

# Neutrino Oscillation Parameters: Results and Prospects – II

Sandhya Choubey

Harish-Chandra Research Institute, Allahabad, India



III International Pontecorvo Neutrino Physics School  
September 16-26, 2007, Alushta, Ukraine

# Determining the Neutrino mass matrix

---



# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$

# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$
- Can be measured in KamLAND or KamLAND-like SPMIN experiments. Next-generation solar neutrino experiments can also measure  $\sin^2 \theta_{12}$ .

# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$
- Can be measured in KamLAND or KamLAND-like SPMIN experiments. Next-generation solar neutrino experiments can also measure  $\sin^2 \theta_{12}$ .
- $\Delta m_{31}^2$  and  $\sin^2 \theta_{23}$ : Channel needed is  $P_{\mu\mu}$

# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$
- Can be measured in KamLAND or KamLAND-like SPMIN experiments. Next-generation solar neutrino experiments can also measure  $\sin^2 \theta_{12}$ .
- $\Delta m_{31}^2$  and  $\sin^2 \theta_{23}$ : Channel needed is  $P_{\mu\mu}$
- Will be measured very well by  $\nu_\mu$  disappearance measurement in accelerator based experiments – MINOS, ICARUS, OPERA, T2K, NO $\nu$ A. It can also be measured very well in atmospheric neutrino experiments.

# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$
- Can be measured in KamLAND or KamLAND-like SPMIN experiments. Next-generation solar neutrino experiments can also measure  $\sin^2 \theta_{12}$ .
- $\Delta m_{31}^2$  and  $\sin^2 \theta_{23}$ : Channel needed is  $P_{\mu\mu}$
- Will be measured very well by  $\nu_\mu$  disappearance measurement in accelerator based experiments – MINOS, ICARUS, OPERA, T2K, NO $\nu$ A. It can also be measured very well in atmospheric neutrino experiments.
- Why disappearance experiments?

# Determining the Neutrino mass matrix

- $\Delta m_{21}^2$  and  $\sin^2 \theta_{12}$ : Channel needed is  $P_{ee}$
- Can be measured in KamLAND or KamLAND-like SPMIN experiments. Next-generation solar neutrino experiments can also measure  $\sin^2 \theta_{12}$ .
- $\Delta m_{31}^2$  and  $\sin^2 \theta_{23}$ : Channel needed is  $P_{\mu\mu}$
- Will be measured very well by  $\nu_\mu$  disappearance measurement in accelerator based experiments – MINOS, ICARUS, OPERA, T2K, NO $\nu$ A. It can also be measured very well in atmospheric neutrino experiments.
- Why disappearance experiments?
- Because statistics there are very large



# The Unanswered Questions

---



# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?



# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?
- Main channels to determine  $\theta_{13}$ 
  - $\nu_e \rightarrow \nu_e$  or  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ :  $P_{ee}$  or  $P_{\bar{e}\bar{e}}$ ; Disappearance Expts
  - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts

# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?
- Main channels to determine  $\theta_{13}$ 
  - $\nu_e \rightarrow \nu_e$  or  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ :  $P_{ee}$  or  $P_{\bar{e}\bar{e}}$ ; Disappearance Expts
  - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
- What is the sign of  $\Delta m_{31}^2$ ?

# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?
  - Main channels to determine  $\theta_{13}$ 
    - $\nu_e \rightarrow \nu_e$  or  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ :  $P_{ee}$  or  $P_{\bar{e}\bar{e}}$ ; Disappearance Expts
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
- What is the sign of  $\Delta m_{31}^2$ ?
  - Main channels to determine  $sign(\Delta m_{31}^2)$ 
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
    - “binned”  $\nu_\mu \rightarrow \nu_\mu$   $P_{\mu\mu}$ ; Disappearance Expts
    - $\nu_e \rightarrow \nu_e$   $P_{ee}$ ; Disappearance Expts

# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?
  - Main channels to determine  $\theta_{13}$ 
    - $\nu_e \rightarrow \nu_e$  or  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ :  $P_{ee}$  or  $P_{\bar{e}\bar{e}}$ ; Disappearance Expts
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
- What is the sign of  $\Delta m_{31}^2$ ?
  - Main channels to determine  $sign(\Delta m_{31}^2)$ 
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
    - “binned”  $\nu_\mu \rightarrow \nu_\mu$   $P_{\mu\mu}$ ; Disappearance Expts
    - $\nu_e \rightarrow \nu_e$   $P_{ee}$ ; Disappearance Expts
- Is there CP violation in the lepton sector?

# The Unanswered Questions ( $\nu$ Oscillations)

- What is the magnitude of  $\theta_{13}$ ?
  - Main channels to determine  $\theta_{13}$ 
    - $\nu_e \rightarrow \nu_e$  or  $\bar{\nu}_e \rightarrow \bar{\nu}_e$ :  $P_{ee}$  or  $P_{\bar{e}\bar{e}}$ ; Disappearance Expts
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
- What is the sign of  $\Delta m_{31}^2$ ?
  - Main channels to determine  $sign(\Delta m_{31}^2)$ 
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
    - “binned”  $\nu_\mu \rightarrow \nu_\mu$   $P_{\mu\mu}$ ; Disappearance Expts
    - $\nu_e \rightarrow \nu_e$   $P_{ee}$ ; Disappearance Expts
- Is there CP violation in the lepton sector?
  - Main channel to see  $\delta_{CP}$ 
    - $\nu_\mu \rightarrow \nu_e$  or  $\nu_e \rightarrow \nu_\mu$ :  $P_{\mu e}$  or  $P_{e\mu}$ ; Appearance Expts
    - Also possible using  $\nu_\mu \rightarrow \nu_\mu$   $P_{\mu\mu}$ ; Disapp Expts

# Measuring $\theta_{13}$ , $\delta_{CP}$ and $\text{sgn}(\Delta m_{31}^2)$

---





# Measuring $\theta_{13}$ with Reactors

---



# Measuring $\theta_{13}$ with Reactors

- $\sin^2 2\theta_{13}$  can be measured in **Reactor** experiments

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + O(\alpha^2)$$

# Measuring $\theta_{13}$ with Reactors

- $\sin^2 2\theta_{13}$  can be measured in **Reactor** experiments

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + O(\alpha^2)$$

- Free of “degeneracies” and matter effects

# Measuring $\theta_{13}$ with Reactors

- $\sin^2 2\theta_{13}$  can be measured in **Reactor** experiments

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + O(\alpha^2)$$

- Free of “degeneracies” and matter effects
- Systematic uncertainty has to be at sub-percent level

# Measuring $\theta_{13}$ with Reactors

- $\sin^2 2\theta_{13}$  can be measured in **Reactor** experiments

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + O(\alpha^2)$$

- Free of “degeneracies” and matter effects
- Systematic uncertainty has to be at sub-percent level
- Proposals include:  
Double Chooz, Daya Bay, Reno, Angra,.....

# Measuring $\theta_{13}$ with Reactors

- $\sin^2 2\theta_{13}$  can be measured in **Reactor** experiments

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \approx 1 - \sin^2 2\theta_{13} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right) + O(\alpha^2)$$

- Free of “degeneracies” and matter effects
- Systematic uncertainty has to be at sub-percent level
- Proposals include:  
**Double Chooz, Daya Bay, Reno, Angra,....**
- Sensitivity (90% C.L.):

$$\sin^2 2\theta_{13} < 0.032 \quad \text{Double Chooz}$$

$$\sin^2 2\theta_{13} < 0.009 \quad \text{Reactor II}$$

# The $\nu_e \rightarrow \nu_\mu$ “Golden” Channel

$$P_{app}^{vac} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta$$
$$\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta$$
$$+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta$$
$$+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta)$$

where  $\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}$ ,  $\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$  and we have expanded the probability in  $\alpha$  and  $\theta_{13}$  keeping only lower order terms

# The $\nu_e \rightarrow \nu_\mu$ “Golden” Channel

$$P_{app}^{vac} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta$$
$$\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta$$
$$+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta$$
$$+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta)$$

where  $\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}$ ,  $\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$  and we have expanded the probability in  $\alpha$  and  $\theta_{13}$  keeping only lower order terms

- First term gives  $\theta_{13}$



# The $\nu_e \rightarrow \nu_\mu$ “Golden” Channel

$$\begin{aligned}
 P_{app}^{vac} &\simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\
 &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\
 &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\
 &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta)
 \end{aligned}$$

where  $\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}$ ,  $\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$  and we have expanded the probability in  $\alpha$  and  $\theta_{13}$  keeping only lower order terms

- First term gives  $\theta_{13}$
- Second term gives CP violating part of the prob

# The $\nu_e \rightarrow \nu_\mu$ “Golden” Channel

$$\begin{aligned}
 P_{app}^{vac} &\simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\
 &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\
 &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\
 &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta)
 \end{aligned}$$

where  $\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}$ ,  $\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$  and we have expanded the probability in  $\alpha$  and  $\theta_{13}$  keeping only lower order terms

- First term gives  $\theta_{13}$
- Second term gives CP violating part of the prob
- Third term gives CP conserving part of the prob

# Bi-Probability Plots

---



# Probability Plots

$$\begin{aligned} P_{app}^{vac} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

# Probability Plots

$$\begin{aligned} P_{app}^{vac} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

$$P_\nu = A \cos \delta_{CP} + B \sin \delta_{CP} + C$$

$$P_{\bar{\nu}} = A \cos \delta_{CP} - B \sin \delta_{CP} + C$$

# Probability Plots

$$P_{app}^{vac} = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta$$
$$\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta$$
$$+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta$$
$$+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta)$$

$$P_\nu = A \cos \delta_{CP} + B \sin \delta_{CP} + C$$

$$P_{\bar{\nu}} = A \cos \delta_{CP} - B \sin \delta_{CP} + C$$

$$P_\nu + P_{\bar{\nu}} - 2C = 2A \cos \delta_{CP}$$

$$P_\nu - P_{\bar{\nu}} = 2B \sin \delta_{CP}$$

# Probability Plots

$$P_{app}^{vac} = \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta$$
$$\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta$$
$$+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta$$
$$+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta)$$

$$P_\nu = A \cos \delta_{CP} + B \sin \delta_{CP} + C$$

$$P_{\bar{\nu}} = A \cos \delta_{CP} - B \sin \delta_{CP} + C$$

$$P_\nu + P_{\bar{\nu}} - 2C = 2A \cos \delta_{CP}$$

$$P_\nu - P_{\bar{\nu}} = 2B \sin \delta_{CP}$$

$$\left( \frac{P_\nu + P_{\bar{\nu}} - 2C}{2A} \right)^2 + \left( \frac{P_\nu - P_{\bar{\nu}}}{2B} \right)^2 = 1$$

# Bi-Probability Plots

$$\begin{aligned}
 P_{app}^{vac} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\
 &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\
 &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\
 &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta)
 \end{aligned}$$

$$P_\nu = A \cos \delta_{CP} + B \sin \delta_{CP} + C$$

$$P_{\bar{\nu}} = A \cos \delta_{CP} - B \sin \delta_{CP} + C$$

$$P_\nu + P_{\bar{\nu}} - 2C = 2A \cos \delta_{CP}$$

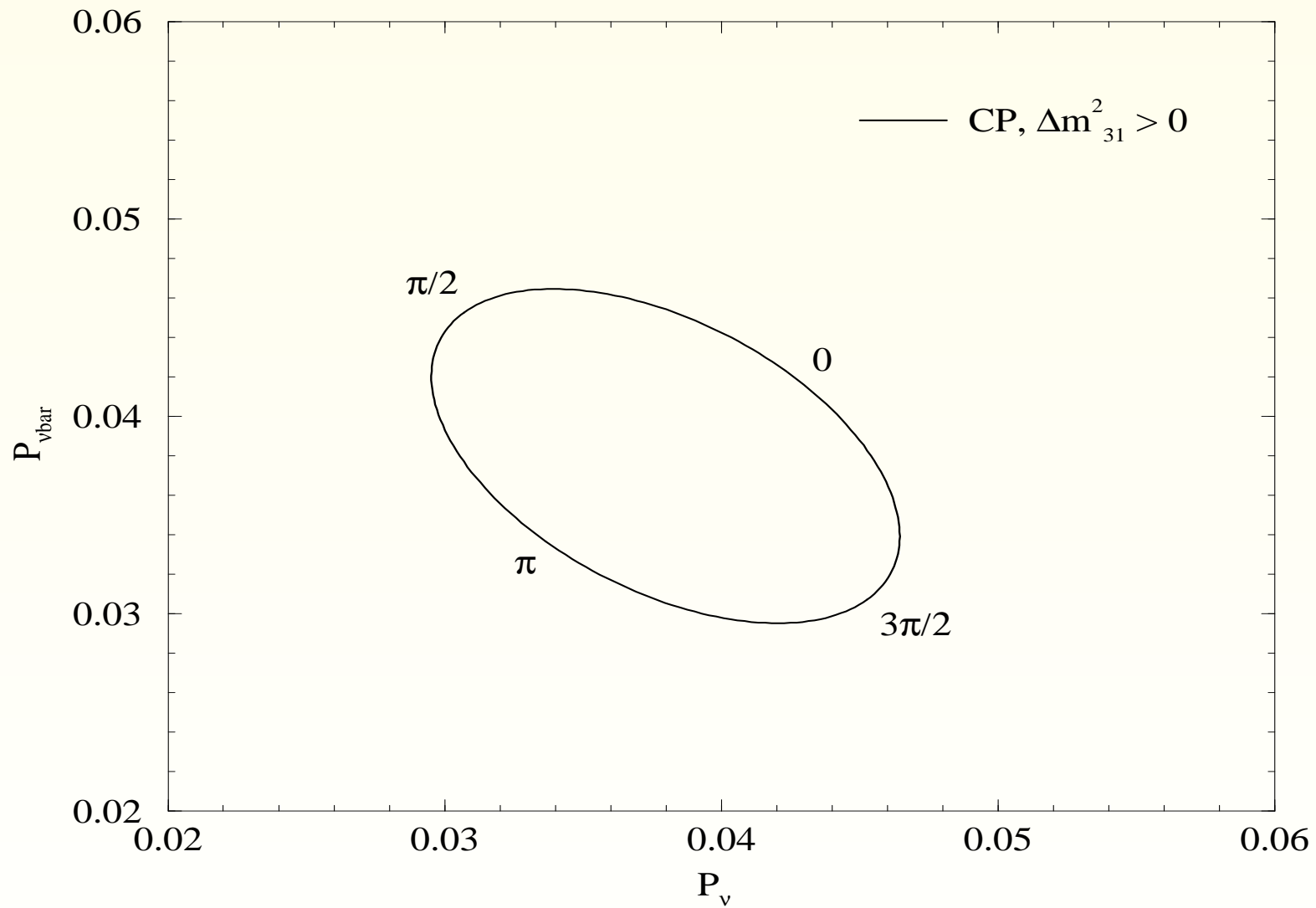
$$P_\nu - P_{\bar{\nu}} = 2B \sin \delta_{CP}$$

$$\left( \frac{P_\nu + P_{\bar{\nu}} - 2C}{2A} \right)^2 + \left( \frac{P_\nu - P_{\bar{\nu}}}{2B} \right)^2 = 1$$

