Majorana neutrinos,

neutrinoless double beta-decay

and Leptogenesis

III International Pontecorvo neutrino physics school

Alushta, September 2007

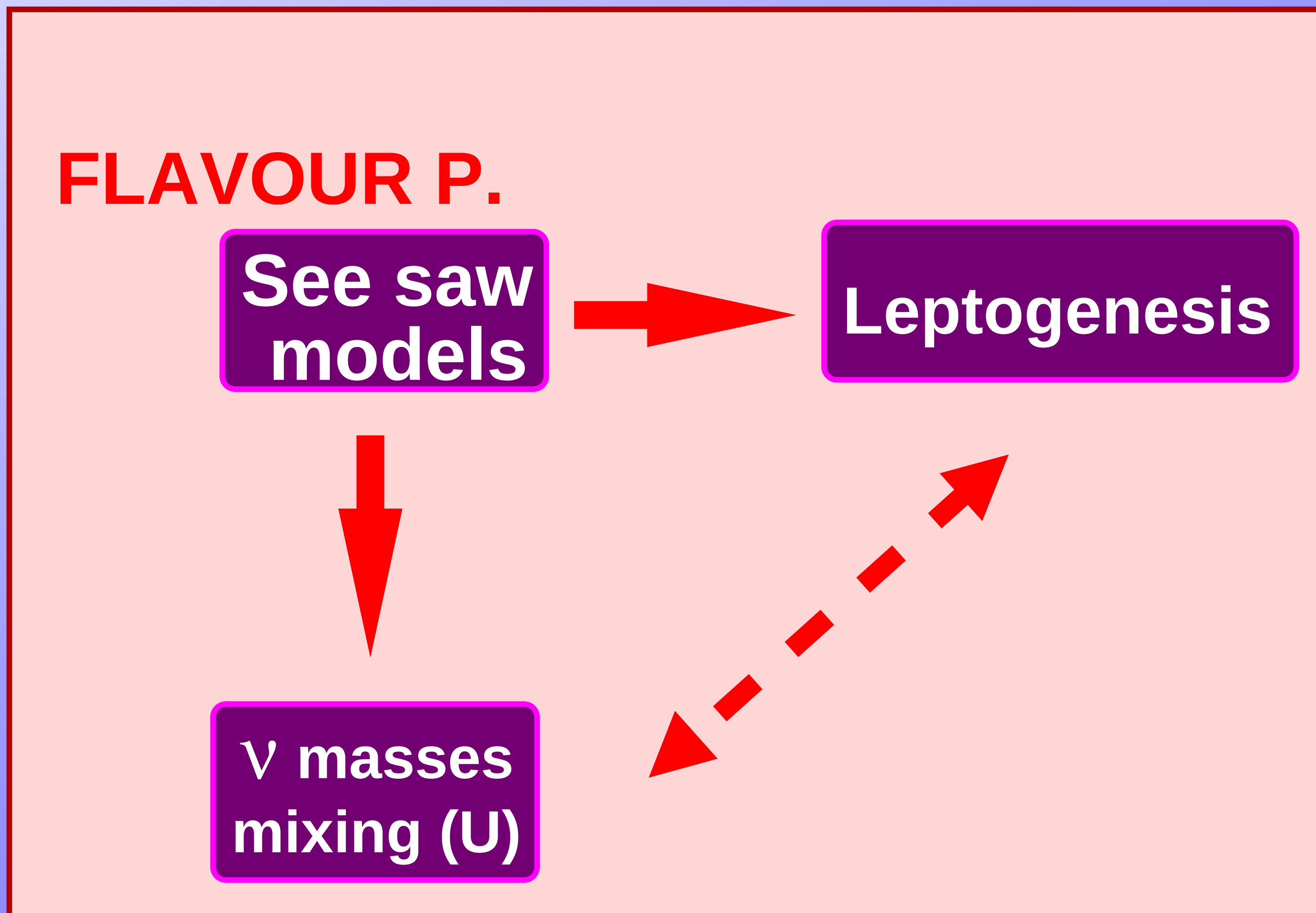
Silvia Pascoli

IPPP - Durham U.

1 – WHAT YOU WILL LEARN FROM THESE LECTURES

- Review of low energy phenomenology, Majorana neutrinos and lepton number violation
- Discuss in detail leptogenesis in the context of the see-saw mechanism of generation of neutrino masses.
- Study the link between the leptogenesis and low energy physics

1 – WHAT YOU WILL LEARN FROM THESE LECTURES



2 – Plan of Lecture 1

From the data to the theory

- 1. Present status of neutrino physics: what we know

neutrino oscillations implies neutrino masses and mixing

- 2. Missing ingredients: what we will measure (hopefully):

we still lack important information about neutrino masses, mixing and CPV

- 3. How to implement neutrino masses and mixing:

Dirac and Majorana mass terms

- 4. Explanation of the mass terms in extensions of the SM:

See-saw mechanism

3 – The SM picture

- Neutrinos are SM particles. They are neutral and transform under $SU(2)$:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- The SM Lagrangian describes neutrino interactions:

$$\boxed{\begin{aligned} \mathcal{L}^{CC}_{\text{lept int}} &= -\frac{g}{\sqrt{2}}\left(\sum_{\alpha=e,\mu,\tau} \bar{\nu}_{\alpha L}\,\gamma_\rho\, l_{\alpha L}\, W^\rho + \text{h.c.}\right) \\ \mathcal{L}^{NC}_{\text{lept int}} &= -\frac{g}{4\cos\theta_W}\left(\sum_{\alpha=e,\mu,\tau} \bar{\nu}_\alpha\,\gamma_\rho\,(1-\gamma_5)\,\nu_\alpha\, Z^\rho + \text{h.c.}\right) \end{aligned}}$$

- In the SM neutrinos are massless.

Notice that this Lagrangian is invariant under a global $U(1)$ transformation for each generation:

$$\boxed{l_\alpha \longrightarrow e^{i\phi}\, l_\alpha \qquad \nu_\alpha \longrightarrow e^{i\phi}\, \nu_\alpha}$$

- This the so called **family lepton number,** L_i.
- The **global lepton number,** L is $L = L_e + L_\mu + L_\tau$:

$$L = \int d^3x \left[\sum_{k=1}^{3} \nu_k^\dagger(x)\nu_k(x) + \sum_{\alpha=e,\mu,\tau} l_\alpha^\dagger(x) l_\alpha(x) \right]$$

- Lepton number violation plays a crucial role in generating a baryon asymmetry in the leptogenesis mechanism.

4 – THE FACTS: neutrino oscillations

- We have compelling **evidence** of neutrino oscillations from atmospheric, solar, reactor and accelerator neutrino experiments.

- Neutrino oscillations require neutrino **masses** ($\Delta m^2 \neq 0$) and neutrino **mixing** ($\theta \neq 0$):

$$P(\nu_a \to \nu_b) = \sin^2 2\theta \; \sin^2 \frac{\Delta m^2 L}{4E}$$

First evidence of Physics beyond the Standard Model.

