{-3}\mathrm{eV}^2$$
$$\sin^2 2\theta_{23} = 1.00^{+0.00}_{-0.08}$$

Perfect agreement with the Super Kamiokande and K2K

Solar neutrinos

Produced mainly in the following reactions of the solar p-p cycle

$$p + p \to d + e^{+} + \nu_e; \quad E \leq 0.42\,\mathrm{MeV}$$
$$e^{-} + {}^{7}\mathrm{Be} \to \nu_{\mathrm{e}} + {}^{7}\mathrm{Li}; \quad \mathrm{E} = 0.86\,\mathrm{MeV}$$
$${}^{8}\mathrm{B} \to {}^{8}\mathrm{Be} + \mathrm{e}^{+} + \nu_{\mathrm{e}}; \quad \mathrm{E} \leq 15\,\mathrm{MeV}$$

Different experiments are sensitive to different sources

Homestake : ^{8}B and ^{7}Be,

GALLEX-GNO and SAGE : $p-p$, ^{7}Be, ^{8}B,

SNO and SK : ^{8}B

In all experiments observed rates are significantly smaller than SSM predicted rates (2-3 times)

Model independent evidence for neutrino oscillations was obtained in the SNO experiment

Solar neutrinos were detected via the observation of three reactions

$$\text{CC} \quad \nu_e + d \to e^- + p + p$$

$$\text{NC} \quad \nu_\text{x} + \text{d} \to \nu_\text{x} + \text{n} + \text{p}$$

$$\text{ES} \quad \nu_\text{x} + \text{e} \to \nu_\text{x} + \text{e}$$

Only high energy ^8B neutrinos can be detected. From CC the flux of ν_e can be inferred. From NC the flux of all active neutrinos ν_e, ν_μ and ν_τ can be determined.

$$\Phi_{\nu_e}^{\text{SNO}} = (1.68 \pm 0.06 \pm 0.09) \cdot 10^6 \ \ cm^{-2}s^{-1}$$

$$\Phi_{\nu_{e,\mu,\tau}}^{\text{SNO}} = (4.94 \pm 0.21 \pm 0.38) \cdot 10^6 \ \ cm^{-2}s^{-1}$$

$$\frac{\Phi_{\nu_e}^{\text{SNO}}}{\Phi_{\nu_{e,\mu,\tau}}^{\text{SNO}}} = 0.340 \pm 0.023 \pm 0.031$$

The total flux measured by SNO is in agreement with the flux predicted by SSM

$$\Phi^{\mathrm{SSM}}_{\nu_e} = (5.69 \pm 0.91) \cdot 10^6 \ cm^{-2} s^{-1}$$

From global analysis of all solar data

$$\Delta m^2_{21} = 6.5^{+4.4}_{-2.3} \cdot 10^{-5} \mathrm{eV}^2; \quad \tan^2\theta_{12} = 0.45 \pm 0.09$$

KamLAND reactor experiment

$\bar{\nu}_e$'s from 53 reactors in Japan are detected via the observation of the reaction