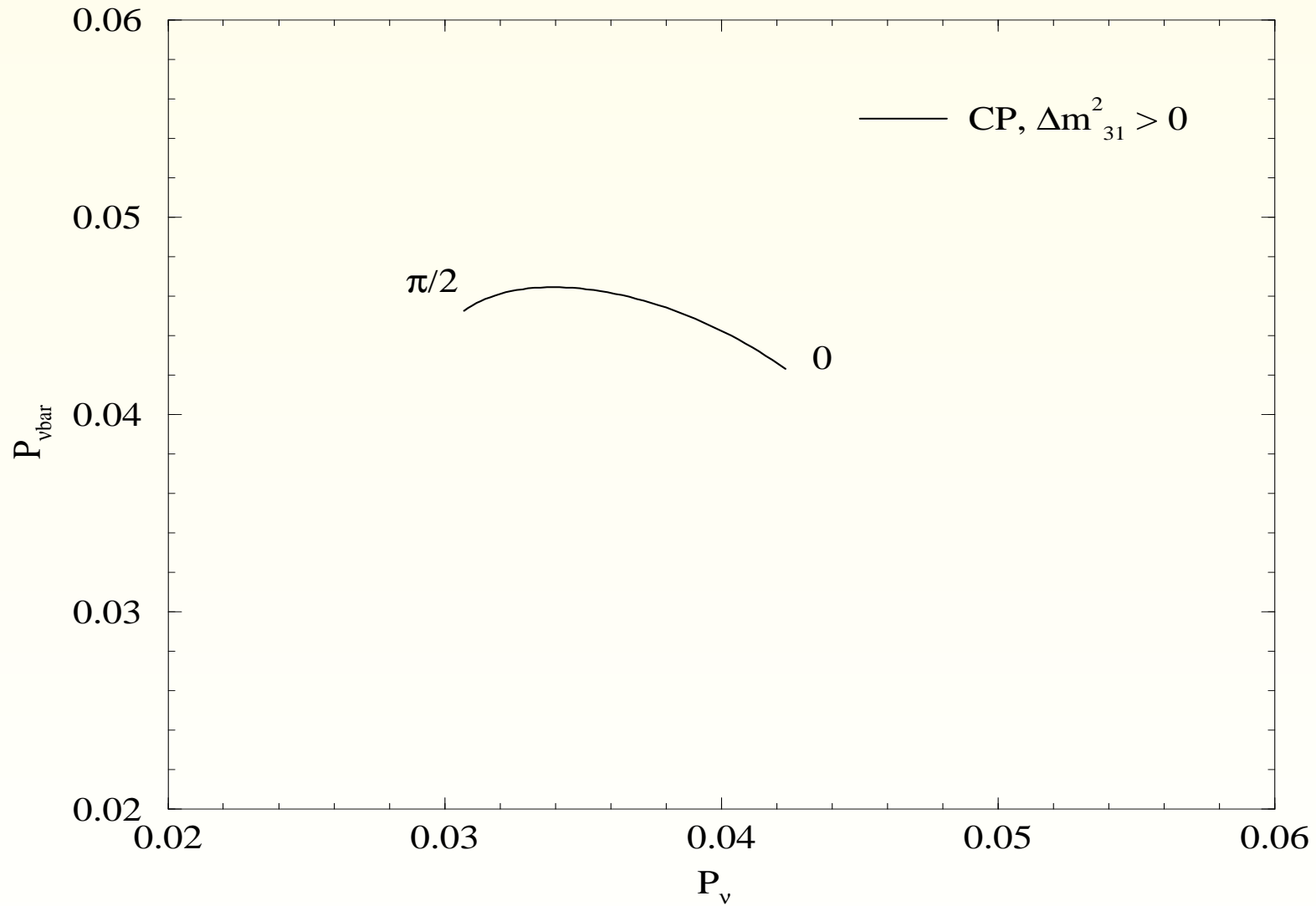
This is the eqn of an ellipse in the  $P_\nu - P_{\bar{\nu}}$  plane. These are called “**bi-probability plots**”. Major (minor) axes measure the amplitude of  $\sin \delta_{CP}$  ( $\cos \delta_{CP}$ ) term