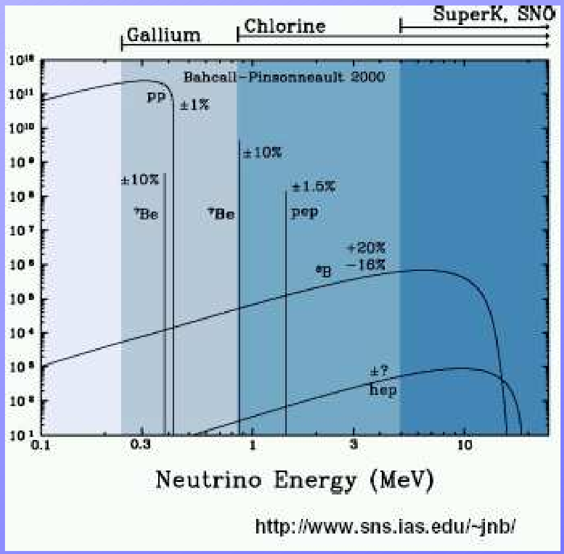
[See also S. Bilenky's and S. Choubey's lectures]

- Solar neutrinos and KamLAND

In the Sun

a large flux of ν_e's

is produced in nuclear reactions.



Many experiments (Homestake, Kamiokande, SAGE, GALLEX/GNO, SuperK, SNO) provide evidence of a depletion of the expected ν_e-flux.

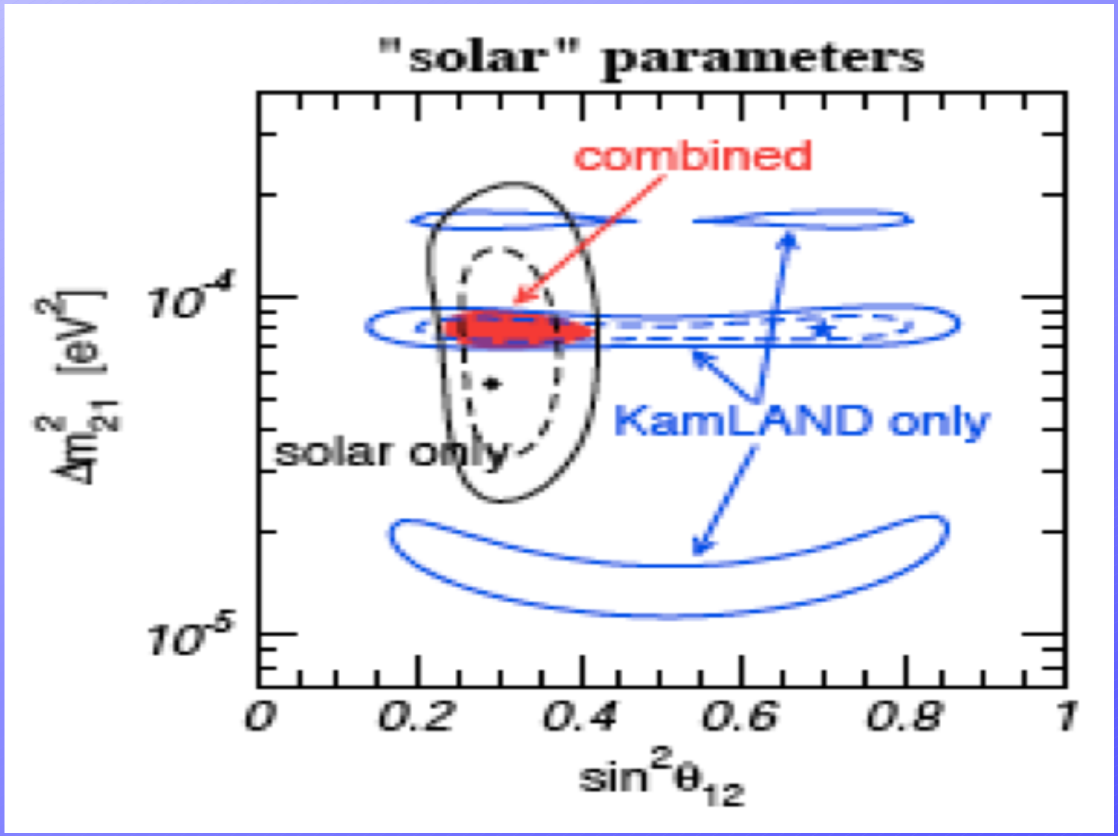
The KamLAND experiment, using reactor $\bar{\nu}_e$, confirmed the disappearance of anti-neutrinos.

[See J. Farine's lecture]

The solar ν_e and reactor $\bar{\nu}_e$ flux depletion can be explained in terms of

$$\boxed{\nu_e \leftrightarrow \nu_{\mu,\tau} \text{ oscillations.}}$$

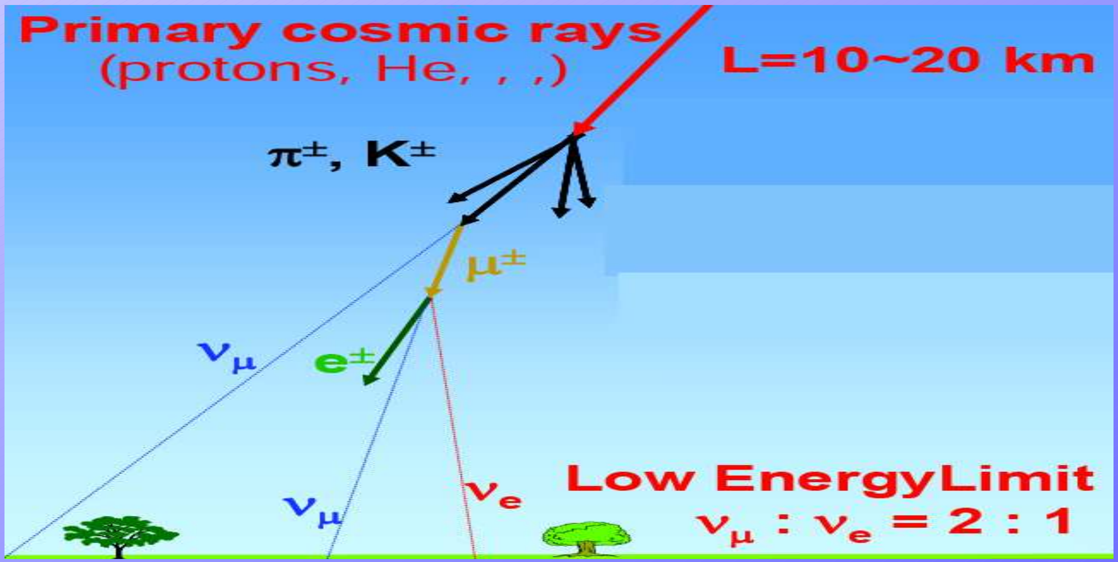
Solar + KamLAND data



[T. Schwetz, hep-ph/0606060]

- Atmospheric neutrinos

Neutrinos are produced in the atmosphere by cosmic rays interactions (π and μ decays).