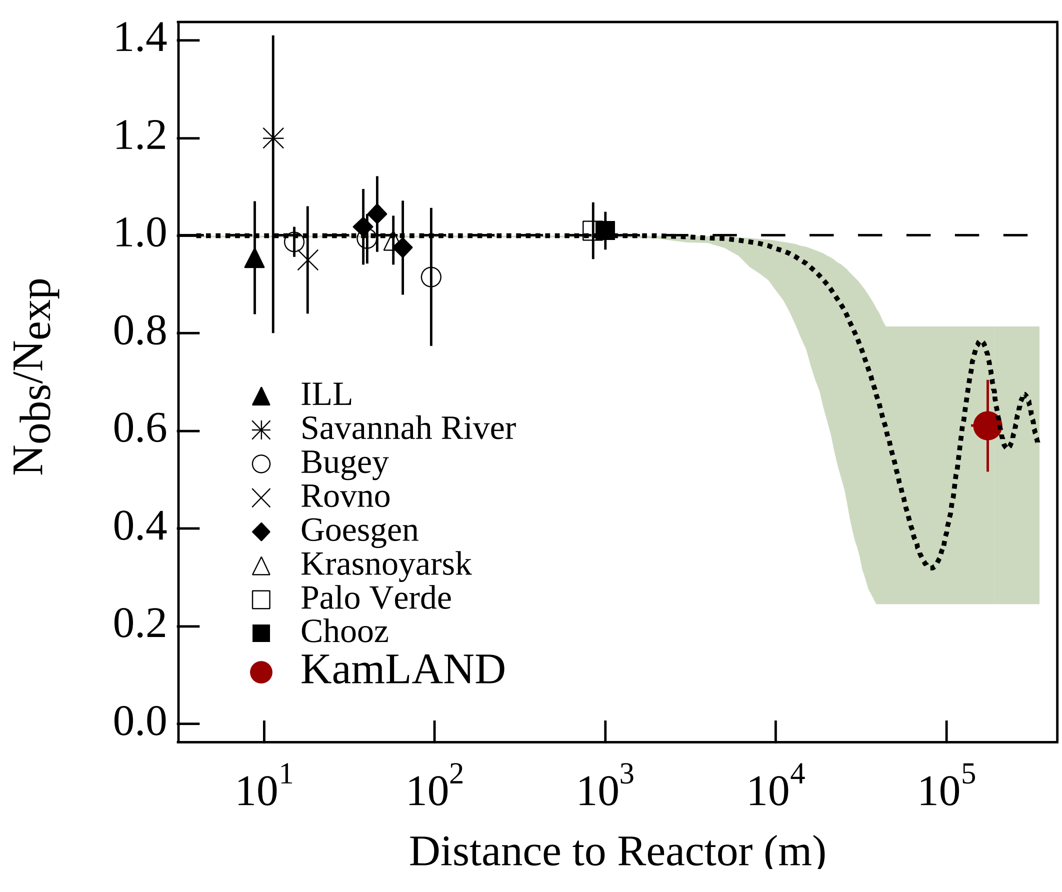
$$\bar{\nu}_e + p \to e^+ + n$$

Average distance from reactors to detector about 170 km

Sensitive to Δm^2_{12}

The expected number of the events (without oscillations) 365.2 ± 23.7. The observed number 258 events. The ratio of the observed and expected events

$$R = 0.658 \pm 0.044 \pm 0.047$$



Distortion of the spectrum of positrons produced in the reaction $\bar{\nu}_e + p \to e^+ + n$ was observed in the KamLAND experiment

From global analysis of all solar and KamLAND data

$$\Delta m^2_{12} = 8.0^{+0.6}_{-0.4}\ 10^{-5}\ \mathrm{eV}^2$$
$$\tan^2\theta_{12} = 0.45^{+0.09}_{-0.07}$$

SUMMARIZING

TODAY

Model independent evidence for neutrino oscillations

Four neutrino oscillation parameters are known

$$\Delta m^2_{12}(\sim 10\%); \quad \tan^2\theta_{12}(\sim 20\%)$$
$$\Delta m^2_{23}\,\mathrm{and}\,\sin^2 2\theta_{23}(\sim 10\%)$$

FUTURE

Improvement of accuracies in the measurement of neutrino oscillation fundamental parameters (MINOS, T2K, Nova, Future neutrino facilities: Superbeam, β-beam, neutrino factory)

The absolute values of neutrino masses

From the measurement of the high-energy part of the β-spectrum of ^{3}H

$$m_\nu < 2.3 \text{ eV}$$

In the future KATRIN experiment the sensitivity $m_\nu \simeq 0.2$ is planned to be reached

