Theory of neutrino masses

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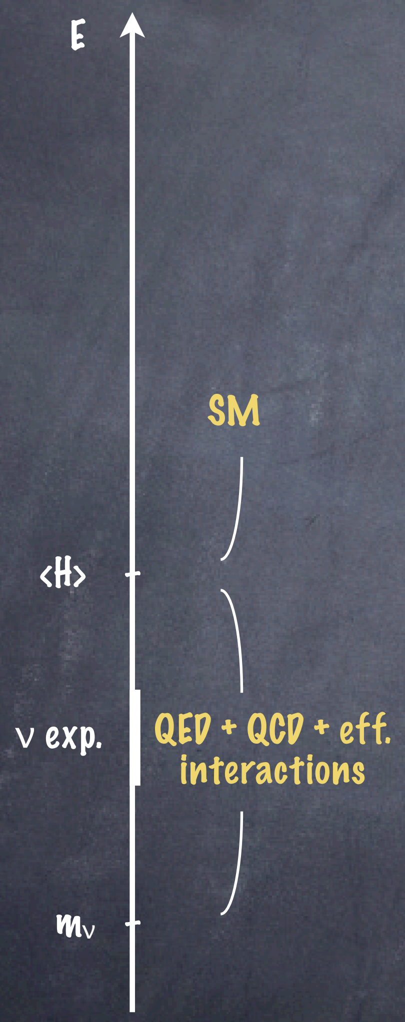
Plan

- Preliminaries
- Origin and description of neutrino masses
  - Description of neutrino masses at $E \ll M_Z$
  - Neutrino and fermion masses in the SM
  - Origin of neutrino masses at $E \gg M_Z$
  - Alternative origin of neutrino masses
- Models of neutrino masses
  - Textures
  - Flavour models

Effective interactions



- Most neutrino esperiments involve E « $M_{Z,W}$
- At E « $M_{Z,W}$: QED + QCD for light dofs + NR terms suppressed by powers of $M_{Z,W}$

$$\mathcal{L}^{\text{eff}}_{E \ll M_Z} = \mathcal{L}^{\text{ren}}_{\text{QED+QCD}} + 4\,\frac{G_F}{\sqrt{2}}\,j^\mu_c j^\dagger_{c\mu} + \text{N.C.} + \ldots$$

$$j^\mu_c = \overline{u_L}\gamma^\mu d_L + \overline{\nu_L}\gamma^\mu e_L + \ldots \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M^2_W}$$

- The effect of NR interactions is suppressed by $(E/M_{Z,W})^n$ at E « $M_{Z,W}$ (only lower dimensional interactions matter)

Effective theories

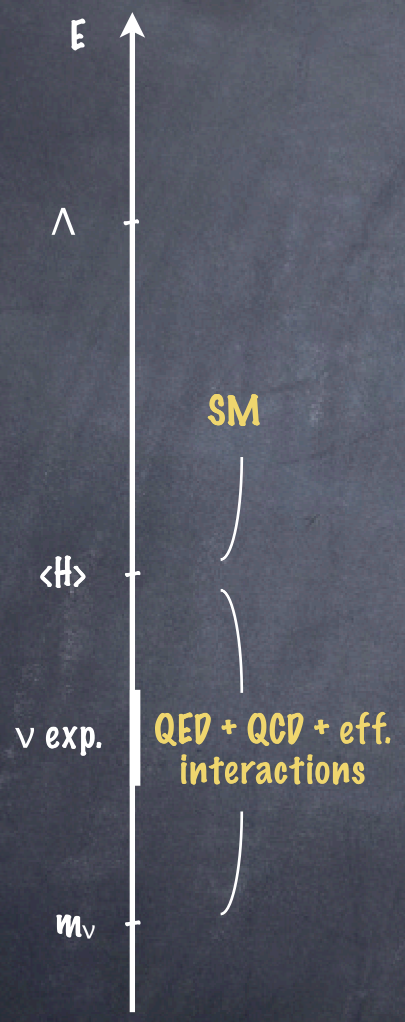
- Provide a general, model-independent, low energy parameterisation of physics at higher E
- If the higher E theory is known, the specific form of the NR remnants can be derived
- If the higher E theory is unknown, the experimental identification of NR interactions provides information on the higher E theory
  - e.g.: Fermi interaction

$$\frac{g^2}{2M_W^2}\bar{\psi}_1\Gamma^A\psi_2\bar{\psi}_3\Gamma_A\psi_4 \Rightarrow \frac{g^2}{8M_W^2}\bar{\psi}_1\gamma^\mu(1-\gamma_5)\psi_2\bar{\psi}_3\gamma_\mu(1-\gamma_5)\psi_4$$

$\Rightarrow$ SM

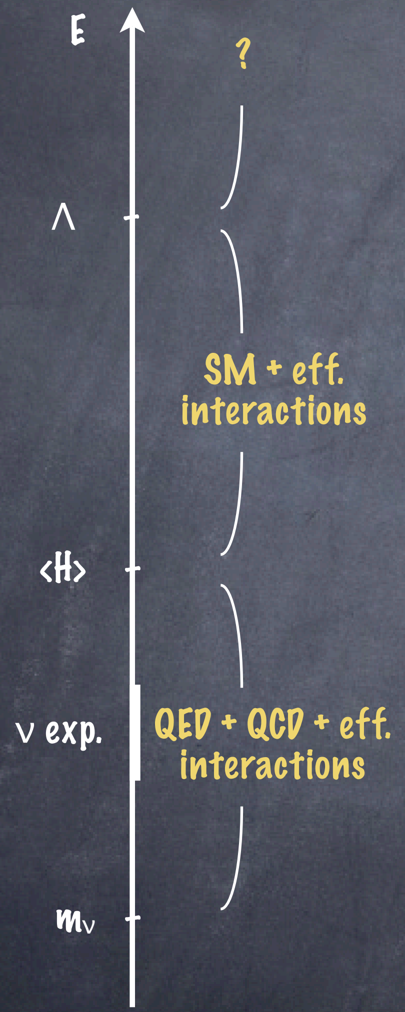
- (btw: renormalizability might well not be a fundamental property of 4D QFT)

The SM as an effective theory



- Analogously...

The SM as an effective theory



- Analogously...

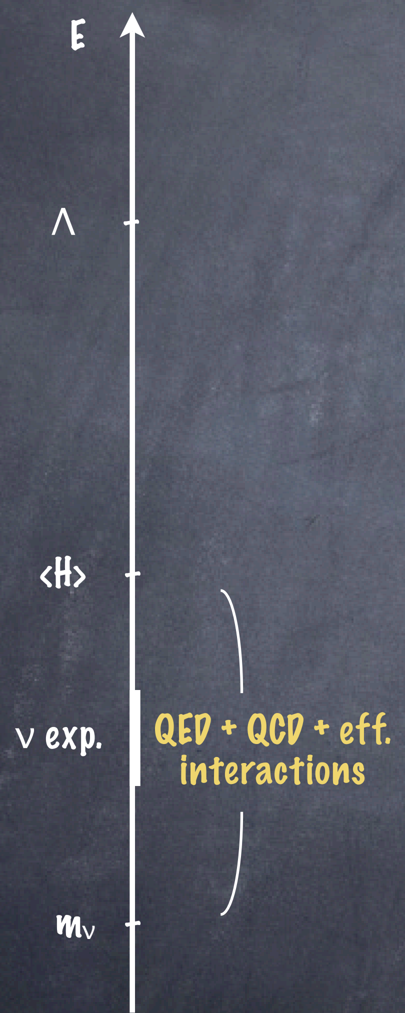
$$\mathcal{L}^{\text{eff}}_{E \ll \Lambda} = \mathcal{L}^{\text{ren}}_{\text{SM}} + \mathcal{L}^{\text{NR}}_{\text{SM}}$$

- No hint of NR interactions from TeV scale

- Only evidence of NR interactions: neutrino masses (see below)

$E \ll M_Z$: description of neutrino masses

Neutrino masses at E « M_Z



- In the broken EW phase, the most general fermion mass term is

$$\frac{m^\nu_{ij}}{2}\nu_i\nu_j + m^E_{ij}e^c_i e_j + m^D_{ij}d^c_i d_j + m^U_{ij}u^c_i u_j + \text{h.c.}$$

$$\Psi \text{ Dirac spinor} \leftrightarrow \Psi_L, \Psi_R \qquad \psi \sim \Psi_L \quad \psi^c \sim \overline{\Psi_R} \text{ Weyl spinors}$$

$$\overline{\Psi_R}\Psi_L \to \psi^c\psi$$

$$\frac{m^\nu_{ij}}{2}\overline{N^c_{iR}}N_{jL} + m^E_{ij}\overline{E_{iR}}E_{jL} + m^D_{ij}\overline{D_{iR}}D_{jL} + m^U_{ij}\overline{U_{iR}}U_{jL} + \text{h.c.}$$

$$\Psi^c \equiv C\overline{\Psi}^T \qquad N^c = N \text{ Majorana}$$

- (a Dirac neutrino mass term would require a ν^c)

(

Weyl spinors

- Dirac spinors are not fundamental: $\Psi = \Psi_L + \Psi_R \approx (0,1/2)+(1/2,0)$

$$\Psi = \begin{pmatrix} \epsilon\psi_c^* \\ \psi \end{pmatrix} \qquad \Psi_L = \begin{pmatrix} 0 \\ \psi \end{pmatrix}, \Psi_R = \begin{pmatrix} \epsilon\,\psi_c^* \\ 0 \end{pmatrix} \qquad \epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\Psi_{L,R} = \frac{1 \mp \gamma_5}{2}\Psi$$

- The gauge group can act differently on Ψ_L Ψ_R
example: in the SM, $(\nu_L, e_L) \approx SU(2)_w$ doublet, $e_R \approx SU(2)_w$ singlet

- In general, the gauge group can mix all (0,1/2) fermions

$$\Psi + \overline{\Psi} \leftrightarrow \underbrace{\Psi_L, \overline{\Psi_R}}_{(0,1/2)} + \underbrace{\Psi_R, \overline{\Psi_L}}_{(1/2,0)} \leftrightarrow \boxed{\psi, \psi_c} + \psi^*, \psi_c^*$$

- In terms of Weyl spinors:

$$\overline{\Psi_1}\Psi_2 = \psi_1^c\psi_2 + (\psi_1\psi_2^c)^* \quad \overline{\Psi_1}\gamma^\mu\Psi_2 = \psi_1^\dagger\sigma^\mu\psi_2 - (\psi_2^c)^\dagger\sigma^\mu\psi_1^c$$

$$(\psi_1\psi_2 = \psi_2\psi_1 = \psi_1^\alpha\epsilon_{\alpha\beta}\psi_2^\beta)$$

- Fundamental objects: Weyl spinors $\psi_1 \ldots \psi_n$ (Dirac if charged + P)

Fermion mass terms

ψ_i Weyl fermions

ψ

Most general mass term: $\frac{m_{ij}}{2}\psi_i\psi_j$

$\frac{m}{2}\psi\psi$

$$\psi_i\psi_j \equiv \psi_i^\alpha \epsilon_{\alpha\beta}\psi_j^\beta$$

"Majorana"
breaks any charge of ψ

Fermion mass terms

ψ_i Weyl fermions

ψ, ψ^c

Most general mass term: $\frac{m_{ij}}{2}\psi_i\psi_j$

$$\underbrace{\frac{m_1}{2}\psi\psi + \frac{m_2}{2}\psi^c\psi^c}_{\text{"Majorana"}} + \underbrace{m\psi^c\psi}_{\text{"Dirac"}}$$

$$\psi_i\psi_j \equiv \psi_i^\alpha \epsilon_{\alpha\beta}\psi_j^\beta$$

"Majorana"

"Dirac"

$Q(\psi) + Q(\psi^c) = 0$

Dirac spinors turn out useful (all the SM fermions except ν)

(e.g. electron mass term: $m\, e^c e$)

)

Neutrino masses at E « M_Z

- Fields: d_i d^c_i u_i u^c_i e_i e^c_i ν (a Dirac neutrino mass term would require a ν^c)
- Most general mass terms:

$$\frac{m^\nu_{ij}}{2}\nu_i\nu_j + m^E_{ij}e^c_i e_j + m^D_{ij}d^c_i d_j + m^U_{ij}u^c_i u_j + \text{h.c.}$$