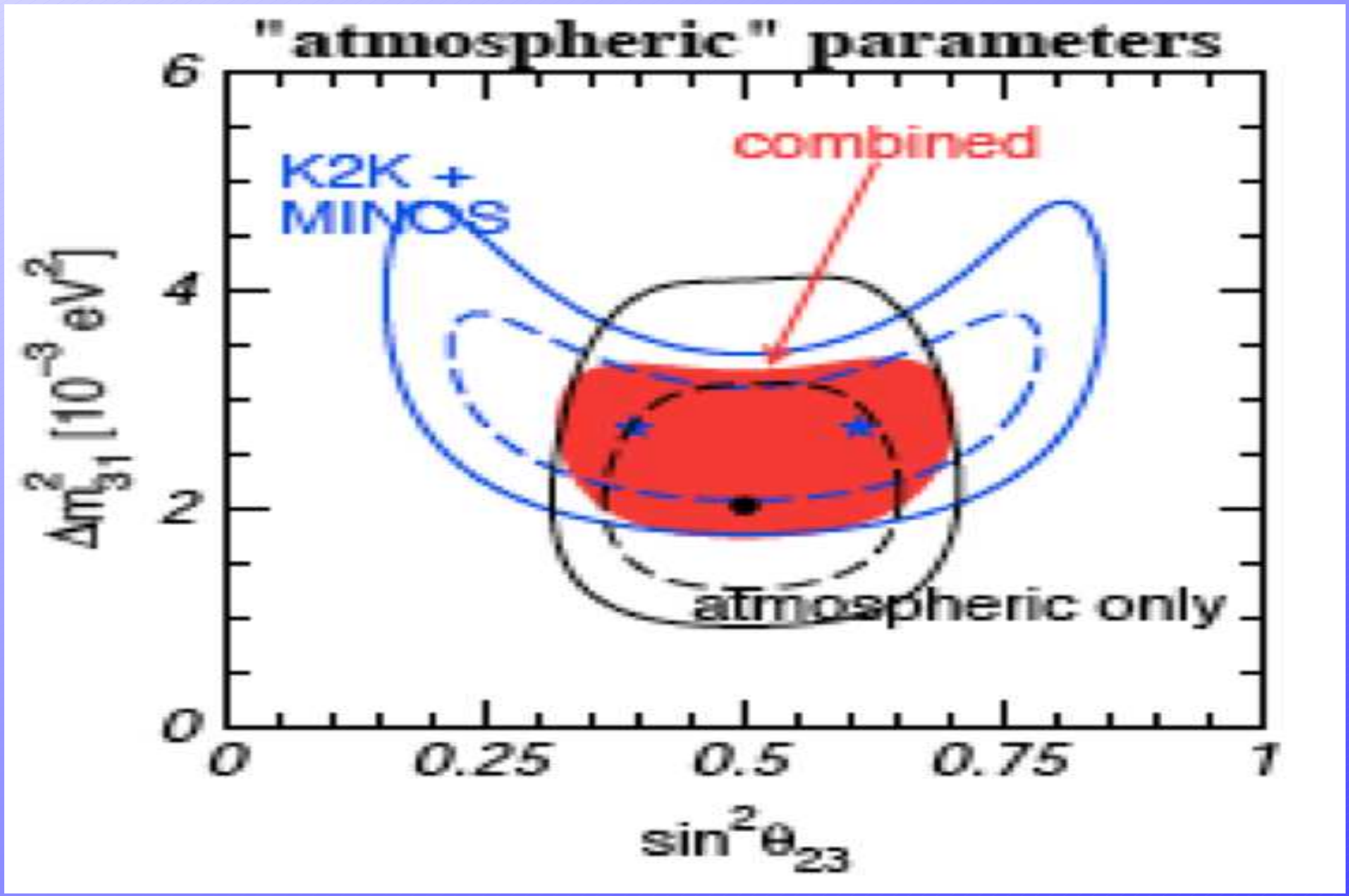


[Thanks to Y. Suzuki @ Taup2005]

The SuperKamiokande (IMB, Kamiokande, MACRO, Soudan2) experiment observes an up-down asymmetry and zenith-angle dependence in the rate of μ-like events.

[See T. Kajita's and K. Nishikawa's lecture]

The K2K and MINOS long-baseline neutrino experiments confirmed ν_μ disappearance while OPERA is searching for $\nu_\mu \to \nu_\tau$ appearance.

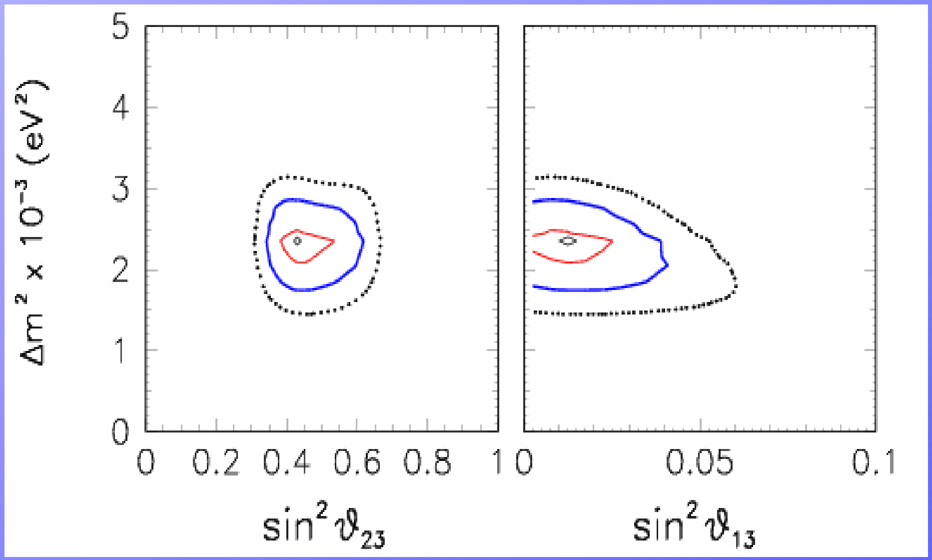


[T. Schwetz, hep-ph/0606060]

- Reactor neutrinos

Reactor $\bar{\nu}_e$ disappearance was searched for in CHOOZ and Palo Verde experiments but no positive signal was found.

Bound on the third mixing angle, θ_{13}:



[Fogli et al., hep-ph/0506083]

New experiments with 2 detectors (D-CHOOZ, Daya Bay) will improve this limit to $\sin^2 2\theta_{13} \sim 0.01 - 0.02$.

[See K. Heeger's lecture]

5 – Present status of neutrino physics

Neutrino oscillations are crucial in our understanding of neutrino physics as they imply that

NEUTRINOS ARE MASSIVE AND THEY MIX.

The explanation of neutrino masses requires physics beyond the Standard Model.

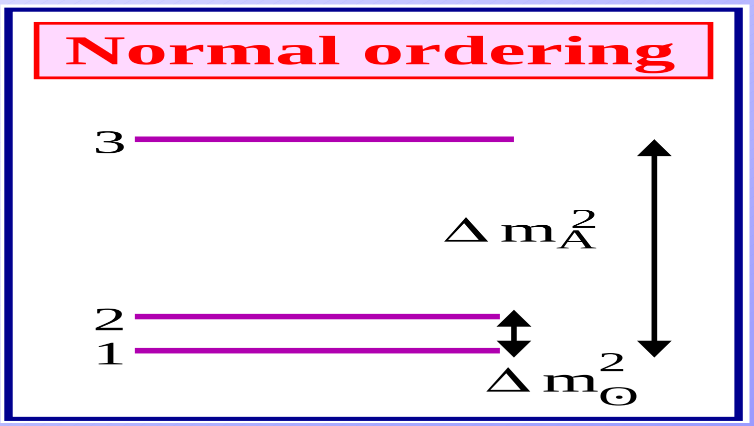
$$\mathcal{L}_{\rm lepton} = \mathcal{L}_{SM} + \textbf{neutrino masses}$$