From cosmological data

$$\sum_i m_i < (02. - 0.7) \text{ eV}$$

Improvement is expected

Nature of neutrinos with definite masses (Majorana or Dirac?) (neutrinoless double β decay)

Character of neutrino mass spectrum

Oscillation data are compatible with two types of neutrino mass spectra

N. Normal spectrum

$$m_1 < m_2 < m_3;\ \Delta m^2_{12} \ll \Delta m^2_{23}$$

I. Inverted spectrum

$$m_3 < m_1 < m_2;\ \Delta m^2_{12} \ll |\Delta m^2_{13}|$$

Can be studied in T2K, Nova and other experiments

The value of the parameter $\sin^2\theta_{13}$

New reactor experiments DOUBLE CHOOZ, Daya Bay and others, accelerator experiments T2K, Nova. Factor 10-20 improvement in the sensitivity to $\sin^2\theta_{13}$ is expected.

CP violation in the lepton sector

$$U_{e3} = \sin\theta_{13}\ e^{-i\delta}$$

Effects of CP violation can be observed if parameter $\sin^2\theta_{13}$ is not too small

Sterile neutrinos?

MiniBooNE (no confirmation of LSND)

BRUNO PONTECORVO AND NEUTRINO

In 1933-34 after famous Pauli idea of neutrino E. Fermi proposed first theory of $\beta - decay$

$$n \to p + e^- + \nu_e$$

Fermi Hamiltonian

$$\mathcal{H}_\beta = G_F \bar{p}\gamma_\alpha n \; \bar{e}\gamma^\alpha \nu_e + \text{h.c.}$$

In 1934 Bethe and Peierls calculated the cross section of the interaction of neutrinos with nuclei

During many years neutrinos were considered as undetectable particles

The first breakthrough was done by B. Pontecorvo in his pioneer paper “Inverse β process” (1946, Chalk River, Canada)

He was the first who understood that a reactor is a high-intensity source of neutrinos (actually antineutrinos).In a reactor $\simeq 10^{20}$ antineutrinos are produced per sec

B. Pontecorvo proposed the first radiochemical method of neutrino detection which could allow to overcome the problem of the smallness of the cross section

One of the realization of the B.P. idea was chlorine-argon radiochemical method which he considered as a very promising

If neutrinos irradiate a large volume of C_2Cl_4 via the Pontecorvo-Davis reaction

$$\nu_e +^{37}\mathrm{Cl} \to \mathrm{e}^- +^{37}\mathrm{Ar}$$

radioactive nuclei ^{37}Ar (with half life about 34 days) will be produced. Several atoms of the noble gas ^{37}Ar, can be extracted from a large detector and their decay can be observed in a counter

In 1948 B.Pontecorvo invented low-background proportional counter with high amplification. The Pontecorvo counter was crucial for neutrino detection

Using a reactor as source of (anti)neutrino, F. Reines and C.L.Cohen in the mid-fifties for the first time detected (anti)neutrino via the observation of the process

$$\bar{\nu}_e + p \to e^{+} + n$$

This was the first realization of the Pontecorvo's ideas of the detection of neutrinos. In 1995 Reines was awarded with the Nobel prize for the discovery of the (first) neutrino.

In 1946 paper B. Pontecorvo also pointed out as a possible high-intensity sources of neutrinos the Sun

In the pioneer Homestake solar neutrino experiment (R.Davis et al) the Pontecorvo Cl-Ar method and the Pontecorvo proportional counter were used

This allowed for the first time to detect solar neutrinos

A new field of research, solar neutrino astronomy, was created

In 2002 R.Davis was awarded the Nobel prize for discovery of the solar neutrinos

The results of six solar neutrino experiments are available at present. In three of them Homestake, GALLEX-GNO, SAGE the Pontecorvo radiochemical method and Pontecorvo proportional counter were used

$\mu - e$ universal weak interaction

In 1947-49 in Canada B. Pontecorvo and E. Hincks made a series of pioneer experiments on the investigation of the muon decay