- Relevant effective interactions:

$$\mathcal{L}^{\text{eff}}_{E\ll M_Z} \supseteq 4\,\frac{G_F}{\sqrt{2}}\,j^\mu_c j^\dagger_{c\mu} + \text{N.C.} + \dots$$

$$j^\mu_c = \overline{u_{iL}}\gamma^\mu d_{iL} + \overline{\nu_{iL}}\gamma^\mu e_{iL} + \dots$$

- Everything needed for a description of most neutrino phenomenology

Masses and mixings: quarks

- Mass eigenstates

$$m^D = U_{d^c}^T m_{\rm diag}^D U_d \qquad m^U = U_{u^c}^T m_{\rm diag}^U U_u$$

$$\begin{cases} d_i^{c\prime} = U_{ij}^{d^c} d_j^c \\ d_i^\prime = U_{ij}^d d_j \end{cases}, \begin{cases} u_i^{c\prime} = U_{ij}^{u^c} u_j^c \\ u_i^\prime = U_{ij}^u u_j \end{cases}$$

$$m_{ij}^D d_i^c d_j + m_{ij}^U u_i^c u_j + \text{h.c.} = m_{d_i} d_i^{c\prime} d_i^\prime + m_{u_i} u_i^{c\prime} u_i^\prime + \text{h.c.} = m_{d_i} \overline{D_i} D_i + m_{u_i} \overline{U_i} U_i$$

- In terms of mass eigenstates:

$$j^\mu_{\rm c,had} = \overline{u_{iL}} \gamma^\mu d_{iL} = V_{ij} \overline{u}^\prime_{iL} \gamma^\mu d^\prime_{jL}$$

$$j^\mu_{\rm n,had} = (j^\mu_{\rm n,had})^\prime$$

$$j^\mu_{\rm em,had} = (j^\mu_{\rm em,had})^\prime$$

$V = U_u U_d^\dagger$ Cabibbo Kobayashi Maskawa (CKM) matrix

Physical parameters in V

$$m_{d_i} d_i^c d_i + m_{u_i} u_i^c u_i \qquad j^{\mu}_{\mathrm{c,had}} = V_{ij} \overline{u}_i \sigma^{\mu} d_j$$

$$V = \underbrace{\begin{pmatrix} e^{i\tau_1} & & \\ & e^{i\tau_2} & \\ & & e^{i\tau_3} \end{pmatrix}}_{\text{unphysical}} \begin{pmatrix} \text{standard par.} \end{pmatrix} \underbrace{\begin{pmatrix} 1 & & \\ & e^{i\sigma} & \\ & & e^{i\rho} \end{pmatrix}}_{\text{unphysical}}$$

9 = 3 + 3 + 1 + 2

With N families

Families	Pars in V	Phys. pars	Angles	Phases
N	N^2	$N^2-(2N-1)$	$N(N-1)/2$	$(N^2-3N+1)/2$
2	4	1	1	0
3	9	4	3	1

Standard parameterizations

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}s^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}s^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Experimentally: V ≈ 1

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda = 0.22$$

$$A, \rho, \eta = \mathcal{O}(1)$$

Masses and mixings: leptons

- Mass eigenstates

$$m^{\nu} = U_{\nu}^{T} m_{\text{diag}}^{D} U_{\nu} \qquad m^{e} = U_{e^c}^{T} m_{\text{diag}}^{E} U_{e}$$

$$\nu_i' = U_{ij}^{\nu} \nu_j, \quad \begin{cases} e_i^{c\prime} = U_{ij}^{e^c} e_j^c \\ e_i' = U_{ij}^{e} e_j \end{cases}$$

$$\frac{m_{ij}^{\nu}}{2} \nu_i \nu_j + m_{ij}^{E} e_i^c e_j + \text{h.c.} = \frac{m_{\nu_i}}{2} \nu_i' \nu_i' + m_{e_i} e_i^{c\prime} e_i' + \text{h.c.}$$

- In terms of mass eigenstates:

$$j_{\text{c,lep}}^{\mu} = \overline{\nu_{iL}} \gamma^{\mu} e_{iL} = U_{ij}^{\dagger} \overline{\nu}'_{iL} \gamma^{\mu} e'_{jL}$$

$$j_{\text{n,lep}}^{\mu} = (j_{\text{n,lep}}^{\mu})'$$

$$j_{\text{em,lep}}^{\mu} = (j_{\text{em,lep}}^{\mu})'$$

$U = U_e U_{\nu}^{\dagger}$ Pontecorvo – Maki Nakagawa Sakata (P-MNS) matrix

Physical parameters in U

$$\frac{m_{\nu_i}}{2}\nu_i\nu_i + m_{e_i} e_i^c e_i \qquad j^\mu_{\text{c,lep}} = U_{ij}\bar{e}_i\sigma^\mu\nu_j$$

$$U = \underbrace{\begin{pmatrix} e^{i\gamma_1} & & \\ & e^{i\gamma_2} & \\ & & e^{i\gamma_3} \end{pmatrix}}_{\text{unphysical}} \begin{pmatrix} \text{standard par.} \end{pmatrix} \underbrace{\begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}}_{\substack{\text{physical} \\ \text{(Majorana)}}}$$

9 = 3 + 3 + 1 + 2

Physical mass and mixing parameters in the lepton sector

$$\frac{m_{\nu_i}}{2}\nu_i\nu_i + m_{e_i}e^c_i e_i \qquad j^\mu_{\rm c,lep} = U^\dagger_{ij}\overline{\nu}_{iL}\gamma^\mu e_{jL}$$

$$m_e,\ m_\mu,\ m_\tau,\ m_{\nu_1},\ m_{\nu_2},\ m_{\nu_3},\ \theta_{23},\ \theta_{12},\ \theta_{13},\ \delta,\ \alpha,\ \beta$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}s^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}s^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$0 \le \theta_{23}, \theta_{12}, \theta_{13} \le \frac{\pi}{2}, \quad 0 \le \delta < 2\pi, \quad 0 \le \alpha, \beta < 2\pi$$

Accessible
to oscillations

Charged
sector

$m_{e,\mu,\tau}$

Δm^2_{12}

$|\Delta m^2_{23}|$

$\mathrm{sign}(\Delta m^2_{23})$

$\theta_{12}, \theta_{23}, \theta_{13}, \delta$

Not accessible
to oscillations

m_{lightest}

α

β

$(\Delta m^2_{ij} \equiv m^2_{\nu_i} - m^2_{\nu_j})$

Accessible
to oscillations

Not accessible
to oscillations

Charged
sector

$$\Delta m^2_{12}$$

$$|\Delta m^2_{23}|$$

$$m_{\rm lightest}$$

$$m_{e,\mu,\tau}$$

$$\alpha$$

$$\mathrm{sign}(\Delta m^2_{23})$$

$$\beta$$

Well known

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta$$

Known

Bounds

$\Delta m^2_{\text{ATM}} \sim 2.5 \times 10^{-3}\,\text{eV}^2 \quad \theta_{23} \sim 45^\circ$ (ATM, K2K)

$\Delta m^2_{\text{SUN}} \sim 0.8 \times 10^{-4}\,\text{eV}^2 \quad \theta_{12} \sim 30^\circ\text{–}35^\circ$ (SUN,KamLAND)

$\theta_{13} < 10^\circ$ (CHOOZ, Palo Verde + ATM)

$|m_{ee}| = |U^2_{ei} m_{\nu_i}| < \mathcal{O}(1) \times 0.4\,\text{eV}$ (Heidelberg-Moscow)

$(m^\dagger m)_{ee} = |U_{ei}|^2 m^2_{\nu_i} < (2.2\,\text{eV})^2$ (Mainz, Troitsk)

$\sum_i m_{\nu_i} < 0.6\,\text{eV} \;\; (\text{priors})$ (Cosmology)

Guidelines for theory:

$$m_{\nu_i} \ll 174\,\text{GeV}$$

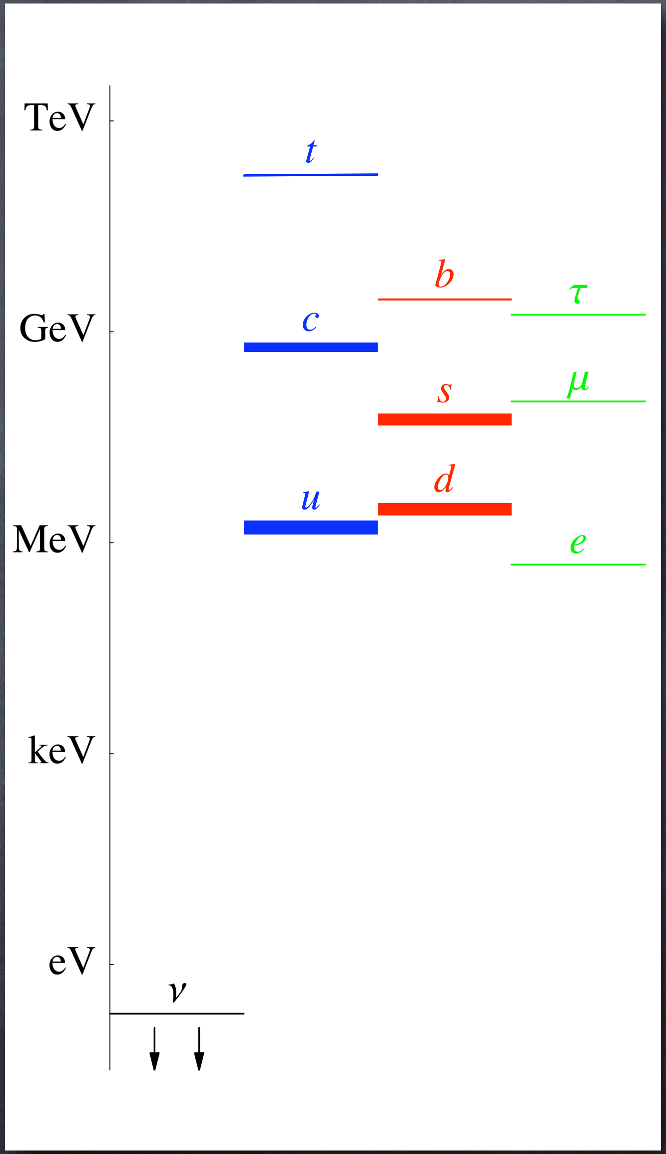
$$\theta_{23} \sim 45^\circ (= 45^\circ ?)$$

$$\theta_{12} \sim 30^\circ\text{–}35^\circ \neq 45^\circ \;\; (> 5\sigma)$$

$$\theta_{13} < 10^\circ$$

$$|\Delta m^2_{12} / \Delta m^2_{23}| \approx 0.035 \ll 1$$

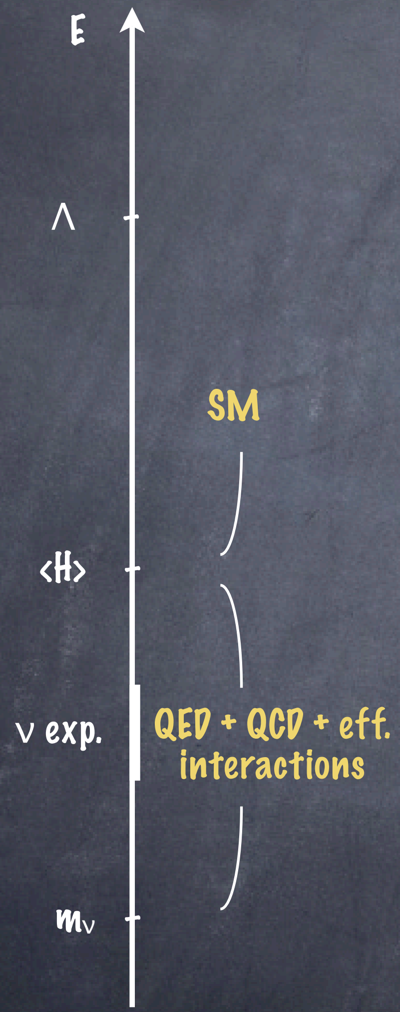
Smallness of neutrino masses



- Natural scale of fermion masses: $\langle H \rangle = 174$ GeV
- Why $m_\nu / \langle H \rangle < 10^{-12}$?
- Must have a different origin than $m_e / \langle H \rangle = 0.3 \times 10^{-5}$
  - larger hierarchy
  - family independent
  - well understood

$E \approx M_Z$: neutrino (and fermion) masses in the SM

SM origin of fermion masses



- What is the form of the fermion mass terms induced by the SM (effective) lagrangian?