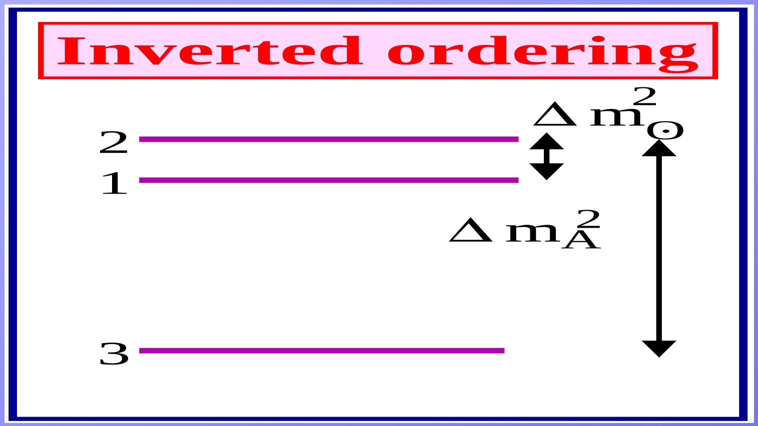
What are the ”ingredients”

necessary for

$\mathcal{L}_{\text{neutrino mass}}$**?**

$\Delta m^2_\odot \ll \Delta m^2_{\rm atm}$ implies at least **3 neutrinos**.

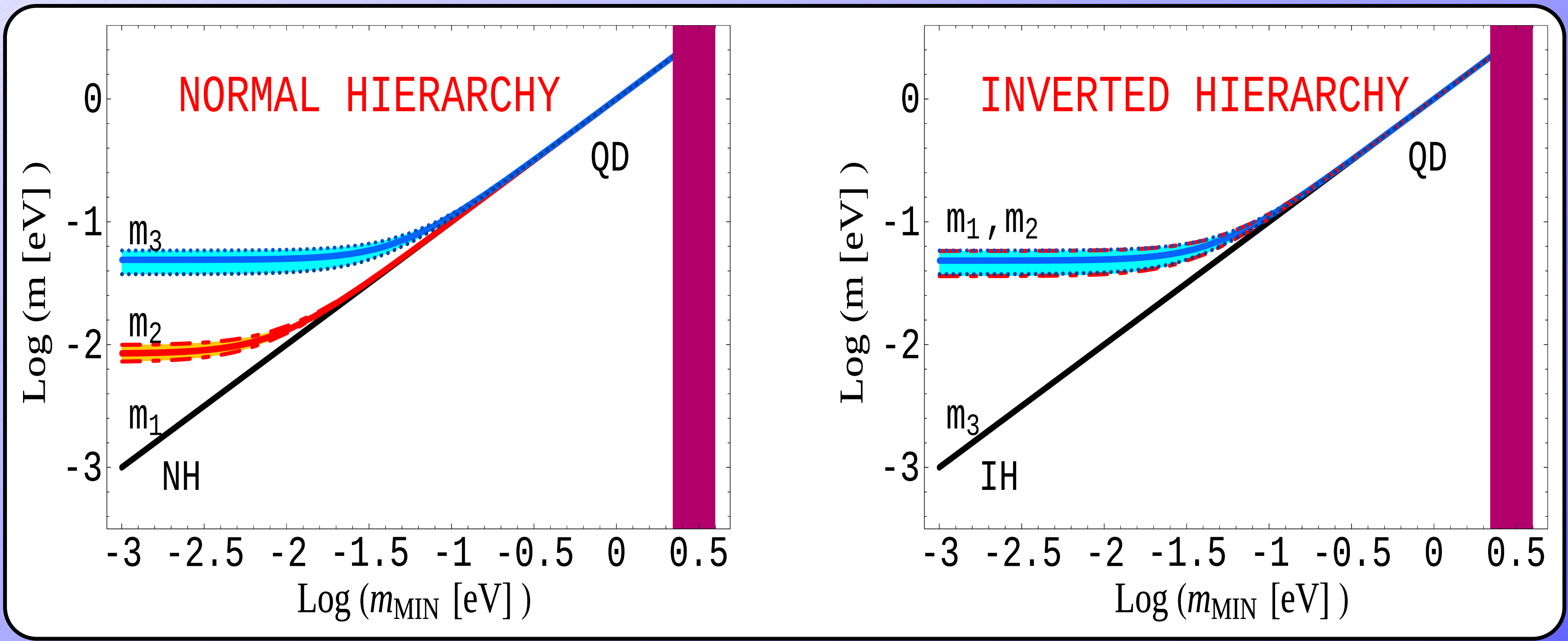




$$m_1 = m_{\rm MIN}$$
$$m_2 = \sqrt{{m_{\rm MIN}}^2 + \Delta m^2_\odot}$$
$$m_3 = \sqrt{{m_{\rm MIN}}^2 + \Delta m^2_{\rm atm}}$$

$$m_3 = m_{\rm MIN}$$
$$m_1 = \sqrt{{m_{\rm MIN}}^2 + \Delta m^2_{\rm atm} - \Delta m^2_\odot}$$
$$m_2 = \sqrt{{m_{\rm MIN}}^2 + \Delta m^2_{\rm atm}}$$

Measuring neutrino masses requires to know $m_{\rm MIN}$ and sign(Δm^2_{31}).



We can identify 3 types of spectra:

NH: $m_1 \ll m_2 \ll m_3$

IH: $m_3 \ll m_1 \simeq m_2$

QD: $m_1 \sim m_2 \sim m_3$.

Mixing is described by a unitary matrix, Pontecorvo-Maki-Nakagawa-Sakata matrix, which enters in $\mathcal{L}^{CC}$:

$$|\nu_l\rangle = \sum_i U_{li} |\nu_i\rangle$$

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \sum_\alpha \sum_k \left(U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + \text{h.c.} \right)$$

$$U = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar, reactor $\theta_\odot \sim 30^o$ **Atm, Acc.** $\theta_A \sim 45^o$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\delta} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{-i\alpha_{31}/2+i\delta} \end{pmatrix}$$

CPV phase **Reactor, Acc.** $\theta < 12^o$ **CPV Majorana phases**

CP-violation

- C-conjugation:

$$U_C \psi(x,t) U_C^{-1} = i\, \eta_C\, \gamma^0\, C\, \bar{\psi}^T(x,t)$$

- Parity

$$U_P \psi(x,t) U_P^{-1} = \eta_P\, \gamma^0\, \psi(-x,t)$$

- CP-parity

$$U_{CP} \psi(x,t) U_{CP}^{-1} = \eta_k\, i\, \gamma^0\, C\, \bar{\psi}^T(-x,t)$$

CP-violation is a crucial element in leptogenesis.

How does CP-violation manifest itself in $\mathcal{L}^{CC}$?

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \sum_\alpha \sum_k \left(U^*_{\alpha k} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho + U_{\alpha k} \bar{l}_{\alpha L} \gamma^\rho \nu_{kL} W^\dagger_\rho \right)$$

$$\Downarrow \text{CP}$$

$$U_{CP} \mathcal{L}^{CC} U^{-1}_{CP} =$$

$$-\frac{g}{\sqrt{2}} \sum_\alpha \sum_k \left(-U^*_{\alpha k} \eta^*_k \xi_{l_\alpha} i \bar{l}_{\alpha L} \gamma^\rho \nu_{kL} W^\dagger_\rho + U_{\alpha k} \eta_k i \xi^*_{l_\alpha} \bar{\nu}_{kL} \gamma^\rho l_{\alpha L} W_\rho \right)$$

where η_k is the ν CP-phase, ξ_{l_α} the lepton one and we used $U_{CP} W_\rho U^{-1}_{CP} = -W^{\rho\dagger}$.