Thinking about muon, a new fascinating particle, B. Pontecorvo paid attention that the constant which characterize μ -capture $\mu^- + (A, Z) \to \nu + (A, Z-1)$ is the same as the Fermi constant , which characterize the electron-capture $e^- + (A, Z) \to \nu + (A, Z-1)$. He came to the fundamental idea of the existence of $\mu - e$ universal weak interaction

Later the idea of the $\mu - e$ universal weak interaction was put forward by Puppi, Klein, Yang and Tiomno

Accelerator neutrinos

In 1958-59 in Dubna B. Pontecorvo came to an idea of the feasability of accelerator neutrino experiments (Markov, Schwartz). He proposed accelerator neutrino experiment which could allow to answer the question

$$\nu_\mu \neq \nu_e \quad \text{or} \quad \nu_\mu \equiv \nu_{\mathrm{e}}$$

The experiment was done at Brookhaven in 1962. It was proved that

$$\nu_\mu \neq \nu_e$$

In 1988 Lederman, Steinberger and Schwartz were awarded with Nobel Prize for the discovery of the second neutrino

NEUTRINO OSCILLATIONS

is the most impressive Pontecorvo idea. He devoted many years of his life to the development of this idea

In the fifties B.Pontecorvo was fascinated by famous phenomenon of $K^0 \rightleftharpoons \overline{K^0}$ oscillations

He believed into hadron-lepton analogy and looked for analogous phenomenon in the lepton world

In 1957 he wrote

> "If the two-component neutrino theory turn out to be incorrect (which at present seems to be rather improbable) and if the conservation law of neutrino charge would not apply, then in principle neutrino $\rightleftharpoons$ antineutrino transitions could take place in vacuum."

In the fifties F. Reines and C. Cowan have been doing a experiment at the Savanah River reactor in which $\bar{\nu}_e$ was discovered

In 1957-58 at the same reactor R.Davis has been doing an experiment on the search for the process

$$\bar{\nu}_e + {}^{37}\mathrm{Cl} \to \mathrm{e}^- + {}^{37}\mathrm{Ar}$$

in which lepton number is violated

A rumor reached B.Pontecorvo that Davis observed such events

Thinking about neutrino oscillations, B.Pontecorvo suggested that $\bar{\nu}_e \to \nu_e$ transitions in vacuum could be the reason

He published his first paper on neutrino oscillations in 1958

In 1958 only one type of neutrinos was known.

B.Pontecorvo assumed that exists some additional interaction between neutrinos which does not conserve the lepton number.

In this case 'neutrino and antineutrino are *mixed* particles, i.e., a symmetric and antisymmetric combination of two truly neutral Majorana particles ν_1 and ν_2 "particles with definite masses. He concluded that under these assumptions transitions of antineutrino into neutrino in vacuum will take place and the "flux of neutral leptons from a reactor at some distance will consist of 1/2 of neutrinos and 1/2 of antineutrinos" “the cross section of the production of neutrons and positrons in the process of the absorption of antineutrinos from a reactor by protons would be smaller than the expected cross section....It would be extremely interesting to perform the Reins-Cowan experiment at different distances from reactor”

In 2002 the effect, predicted by B.Pontecorvo in 1958, was observed in the reactor KamLAND experiment

In the first 1958 paper on neutrino oscillations B.Pontecorvo could not exclude that ^{37}Ar was produced in the reaction which was searched for by R.Davis. Later when two-component neutrino theory, in which neutrino is a left-handed particle and antineutrino is a right-handed particle, was fully established he understood that $\bar{\nu}_e$ can transfer only into right-handed ν_{eR} which is a sterile particle

In fact, B.Pontecorvo was the first who introduced the notion of sterile neutrinos so popular nowadays.