The Standard Model (at the ren level)

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar{\Psi}_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar{\Psi}_i\Psi_j H & \text{flavor} \end{array}$$

- An extremely successful synthesis of particle physics
- (in compact notations)
- i = 1,2,3: family index

SM fermion quantum numbers

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar{\Psi}_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar{\Psi}_i\Psi_j H & \text{flavor} \end{array}$$

SM fermion quantum numbers

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$G = SU(3)_C \times SU(2)_W \times U(1)_Y$

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad Q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$q = q_a$$
$$a = 1, 2, 3 \text{ (color)}$$

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
e^c_i	1	1	1
Q_i	3	2	1/6
u^c_i	3*	1	1/3
d^c_i	3*	1	-2/3
			Y

SM fermion quantum numbers

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar\Psi_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar\Psi_i\Psi_j H & \text{flavor} \end{array}$$

The representation is "chiral": no gauge invariant mass term is allowed → SM fermion masses protected by the EW symmetry

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
e^c_i	1	1	1
Q_i	3	2	1/6
u^c_i	3^*	1	1/3
d^c_i	3^*	1	-2/3
			Y

Fermion masses (at the ren. level)

- Fermion masses are induced by

$$\mathcal{L}_{\rm SM}^{\rm flavor} = \lambda^E_{ij} e^c_i L_j H^\dagger + \lambda^D_{ij} d^c_i Q_j H^\dagger + \lambda^U_{ij} u^c_i Q_j H + \text{h.c.}$$

after EWSB:

$$H = \begin{pmatrix} G^+ \\ v + \dfrac{h + iG^0}{\sqrt{2}} \end{pmatrix}$$

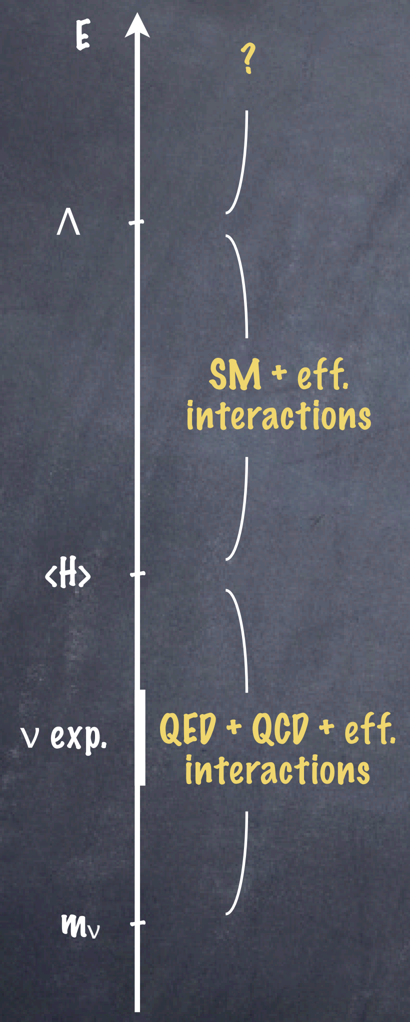
$$\begin{aligned} \mathcal{L}_{\rm SM}^{\rm flavor} &= \lambda^E_{ij} e^c_i L_j H^\dagger + \lambda^D_{ij} d^c_i Q_j H^\dagger + \lambda^U_{ij} u^c_i Q_j H + \text{h.c.} \\ &= m^E_{ij} e^c_i e_j \quad + m^D_{ij} d^c_i d_j \quad + m^U_{ij} u^c_i u_j \quad + \text{h.c.} + \ldots \end{aligned}$$

- We then get $\dfrac{m^\nu_{ij}}{2} \nu_i \nu_j + m^E_{ij} e^c_i e_j + m^D_{ij} d^c_i d_j + m^U_{ij} u^c_i u_j + \text{h.c.}$

with $m^E_{ij} = \lambda^E_{ij} v \quad m^D_{ij} = \lambda^D_{ij} v \quad m^U_{ij} = \lambda^U_{ij} v$

$m^\nu_{ij} = 0$

The SM as an effective theory



- $$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \mathcal{L}^{\rm NR}_{\rm SM} = \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{2\Lambda}(HL_i)(HL_j) + \ldots$$
- $$m^{E,D,U}_{ij} = \lambda^{E,D,U}_{ij} v \quad m^{\nu}_{ij} = h_{ij} v \times \frac{v}{\Lambda}$$
  $$\Lambda \sim 0.5 \times 10^{15}\,{\rm GeV} h \left(\frac{0.05\,{\rm eV}}{m_\nu}\right)$$
- $M_{GUT} \approx 2 \times 10^{16}$ GeV (double see-saw?)
- Room for leptogenesis
- L^{eff} is sensitive to the GUT scale only through L- and B-violating operators
- $\Lambda_L \sim 10^{15}$ GeV, $\Lambda_B > 4 \times 10^{15}$ GeV (no or small L, B violation at TeV scale)

Right-handed neutrinos

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{matrix} u^c \\ d^c \end{matrix} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad e^c$$

$SU(3)_C \times SU(2)_W \times U(1)_Y$

Right-handed neutrinos

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} u^c \\ d^c \end{pmatrix} \quad \begin{pmatrix} \nu \\ e \end{pmatrix} \quad \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}$$

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Right-handed neutrinos

$$\begin{pmatrix} u & u^c & \nu & \nu^c \\ d & d^c & e & e^c \end{pmatrix}$$

SO(10)

Right-handed neutrinos

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{matrix} u^c \\ d^c \end{matrix} \qquad \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \begin{matrix} \nu^c \\ e^c \end{matrix}$$

$SU(3)_C \times SU(2)_W \times U(1)_Y$

$$\lambda \nu_c L H \rightarrow m_\nu = \lambda_\nu v$$

(like the other fermions)

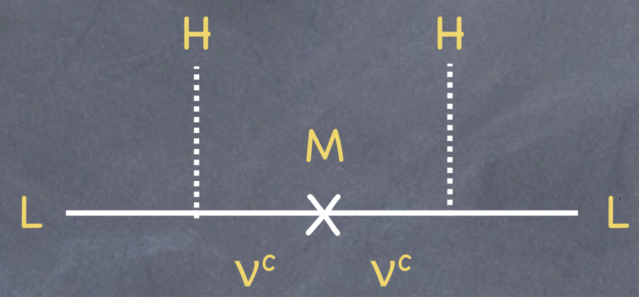
ν_c is a SM singlet and can therefore be heavy

$$\mathcal{L}_{\rm HE} \supset -\frac{M}{2}\nu^c \nu^c$$

(unlike the other fermions)

See-saw

Integrate out ν^c:



$$\frac{h}{\Lambda}(HL)(HL)$$

$$\frac{h}{\Lambda} \to -\lambda^T \frac{1}{M} \lambda$$

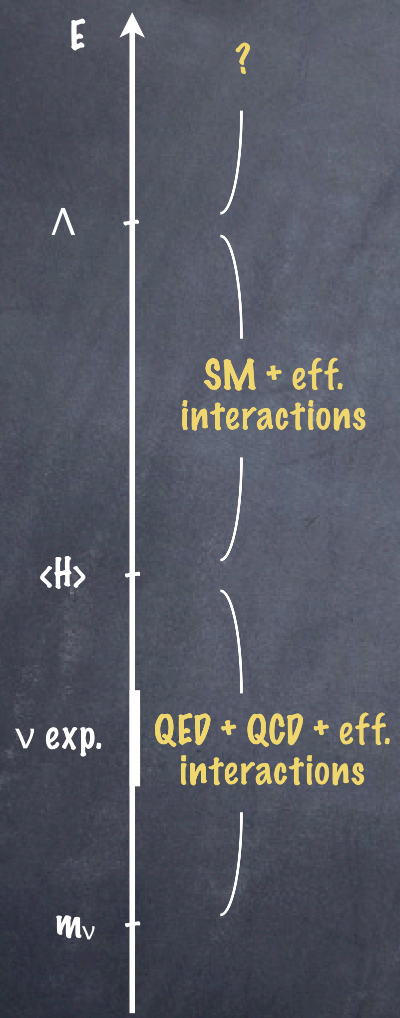
$$m_\nu = -m_{\rm D}^T \frac{1}{M} m_{\rm D}$$

Majorana

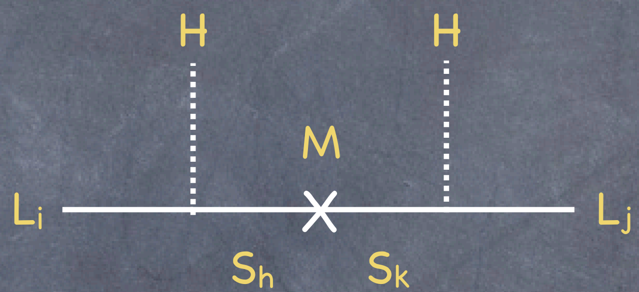
$E \gg M_Z$:
origin of neutrino masses

Renormalizable origin of LLHH

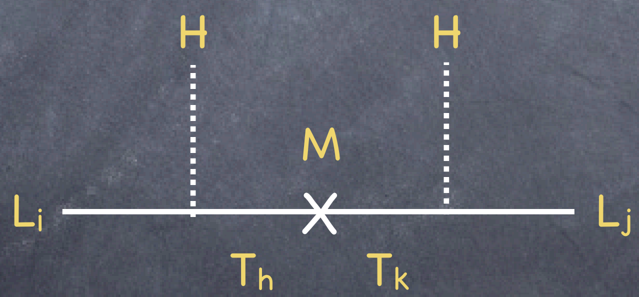
$$\mathcal{L}^{\text{eff}}_{E\ll\Lambda} = \mathcal{L}^{\text{ren}}_{\text{SM}} + \frac{h_{ij}}{\Lambda}(HL_i)(HL_j) + \dots$$



$L_i H \approx (1,2,-\frac{1}{2}) \otimes (1,2,\frac{1}{2}) = (1,1,0) \oplus (1,3,0)$

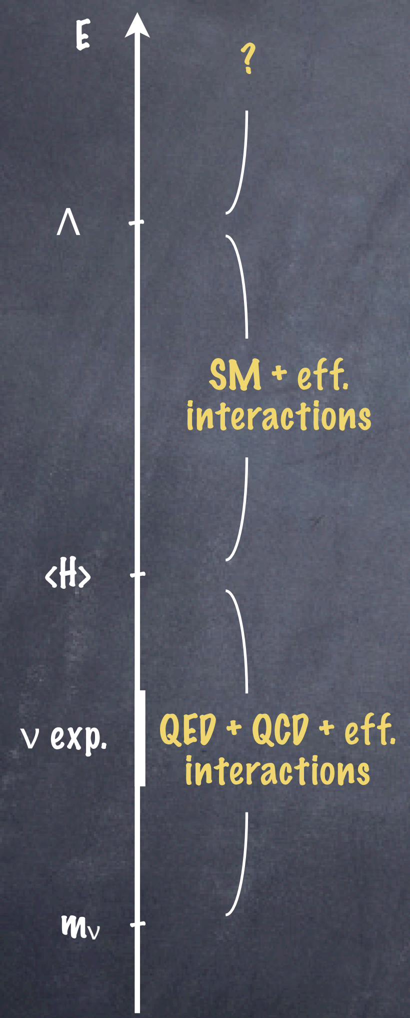


See-saw type I



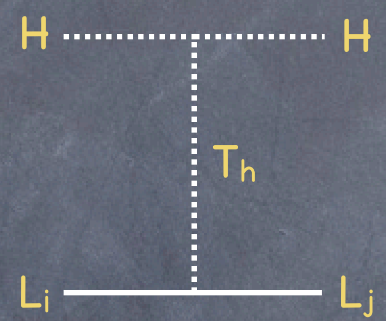
See-saw type II

Renormalizable origin of LLHH



$$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{\Lambda}(HL_i)(HL_j) + \ldots$$