- CP-conservation requires:

$$U_{\alpha k}\, \eta_k i \xi^*_{l_\alpha} = U^*_{\alpha k}.$$

We notice that any unitary matrix can be rewritten as:

$$U = \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})\, U^D\, \mathrm{diag}(e^{i\beta_4}, e^{i\beta_5}, 1).$$

Therefore, CP-conservation only if U^D is real $\Rightarrow \delta = 0$.

For Majorana neutrinos, two additional phases are physical and violate CP.

Summary of point 1: What we know

- Experimental evidence of neutrino oscillations from solar, atmospheric, reactor and accelerator neutrino experiments.

- Neutrinos have mass and mix.

- $\Delta m^2_{\odot} = 8.0 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{\mathrm{atm}} = 2.5 \times 10^{-3}\,\mathrm{eV}^2$ have been measured. m_{MIN} and sgn(Δm^2_{31}) still unknown.

- $\theta_{\odot} \sim 30^o$ and $\theta_A \sim 45^o$ measured. θ_{13} constrained to be small. CP-violation?

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we still lack important information about neutrino masses, mixing and CPV

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See-saw mechanism

7 – Questions for the future

$$\mathcal{L}_{\text{lepton}} = \mathcal{L}_{SM} + \textbf{neutrino masses ???}$$

In order to fully describe neutrino masses and mixing, we still need to answer some fundamental questions:

NATURE

DETERMINE

$\delta, \alpha_{12}, \alpha_{13}$

N of ν

$\theta_{12}, \theta_{23}, \theta_{13}$

$m_{1,2,3}$

ORIGIN OF ν MASSES AND OF FLAVOUR STRUCTURE (see−saw meachanism...)

CONNECTION WITH ASTROPHYSICS AND COSMOLOGY (leptogenesis)

QUESTION 1: Nature of neutrinos.

Neutrino can be either

Dirac or **Majorana**

lepton number conserved — lepton number broken

- A Majorana neutrino satisfies the **Majorana condition**:

$$\boxed{\nu = C\bar{\nu}^T}$$

We can rewrite:

$$\nu = \nu_L + \nu_L^c$$

- Notice that this condition is not invariant under a $U(1)_{\rm lept}$ transformation:

$$e^{i\alpha}\nu = e^{-i\alpha}C\bar{\nu}^T$$

For Majorana neutrinos, lepton number is not a conserved symmetry.

- Majorana neutrinos have only 2 degrees of freedom (instead of 4 as Dirac neutrinos).

Dirac: $\nu(p,h)$, $\nu(p,-h)$, $\bar{\nu}(p,h)$, $\bar{\nu}(p-h)$

Majorana: $\nu(p,h)$, $\nu(p,-h)$

- Quantised Majorana field:

$$\nu(x) = \int \frac{d^3p}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a^h(p)\, u^h(p) \exp^{-ip\cdot x} + (a^h(p))^\dagger\, v^h(p) \exp^{ip\cdot x} \right]$$

- Notice that $a^h(p) = b^h(p)$ as we expected.

There is no distinction between neutrino and antineutrino as there is no conserved quantum number which identifies them.

- Commonly, we refer to *neutrinos* as to the Majorana neutrinos with negative helicities and to *antineutrinos* to the ones with positive helicity.

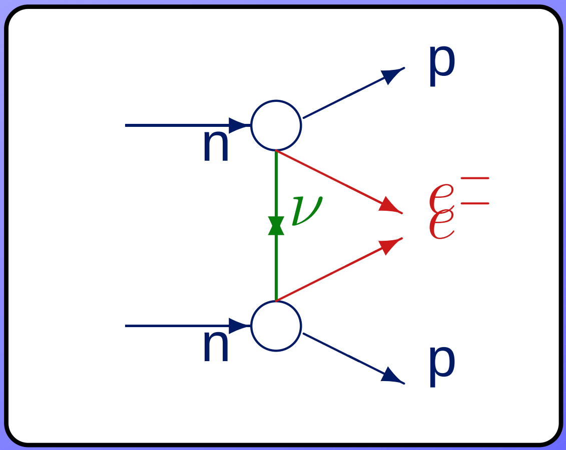
- The **nature of neutrinos** is directly related to the **fundamental symmetries** of elementary particles interactions.

- It provides important information on the **origin of neutrino masses**: in the see-saw mechanism neutrinos are predicted to be Majorana particles.

- Lepton number violation is one of the key ingredients of **leptogenesis** as the mechanism for generating the baryon asymmetry of the Universe.

Neutrino oscillations are not sensitive to the nature of neutrinos.

Processes which violate the lepton number by two units are required. The most sensitive of all is $(\beta\beta)_{0\nu}$**-decay** ($(A, Z) \rightarrow (A, Z+2) + 2e^-$). It can proceed through the exchange of light Majorana neutrinos:



[See F. Simkovic's lecture on Neutrinoless double beta decay]

The half-life time, $\mathrm{T}^{1/2}_{0\nu}$, of the $(\beta\beta)_{0\nu}$-decay can be factorized, for light Majorana neutrinos, as:

$$\boxed{\left[\mathrm{T}^{1/2}_{0\nu}(0^+ \to 0^+)\right]^{-1} \propto |M_F - g_A^2 M_{GT}|^2 \; |{<}\, m \,{>}|^{\,2}}$$

- $|{<}\, m \,{>}|$ **is the effective Majorana mass parameter**:

$$\boxed{|{<}\, m \,{>}| \;\equiv |\, m_1 |U_{\mathrm{e}1}|^2 + m_2 |U_{\mathrm{e}2}|^2 e^{i\alpha_{21}} + m_3 |U_{\mathrm{e}3}|^2 e^{i\alpha_{31}} \,| \,,}$$

The present bound on $|{<}\, m \,{>}|$ reads $|{<}\, m \,{>}| \; < (0.35 - 1.05) \;\; \mathrm{eV}$ (Heidelberg-Moscow), $|{<}\, m \,{>}| \; < (0.20 - 1.05) \;\; \mathrm{eV}$ (Cuoricino) and $|{<}\, m \,{>}| \; < (0.8 - 1.3) \;\; \mathrm{eV}$ (NEMO3).

In 2002 and with a new more detailed analysis, evidence of $(\beta\beta)_{0\nu}$-decay was reported [Klapdor-Kleingrothaus et al., PLB 586 198 (2004)]:

$$|{<}\, m \,{>}|_{\,BF} = 440 \,\mathrm{meV} \qquad 120 \,\mathrm{meV} \;\leq |{<}\, m \,{>}| \;\leq 700 \,\mathrm{meV}.$$

The next generation of $(\beta\beta)_{0\nu}$-decay exp (SuperNEMO, EXO, Majorana, CUORE, GERDA, Cobra ...) aim to $|{<}\, m \,{>}| \;\sim 10 - -30 \;\; \mathrm{meV}$.

The predictions for $|<m>|$ depend strongly on the type of spectrum.