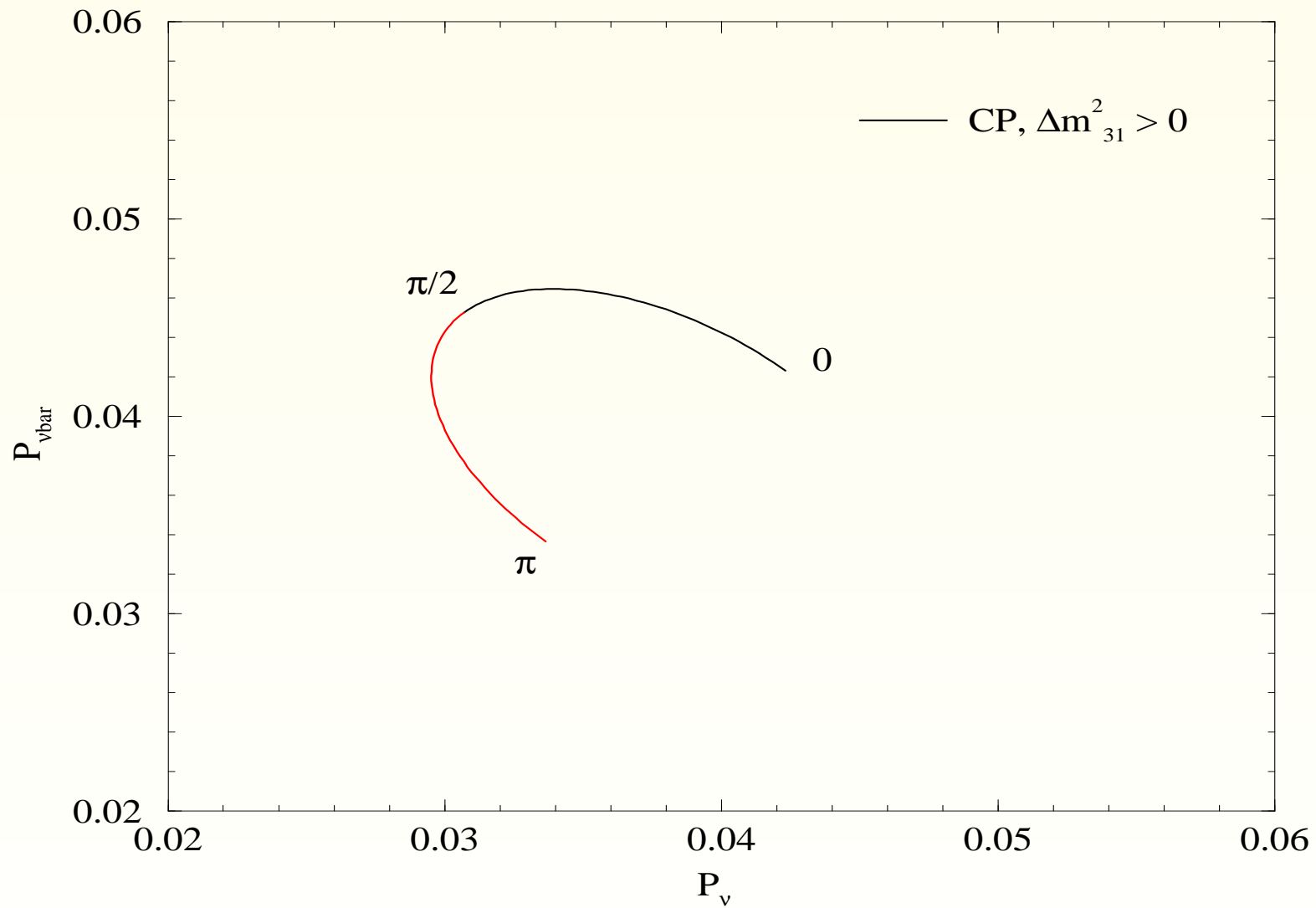
# Bi-Probability Plots



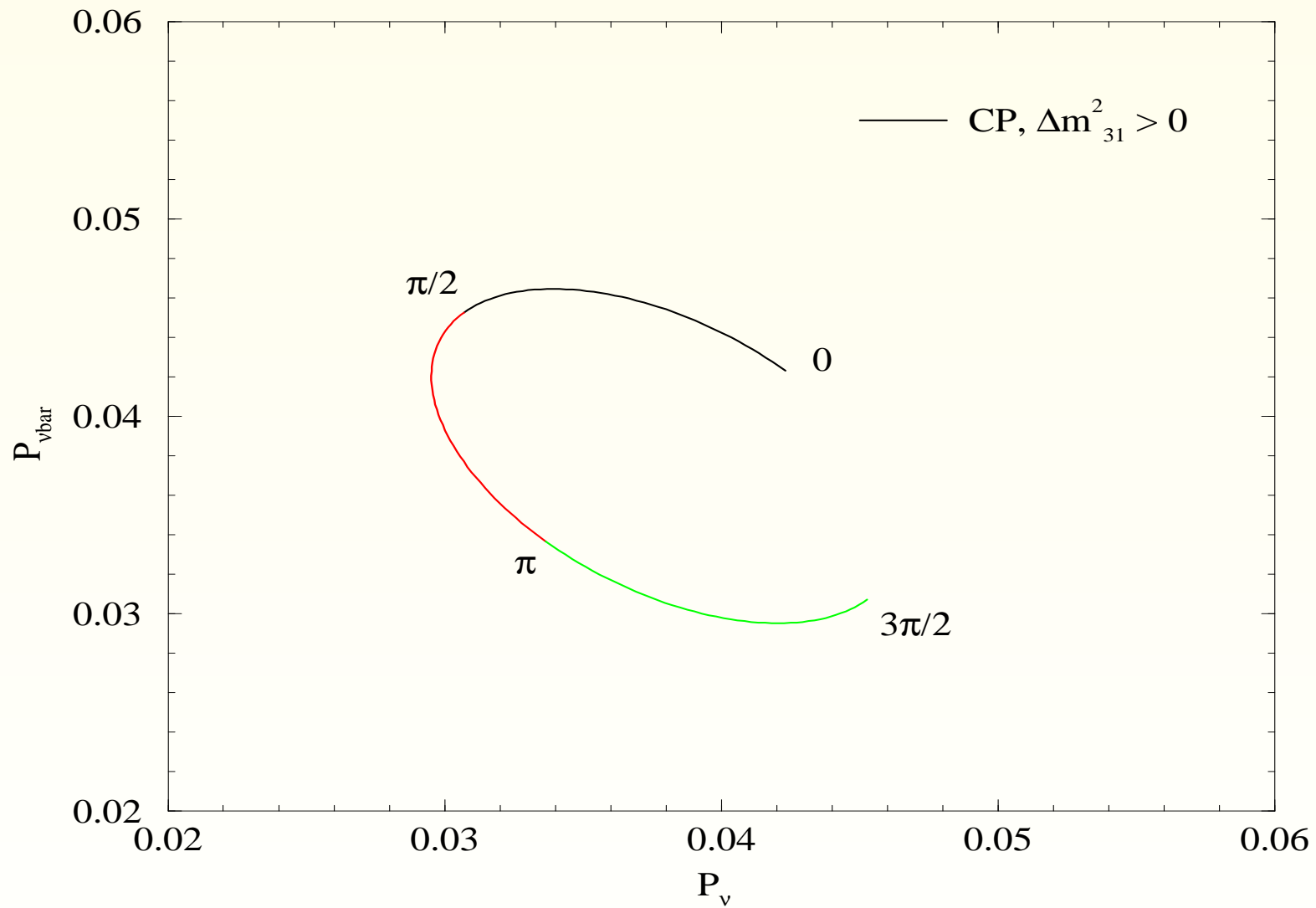
# Bi-Probability Plots



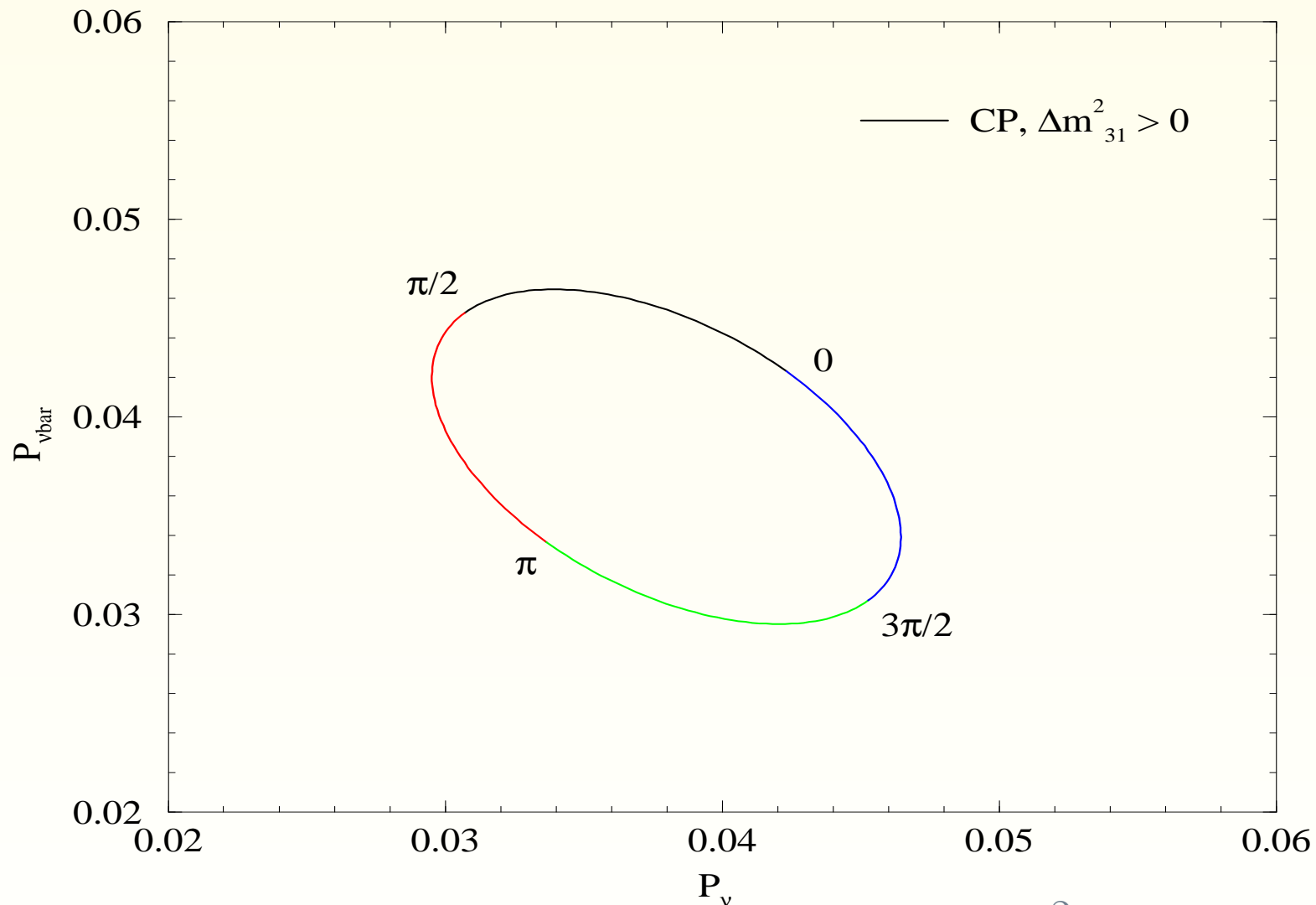
# Bi-Probability Plots



# Bi-Probability Plots

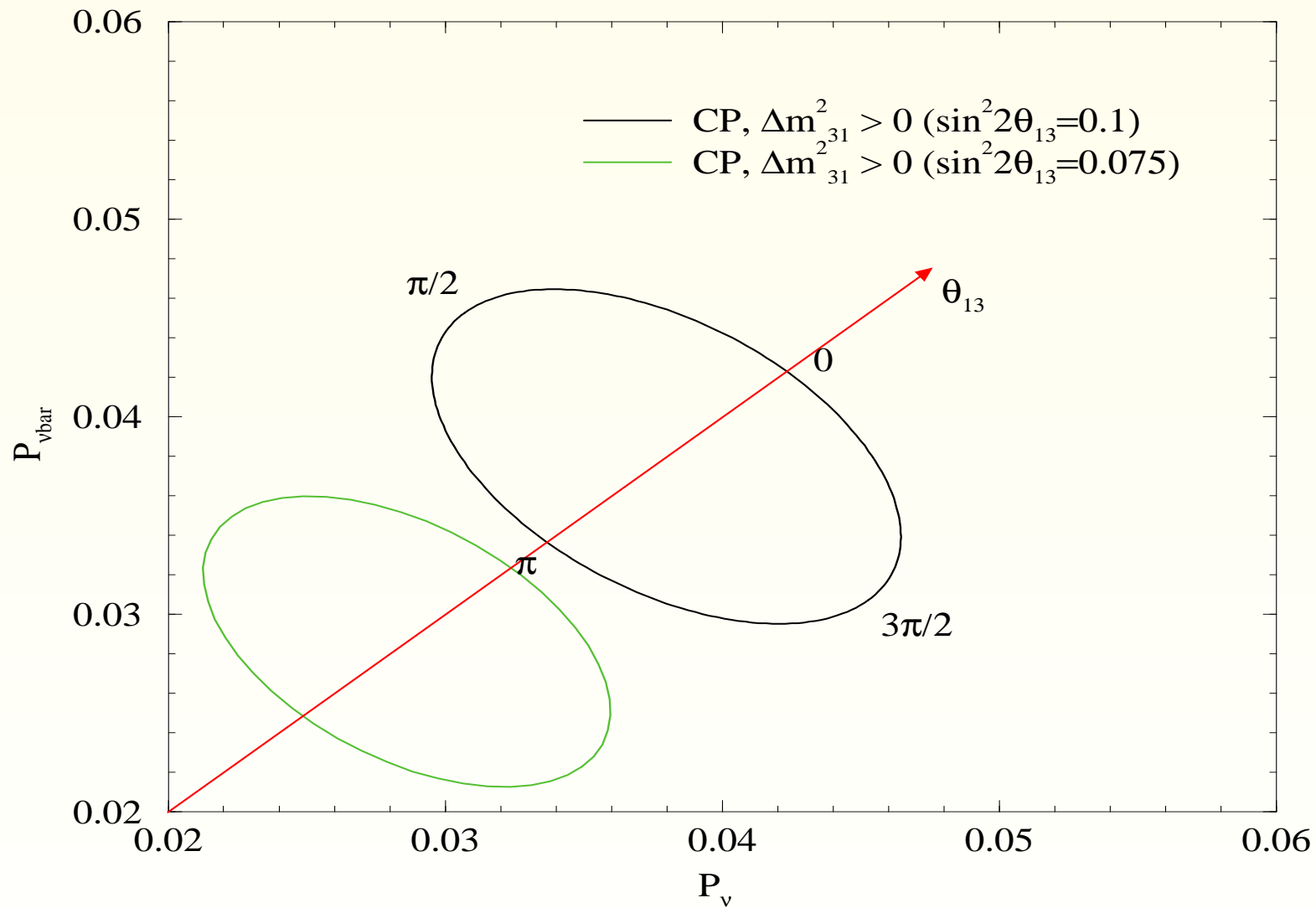


# Bi-Probability Plots



- The ellipse moves anticlockwise for  $\Delta m^2_{31} > 0$ .
- You can check that it goes clockwise for  $\Delta m^2_{31} < 0$ .

# Bi-Probability Plots



● Smaller  $\theta_{13} \Rightarrow \delta_{CP}$  measurement more difficult .

# Degeneracies

---



# $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta) \end{aligned}$$



# $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta) \end{aligned}$$

- Do the simultaneous transformation

$$\begin{aligned} \delta_{CP} &\rightarrow \pi - \delta_{CP} \\ \Delta m_{31}^2 &\rightarrow -\Delta m_{31}^2 \end{aligned}$$

# $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta) \end{aligned}$$

- Do the simultaneous transformation

$$\begin{aligned} \delta_{CP} &\rightarrow \pi - \delta_{CP} \\ \Delta m_{31}^2 &\rightarrow -\Delta m_{31}^2 \end{aligned}$$

- The expression for  $P_{\text{appearance}}$  remains invariant

# $Sgn(\Delta m_{31}^2)$ Degeneracy



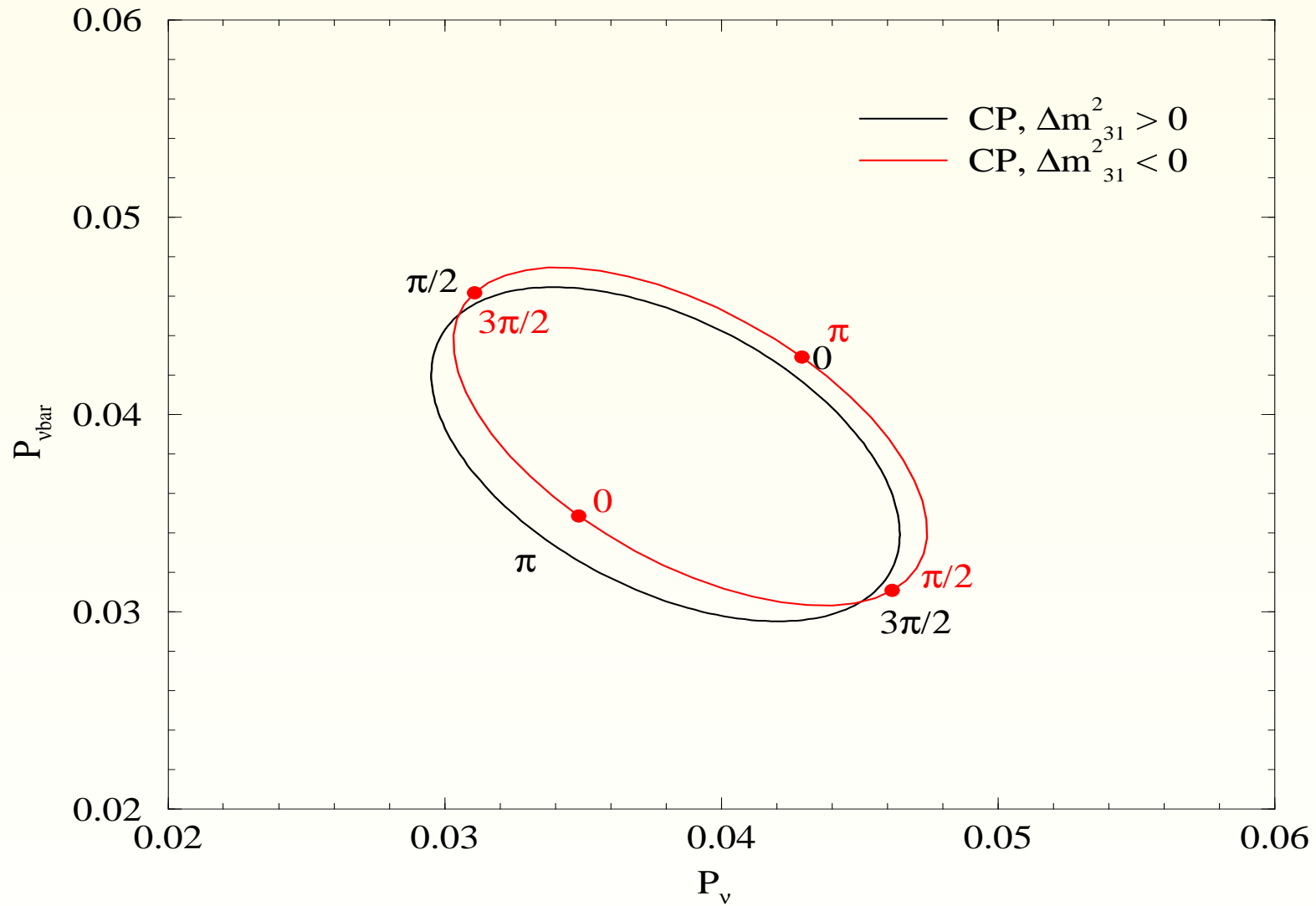
$$\begin{aligned} P_{appearance} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha \Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha \Delta) \end{aligned}$$

- Do the simultaneous transformation

$$\begin{aligned} \delta_{CP} &\rightarrow \pi - \delta_{CP} \\ \Delta m_{31}^2 &\rightarrow -\Delta m_{31}^2 \end{aligned}$$

- The expression for  $P_{appearance}$  remains invariant
- Since  $Sgn(\Delta m_{31}^2)$  is unknown, there will always be an ambiguity in the measured value of  $\delta_{CP}$

# $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy



# Octant of $\theta_{23}$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

# Octant of $\theta_{23}$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

- We only have measurement on  $\sin^2 2\theta_{23}$ .

# Octant of $\theta_{23}$ Degeneracy



$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

- We only have measurement on  $\sin^2 2\theta_{23}$ .
- For every non-maximal  $\sin^2 2\theta_{23}$ , there are 2 possible  $\sin^2 \theta_{23}$

# Octant of $\theta_{23}$ Degeneracy

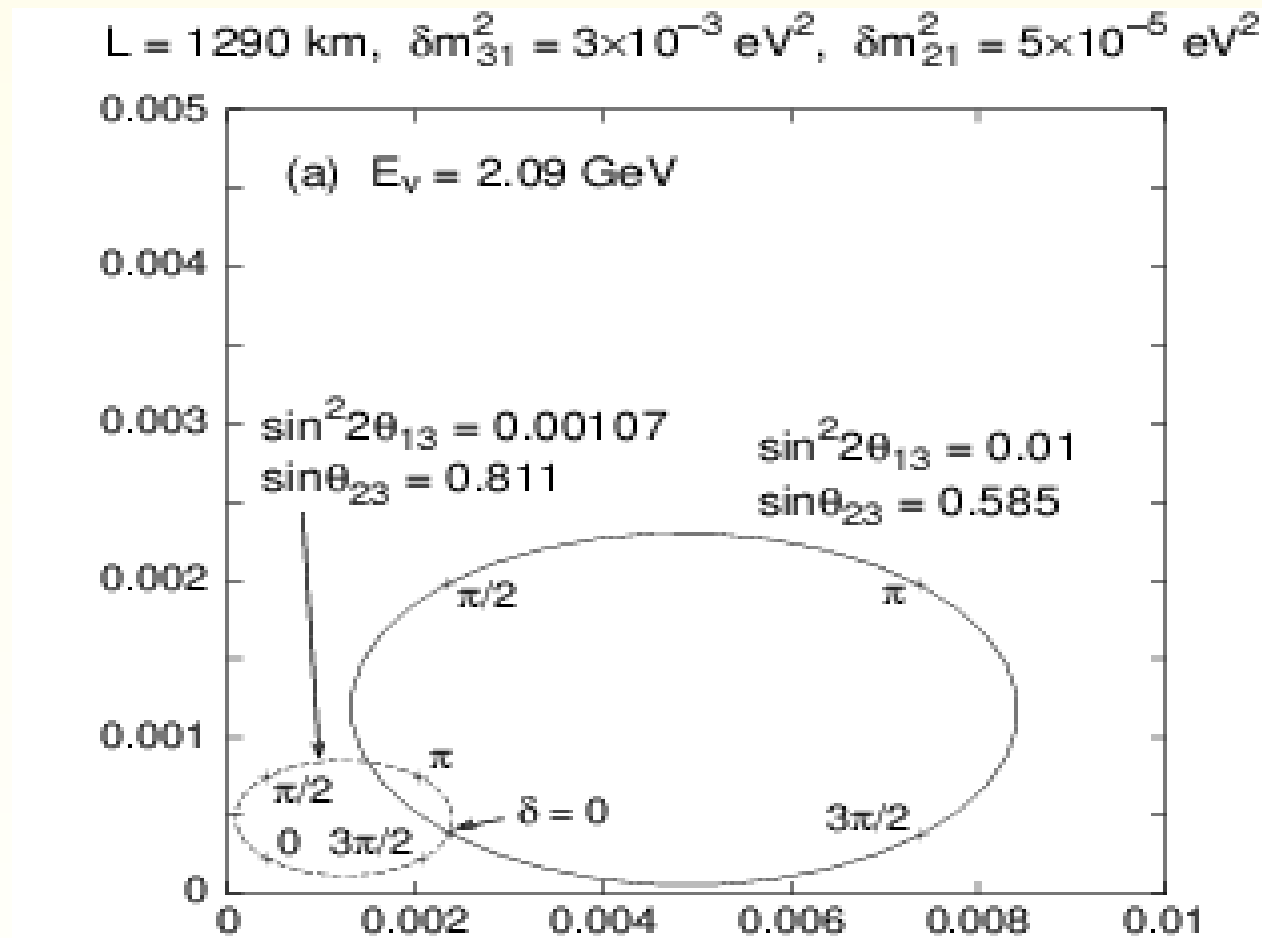


$$\begin{aligned} P_{\text{appearance}} &= \sin^2 \theta_{23} \sin^2 2\theta_{13} \sin^2 \Delta \\ &\pm (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin \Delta \sin \Delta \\ &+ (\alpha\Delta) \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \sin \Delta \\ &+ \cos^2 \theta_{23} \sin^2 2\theta_{12} \sin^2(\alpha\Delta) \end{aligned}$$

- We only have measurement on  $\sin^2 2\theta_{23}$ .
- For every non-maximal  $\sin^2 2\theta_{23}$ , there are 2 possible  $\sin^2 \theta_{23}$
- This will give 2 disjoint fitted value for  $\theta_{13}$ .