HH ≈ $(1,2,-\tfrac{1}{2}) \otimes (1,2,-\tfrac{1}{2}) = (1,\not{1},-1)_a \oplus (1,3,-1)_s$



See-saw type III

- Any number of S_h, T_h
- No loops if low energy supersymmetry

Alternative origin of neutrino masses

- Standard paradigm:

$$\mathcal{L}^{\rm eff}_{E\ll\Lambda} = \mathcal{L}^{\rm ren}_{\rm SM} + \frac{h_{ij}}{\Lambda}(HL_i)(HL_j) + \ldots$$

$$\Lambda \sim 0.5\times 10^{15}\,{\rm GeV} h \left(\frac{0.05\,{\rm eV}}{m_\nu}\right) \gg {\rm TeV}$$

- Alternative: the SM extension accounting for neutrino masses arises at a scale Λ < TeV (the EFT description does not hold)

Example: Dirac neutrinos

- Lepton number "exactly" conserved: no $\nu^c\nu^c$ mass term, $h_{ij} = 0$
- Neutrino masses then need an L = -1 neutrino ν^c

$$m^N_{ij}\nu^c_i\nu_j + m^E_{ij}e^c_i e_j + m^D_{ij}d^c_i d_j + m^U_{ij}u^c_i u_j + \text{h.c.}$$

- In the SM:

$$\begin{aligned}\mathcal{L}^{\text{flavor}}_{\text{SM}} &= \lambda^N_{ij}\nu^c_i L_j H + \lambda^E_{ij}e^c_i L_j H^\dagger + \lambda^U_{ij}u^c_i Q_j H + \lambda^D_{ij}d^c_i Q_j H^\dagger + \text{h.c.} \\ &= m^N_{ij}\nu^c_i\nu_j \quad + m^E_{ij}e^c_i e_j \quad + m^U_{ij}u^c_i u_j \quad + m^D_{ij}d^c_i d_j \quad + \text{h.c.} + \ldots\end{aligned}$$

$$m^N_{ij} = \lambda^N_{ij}v \quad m^E_{ij} = \lambda^E_{ij}v \quad m^D_{ij} = \lambda^D_{ij}v \quad m^U_{ij} = \lambda^U_{ij}v$$

- Needs L and $\lambda^N < 10^{-11}$: why?

$\lambda^N < 10^{-11}$ (1)

- L is conserved + $\lambda\,\nu^c LH$ forbidden by a symmetry, e.g. because it is charged under a U(1) symmetry:

$$\lambda\nu^c LH \to \lambda\left(\frac{\phi}{M}\right)^n \nu^c LH, \quad \lambda_{\text{eff}} = \lambda\left(\frac{\langle\phi\rangle}{M}\right)^n$$

[Chacko Hall Okui Oliver ph/0312267
Chacko, Hall Oliver Perelstein ph/0405067
Davoudiasl Kitano Kribs Murayama ph/0502176]

- interesting (model dependent) consequences for cosmology (and LSND), no consequences for LHC:

$$\frac{\langle H\rangle}{M} \sim \frac{m_\nu}{\langle\phi\rangle} \sim g_{\phi\nu\nu^c} \lesssim 10^{-5} \quad \text{(BBN)}$$

$\lambda^N < 10^{-11}$ (2)

- L is conserved + λ^N originates in extra-dimensions
  - ν^c lives in the flat bulk of large extra dimensions:

$$\lambda_{\rm eff} = \frac{\lambda}{(2\pi R M_*)^{\delta/2}} = \lambda \frac{M_*}{M_{\rm Pl}}$$

[Arkani-Hamed et al. ph/9811448
Dienes Dudas Ghergetta ph/9811428]

    - 5D $\nu^c \leftrightarrow$ 4D $(\nu^c)_n$ $\quad M_n \approx n/R$ (large n)
    - Brane-bulk mixing: $m \approx \lambda_{\rm eff} \langle H \rangle$

$$\nu_i = U_{ik}\hat{\nu}_k + \frac{m_i}{M_n} N_n$$

In the presence of bulk mass terms

[Lukas Ramond R Ross ph/0008049, ph/0011295]

$U_{ik}\hat{\nu}_k$: Standard 3 (mainly) active neutrino mixing

$\frac{m_i}{M_n} N_n$: Small (mainly) sterile component

  - ν^c and L are localized in distant points of a (warped) extra dimension:

$$\lambda \propto e^{-(\text{superposition of the wave functions})}$$

Low scale lepton number violation

$$\mathcal{L}^{\text{eff}}_{E\ll\Lambda} = \mathcal{L}^{\text{ren}}_{\text{SM}} + \frac{h_{ij}}{\Lambda}(HL_i)(HL_j) + \dots$$

- h ≈ 10^{-13}-10^{-11} allows Λ < TeV
- Why? E.g. $h\frac{LLHH}{\Lambda} \to h\left(\frac{\phi}{M}\right)^n \frac{LLHH}{\Lambda}, \; h_{\text{eff}} = h\left(\frac{\langle\phi\rangle}{M}\right)^n$ (as before)
- How is (HLHL) generated? Origin of L-violation?

Low-scale origin of L-violation (1)

- TeV-scale see-saw
  - ν^c with M ≈ TeV
  - Probe ν^c through $\lambda\, \nu^c L H$: $m_\nu = -m_D^T \frac{1}{M} m_D, \quad m_D = \lambda \langle H \rangle$
  - M ~ TeV ⇒ $\lambda = \frac{m_D}{\langle H \rangle} \sim 10^{-6} \left(\frac{m_\nu}{0.05\,\text{eV}} \right)^{1/2} \left(\frac{M}{\text{TeV}} \right)^{1/2}$ too small for LHC
  - Unless λ » 10^{-7} + cancellations in $m_\nu = -m_D^T \frac{1}{M} m_D$ (2 or more ν^c's)
    - "magical", e.g.: $m_\nu = 0 + \text{corrections}$ if [Buchmuller Greub NPB363]

$$m^D_{nj} = \alpha_n \beta_j\, m_0, \quad M_R = \text{Diag}(M_1 \dots M_n), \quad \sum_n \alpha_n^2 M_n = 0$$

    - natural, e.g.:

$L_e, L_\mu, L_\tau, (\nu_R)_1 \equiv N$ have L = 1, $(\nu_R)_2 \equiv N'$ has L = -1 then $-\mathcal{L}^{\nu}_{\text{mass}} = (\nu_e, \nu_\mu, \nu_\tau, N, N')\, \mathcal{M}\, (\nu_e, \nu_\mu, \nu_\tau, N, N')^T$

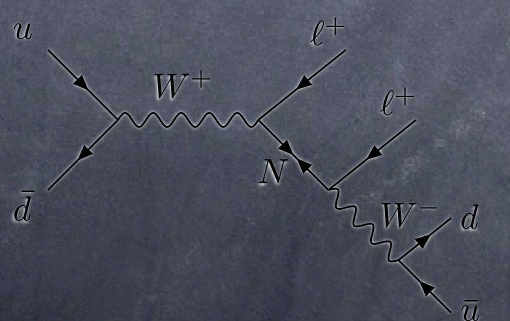
$$\mathcal{M} = \begin{pmatrix} & \mathbf{0} & & 0 & m_e \\ & & & 0 & m_\mu \\ & & & 0 & m_\tau \\ 0 & 0 & 0 & 0 & M \\ m_e & m_\mu & m_\tau & M & 0 \end{pmatrix}$$

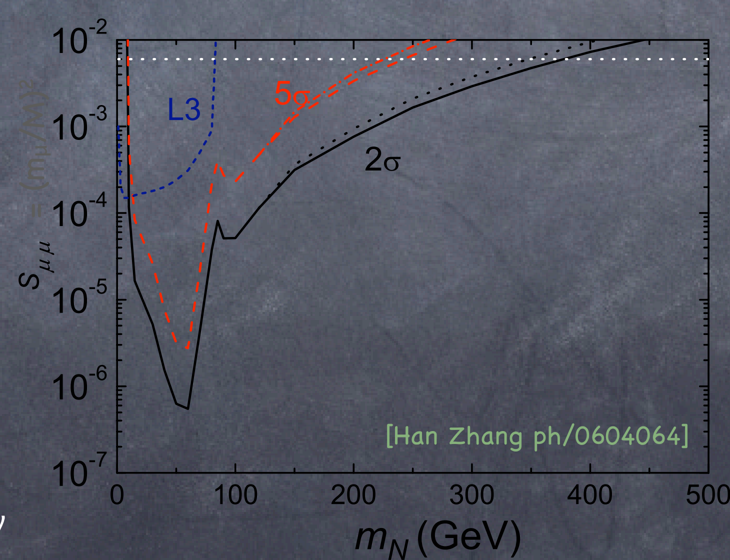
- rank = 2: 3 massless neutrinos independently of the size of m_i
- $\nu_i = U_{ik}\hat{\nu}_k + \frac{m_i}{M}\hat{N}$

- Constraints: $(m_e/M)^2 < 0.006$, $(m_\mu/M)^2 < 0.003$, $(m_\tau/M)^2 < 0.003$

[Nardi Roulet Tommasini NPB386 (1992), ph/9402224, ph/9409310 Bergmann Kagan, ph/9803305]

- LNV at LHC (!): $qq' \to \mu^\pm\mu^\pm W^\mp$ cleanly probes m_μ/M





- No connection with m_ν

[Dicus Karatas Roy PRD44 (1991)
Datta Guchait Pilaftsis ph/9311257
Almeida et al ph/0002024
Ali Borisov Zamorin ph/0104123
Panella et al ph/0107308
Han Zhang ph/0604064
Aguila Aguilar-Saavedra Pittau ph/0606198
Bar-Shalom et al ph/0608309
Atwood Bar-Shalom Soni ph/0701005
Bray Lee Pilaftsis ph/0702294]

Low-scale origin of L-violation (2)

- (R_P-violating) supersymmetry
- Supersymmetry does not guarantee (accidental) L (or B) conservation, unlike the SM: $H_d \approx L_i$

$$W = \lambda^U_{ij} u^c_i Q_j H_u + \lambda^D_{ij} d^c_i Q_j H_d + \lambda^E_{ij} e^c_i L_j H_d + \mu H_u H_d$$
$$+ \lambda''_{ijk} u^c_i d^c_j d^c_k + \lambda'_{ijk} L_i Q_j d^c_k + \lambda_{ijk} L_i L_j e^c_k + \mu_i H_u L_i$$

$$\mathcal{L}_{\rm soft} = A^U_{ij} \tilde{u}^c_i \tilde{Q}_j H_u + A^D_{ij} \tilde{d}^c_i \tilde{Q}_j H_d + A^E_{ij} \tilde{e}^c_i \tilde{L}_j H_d + B\mu H_u H_d$$
$$+ A''_{ijk} \tilde{u}^c_i \tilde{d}^c_j \tilde{d}^c_k + A'_{ijk} \tilde{L}_i \tilde{Q}_j \tilde{d}^c_k + A_{ijk} \tilde{L}_i \tilde{L}_j \tilde{e}^c_k + (B\mu)_i H_u \tilde{L}_i$$
$$+ \tilde{m}^2_Q \tilde{Q}^\dagger \tilde{Q} + (\tilde{m}^2_i H^\dagger_d \tilde{L}_i + \text{h.c.}) + \text{gaugino masses}$$

- L and B violating terms controlled by $R_P = (-1)^{3(B-L)+2s}$
- A small R_P breaking:
  - induces $(h_{ij}/\Lambda) L_i L_j H H$, with $\Lambda = m$, $h \leftrightarrow$ small R_P breaking
  - makes the LSP unstable (could be any susy partner)