After the second neutrino ν_μ was discovered in Brookhaven and CERN experiments it was natural and not difficult for B.Pontecorvo to generalize his idea of neutrino oscillations for the case of two types of neutrinos

He published his second paper on neutrino oscillations in 1967

He discussed possible oscillations of accelerator and solar neutrinos

Before the first result of the Davis solar neutrino experiment was published (in 1968) B. Pontecorvo envisaged the solar neutrino problem “. ....direct oscillations will be smeared out and unobservable. The only effect on the earth’s suffice would be that the flux of observable solar neutrinos must be two times smaller than the total neutrino flux”.

In 1969 B.Pontecorvo and V. Gribov published important paper. They proposed a scheme with two Majorana neutrinos (without sterile). Majorana mass term. They applied the developed formalism to solar neutrinos

Maki, Nakagawa and Sakata 1962

Neutrino masses was introduced by MNS in 1962 in the framework of the Nagoya model in which nucleons were considered as bound states of a new sort of matter B^+ and leptons.

In addition to ν_e and ν_μ, the authors introduced massive neutrinos ν_1 and ν_2 (the proton was considered as a bound state of a B^+ and ν_1 etc.). They assumed

$$\begin{aligned} \nu_1 &= +\nu_e \cos\delta + \nu_\mu \sin\delta \\ \nu_2 &= -\nu_e \sin\delta + \nu_\mu \cos\delta \end{aligned} \tag{5}$$

MNS shortly discussed the possibility to see effects of $\nu_\mu \to \nu_e$ transitions in Brookhaven neutrino experiment.

"In the present case, however, weak neutrinos are *not stable* due to occurrence of virtual transmutation $\nu_\mu \to \nu_e$ ..." Therefore a chain of reactions

$$\begin{aligned} \pi^+ &\to \mu^+ \nu_\mu \\ \nu_\mu + Z\,(nucleus) &\to Z' + \left(\mu^- \;/\; e^-\right) \end{aligned} \quad (6)$$

is useful to check the two-neutrino hypothesis only when $|m_{\nu_2} - m_{\nu_1}| \le 10^{-6}$ MeV under the conventional geometry of the experiments . Conversely, the absence of e^- in the reaction (5) will be able not only to verify two-neutrino hypothesis but also to provide an upper limit of the mass of the second neutrino ν_2 if the present scheme should be accepted."

We started our collaboration on neutrino oscillations in 1975 in a car ...

We published about 25 papers and first review on neutrino oscillations . The last paper(for Italian Encyclopedia) was written in 1987

We consider the D, M and D+M mass terms for general n types of neutrinos, developed neutrino oscillation formalism, considered different possible experiments.....

The years of work and friendship with Bruno Pontecorvo were the happiest and unforgettable years in my life

His wide and profound knowledge of physics, his love of physics, his ingenious intuition and his ability to understand complicated problems in a clear and simple way were gifts of God

Bruno Pontecorvo was a true scientist in the best, classical sense of the word. When he thought about some problem he thought about it continuously from early morning till late evening

He devoted all his resources and great intellect to science, and though he was not indifferent to the recognition of his contribution to physics, his main stimulus was search for the truth

More than ten last years were for Bruno Pontecorvo years of courages struggle against Parkinson illness. His love to physics and to neutrino helped him to overcome difficult problems of the illness. He never stooped to work, to think about neutrinos and to continue active life.

Two days before his death he was in his office at the second floor of the Laboratory of

Nuclear Problem in Dubna, where he spent 43 years. When he was leaving the Laboratory for the last time he looked into window and said to his secretary Irina Pokrovskaja: “Look how beautiful are these colours....” It was beautiful Russian gold autumn, September 1993

It required many years and heroic efforts of many experimental groups to reveal effects of tiny neutrino masses

The discovery of neutrino oscillations was real triumph of Bruno Pontecorvo who proposed neutrino oscillations and pursued the idea of oscillations for many years, when the general opinion favored massless neutrinos and no neutrino oscillations