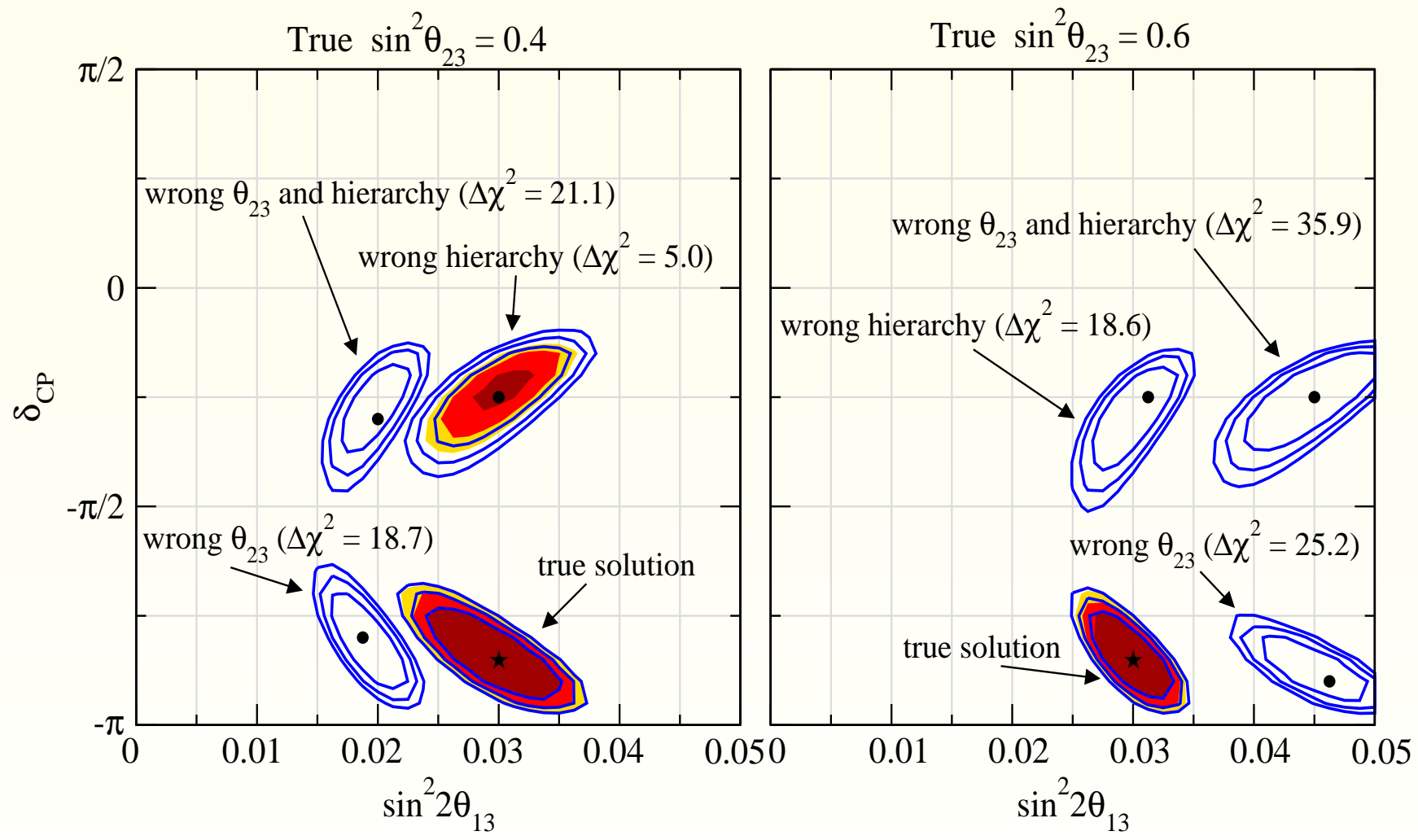


# Octant of $\theta_{23}$ Degeneracy



Barger et al, hep-ph/0112119

- If  $\sin^2 2\theta_{23} = 0.9$  then both  $\sin^2 2\theta_{13} = 0.01$  and  $0.0011$  would fit the data from a given LBL expt.



Huber et al, hep-ph/0501037

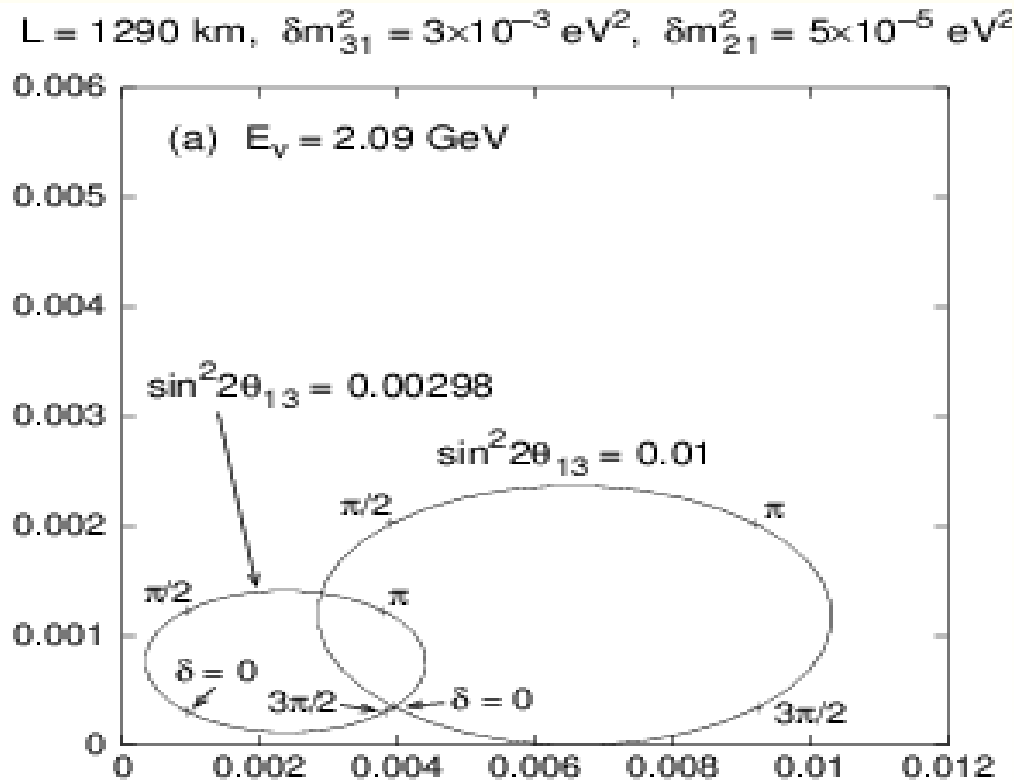
# Intrinsic $(\delta_{CP}, \theta_{13})$ Degeneracy

- For a fixed value of  $E$ , it might happen that

$$P_{app}(\theta_{13}, \delta_{CP}) = P_{app}(\theta'_{13}, \delta'_{CP})$$

$$\bar{P}_{app}(\theta_{13}, \delta_{CP}) = \bar{P}_{app}(\theta'_{13}, \delta'_{CP})$$

- $(\theta_{13}, \delta_{CP})$  (true solution) &  $(\theta'_{13}, \delta'_{CP})$  (fake solution)





# Up to Eight-Fold Degeneracy Expected

# The $\nu_e \rightarrow \nu_\mu$ Channel in Matter

---



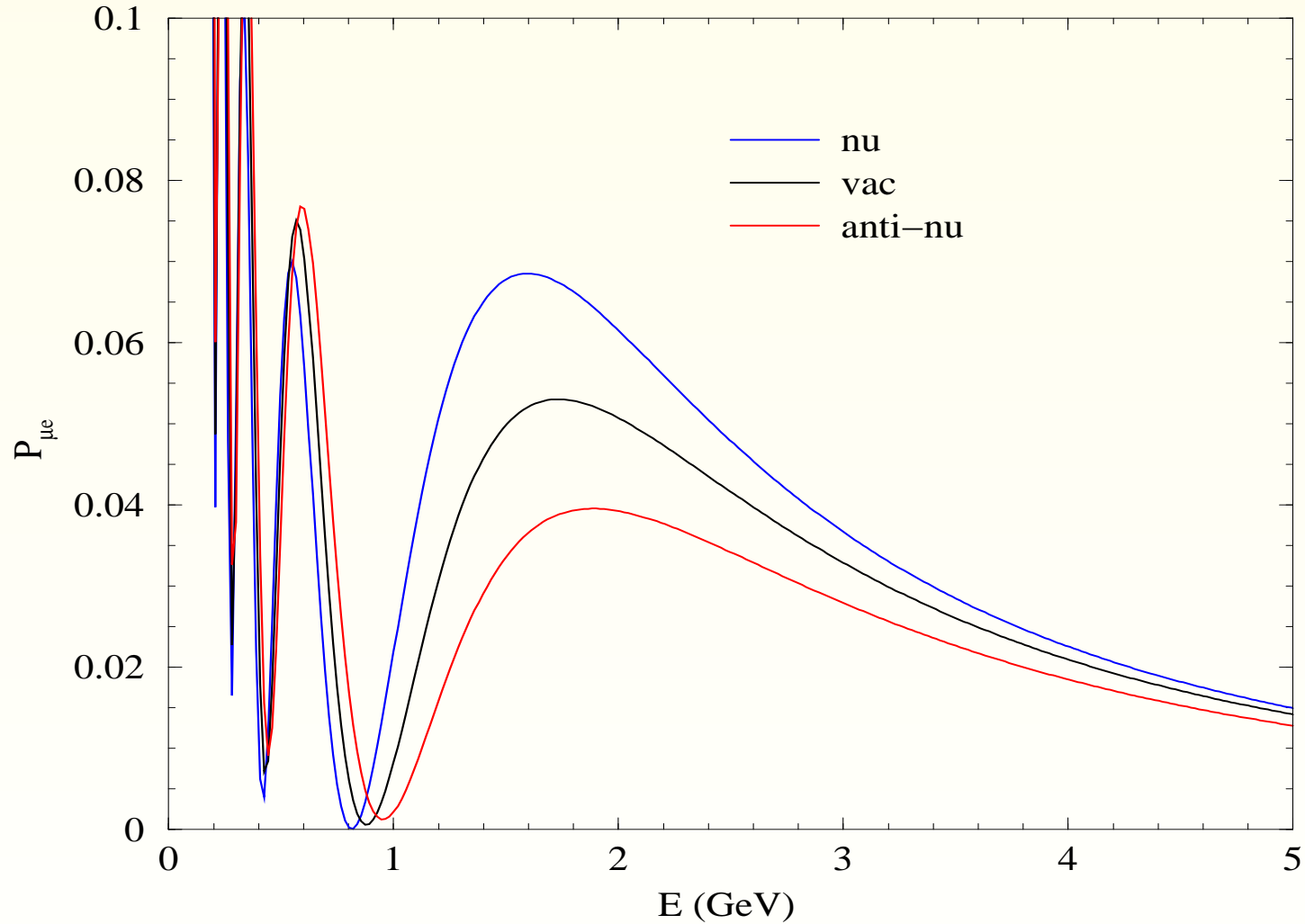
# The $\nu_e \rightarrow \nu_\mu$ Channel in Matter

$$\begin{aligned}
 P_{app} \simeq & \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \\
 & \pm \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \\
 & + \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \\
 & + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
 \end{aligned}$$

- $\hat{A} = \frac{\pm A}{\Delta m_{31}^2}$
- $A$  is positive for neutrinos
- $A$  is negative for antineutrinos

# The $\nu_e \rightarrow \nu_\mu$ Channel in Matter

$L=1000\text{km}$ ,  $\Delta m_{31}^2=0.002$ ,  $\Delta m_{21}^2=8 \times 10^{-5}$ ,  $s_{23}^2=0.5$ ,  $s_{12}^2=0.31$ ,  $\sin^2 \theta_{13}=0.1$ ,  $\delta=0$