[Hall Suzuki NPB231 (1984), Lee PLB138 (1984), NPB246 (1984), Dawson NPB261 (1985), Hempfling ph/9511288, Nilles Polonsky ph/9606388, Kaplan Nelson ph/9901254, Chun Kang ph/9909429, Hirsch Diaz Porod Romao Valle ph/0004115, Joshipura Vempati ph/9808232, Takayama Yamaguchi ph/9910320, Joshipura Vaidya Vempati ph/0203182, Chun Jung Park ph/0211310]
[Aulakh Mohapatra PLB119, Ellis et al PLB150, Ross Valle PLB151, Chikashige Mohapatra Peccei PLB98]

- Bilinear R_P-violation
- In an appropriate basis for $L_\alpha = (H_d, L_e, L_\mu, L_\tau)$:
  - No R_P-violating trilinear terms
  - $W \supset \mu H_u H_d + \mu_i H_u L_i$, $\mathcal{L}_{soft} \supset B\mu H_u H_d + (B\mu)_i H_u L_i$
  - Predictive + might follow from spontaneous R_P breaking
  - $\langle H_u \rangle_2 = v_u$, $\langle H_d \rangle_1 = v_d$, $\langle L_i \rangle_2 = v_i$ → neutrino-neutralino mixing → m_ν
  - Tree level: $\frac{h_{ij}}{\Lambda} \approx \frac{g^2}{2M_2}\xi_i\xi_j \to (m_\nu)_{ij} \approx \frac{M_Z^2}{M_2}\xi_i\xi_j$ controlled by $\xi_i = \frac{v_i\mu - \mu_i v_d}{\mu v_d}$ $\qquad \xi = |\vec{\xi}| \approx 2.5 \times 10^{-6}/\cos\beta\left(\frac{M_2}{\text{TeV}}\right)$
    - $m_1 = m_2 = 0$ (normal hierarchy), $\tan\theta_{23} = \xi_2/\xi_3$, $\tan\theta_{13} = \xi_1/(\xi^2_2+\xi^2_3)^{1/2}$
  - $(\Delta m^2)_{12}$ and θ_{12} at 1-loop, controlled by μ_i/μ
  - LHC: production and decay to LSP almost unaffected
  - Small R_P breaking effect ξ_i, μ_i visible through LSP decay

The origin of the neutrino flavour structure

$$\Delta m^2_{\mathrm{ATM}} \sim 2.5 \times 10^{-3}\,\mathrm{eV}^2 \quad \theta_{23} \sim 45^\circ$$

(ATM, K2K)

$$\Delta m^2_{\mathrm{SUN}} \sim 0.8 \times 10^{-4}\,\mathrm{eV}^2 \quad \theta_{12} \sim 30^\circ\text{–}35^\circ$$

(SUN,KamLAND)

$$\theta_{13} < 10^\circ$$

(CHOOZ, Palo Verde + ATM)

$$|m_{ee}| = |U^2_{ei} m_{\nu_i}| < \mathcal{O}(1) \times 0.4\,\mathrm{eV}$$

(Heidelberg-Moscow)

$$(m^\dagger m)_{ee} = |U_{ei}|^2 m^2_{\nu_i} < (2.2\,\mathrm{eV})^2$$

(Mainz, Troitsk)

$$\sum_i m_{\nu_i} < 0.6\,\mathrm{eV} \ (\text{priors})$$

(Cosmology)

$$m_{\nu_i} \ll 174\,\mathrm{GeV}$$

Guidelines for theory:

$$\theta_{23} \sim 45^\circ (= 45^\circ?)$$

$$\theta_{12} \sim 30^\circ\text{–}35^\circ \neq 45^\circ \ (> 5\sigma)$$

$$\theta_{13} < 10^\circ$$

$$|\Delta m^2_{12} / \Delta m^2_{23}| \approx 0.035 \ll 1$$

The flavour structure of the SM

The flavour puzzle in the SM

- 3 families, or $U(3)^5$ symmetry of the fermion gauge lagrangian

	1	2	3
L	L_1	L_2	L_3
e^c	$(e^c)_1$	$(e^c)_2$	$(e^c)_3$
Q	Q_1	Q_2	Q_3
u^c	$(u^c)_1$	$(u^c)_2$	$(u^c)_3$
d^c	$(d^c)_1$	$(d^c)_2$	$(d^c)_3$

family number (horizontal) not understood

gauge irreps (vertical) well understood

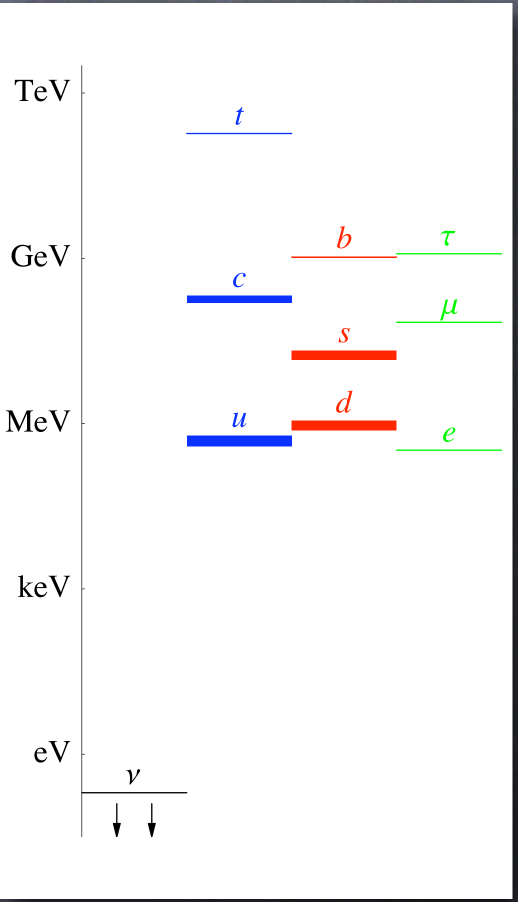
The flavour puzzle in the SM

- 3 families, or $U(3)^5$ symmetry of the fermion gauge lagrangian
- Pattern of $U(3)^5$ breaking from Yukawa sector (most SM pars)

$$\mathcal{L}_{\rm SM}^{\rm flavor} = \lambda^E_{ij} e^c_i L_j H^\dagger + \lambda^D_{ij} d^c_i Q_j H^\dagger + \lambda^U_{ij} u^c_i Q_j H + \text{h.c.}$$

The flavour puzzle in the SM

- 3 families, or $U(3)^5$ symmetry of the fermion gauge lagrangian
- Pattern of $U(3)^5$ breaking from Yukawa sector (most SM pars)
  - (horizontal) hierarchy of fermion masses: $1 \ll 2 \ll 3$
  - CKM mixing angles $\ll 1$
  - U vs D vs E
    - different hierarchies: $U \gg D, E$
    - $m_b \approx m_\tau$, $3m_s \approx m_\mu$, $m_d \approx 3m_e$ @ M_G
  - mass hierarchy vs mixing hierarchy
    - $|V_{cb}| \sim m_s/m_b$, $|V_{us}| \approx (m_d/m_s)^{1/2}$
  - neutrino sector
    - see above



Family replication

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar\Psi_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar\Psi_i\Psi_j H & \text{flavor} \end{array}$$

$\Psi_i = (L_i\ e^c_i\ Q_i\ u^c_i\ d^c_i) \leftrightarrow$ 1 family

3 families $\leftrightarrow$ 3 identical copies

of the same (reducible) repr

WHY?

	SU(3)	SU(2)	U(1)
L_i	1	2	-1/2
e^c_i	1	1	1
Q_i	3	2	1/6
u^c_i	3^*	1	1/3
d^c_i	3^*	1	-2/3
			Y

U(3)^5

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar\Psi_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar\Psi_i\Psi_j H & \text{flavor} \end{array}$$

Family replication ↔ the gauge lagrangian cannot tell families ↔ is U(3)^5 invariant:

$$U(3)^5: \begin{array}{l} L_i \to U^L_{ij} L_j \\ e^c_i \to U^{e^c}_{ij} e^c_j \\ Q_i \to U^Q_{ij} Q_j \\ u^c_i \to U^{u^c}_{ij} u^c_j \\ d^c_i \to U^{d^c}_{ij} d^c_j \end{array} \Rightarrow \mathcal{L}_{\rm SM}^{\rm gauge} \to \mathcal{L}_{\rm SM}^{\rm gauge}$$

(U(3)^5 → U(3) in SO(10) gauge-unified models)

U(3)^5

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar\Psi_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar\Psi_i\Psi_j H & \text{flavor} \end{array}$$

The symmetry breaking lagrangian is U(3)^5 invariant:

$$U(3)^5: \begin{array}{l} L_i \to U^L_{ij} L_j \\ e^c_i \to U^{e^c}_{ij} e^c_j \\ Q_i \to U^Q_{ij} Q_j \\ u^c_i \to U^{u^c}_{ij} u^c_j \\ d^c_i \to U^{d^c}_{ij} d^c_j \end{array} \Rightarrow \mathcal{L}_{\rm SM}^{\rm SB} \to \mathcal{L}_{\rm SM}^{\rm SB}$$

The symmetry breaking itself $H = \begin{pmatrix} G^+ \\ v + \frac{h + iG^0}{\sqrt{2}} \end{pmatrix}$ is also U(3)^5 invariant

U(3)⁵

$$\mathcal{L}_{\rm SM}^{\rm ren} = \begin{array}{ll} \bar\Psi_i i\hat{D}\Psi_i - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} & \text{gauge} \\ +|D_\mu H|^2 - V(H) & \text{symmetry breaking} \\ +\lambda_{ij}\bar\Psi_i\Psi_j H & \text{flavor} \end{array}$$

The flavour lagrangian is is not $U(3)^5$ invariant (unless $\lambda_{ij}=0$)

$$U(3)^5: \begin{array}{l} L_i \to U^L_{ij} L_j \\ e^c_i \to U^{e^c}_{ij} e^c_j \\ Q_i \to U^Q_{ij} Q_j \\ u^c_i \to U^{u^c}_{ij} u^c_j \\ d^c_i \to U^{d^c}_{ij} d^c_j \end{array} \Rightarrow \begin{array}{l} \lambda_E \to U^T_{e^c}\lambda_E U_L \\ \lambda_D \to U^T_{d^c}\lambda_D U_Q \\ \lambda_U \to U^T_{u^c}\lambda_U U_Q \end{array}$$

$$\mathcal{L}_{\rm SM}^{\rm flavor} = \lambda^E_{ij} e^c_i L_j H^\dagger + \lambda^D_{ij} d^c_i Q_j H^\dagger + \lambda^U_{ij} u^c_i Q_j H + \text{h.c.}$$

The flavor sector (at the ren. level)

$$m^D = U^T_{d^c} m^D_{\text{diag}} U_d \qquad m^U = U^T_{u^c} m^U_{\text{diag}} U_u$$

$$\begin{cases} u_i^{c\prime} = U^{u^c}_{ij} u^c_j \\ d_i^{c\prime} = U^{d^c}_{ij} d^c_j \\ Q'_j = U^d_{ij} Q_j \end{cases} \rightarrow \begin{array}{l} \lambda^D_{ij} d^c_i Q_j H^\dagger = \lambda_{d_i} d_i^{c\prime} Q'_i H^\dagger \\ \lambda^U_{ij} u^c_i Q_j H^\dagger = \lambda_{u_i} V_{ij} u_i^{c\prime} Q'_i H \end{array}$$

$$m^E = U^T_{e^c} m^E_{\text{diag}} U_e \qquad \begin{cases} e_i^{c\prime} = U^{e^c}_{ij} e^c_j \\ L'_j = U^e_{ij} L_j \end{cases} \rightarrow \lambda^E_{ij} e^c_i L_j H^\dagger = \lambda_{e_i} e_i^{c\prime} L'_i H^\dagger$$

- Individual lepton numbers: e.g. L_e corresponds to $e^c \rightarrow e^{-i\alpha} e^c, \quad L_e \rightarrow e^{i\alpha} L_e,$

  Total lepton number $L = L_e+L_\mu+L_\tau$: corresponds to $e^c_i \rightarrow e^{-i\alpha} e^c_i, \quad L_i \rightarrow e^{i\alpha} L_i \quad (\forall\, i)$