- NH: $\boxed{|<m>| \simeq \left|\sqrt{\Delta m^2_\odot}\cos^2\theta_{13}\sin^2\theta_\odot + \sqrt{\Delta m^2_{\text{atm}}}\sin^2\theta_{13}e^{i\alpha_{32}}\right|}$

- IH:

$$\boxed{\sqrt{\Delta m^2_{\text{atm}}}\cos 2\theta_\odot \leq |<m>| \simeq \sqrt{\left(1-\sin^2(2\theta_\odot)\sin^2\frac{\alpha_{21}}{2}\right)\Delta m^2_{\text{atm}}} \leq \sqrt{\Delta m^2_{\text{atm}}}}$$

$|<m>|$ has a significant lower bound

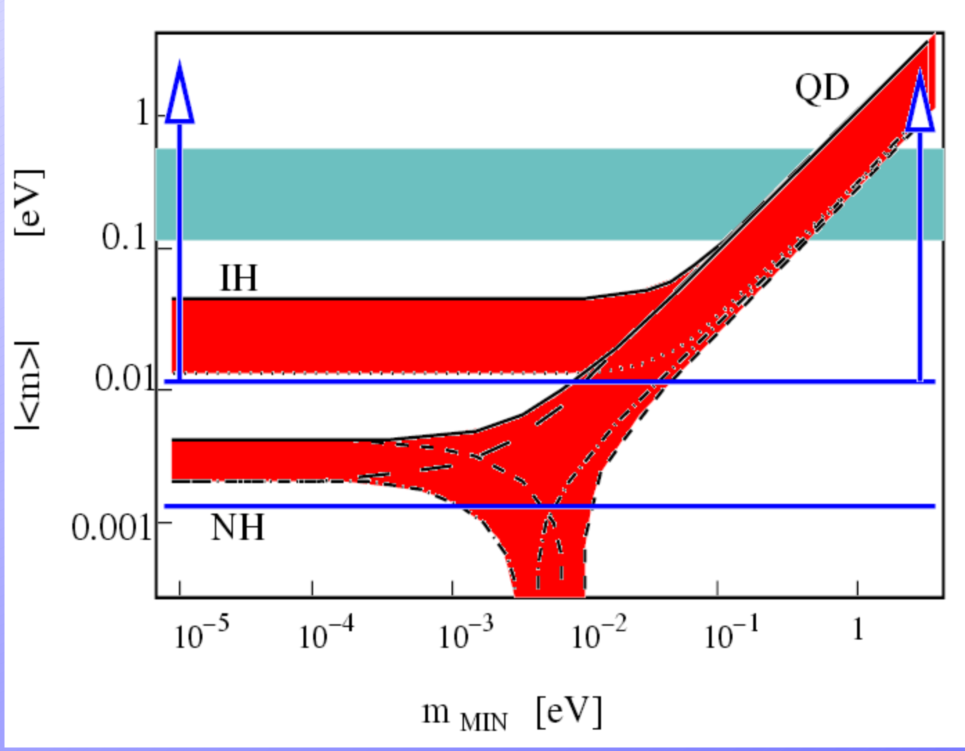
$$0.01\ \text{eV} \lesssim |<m>| \lesssim 0.06\ \text{eV}$$

$|<m>|$ is in the range of sensitivity of the upcoming $(\beta\beta)_{0\nu}$-decay experiments.

- QD:

$$\boxed{|<m>| \simeq m_{\bar{\nu}_e}\left|\left(\cos^2\theta_\odot + \sin^2\theta_\odot e^{i\alpha_{21}}\right)\cos^2\theta_{13} + \sin^2\theta_{13}e^{i\alpha_{31}}\right|}$$

All the allowed range for $|<m>|$ is in the range of sensitivity of present and upcoming $(\beta\beta)_{0\nu}$-decay experiments.



[S.T. Petcov, A. Yu. Smirnov, PLB; S.M. Bilenky, S. P., S.T. Petcov, PRD; S.P., S.T. Petcov, PLB; S.P., S.T. Petcov, T. Schwetz, PLB]

- For the IH and QD spectra, present and/or future exp will probe the allowed values of $|{<}m{>}|$.

- As $\cos 2\theta_{\odot} > 0.25$, $\boxed{|{<}m{>}|_{NH} < |{<}m{>}|_{IH}}$.

$(\beta\beta)_{0\nu}$**-decay exp can distinguish the NH vs IH and QD cases.** This is independent from the value of θ_{13}. [S.P., Petcov, PLB2003; S.P., Petcov, Rodejohann, 2003]

All the allowed values for $|<m>|$ in the IH and QD spectra are at reach of present and/or upcoming $(\beta\beta)_{0\nu}$-decay experiments.

A positive signal in present or future exp would imply that

- the lepton number is not a conserved symmetry in nature,
- neutrinos are Majorana particles,

and it could give important information

- on the neutrino mass spectrum
- and on the Majorana CP-V phases.

[See also F. Simkovic and S. Schoenert lectures]

QUESTION 2: Neutrino masses.

m_1, m_2, m_3.

As we know only $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$, we need:

- the absolute mass scale ($m_{\rm MIN}$).
- the type of hierarchy ($sgn(\Delta m^2_{31})$).

Knowing the type of spectrum (NH, IH, QD) is crucial for understanding the origin of neutrino masses in the context of models Beyond the Standard Model.

Absolute mass scale

Neutrino oscillations are sensitive only to mass squared differences:

$$\Delta m^2_{\odot} \ll \Delta m^2_{\rm atm}.$$

- Direct mass searches in **tritium beta decay** experiments. The present limit is $m_0 < 2.2$ eV (Troiztk and Mainz). KATRIN can reach a sensitivity to $m_0 \sim 0.2$ eV, covering all the QD spectrum.

- Cosmological observations. Exploiting the effects of massive neutrinos on structure formation in the Early Universe it is possible to constrain the sum of neutrino masses. Dependence on other parameters and degeneracies. Prospects for $\Sigma \sim 0.1$ eV.

[See F. Glueck's lecture]

Type of hierarchy

- $(\beta\beta)_{0\nu}$decay: it can distinguish between different types of spectra (NH, IH, QD). The role of $\sin^2\theta_{13}$ is subdominant.

- atmospheric neutrino experiments: exploiting matter effects in magnetized or not detectors (Hyper-Kamiokande, INO).

- long baseline neutrino oscillation experiments exploiting matter effects. It depends crucially on the value of $\sin^2\theta_{13}$. Degeneracies arise with the CP-violating phase δ.

Neutrinos travelling through the Earth are affected by matter and a potential V appears in the Hamiltonian ($V = \sqrt{2} G_F (N_e - N_n/2)$).

The probability can be approximated as:

$$\boxed{P_{e\mu} = \sin^2\theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{\Delta_{13}^m L}{2}}$$

The mixing angle changes with respect to the vacuum case:

$$\boxed{\sin 2\theta_m = \frac{(\Delta m^2/2E)\sin 2\theta}{\sqrt{\left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta - V\right)^2}}}$$

and $\Delta_{13}^m = \sqrt{\left(\frac{\Delta m^2}{2E}\sin 2\theta\right)^2 + \left(\frac{\Delta m^2}{2E}\cos 2\theta - V\right)^2}$.

[See S. Mikheev's lecture]

QUESTION 4: CP-violation.

If U is complex we have CP-violation:

$$\boxed{P(\nu_l \to \nu_{l'}) \neq P(\bar{\nu}_l \to \bar{\nu}_{l'})}$$

- Establishing leptonic CP-V is a fundamental and challenging task.

- There are:

1 Dirac phase (measurable in long base-line experiments)

and 2 Majorana phases (one might be determined in neutrinoless double beta decay).