# Resolving the Eight-Fold Degeneracy

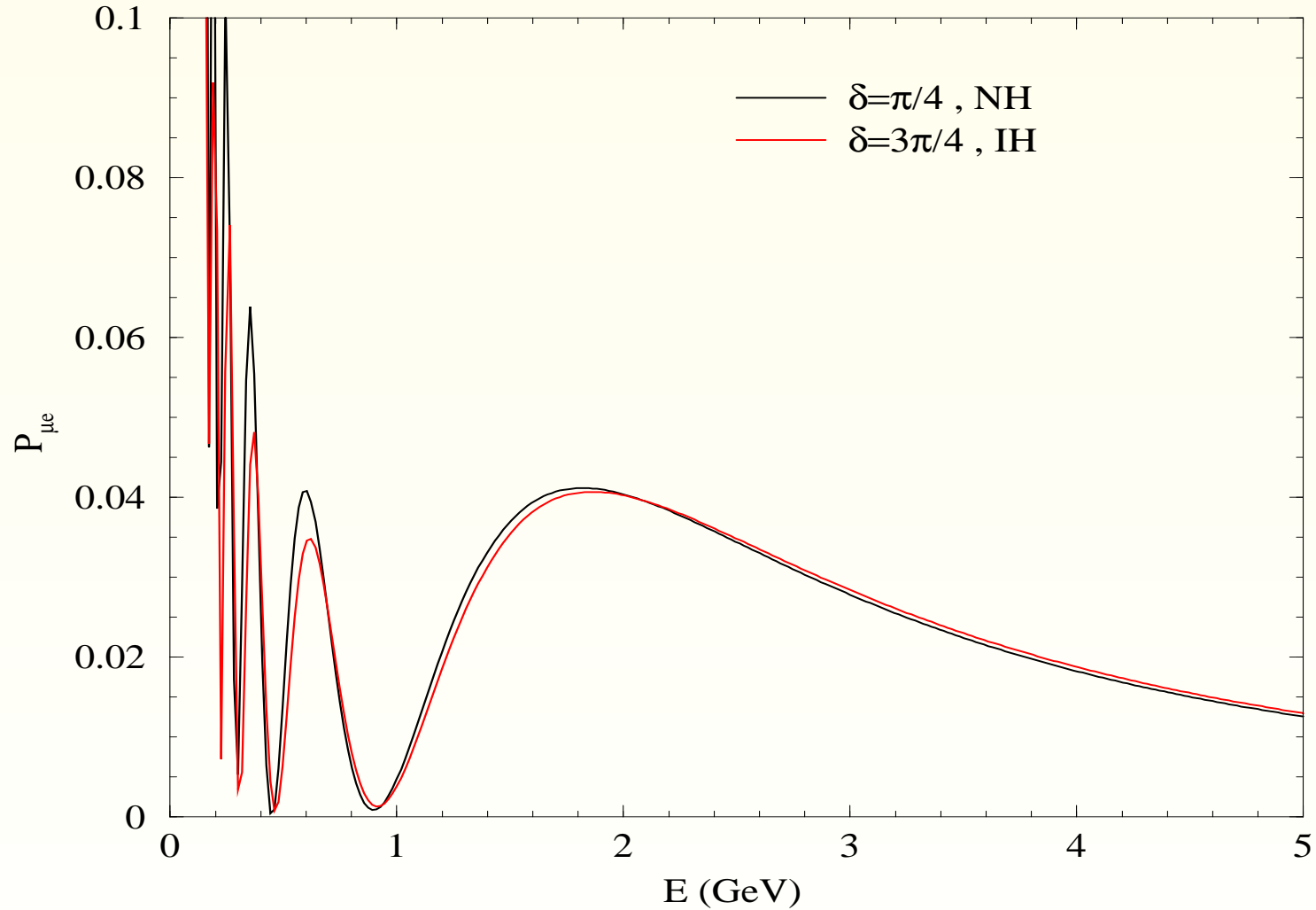
---





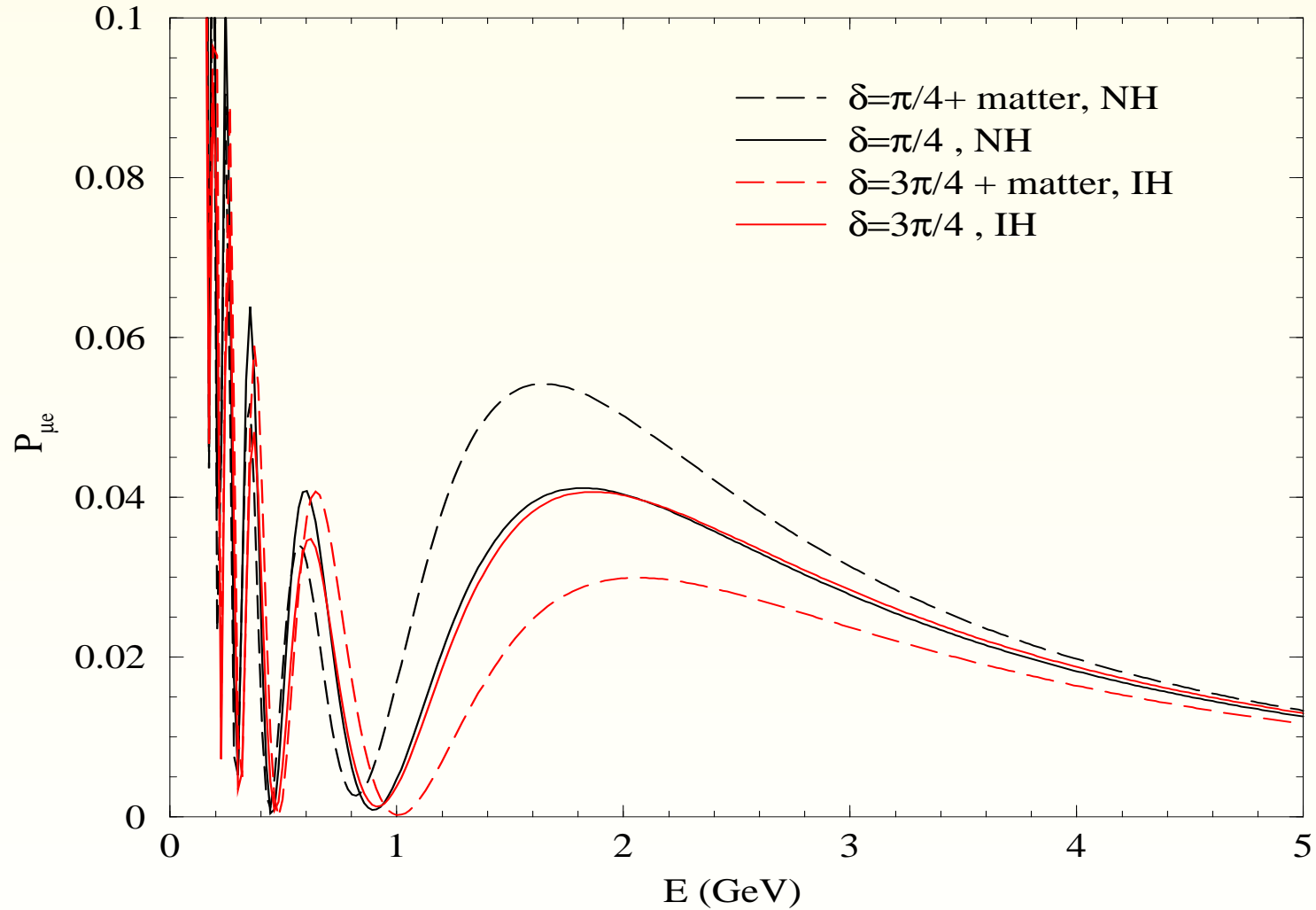
# Resolving the $Sgn(\Delta m_{31}^2)$ Degeneracy

$L=1000\text{km}$ ,  $|\Delta m_{31}^2|=0.002$ ,  $\Delta m_{21}^2=8\times 10^{-5}$ ,  $s_{23}^2=0.5$ ,  $s_{12}^2=0.31$ ,  $\sin^2\theta_{13}=0.1$



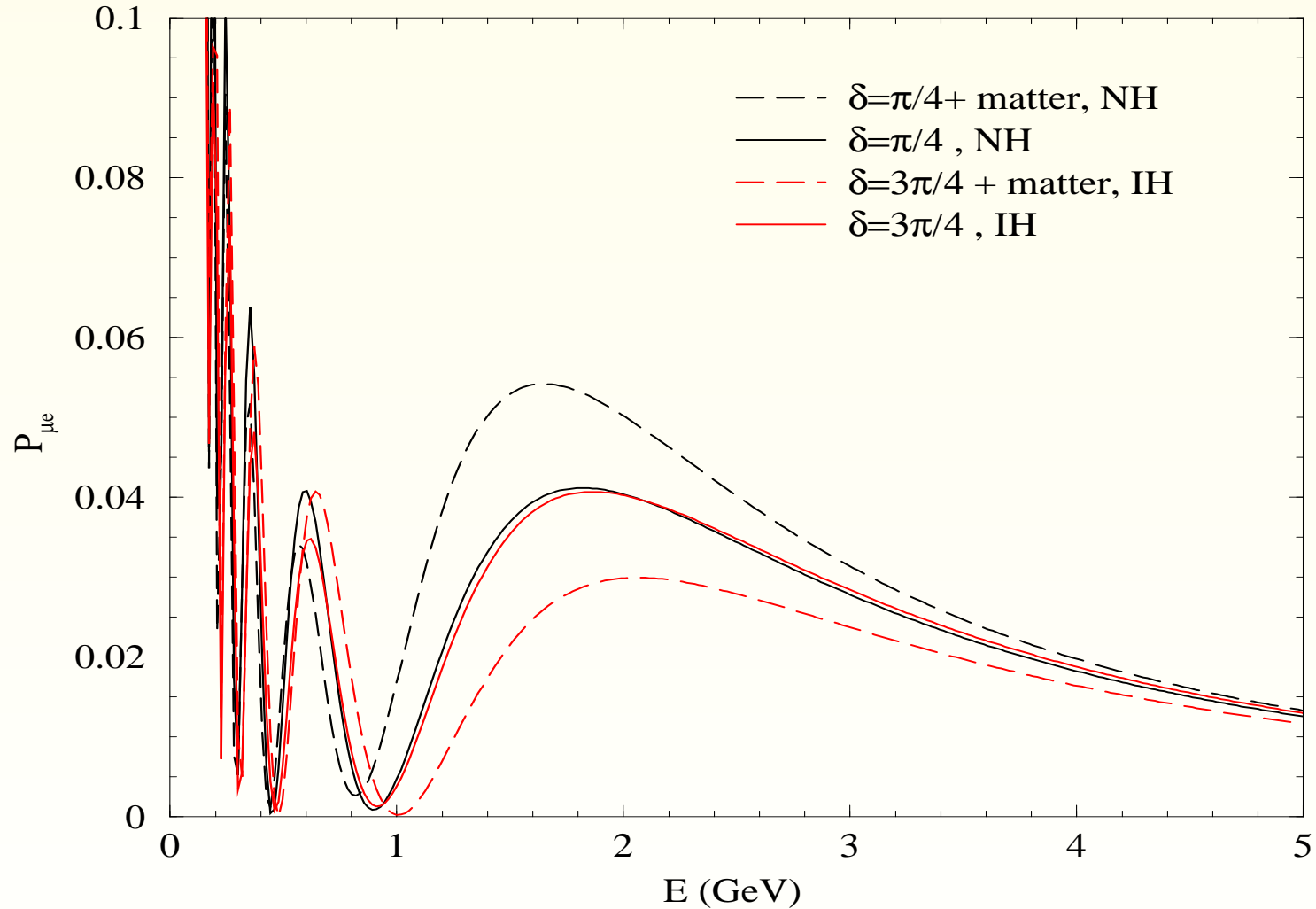
# Resolving the $Sgn(\Delta m_{31}^2)$ Degeneracy

$L=1000\text{km}$ ,  $|\Delta m_{31}^2|=0.002$ ,  $\Delta m_{21}^2=8\times 10^{-5}$ ,  $s_{23}^2=0.5$ ,  $s_{12}^2=0.31$ ,  $\sin^2\theta_{13}=0.1$



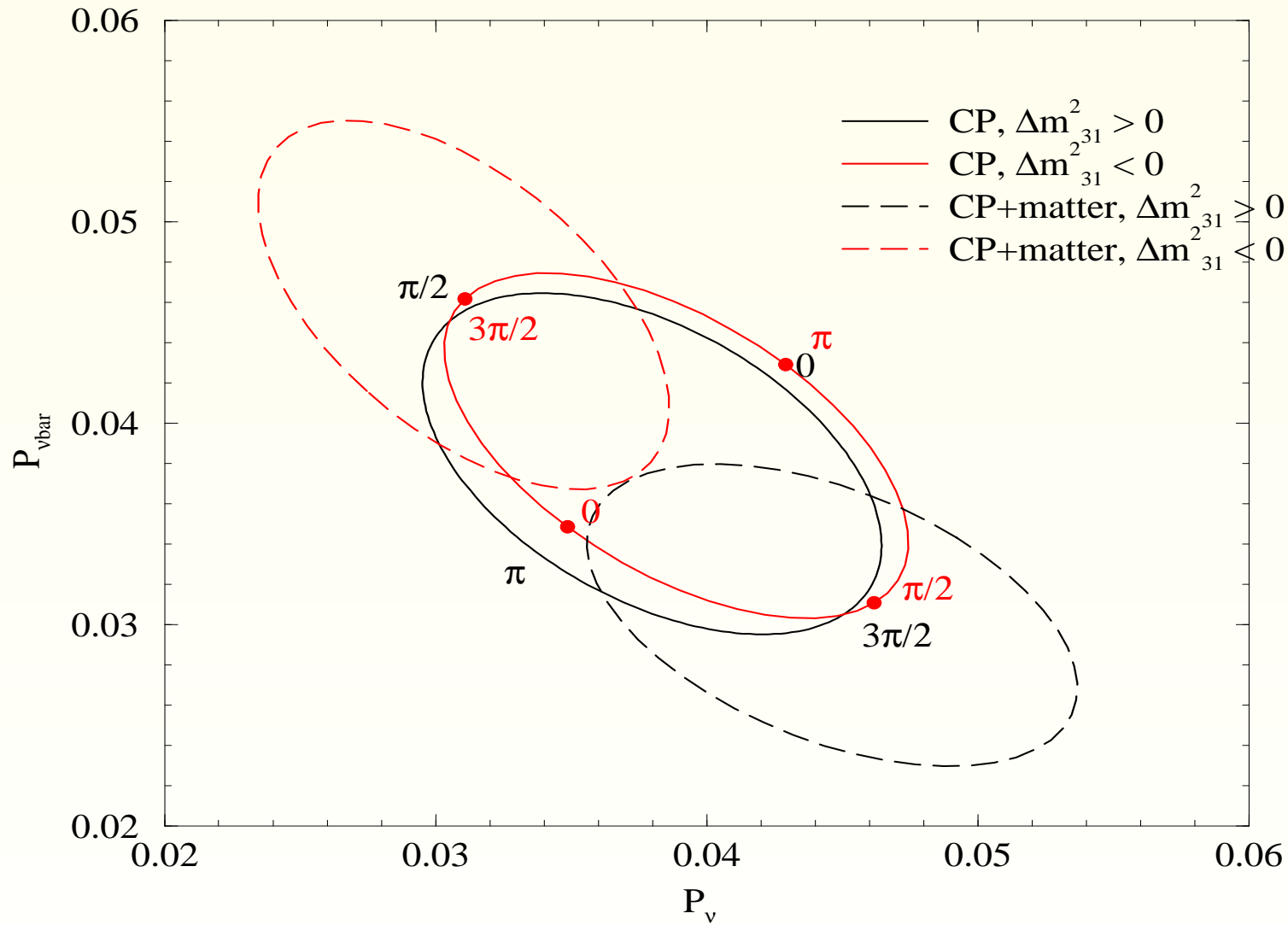
# Resolving the $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy

$L=1000\text{km}, |\Delta m_{31}^2|=0.002, \Delta m_{21}^2=8 \times 10^{-5}, s_{23}^2=0.5, s_{12}^2=0.31, \sin^2 \theta_{13}=0.1$



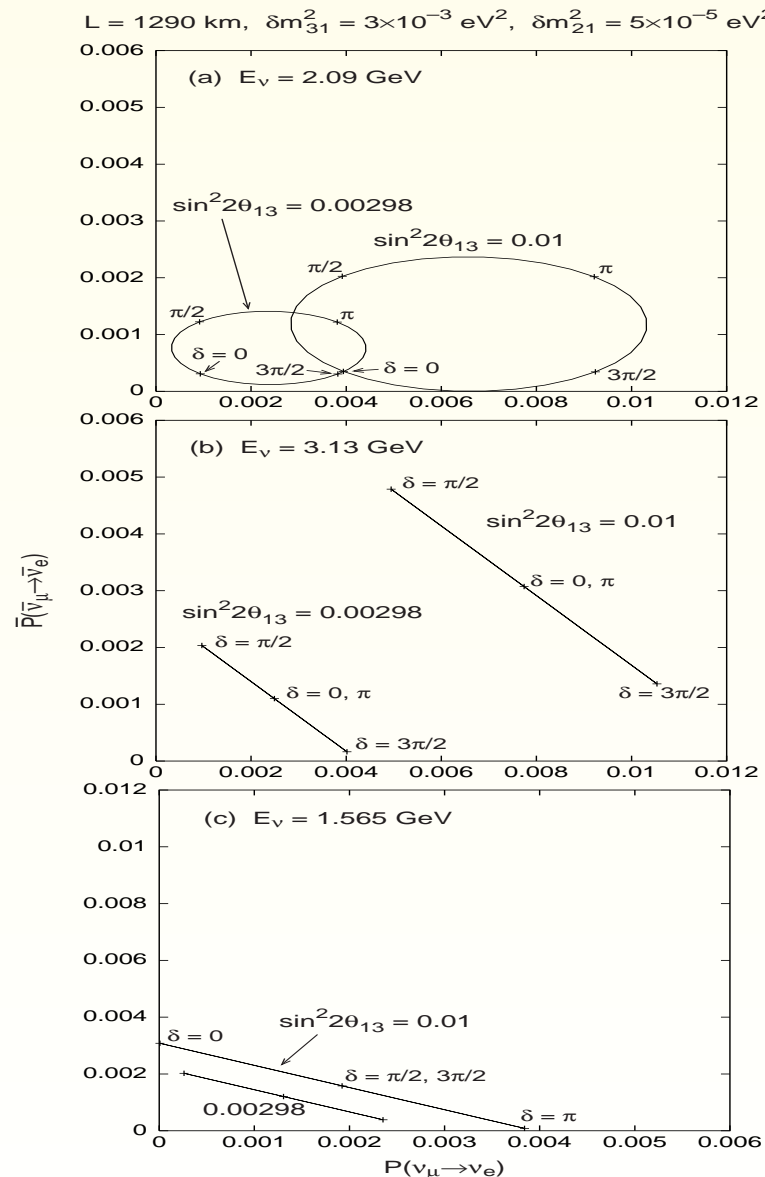
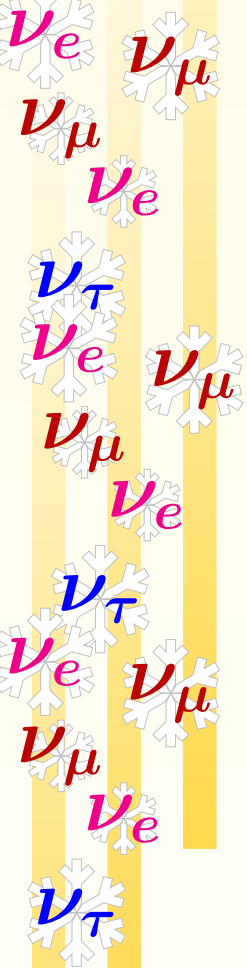
● Matter effects break the  $\text{Sgn}(\Delta m_{31}^2)$  Degeneracy

# Resolving the $\text{Sgn}(\Delta m_{31}^2)$ Degeneracy



● Matter effects break the  $\text{Sgn}(\Delta m_{31}^2)$  Degeneracy

# Resolving the Intrinsic $(\delta_{CP}, \theta_{13})$ Degeneracy



- Choose  $\Delta = m\pi/2$ .
- The  $\cos \delta_{CP}$  term vanishes for  $\Delta = (n - \frac{1}{2})\pi$ .
- The  $\sin \delta_{CP}$  term vanishes for  $\Delta = n\pi$ .
- Ellipse collapse to a line
- Ambiguity resolved.
- Better to work with  $\Delta = (n - \frac{1}{2})\pi$ .
- That is where we will directly see CPV.
- That's where we get the oscillation maxima.

Barger et al, hep-ph/0112119

# Resolving the Octant of $\theta_{23}$ Degeneracy

---



# Resolving the Octant of $\theta_{23}$ Degeneracy

- Using atmospheric neutrino data in
  - ✓ Megaton water detectors:
    - ★  $\Delta m_{21}^2$  driven osc effects in sub-GeV electrons
    - ★  $\theta_{13}$  driven matter effects in multi-GeV electrons
  - ✓ Large magnetized iron detectors:
    - ★  $\theta_{13}$  driven matter effects in multi-GeV muons

# Resolving the Octant of $\theta_{23}$ Degeneracy

- Using atmospheric neutrino data in
  - ✓ Megaton water detectors:
    - ★  $\Delta m_{21}^2$  driven osc effects in sub-GeV electrons
    - ★  $\theta_{13}$  driven matter effects in multi-GeV electrons
  - ✓ Large magnetized iron detectors:
    - ★  $\theta_{13}$  driven matter effects in multi-GeV muons
- Using data from next generation reactor neutrinos



# Resolving the Octant of $\theta_{23}$ Degeneracy

- Using atmospheric neutrino data in
  - ✓ Megaton water detectors:
    - ★  $\Delta m_{21}^2$  driven osc effects in sub-GeV electrons
    - ★  $\theta_{13}$  driven matter effects in multi-GeV electrons
  - ✓ Large magnetized iron detectors:
    - ★  $\theta_{13}$  driven matter effects in multi-GeV muons
- Using data from next generation reactor neutrinos
- Other ways using LBL expts have been suggested.

# Correlations

---

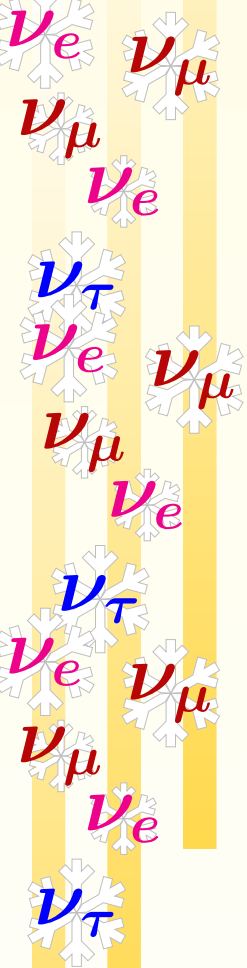


# Correlations

- Suppose you measure a quantity  $z$  experimentally.

# Correlations

- Suppose you measure a quantity  $z$  experimentally.
- And suppose  $z$  is parametrized in terms of 2 parameters  $x$  and  $y$  such that  $z = x + y$ .



# Correlations

- Suppose you measure a quantity  $z$  experimentally.
- And suppose  $z$  is parametrized in terms of 2 parameters  $x$  and  $y$  such that  $z = x + y$ .
- This means that measurement of  $z$  only gives information on a particular combination of  $x$  and  $y$  and not  $x$  and  $y$  themselves.

# Correlations

- Suppose you measure a quantity  $z$  experimentally.
- And suppose  $z$  is parametrized in terms of 2 parameters  $x$  and  $y$  such that  $z = x + y$ .
- This means that measurement of  $z$  only gives information on a particular combination of  $x$  and  $y$  and not  $x$  and  $y$  themselves.
- We say that the 2 params  $x$  and  $y$  are “correlated”.

# Correlations

- Suppose you measure a quantity  $z$  experimentally.
- And suppose  $z$  is parametrized in terms of 2 parameters  $x$  and  $y$  such that  $z = x + y$ .
- This means that measurement of  $z$  only gives information on a particular combination of  $x$  and  $y$  and not  $x$  and  $y$  themselves.
- We say that the 2 params  $x$  and  $y$  are “correlated”.
- Oscillation probabilities come in such forms:

$$P_{ee} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

# Correlations

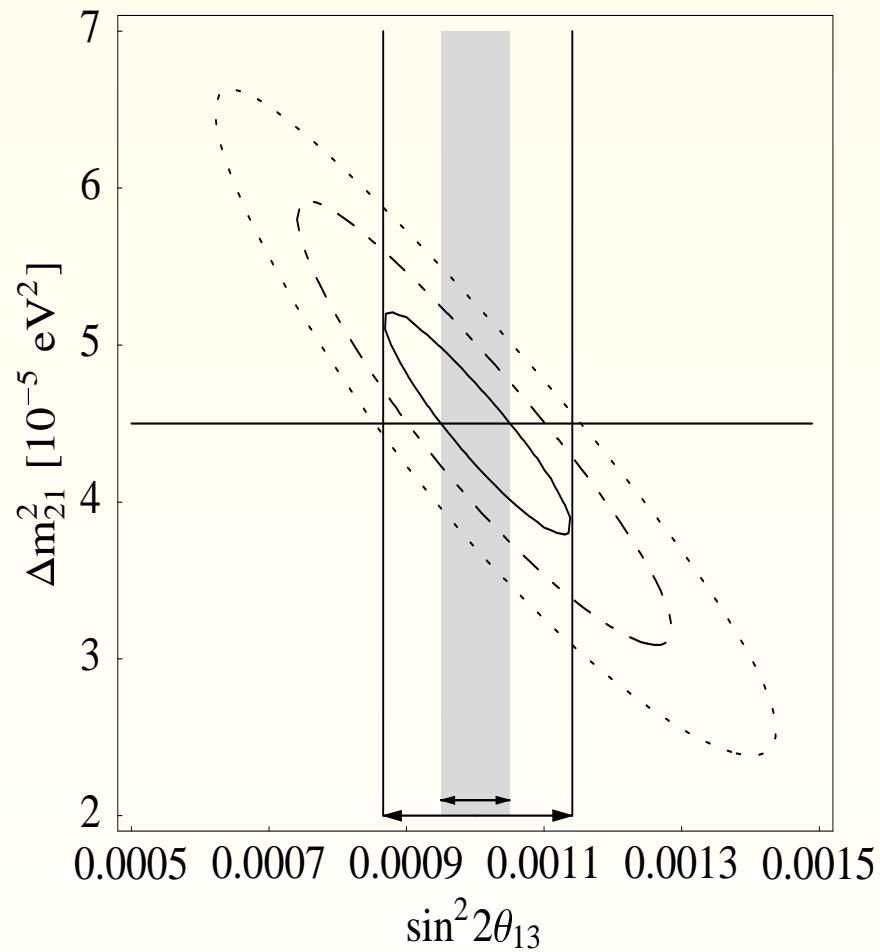
- Suppose you measure a quantity  $z$  experimentally.
- And suppose  $z$  is parametrized in terms of 2 parameters  $x$  and  $y$  such that  $z = x + y$ .
- This means that measurement of  $z$  only gives information on a particular combination of  $x$  and  $y$  and not  $x$  and  $y$  themselves.
- We say that the 2 params  $x$  and  $y$  are “correlated”.
- Oscillation probabilities come in such forms:

$$P_{ee} = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}$$

- This correlation between parameters leads to increase in the error in the measured value of the individual parameters.