  Baryon number B: corresponds to $u^c_i \rightarrow e^{-i\alpha} u^c_i, \quad d^c_i \rightarrow e^{-i\alpha} d^c_i, \quad Q_i \rightarrow e^{i\alpha} Q_i \quad (\forall\, i)$
- The L_e L_μ L_τ B transformations are all part of $U(3)^5$

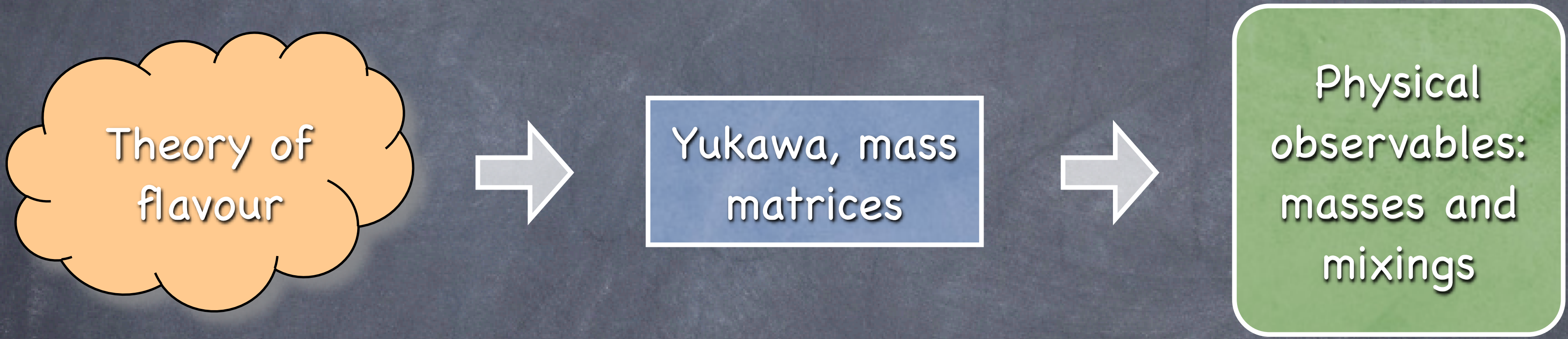
B & L

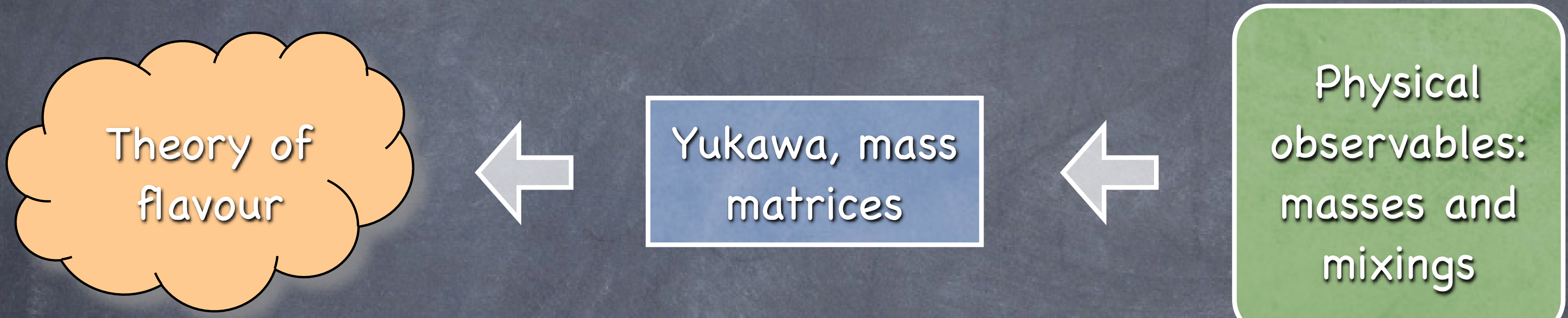
$$\mathcal{L}_{\rm SM}^{\rm flavor} = \lambda_{e_i} e_i^c L_i H^\dagger + \lambda_{d_i} d_i^c Q_i H^\dagger + \lambda_{u_i} V_{ij} u_i^c Q_j H + \text{h.c.}$$

$$L_i \to e^{i\alpha_L} L_i$$
$$e_i^c \to e^{-i\alpha_L} e_i^c$$

$$Q_i \to e^{i\alpha_B} Q_i$$
$$u_i^c \to e^{-i\alpha_B} u_i^c$$
$$d_i^c \to e^{-i\alpha_B} d_i^c$$

are both symmetries of $\mathcal{L}_{\rm SM}^{\rm flavor}$


Theory of flavour
Yukawa, mass matrices
Physical observables: masses and mixings



Quarks:
36 parameters

Quarks:
10 parameters

Textures


Yukawa, mass matrices
Physical observables: masses and mixings

Origin of large mixings

$$m_U = U_{u^c}^T m_U^{\text{diag}} U_u$$
$$m_D = U_{d^c}^T m_D^{\text{diag}} U_d$$

$$V = U_u U_d^\dagger$$

$$m_\nu = U_\nu^T m_\nu^{\text{diag}} U_\nu$$
$$m_E = U_{e^c}^T m_E^{\text{diag}} U_e$$

$$U = U_e U_\nu^\dagger$$

The large mixing angles can in principle originate from both m_E, m_ν

Does the distinction makes sense?

SM: U(3)[5]

$$\begin{cases} e_i^{c\prime} = U_{ij}^{e^c} e_j^c \\ L_j' = U^e{}_{ij} L_j \end{cases} \rightarrow \begin{cases} m_\nu \rightarrow U^* m_\nu^{\text{diag}} U^\dagger \\ m_E \rightarrow m_E^{\text{diag}} \end{cases} \quad \begin{cases} e_i^{c\prime} = U_{ij}^{e^c} e_j^c \\ L_j' = U^\nu{}_{ij} L_j \end{cases} \rightarrow \begin{cases} m_\nu \rightarrow m_\nu^{\text{diag}} \\ m_E \rightarrow m_E^{\text{diag}} U \end{cases}$$

Yes, in terms of the physics giving rise to the mass matrices

Assumption: there exists a privileged basis in flavour space in which correlations among entries of the mass matrices only arise from symmetries of the underlying theory or accidents

Origin of θ_{23}

- (From m_ν in the case of degenerate neutrinos)
- From m_ν in the case of normal hierarchy
- From m_ν in the case of inverse hierarchy
- From m_E
- (Anarchy)

Large angles?

- $\vartheta_q\ \vartheta_l \ll 1 \not\Rightarrow \vartheta_\nu \ll 1$: Dirac and Majorana mass terms transform differently under symmetries

- Example: $L_\mu - L_\tau$. In the symmetric limit:

$$m_E \propto \begin{pmatrix} & & \\ & a & 0 \\ & 0 & 1 \end{pmatrix} \qquad m_\nu \propto \begin{pmatrix} & & \\ & 0 & 1 \\ & 1 & 0 \end{pmatrix}$$

$$\theta_l = 0^\circ \qquad\qquad \theta_\nu = 45^\circ$$

- However, it only works with degenerate ν's:
  - $m_2 \approx m_3$, $(\Delta m^2)_{12} \ll (\Delta m^2)_{23} \Rightarrow m_1 \approx m_2 \approx m_3$
  - Example: $m_\nu \propto \begin{pmatrix} 1 & & \\ & & 1 \\ & 1 & \end{pmatrix}$

A large ϑ_{23} from m_ν - normal hierarchy

- ϑ_{23} large and $m_2 \ll m_3$ seems unnatural:

$$m_\nu \propto \begin{pmatrix} & & \\ & C & B \\ & B & A \end{pmatrix}$$

ϑ_{23} large: $A \sim B \sim C$

$m_2 \ll m_3$: $AC - B^2 \ll 1$

(Semi-)anarchy?

- However, in a see-saw context A, B, C are not fundamental parameters $m_\nu = -m_{\rm D}^T M^{-1} m_{\rm D}$

$$[M]_{23} = \begin{pmatrix} M_2 & \\ & M_3 \end{pmatrix}, \quad [m_\nu]_{23} = \frac{1}{M_2}\begin{pmatrix} m_{22}^2 & m_{22}m_{23} \\ m_{22}m_{23} & m_{23}^2 \end{pmatrix} + \frac{1}{M_3}\begin{pmatrix} m_{32}^2 & m_{32}m_{33} \\ m_{32}m_{33} & m_{33}^2 \end{pmatrix}$$

det ≠ 0 det = 0 det = 0

- Natural option: $M_2 \ll M_3, \quad m_{22} \sim m_{23}$ [King; Altarelli Feruglio Masina]

A large ϑ_{23} from m_ν - inverse hierarchy

- ϑ_{23} large and $m_1 \approx m_2$ + no correlations: $m_\nu \propto \begin{pmatrix} & A & B \\ A & & \\ B & & \end{pmatrix} + \text{corr.}$

  (no correlations ⇒ stable under rad corrs)

- $\tan \vartheta_{23} = B/A$

- Bonus: ϑ_{12} automatically large

- Potential problem: $(\vartheta_{12})_\nu = 45^0$ (see below)

A large ϑ_{23} from m_E

$$m_E \propto \begin{pmatrix} & & \\ & & \epsilon' \\ & A & 1 \end{pmatrix} \qquad A = 1.0 \pm 0.3$$

$$m_D \propto \begin{pmatrix} & & \\ & & A' \\ & \epsilon & 1 \end{pmatrix} \qquad \epsilon \sim 0.04$$

Not incompatible even in SU(5), where $m_E \leftrightarrow (m_D)^T$ (up to JG factors)

[e.g. Altarelli Feruglio and refs]

Comments:

- m_s/m_b: $m_s/m_b = A' \epsilon$, $m_\mu/m_\tau = A\ \epsilon' \rightarrow A' \approx 1/3$?

- V_{ub}: $V_{ub} \sim s^U_{12} V_{cb} + A' V_{us} \dfrac{m_s}{m_b} + \text{“13” contributions}$

  The A' contribution fixes the "texture zero" prediction for V_{ub} when $A' \approx 1/3$

- m_b-m_τ supersymmetric unification:

$$\left.\frac{m_\tau}{m_b}\right|_{\rm GUT} = \frac{\sqrt{1+A^2}}{\sqrt{1+A'^2}} \neq 1 \quad \text{if} \quad A' \neq A \sim 1$$

  The low energy value of m_b-m_τ is in better agreement with $A' = 1/3$ than $A' = 1$

- Asymmetric textures: $m^D_{23} \gg m^D_{32}$ but $m^D_{12} \sim m^D_{21}$

- Supersymmetry: $\theta_{s_R \tilde{b}_R} \gtrsim A'$ (barring alignment)

  Sizeable effects in $b \leftrightarrow s$ transitions: Δm_{Bs}, $b \to s\gamma$...

  (moderate $\tan\beta$ safer)

Is ϑ_{23} large or maximal?

- Large = $O(\pi/4)$; maximal = $\pi/4 \pm$ correction $\ll 1$
- SK: $\sin^2 2\vartheta_{23} > 0.9$ - not enough

$$\tan \vartheta_{23} = B/A;\ A \sim B \leftrightarrow \text{large};\ A = B \leftrightarrow \text{maximal}$$

$$1 - \epsilon < B/A < 1 + \epsilon \quad \Rightarrow \sin^2 2\vartheta_{23} > 1 - \epsilon^2$$

$$0.7 < B/A < 1.4 \Rightarrow \sin^2 2\vartheta_{23} > 0.9$$

$$0.9 < B/A < 1.1 \Rightarrow \sin^2 2\vartheta_{23} > 0.99$$

- Obtaining a maximal atm angle in a 3 neutrino context is non-trivial. A maximal angle would set a powerful constraint on the origin of lepton mixing (non-abelian horizontal symmetries?)