- Leptogenesis takes place in the context of see-saw models, which explain the origin of neutrino masses.

The observation of neutrinoless double beta decay (L violation) and of CPV in the lepton sector would be an indication, even if not a proof, of leptogenesis as the explanation for the observed baryon asymmetry of the Universe.

Determining CP-V and the type of hierarchy in LBL exp

The CP-asymmetry and the type of hierarchy will be searched for in future long base-line experiments, looking for $\nu_\mu \longrightarrow \nu_e (\bar{\nu}_\mu \longrightarrow \bar{\nu}_e)$.

These oscillations take place in matter (Earth). The Earth matter is not charge-symmetric as it is given only by e^-, p and n. Matter effects in oscillations are not CP-invariant ($P \neq \bar{P}$).

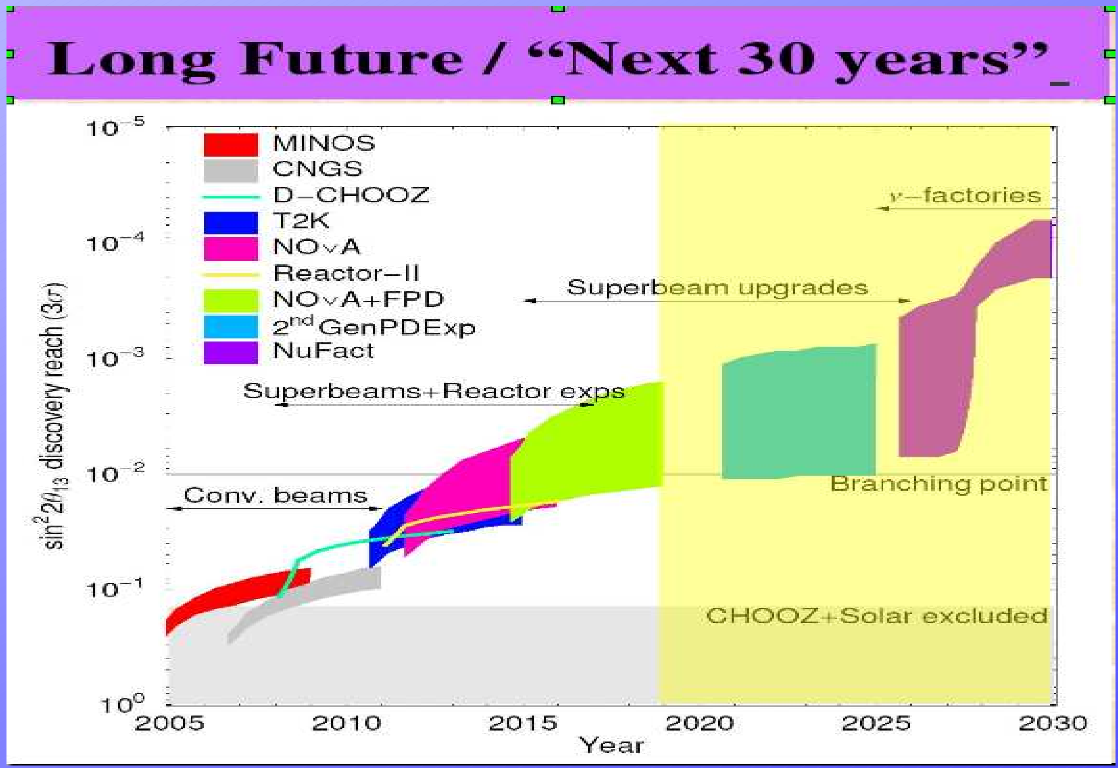
It is necessary to disentangle true CP-V effects due to the δ phase from the ones induced by matter: **degeneracies**.

There are degenerate solutions which need to be solved:

$$\boxed{\begin{array}{l} \Delta m^2_{31}\ ,\ \theta_{13}\ ,\ \delta\ ,\ \theta_{23} \\ \Delta m^{2\,\prime}_{31},\ \theta'_{13}\ ,\ \delta'\ ,\ \theta'_{23} \end{array} \longrightarrow P, \bar{P}}$$

Many future long base-line neutrino oscillation experiments will do precision studies of neutrino oscillations.

1. Superbeams: T2K, NOνA.
2. Neutrino factories.
3. Beta-beams.



[See J. Bernabeu's lecture]

With this wide experimental program in neutrino physics, we can hope to have the ”ingredients” needed to describe neutrino masses and mixing in the not-too-far future:

3 masses, 3 angles, 1 (3) CP-violating phases

The neutrino mass term can be either:

- Dirac
- Majorana

Neutrinoless double beta decay experiments play a crucial role as they will guide us in choosing one option.

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we still lack important information about neutrino masses, mixing and CPV

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Dirac and Majorana mass terms

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See-saw mechanism

9 – Implementing neutrino masses

Dirac mass term

- Let's assume ν_R exist.
- The Dirac mass term, in analogy to the other fermions, reads:

$$\boxed{\mathcal{L}^D_{\mathrm{mass}} = -\bar{\nu}_R M_D \nu_L + \mathrm{h.c.}}$$

- This mass term preserves lepton number.
- It can arise as for the other fermions by a Yukawa coupling with the Higgs: $m^D = y_\nu v$
- Problem: why the coupling is so small?

$$h_\nu = \frac{m_\nu}{v} \sim \frac{0.1\ \mathrm{eV}}{100\ \mathrm{GeV}} \sim 10^{-12}$$

Majorana mass term

- Using only ν_L, one can write a Majorana mass term as:

$$\boxed{\mathcal{L}^M_{\mathrm{mass}} = \frac{1}{2}{\nu_L}^T C^\dagger M_M \nu_L + \mathrm{h.c.}}$$

- This term breaks explicitly lepton number conservation by 2 units.

- It is forbidden in the Standard Model by gauge symmetries but could arise as an effective term in the Lagrangian.

- Neutrinos are Majorana particles.

Dirac+Majorana mass term

$$\boxed{\mathcal{L}_{\text{mass}} = \mathcal{L}^{ML}_{\text{mass}} + \mathcal{L}^{MR}_{\text{mass}} + \mathcal{L}^{D}_{\text{mass}}}$$

It is useful to define:

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} = \begin{pmatrix} \nu_L \\ C\bar{\nu_R}^T \end{pmatrix}$$

The Lagrangian can be rewritten as:

$$\mathcal{L}_{\text{mass}} = \tfrac{1}{2} n_L^T C^\dagger \mathcal{M} n_L + \text{h.c.}$$

The mass matrix $\mathcal{M}$ is: $\boxed{\mathcal{M} = \begin{pmatrix} M_{ML} & M_D \\ M_D^T & M_{MR} \end{pmatrix}}$

It is symmetric and can be diagonalised by a congruent transformation, $U\mathcal{M}U^T = \mathrm{diag}(m_i)$.

So we have $n_L = U\nu_{k\,L}$, where $\nu_k = \nu_{kL} + \nu_{kL}^c$ are the massive states.

The Lagrangian becomes

$$\mathcal{L}_{\mathrm{mass}} = \frac{1}{2} m_k \nu_{kL}^T C^\dagger \nu_{kL} + \mathrm{h.c.}$$

For a Dirac+Majorana mass term,

massive neutrinos are Majorana particles.

Summary of part 3: implementing neutrino masses.

- Neutrino masses can be of Dirac or Majorana type.

- In the case of Dirac + Majorana mass term, massive neutrinos are Majorana particles.