# Correlations

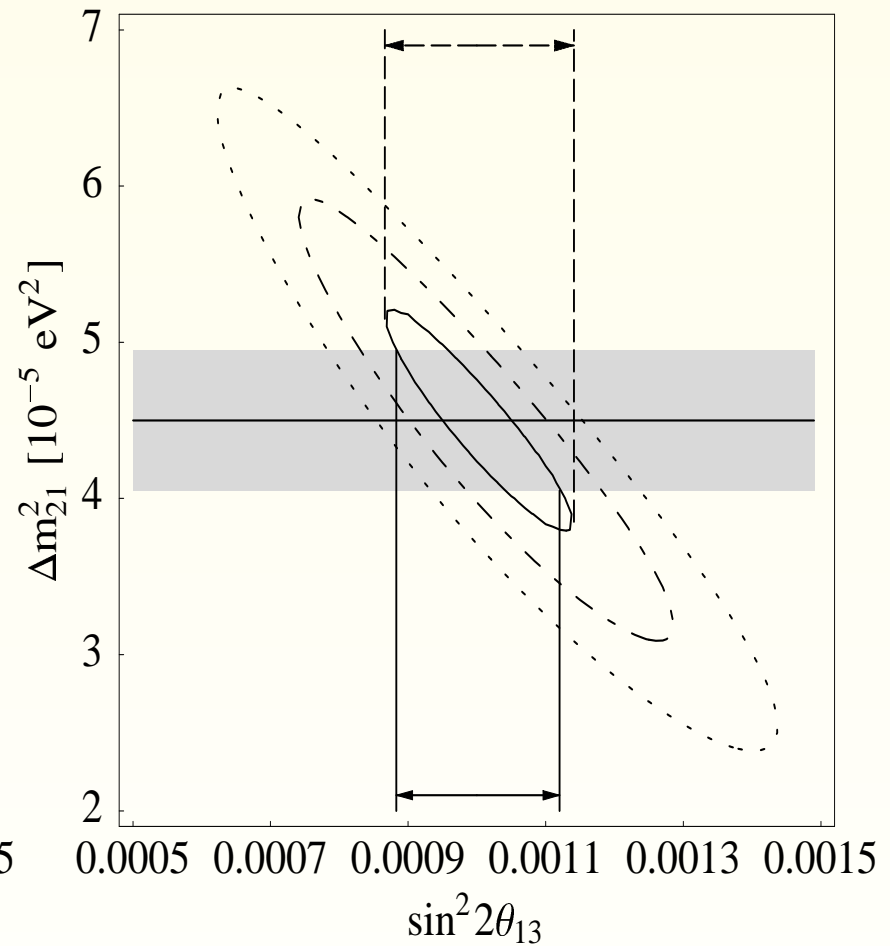
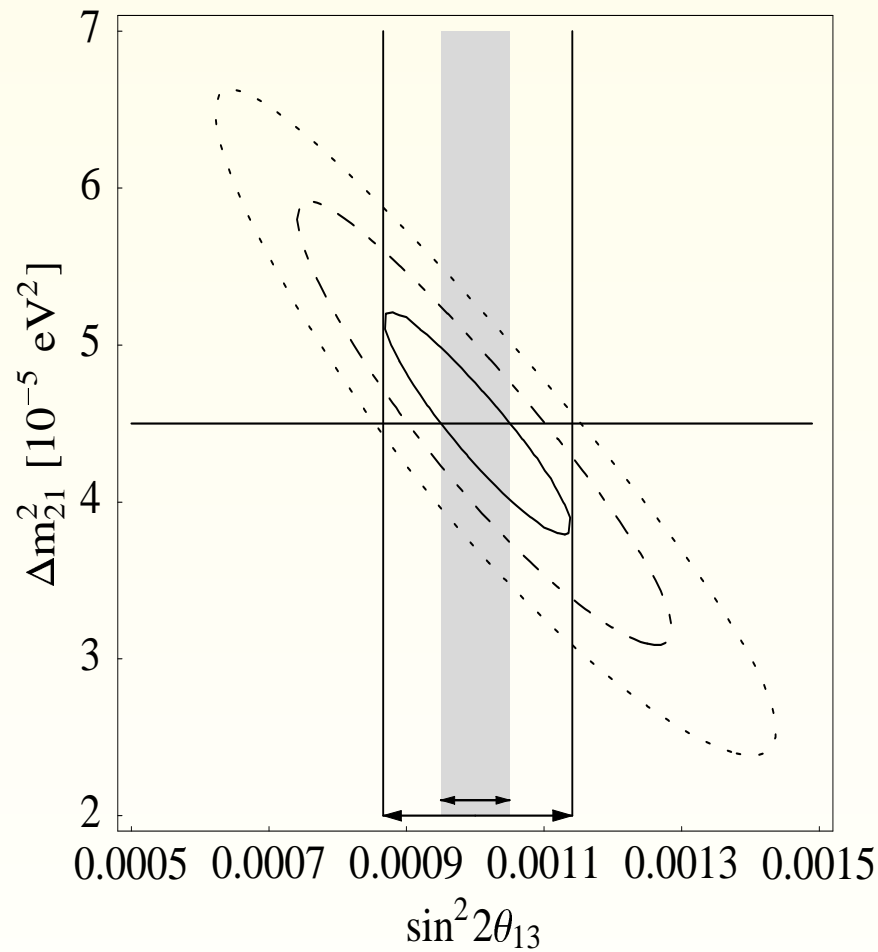


Huber et al, hep-ph/0204352

- Note how correlations are increasing the spread.



# Correlations



- Input from an external measurement helps. [Huber et al, hep-ph/0204352](https://arxiv.org/abs/hep-ph/0204352)
- Note how correlations are increasing the spread.

# Differentiating Correlations and Degeneracies

- Correlations, as I showed in the previous case, come from **continuous** uncertainty in the measured value of the parameters which appear together in the expression for the probability.

# Differentiating Correlations and Degeneracies

- Correlations, as I showed in the previous case, come from **continuous** uncertainty in the measured value of the parameters which appear together in the expression for the probability.
- **Degeneracies** are usually **disjoint** and appear because the same probability can be given by two very different sets of parameter values

# Differentiating Correlations and Degeneracies

- Correlations, as I showed in the previous case, come from **continuous** uncertainty in the measured value of the parameters which appear together in the expression for the probability.
- **Degeneracies** are usually **disjoint** and appear because the same probability can be given by two very different sets of parameter values
- **Both** result in **loss of sensitivity** of the experiment and we have to find ways to overcome/reduce them.

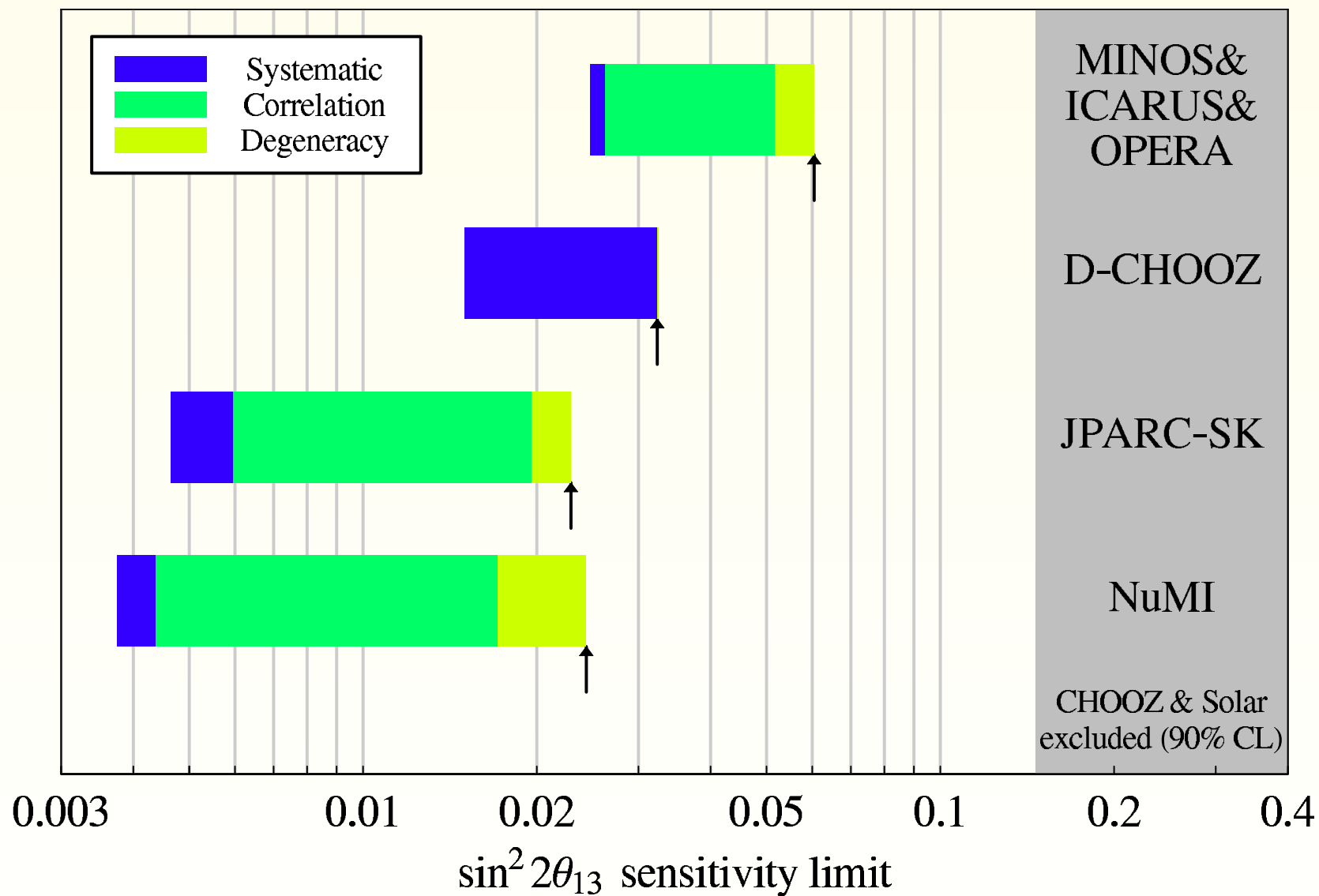
# Differentiating Correlations and Degeneracies

- Correlations, as I showed in the previous case, come from **continuous** uncertainty in the measured value of the parameters which appear together in the expression for the probability.
- **Degeneracies** are usually **disjoint** and appear because the same probability can be given by two very different sets of parameter values
- **Both** result in **loss of sensitivity** of the experiment and we have to find ways to overcome/reduce them.
- We discussed some ways to tackle degeneracies.

# Differentiating Correlations and Degeneracies

- Correlations, as I showed in the previous case, come from **continuous** uncertainty in the measured value of the parameters which appear together in the expression for the probability.
- **Degeneracies** are usually **disjoint** and appear because the same probability can be given by two very different sets of parameter values
- **Both** result in **loss of sensitivity** of the experiment and we have to find ways to overcome/reduce them.
- We discussed some ways to tackle degeneracies.
- The only way to reduce impact of correlations is to combine experiments with different characteristics.

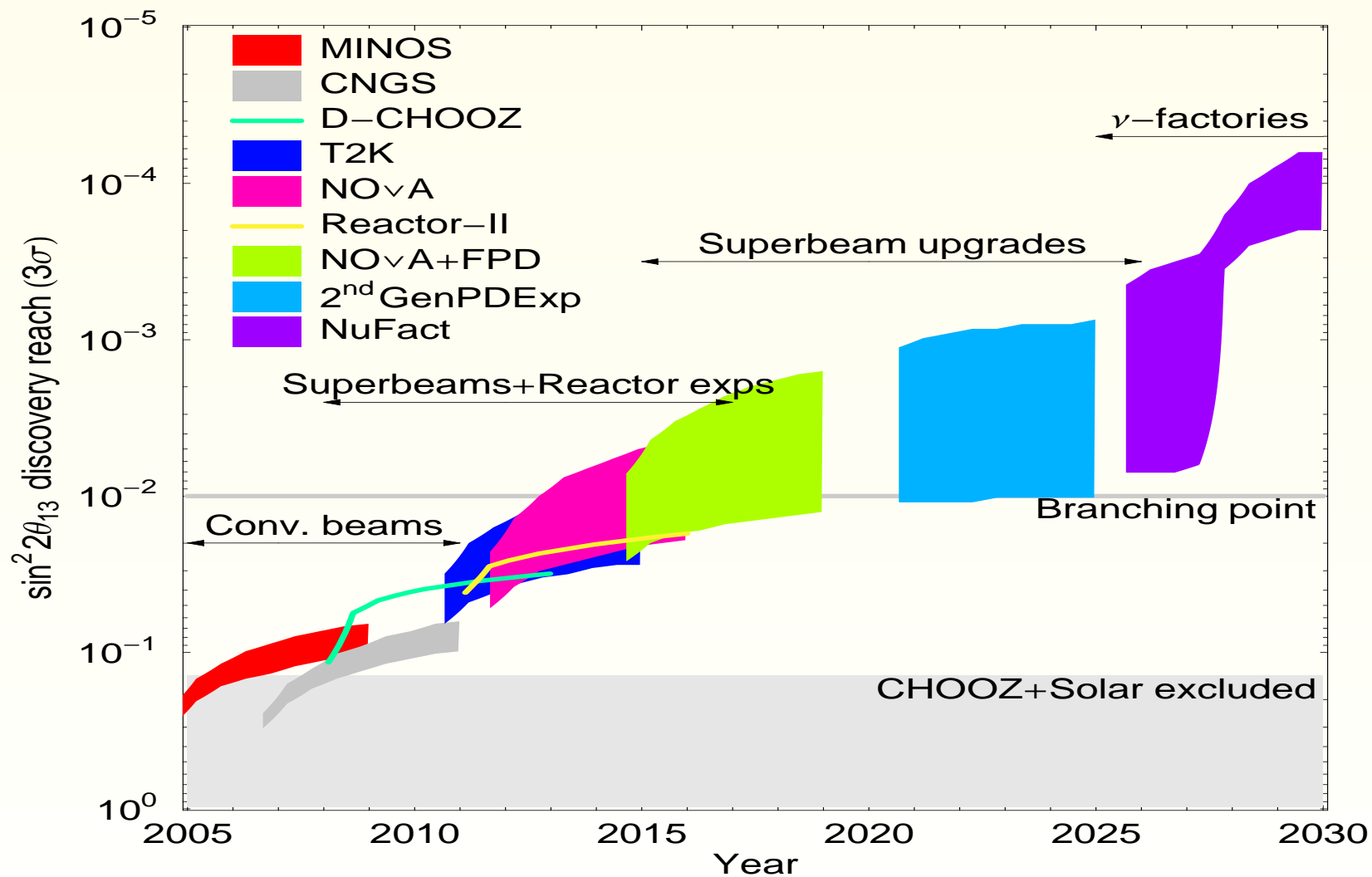
# Sensitivity of Near-Future Experiments to $\theta_{13}$