θ_{12}

For each of the above ways to explain $\theta_{12} \sim 1$, the measured values of θ_{12} provides an additional constraint on m_ν or m_E (see below)

General expectations for θ_{13}

- Inverse Hierarchy: barring tunings or cancellations, θ_{13} must be close to the experimental limit

  In fact:

  - an inverse hierarchy requires, barring tunings, a correction to θ_{12} from m_E
  - a correction to θ_{12} from m_E contributes to θ_{13}

- An inverse hierarchy requires, barring tunings, a correction to θ_{12} from m_E:
  - $$m_\nu = \begin{pmatrix} & A & B \\ A & & \\ B & & \end{pmatrix} \xrightarrow{\theta_{23}\ \text{rotation}} m_\nu \propto \begin{pmatrix} & 1 & \\ 1 & & \\ & & \end{pmatrix} \to \theta_{12} = 45^o$$
  - Correction from m_ν:

$$m_\nu \propto \begin{pmatrix} a & 1 & \\ 1 & b & \\ & & \end{pmatrix}$$

$\theta^\nu_{12} \sim 30^o$–$35^o$ if $a \sim b \sim 1$

$m_1 \approx m_2$ if $a, b \ll 1$ or $a \approx b$

  - Correction from m_E:

requires $\theta^e_{12} \sim \dfrac{45^o - \theta_{12}}{\sqrt{2}}$

- A correction to θ_{12} from m_E contributes to θ_{13}:

$$U \supseteq \begin{pmatrix} c^e_{12} & s^e_{12} & \\ -s^e_{12} & c^e_{12} & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & \\ -1/\sqrt{2} & 1/\sqrt{2} & \\ & & 1 \end{pmatrix}$$

$$\theta^e_{12} \sim \frac{45^o - \theta_{12}}{\sqrt{2}} \Rightarrow s_{13} \supseteq s^e_{12} s_{23} \sim \frac{45^o - \theta_{12}}{2} \sim \text{exp. limit}$$

- Taking into account phases, $(45^0 - \theta_{12})/2$ becomes a lower limit

- $\theta^e_{12} \sim \theta_{12} \Rightarrow s_{13} \supseteq s^e_{12} s_{23} \sim 0.5 > \text{exp. limit}$

General expectations for θ_{13}

- Unification + Gatto-Sartori-Tonin
  - In all cases, θ^e_{12} contributes to θ_{13}
  - θ^e_{12} is also model dependent, but is related to charged fermions
  - $m_D \propto \begin{pmatrix} 0 & \epsilon' & \\ \epsilon' & \epsilon & \\ & & 1 \end{pmatrix}$ is successful: $\theta_c \approx \sqrt{\frac{m_d}{m_s}}$ (precise) [Gatto Sartori Tonin 68]
  - Implementing the same pattern in m_E: $\theta^e_{12} \approx \sqrt{\frac{m_e}{m_\mu}} \approx \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \approx \frac{\theta_c}{3}$

$$\theta_{13} \supseteq s_{23}\sqrt{\frac{m_e}{m_\mu}} \approx \frac{1}{3} \times \text{exp. limit} \sim 3^o$$

  - Central value observable with suberbeams (but > O(1) uncertainty)

A comment on complementarity

- $\theta_c + \theta_{12} = \pi/2$? "complementarity"

- Naive understanding:
  - $\theta_c \equiv \theta^q_{12} = \theta^e_{12}$
  - $\theta_{12} = \pi/2 - \theta^e_{12}$

- However:
  - $\theta^q_{12} = 3\ \theta^e_{12}$ is more appealing
  - $\theta_{12} = (\pi/2 - \theta^e_{12})/\sqrt{2}$

General expectations for θ_{13}

- Normal hierarchy, θ_{23} from m_ν
  - $m_\nu \propto \begin{pmatrix} & \epsilon & \\ \epsilon & 1 & 1 \\ & 1 & 1 \end{pmatrix}$, $\epsilon \leftrightarrow \Delta m^2_{12}/\Delta m^2_{23}$, θ_{12}
  - The diagonalisation of the 23-block rotates ϵ into the 13 entry

$$\theta_{13} \supseteq \epsilon s_{23} \sim \frac{m_2}{m_3} s_{23} \approx \sqrt{\frac{\Delta m^2_{12}}{\Delta m^2_{23}}} s_{23}$$

General expectations for θ_{13}

- Normal hierarchy, θ_{23} from m_E

?

Minimal models

- Use the minimal number of "effective" parameters needed to account for the data: 4+1

- Produce 2 relations among θ_{23}, θ_{12}, θ_{13}, δ, Δm^2_{12}, Δm^2_{23}, m_{ee}

i.e. a prediction for θ_{13}, m_{ee}

Reducing the number of parameters

- Simplest possibility: assume the presence of (2) zeros in the neutrino mass matrix written in the flavor basis, $(m_\nu)_{e_i e_j}$

[Frampton, Glashow, Marfatia]

- However, the parameters in $(m_\nu)_{e_i e_j}$ are only combinations of the parameters in the basic lagrangian
- Assume instead:
  - the relative smallness (vanishing) of some parameters in the basic lagrangian (m_E, m_N, M)
  - the absence of correlations among those parameters (non-abelian symmetries could give rise to further possibilities)
- There are only 5 possible predictions

Example

- Only two singlets are relevant: N_1, N_2

[Frampton, Glashow, Yanagida]
[Raidal, Strumia]

- Their mass matrix is diagonal: $M = \mathrm{diag}(M_1\ M_2)$
- The Dirac mass term is minimal:
  $\mu_1 N_1 (c\, \nu_\mu + s\, \nu_\tau) + \mu_2 N_2 (c'\, \nu_e + e^{i\varphi} s'\, \nu_\mu)$

- 5 parameters: θ, θ', φ, μ^2_1/M_1, μ^2_2/M_2
- 7 observables: θ_{23}, θ_{12}, θ_{13}, δ, Δm^2_{12}, Δm^2_{23}, m_{ee}
- 2 predictions:

$$\theta_{13} = \frac{\tan\theta_{23}}{2} \sin 2\theta_{12} \left(\frac{\Delta m^2_{12}}{\Delta m^2_{23}}\right)^{1/2}, \quad m_{ee} = \sin^2\theta_{12} \left(\frac{\Delta m^2_{12}}{\Delta m^2_{23}}\right)^{1/2}$$

- (also: $\Delta m^2_{23} > 0$)

Not the only possibility:

- 3 singlets: N_1, N_2, N_3
- Their mass matrix: $\frac{M_1}{2} N_1 N_1 + M_{23} N_2 N_3$
- Dirac mass term:
  $\mu_1 N_1 (c\, \nu_\mu + s\, \nu_\tau) + \mu_2 N_2 (c'\, \nu_e + e^{i\varphi} s'\, \nu_\mu) + \mu_3 N_3 \nu_\tau$
- 5 parameters: θ, θ', φ, μ^2_1/M_1, $\mu_2\mu_3/M_2$
- 7 observables: θ_{23}, θ_{12}, θ_{13}, δ, Δm^2_{12}, Δm^2_{23}, m_{ee}
- 2 predictions:

$$\theta_{13} = \frac{\tan\theta_{23}}{2} \sin 2\theta_{12} \left(\frac{\Delta m^2_{12}}{\Delta m^2_{23}}\right)^{1/2}, \quad m_{ee} = \sin^2\theta_{12} \left(\frac{\Delta m^2_{12}}{\Delta m^2_{23}}\right)^{1/2}$$

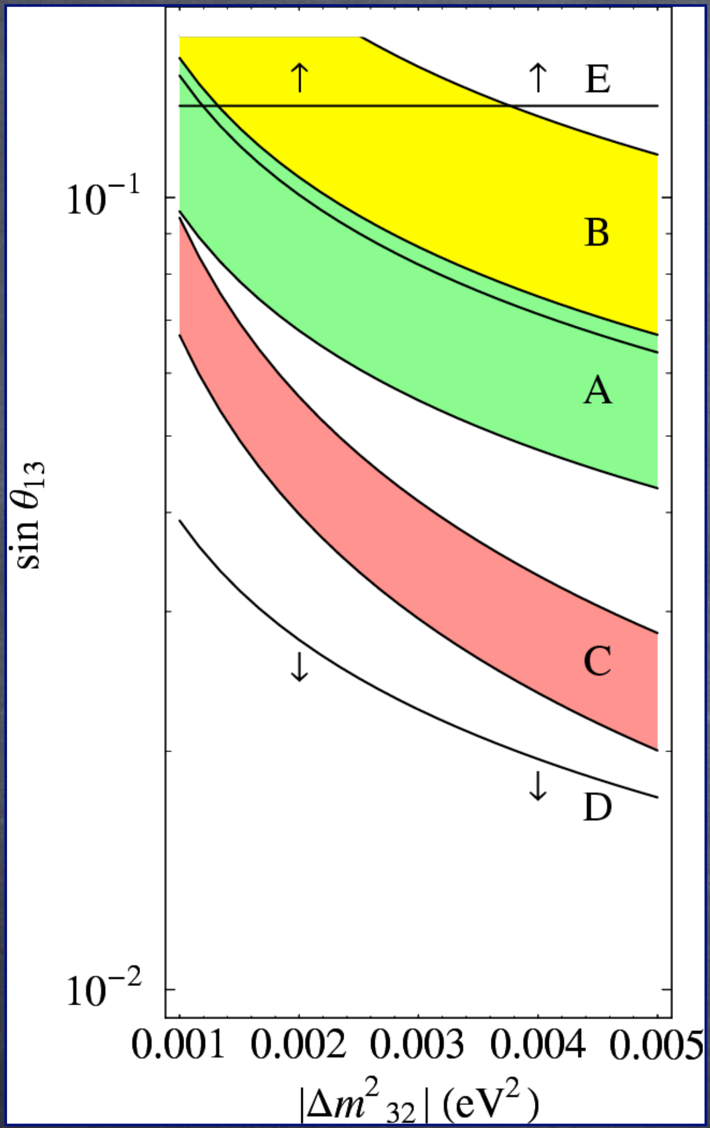
- (also: $\Delta m^2_{23} > 0$)

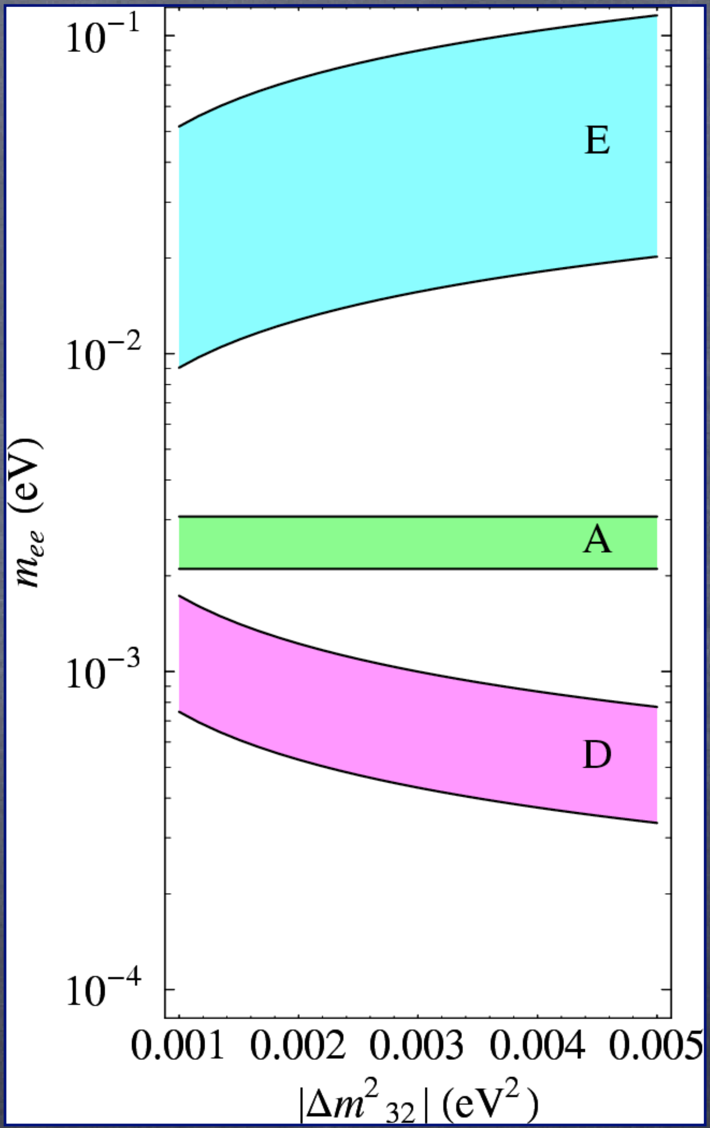
Moreover:

Cases with non vanishing rotations in the charged lepton sector

(1 parameter less available in the neutrino lagrangian)

Predictions for θ_{13}, m_{ee}

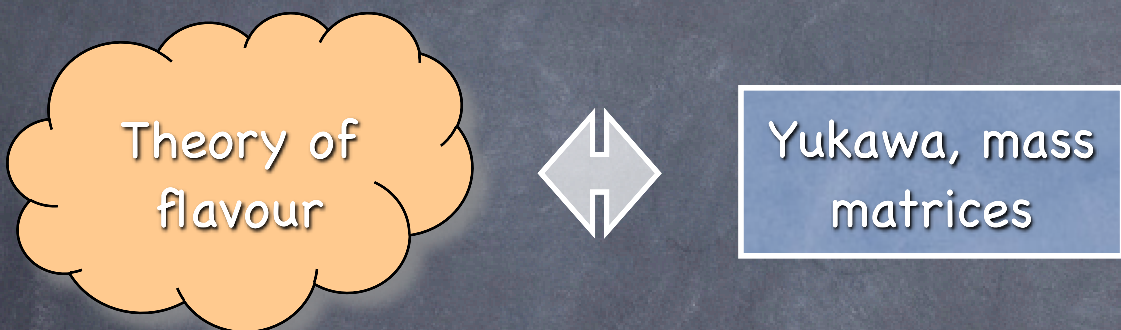




Barbieri, Hambye, AR

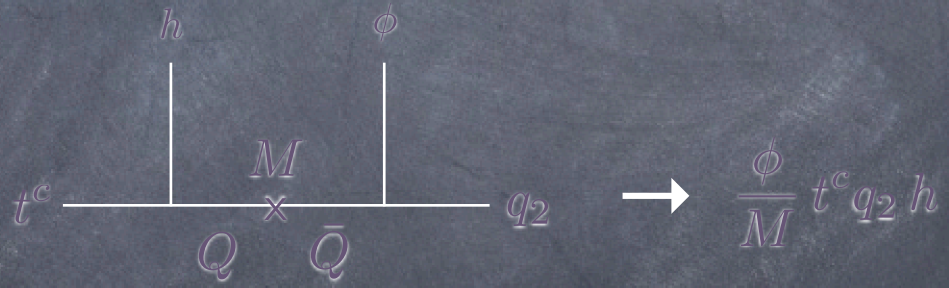
- E is the only case which corresponds to IH and in which the predictions depend on δ (hence the lower limit and the constraint $\cos\delta > 0.8$)
- In case D, $\theta_{13} \propto 45^o - \theta_{23}$ (hence the upper limit)
- Cases A, B, E are within the sensitivity of superbeams; case C requires SB + BB; case D has chances with a nu-factory.
- Cases A, B, C, D assume no "12" rotation in the charged lepton sector
- There are good prospects for $0\nu2\beta$ decay only in the IH case (E), but as long as δ is not known, there is no special prediction.
- Case A has been first studied by Frampton, Glashow, Yanagida.

Flavour models


Theory of flavour
Yukawa, mass matrices

Flavour symmetries

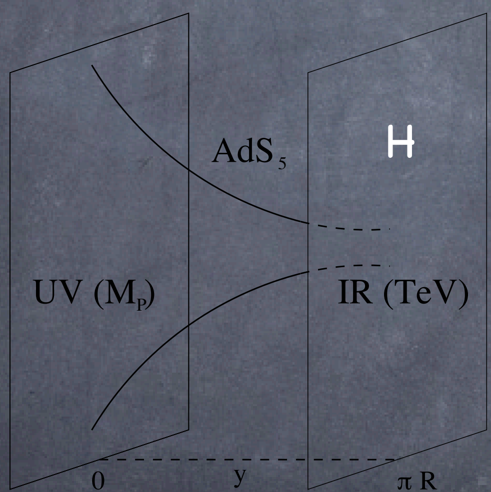
- "Flavour symmetries" acting on family indexes (subgroup of $U(3)^5$)
  - symmetric limit: only O(1) Yukawas possibly allowed: λ_t (λ_b λ_τ)
    - e.g. t^c, q_3, h neutral under a U(1): $Y_{33} t^c Q_3 H$ is allowed
  - spontaneous breaking of U(1) by SM singlets ϕ at high scale
    - e.g. $Q(q_2) = 1$, $Q(\phi) = -1$: $\frac{\phi}{M} t^c Q_2 H$ is allowed $\Rightarrow$ $Y_{32} = \frac{\langle \phi \rangle}{M}$
  - breaking communicated to SM fermions by heavy messengers (M = mass)
    - at E « M



  - gauge/global, continuous/discrete, abelian/non-abelian

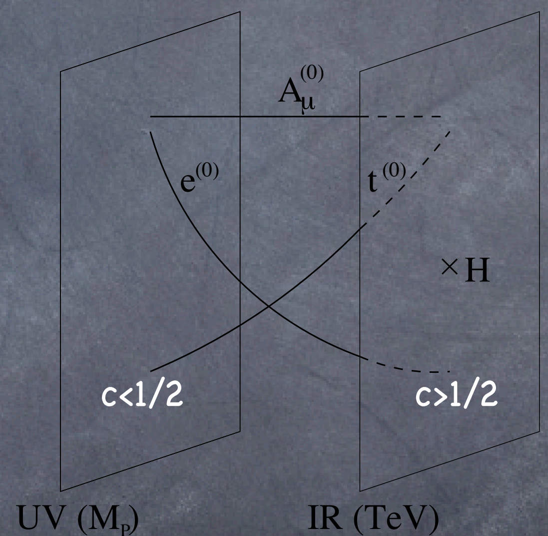
Localisation in extra-dimensions

- Extra-dimension mechanisms
  - flavour symmetry breaking with boundary conditions
  - localized fermions
    - e.g. in RS-type models:



$$m_H \sim M_5 e^{-\pi k R}$$

$k =$ curvature



$$\psi_i(y) \propto e^{(1/2-c_i)ky}$$

$c =$ bulk mass in k units

$$\lambda_{ij} \propto e^{(1-c_i-c_j)\pi k R}$$

Flavour symmetries: rules of the game

	SU(3)	SU(2)	U(1)
(L_1,L_2,L_3)	1	2	-1/2
(e^c_1,e^c_2,e^c_3)	1	1	1
(Q_1,Q_2,Q_3)	3	2	1/6
(u^c_1,u^c_2,u^c_3)	3^*	1	1/3
(d^c_1,d^c_2,d^c_3)	3^*	1	-2/3
H	1	2	1/2

Flavour symmetries: rules of the game

	SU(3)	SU(2)	U(1)	G_f
(L_1,L_2,L_3)	1	2	-1/2	n_L
(e^c_1,e^c_2,e^c_3)	1	1	1	n_e
(Q_1,Q_2,Q_3)	3	2	1/6	n_Q
(u^c_1,u^c_2,u^c_3)	3^*	1	1/3	n_u
(d^c_1,d^c_2,d^c_3)	3^*	1	-2/3	n_d
H	1	2	1/2	n_H
ϕ	1	1	0	n_ϕ
ϕ'	1	1	0	$n_{\phi'}$

$\langle\phi\rangle \gg \langle H\rangle$

Flavour symmetries: rules of the game

	SU(3)	SU(2)	U(1)	$G_F = U(1)$
(L_1, L_2, L_3)	1	2	-1/2	$(q^L{}_1, q^L{}_1, q^L{}_1)$
$(e^c{}_1, e^c{}_2, e^c{}_3)$	1	1	1	$(q^e{}_1, q^e{}_1, q^e{}_1)$
(Q_1, Q_2, Q_3)	3	2	1/6	$(q^Q{}_1, q^Q{}_1, q^Q{}_1)$
$(u^c{}_1, u^c{}_2, u^c{}_3)$	3^*	1	1/3	$(q^u{}_1, q^u{}_1, q^u{}_1)$
$(d^c{}_1, d^c{}_2, d^c{}_3)$	3^*	1	-2/3	$(q^d{}_1, q^d{}_1, q^d{}_1)$
H	1	2	1/2	q_H
ϕ	1	1	0	q_ϕ
ϕ'	1	1	0	$q_{\phi'}$

$\langle\phi\rangle \gg \langle H\rangle$

Flavour symmetries: rules of the game

	SU(3)	SU(2)	U(1)	$G_F = U(1)$
(L_1, L_2, L_3)	1	2	-1/2	$(q^L{}_1, q^L{}_1, q^L{}_1)$
$(e^c{}_1, e^c{}_2, e^c{}_3)$	1	1	1	$(q^e{}_1, q^e{}_1, q^e{}_1)$
(Q_1, Q_2, Q_3)	3	2	1/6	$(q^Q{}_1, q^Q{}_1, q^Q{}_1)$
$(u^c{}_1, u^c{}_2, u^c{}_3)$	3^*	1	1/3	$(q^u{}_1, q^u{}_1, q^u{}_1)$
$(d^c{}_1, d^c{}_2, d^c{}_3)$	3^*	1	-2/3	$(q^d{}_1, q^d{}_1, q^d{}_1)$
H	1	2	1/2	q_H
ϕ	1	1	0	q_ϕ
ϕ'	1	1	0	$q_{\phi'}$

$\langle\phi\rangle \gg \langle H\rangle$

- Write the most general lagrangian (including powers of ϕ/M) invariant under G_F
- Substitute $\phi/M \to \langle\phi\rangle/M$
- Write the corresponding mass matrices

Example: ϑ_{23} from m_E

	SU(3)	SU(2)	U(1)	U(1)
(L_1,L_2,L_3)	1	2	-1/2	(1,0,0)
(e^c_1,e^c_2,e^c_3)	1	1	1	(3,2,0)
(Q_1,Q_2,Q_3)	3	2	1/6	(3,2,0)
(u^c_1,u^c_2,u^c_3)	3^*	1	1/3	(3,2,0)
(d^c_1,d^c_2,d^c_3)	3^*	1	-2/3	(1,0,0)
H	1	2	1/2	0
ϕ	1	1	0	-1

(SU(5) invariant)

$\eta^E_{33} e^c_3 L_3 H^*$ q = 0 $\lambda^E_{33} = \eta^E_{33} = \mathcal{O}(1)$

$\eta^E_{32} e^c_3 L_2 H^*$ q = 0 $\lambda^E_{32} = \eta^E_{32} = \mathcal{O}(1)$

$\eta^E_{23} \frac{\phi^2}{M^2} e^c_2 L_3 H^*$ q = 0 $\lambda^E_{23} = \eta^E_{23} \frac{\langle\phi^2\rangle}{M^2} = \mathcal{O}(\epsilon^2)$

$\eta^E_{22} \frac{\phi^2}{M^2} e^c_2 L_2 H^*$ q = 0 $\lambda^E_{22} = \eta^E_{22} \frac{\langle\phi^2\rangle}{M^2} = \mathcal{O}(\epsilon^2)$

...

$$m^E_{ij} = v\lambda^E_{ij} = v \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix} + \mathcal{O}(1) \text{ factors}$$

($\epsilon \equiv \langle\phi\rangle/M$)

- Same for the neutrino mass

- In a see-saw context: depending on whether $M_{flavour} \gtrless M_R$, the game applies to the light neutrino mass matrix or separately to the right-handed neutrino Yukawa and mass matrices

- Must also account for quark masses and mixings

- Unification?

Summary

- Compelling understanding of the smallness of neutrino masses in terms of the high scale breaking of an accidentally conserved lepton number, compatible with GUTs, leptogenesis

- Interesting alternatives are available, some of them offer the opportunity to probe such an origin at the LHC

- Detailed analysis of flavour structure: long list of interesting ideas, but no unique compelling understanding

- Measurement of remaining parameters will shed more light, hopefully more handles on the flavour problem will come from complementary experiments (low E and LHC, ILC)