- Neutrinoless double beta decay, providing information on the nature of neutrinos, plays a crucial role in extending the SM to include neutrino masses.

10 – Plan of Lecture 1

From the data to the theory

- 1. Present status of neutrino physics: what we know

neutrino oscillations implies neutrino masses and mixing

- 2. Missing ingredients: what we will discover (hopefully)

we still lack important information about neutrino masses, mixing and CPV

- 3. How to implement neutrino masses and mixing

Dirac and Majorana mass terms

- 4. **Explanation of the mass terms in extensions of the SM**

See-saw mechanism

11 – Explaining neutrino masses and mixing

It is necessary to understand the origin of neutrino masses, extending the SM.

- For Dirac neutrinos, simply add ν_R but the explanation of the smallness of neutrino masses lacks.
- Add a triplet Δ. The term $\nu^T \nu \Delta$ is $SU(2)$ preserving. Once $< \Delta >= d$ neutrino acquire a Majorana mass.
- Add a singlet N_R which couples with ν_L and has a very heavy mass. A Dirac+Majorana mass term arises. This is what happens in the **see-saw mechanism**.

[For a detailed discussion, see A. Romanino lectures.]

See-saw mechanism, type I

- We extend the SM by assuming the existence of N_R, singlets with respect to the SM gauge group.
- They have a Majorana mass term. Lepton number is explicitly broken.

$$\boxed{\mathcal{L} = -y_\nu(\bar{N}_R \tilde{\Phi}^\dagger L_L) + \text{h.c.} + \frac{1}{2} M_{MR}(N_R^T C^\dagger N_R + N_R^\dagger C N_R^*)}$$

- M_{MR} is typically at a very heavy scale, TeV–10^{16} GeV.

Once $\tilde{\Phi}$ gets a vev, Dirac masses appear in the Lagrangian.

Similarly to the Dirac+Majorana case discussed previously, we have to diagonalise: $\mathcal{M} = \begin{pmatrix} 0 & m_D \\ (m_D)^T & M_{MR} \end{pmatrix}$

whose eigenvalues are:

$$m_1 \simeq -\frac{m_D^2}{M_R} \sim \frac{1\ \mathrm{GeV}^2}{10^{10}\ \mathrm{GeV}} \sim 0.1\ \mathrm{GeV}$$

$$m_2 \simeq M_R$$

The Majorana massive neutrinos are: $\begin{matrix} \nu_{1L} \simeq -i\nu_L \\ \nu_{2L} \simeq N_L^c \end{matrix}$

- Light neutrinos acquire naturally a small neutrino mass.

N generation see-saw mechanism

For 3 ν_L and 3 N_R, M_D, M_R are 3×3 matrices.

The **see-saw formula** applies in matrix form:

$$\boxed{\begin{aligned} m_\nu &\simeq -M_D^\dagger \, M_R^{-1} \, M_D^* \, , \\ &= U_{\text{PMNS}} \, D_m \, U_{\text{PMNS}}^T . \end{aligned}}$$

- At high energy there are in $\mathcal{L}_{\text{see}-\text{saw}}$:

3 masses in M_R, 9 real parameters and 6 phases in y_ν.

It is useful to use various parametrizations of $M_D = y_\nu v$:

- Bi–unitary parametrization:

$$\boxed{M_D = U_R^\dagger \, m_D^{diag} \, U_L \ ,}$$

where U_L and U_R are unitary 3×3 matrices and M_D^{diag} is a real diagonal matrix.

- Orthogonal parametrization.

By the see-saw formula, we can express M_D as:
$-M_D^\dagger \, M_R^{-1/2} \, M_R^{-1/2} M_D^* = U_{\mathrm{PMNS}} \, D_m^{1/2} \, R^T R \, D_m^{1/2} \, U_{\mathrm{PMNS}}^T$:

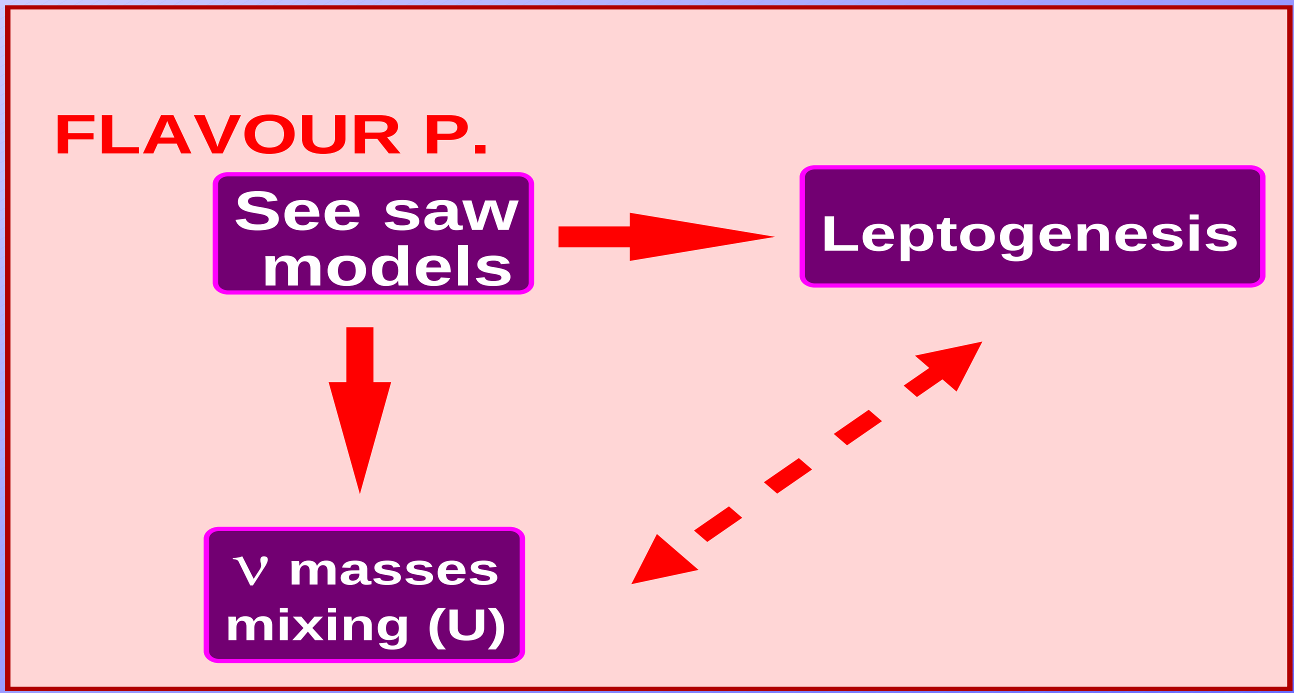
$$\boxed{M_D \simeq i \sqrt{M_R} \, R \, D_m^{1/2} \, U_{\mathrm{PMNS}}^\dagger \ ,}$$

where R is a complex orthogonal matrix, [Casas, Ibarra].

Summary of part 4: explaining neutrino masses.

- The see-saw mechanism requires the existence of very heavy N_R.
- Light neutrinos emerge naturally with masses in the right range.
- Light neutrinos are Majorana particles.
- See-saw models can be embedded in GUT theories, as $SO(10)$.

12 – Conclusions and outlook



On Wednesday, we are going to discuss leptogenesis in the context of see-saw models and its connection with the parameters we can measure at low energy.