Huber *et al.*, hep-ph/0403068



# Sensitivity of Near-Future Experiments to $\theta_{13}$



Albrow *et al.*, hep-ex/0509019

# Sensitivity of Far-Future LBL Experiments

● SuperBeam  $\Rightarrow P_{\mu e}$

# Sensitivity of Far-Future LBL Experiments

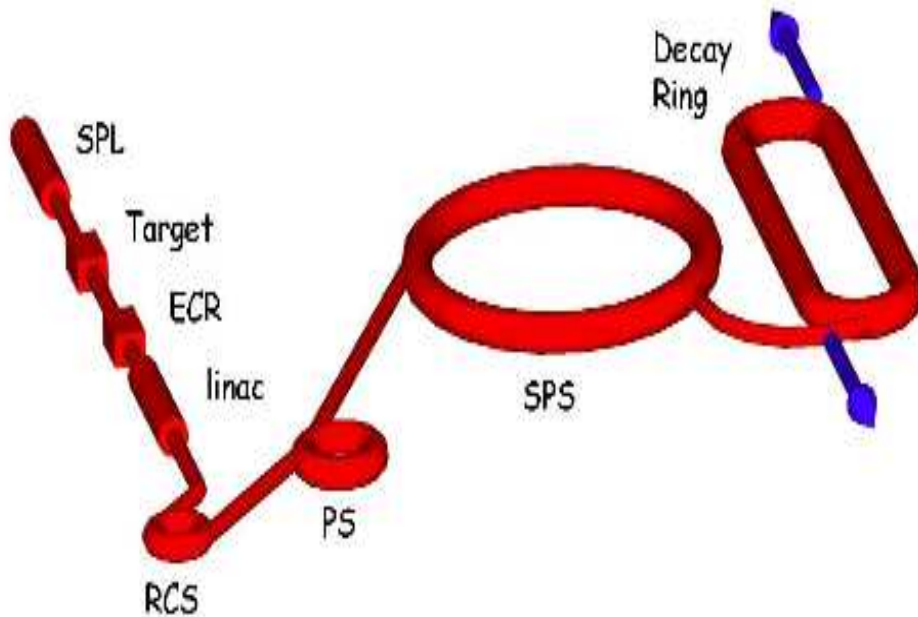
- SuperBeam  $\Rightarrow P_{\mu e}$
- Beta-Beam  $\Rightarrow P_{e\mu}$

# Sensitivity of Far-Future LBL Experiments

- SuperBeam  $\Rightarrow P_{\mu e}$
- Beta-Beam  $\Rightarrow P_{e\mu}$
- Neutrino Factory  $\Rightarrow P_{e\mu}$

# BetaBeams

● Beta-Beams are produced from beta decay of accelerated radioactive ions, circulating in a storage ring



● Proton Driver –

SPL ( $\approx 4$  GeV)

● Target

● Ion Source –

Pulsed ECR

● Accelerators –

linac, RCS, PS, SPS

● Storage Ring –

7000m; 2500m straight

Zucchelli, PLB 532, 166, (2002)

# BetaBeams

- Pure beam with just one flavor

# BetaBeams

- Pure beam with just one flavor
- Very intense beam

# BetaBeams

- Pure beam with just one flavor
- Very intense beam
- Completely known



# BetaBeams

- Pure beam with just one flavor
- Very intense beam
- Completely known
- Determined by just the end-point beta decay energy and  $\gamma$

# BetaBeams

- Pure beam with just one flavor
- Very intense beam
- Completely known
- Determined by just the end-point beta decay energy and  $\gamma$
- Flux normalization given by number of radioactive ions circulating in the ring

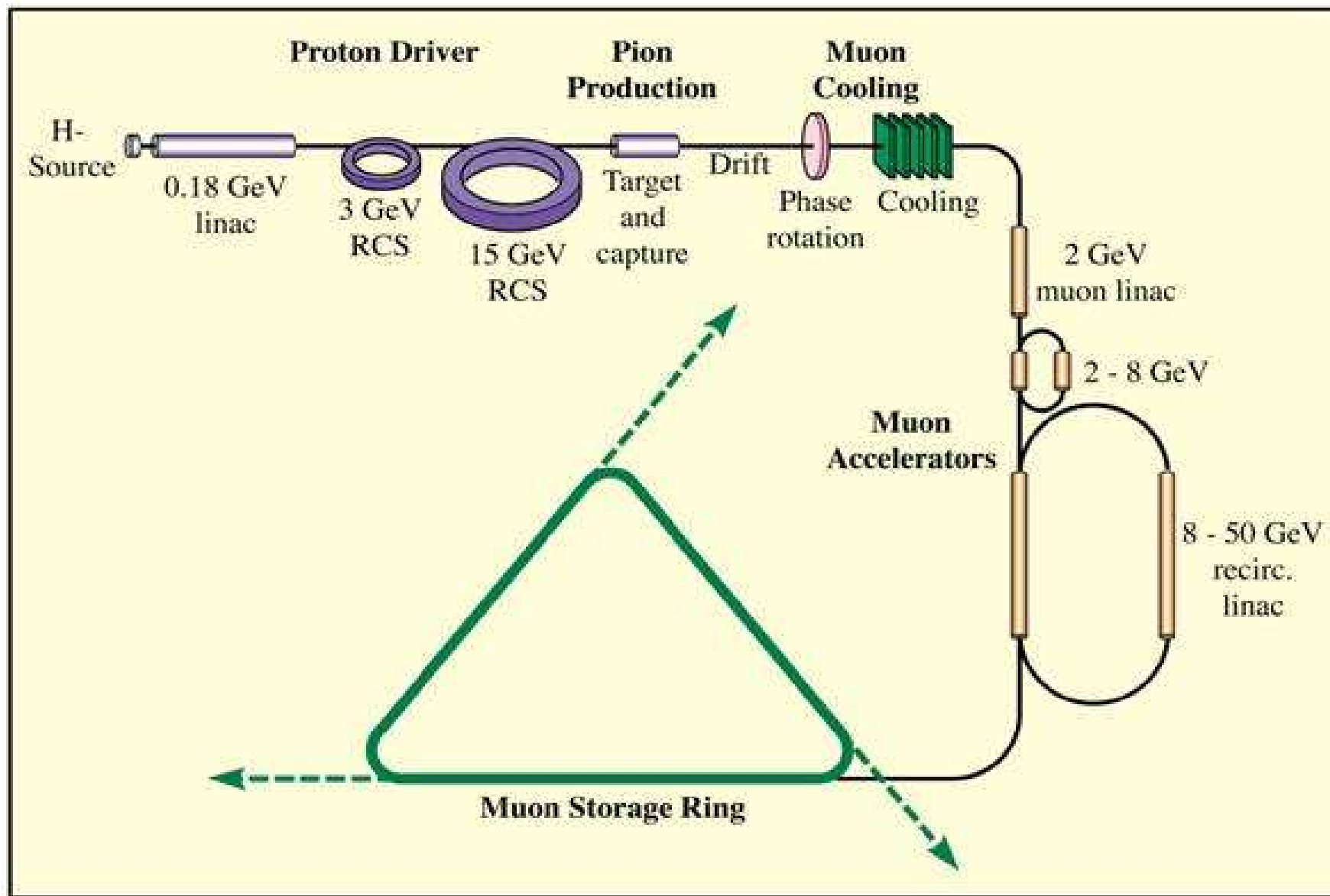
# BetaBeams

- Pure beam with just one flavor
- Very intense beam
- Completely known
- Determined by just the end-point beta decay energy and  $\gamma$
- Flux normalization given by number of radioactive ions circulating in the ring
- Beam divergence given by  $\gamma$ : higher boost gives higher collimation

# BetaBeams

- Pure beam with just one flavor
- Very intense beam
- Completely known
- Determined by just the end-point beta decay energy and  $\gamma$
- Flux normalization given by number of radioactive ions circulating in the ring
- Beam divergence given by  $\gamma$ : higher boost gives higher collimation
- Can produce either  $\nu_e$  OR  $\bar{\nu}_e$  FLUX

# Neutrino Factory



Neutrino Factory at RAL

# Neutrino Factory

- Produced by decay of accelerated muons circulating in a storage ring:



# Neutrino Factory

- Produced by decay of accelerated muons circulating in a storage ring:



- Pure, Intense, Collimated, Known beam

# Neutrino Factory

- Produced by decay of accelerated muons circulating in a storage ring:



- Pure, Intense, Collimated, Known beam
- For a given muon sign in the ring, we will have muons of both signs in the detector. For a  $\mu^+$  source  $\bar{\nu}_\mu$  in the original beam will give rise to  $\mu^+$  in the detector (right sign muons), while  $\nu_e \rightarrow \nu_\mu$  oscillations will give  $\mu^-$  (wrong sign muons).



# Neutrino Factory

- Produced by decay of accelerated muons circulating in a storage ring:



- Pure, Intense, Collimated, Known beam
- For a given muon sign in the ring, we will have muons of both signs in the detector. For a  $\mu^+$  source  $\bar{\nu}_\mu$  in the original beam will give rise to  $\mu^+$  in the detector (right sign muons), while  $\nu_e \rightarrow \nu_\mu$  oscillations will give  $\mu^-$  (wrong sign muons).
- Charge ID is a MUST.

# Neutrino Factory

- Produced by decay of accelerated muons circulating in a storage ring:



- Pure, Intense, Collimated, Known beam
- For a given muon sign in the ring, we will have muons of both signs in the detector. For a  $\mu^+$  source  $\bar{\nu}_\mu$  in the original beam will give rise to  $\mu^+$  in the detector (right sign muons), while  $\nu_e \rightarrow \nu_\mu$  oscillations will give  $\mu^-$  (wrong sign muons).
- Charge ID is a MUST.
- Need magnetised detectors

# Sensitivity of Far-Future LBL Experiments

- SuperBeam  $\Rightarrow P_{\mu e}$
- Beta-Beam  $\Rightarrow P_{e\mu}$
- Neutrino Factory  $\Rightarrow P_{e\mu}$
- One has to find ways to kill the clone solutions
- Various ways have been suggested in the literature for this

# Killing the Clones

● Combining data from appearance experiments at different  $L$  and/or different  $E$ :

Barger, Marfatia, Whisnant, hep-ph/0206038

Barger, Marfatia, Whisnant, hep-ph/0210428

Burguet-Castell, Gavela, Gomez-Cadenas, Hernandez, Mena, hep-ph/0103258

Huber, Lindner, Winter, hep-ph/0211300

Mena and Parke hep-ph/0408070

Mena, Palomares-Ruiz, Pascoli, hep-ph/0504015

Mena, Palomares-Ruiz, Pascoli, hep-ph/0510182

Mena, Nunokawa, Parke, hep-ph/0609011

Minakata, Nunokawa, hep-ph/9706281

Minakata, Nunokawa, Parke, hep-ph/0301210

Ishitsuka, Kajita, Minakata, Nunokawa, hep-ph/0504026

Hagiwara, Okamura, Senda, hep-ph/0607255

# Killing the Clones

- Combining data from different channels:

The Silver Channel  $P_{e\tau}$

Autiero *et al.*, hep-ph/0305185

Donini, Meloni, Migliozi, hep-ph/0206034

Disappearance Channel  $P_{\mu\mu}$

Donini, Fernandez-Martinez, Meloni, Rigolin, hep-ph/0512038

Donini, Fernandez-Martinez, Rigolin, hep-ph/0411402

The Platinum Channel  $P_{\mu e}$



# Killing the Clones

---

- Combining LBL data with data from other experiments:

Adding reactor antineutrino data

Huber, Lindner, Schwetz, Winter, hep-ph/0303232

Adding atmospheric neutrino data

Huber, Maltoni, Schwetz, hep-ph/0501037

Campagne, Maltoni, Mezzetto, Schwetz, hep-ph/0603172

# Killing the Clones at The Magic Baseline

---



# The Magic baseline

$$P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$
$$\pm \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$
$$+ \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$
$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$



# The Magic baseline

$$P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

$$\pm \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin \delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$

$$+ \alpha \sin 2\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos \delta_{CP} \cos \Delta \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$

$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$

● If  $\sin(\hat{A}\Delta) \simeq 0$

# The Magic baseline

$$P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

# The Magic baseline

$$P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

- All  $\delta_{CP}$  dependent terms drop out
- $(\delta_{CP}, \theta_{13})$  and  $(\delta_{CP}, \text{sgn}(\Delta m_{31}^2))$  degeneracies vanish
- “Clean” measurement of  $\theta_{13}$  and  $\text{sgn}(\Delta m_{31}^2)$

# The Magic baseline

$$P_{e\mu} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

- All  $\delta_{CP}$  dependent terms drop out
- $(\delta_{CP}, \theta_{13})$  and  $(\delta_{CP}, \text{sgn}(\Delta m_{31}^2))$  degeneracies vanish
- “Clean” measurement of  $\theta_{13}$  and  $\text{sgn}(\Delta m_{31}^2)$

$$\sin(\hat{A}\Delta) \simeq 0$$

$\Rightarrow$

$$L_{magic} \simeq 7690 \text{ km}$$

Barger, Marfatia, Whisnant, hep-ph/0112119

Huber, Winter, hep-ph/0301257

Smirnov, hep-ph/0610198

# Near-Resonant Matter Effects

---



# Near-Resonant Matter Effects

- Large Distance  $\Rightarrow$  Large Matter effects



# Near-Resonant Matter Effects

- Large Distance  $\Rightarrow$  Large Matter effects
- Resonance energy

$$E_{res} = \frac{|\Delta m_{31}^2| \cos 2\theta_{13}}{2\sqrt{2}G_F N_e}$$

# Near-Resonant Matter Effects

- Large Distance  $\Rightarrow$  Large Matter effects
- Resonance energy

$$E_{res} = \frac{|\Delta m_{31}^2| \cos 2\theta_{13}}{2\sqrt{2}G_F N_e}$$

- For  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{13} = 0.1$  and the PREM profile  $\rho_{av} = 4.13 \text{ gm/cc}$ ,  $E_{res} \simeq 7.5 \text{ GeV}$



# Near-Resonant Matter Effects

- Large Distance  $\Rightarrow$  Large Matter effects
- Resonance energy

$$E_{res} = \frac{|\Delta m_{31}^2| \cos 2\theta_{13}}{2\sqrt{2}G_F N_e}$$

- For  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{13} = 0.1$  and the PREM profile  $\rho_{av} = 4.13 \text{ gm/cc}$ ,  $E_{res} \simeq 7.5 \text{ GeV}$
- Maximal oscillations when  $\sin^2 2\theta_{13}^m \simeq 1$  and  $\sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \simeq 1$  simultaneously

Gandhi et al, hep-ph/0408361

# Near-Resonant Matter Effects

- Large Distance  $\Rightarrow$  Large Matter effects
- Resonance energy

$$E_{res} = \frac{|\Delta m_{31}^2| \cos 2\theta_{13}}{2\sqrt{2}G_F N_e}$$

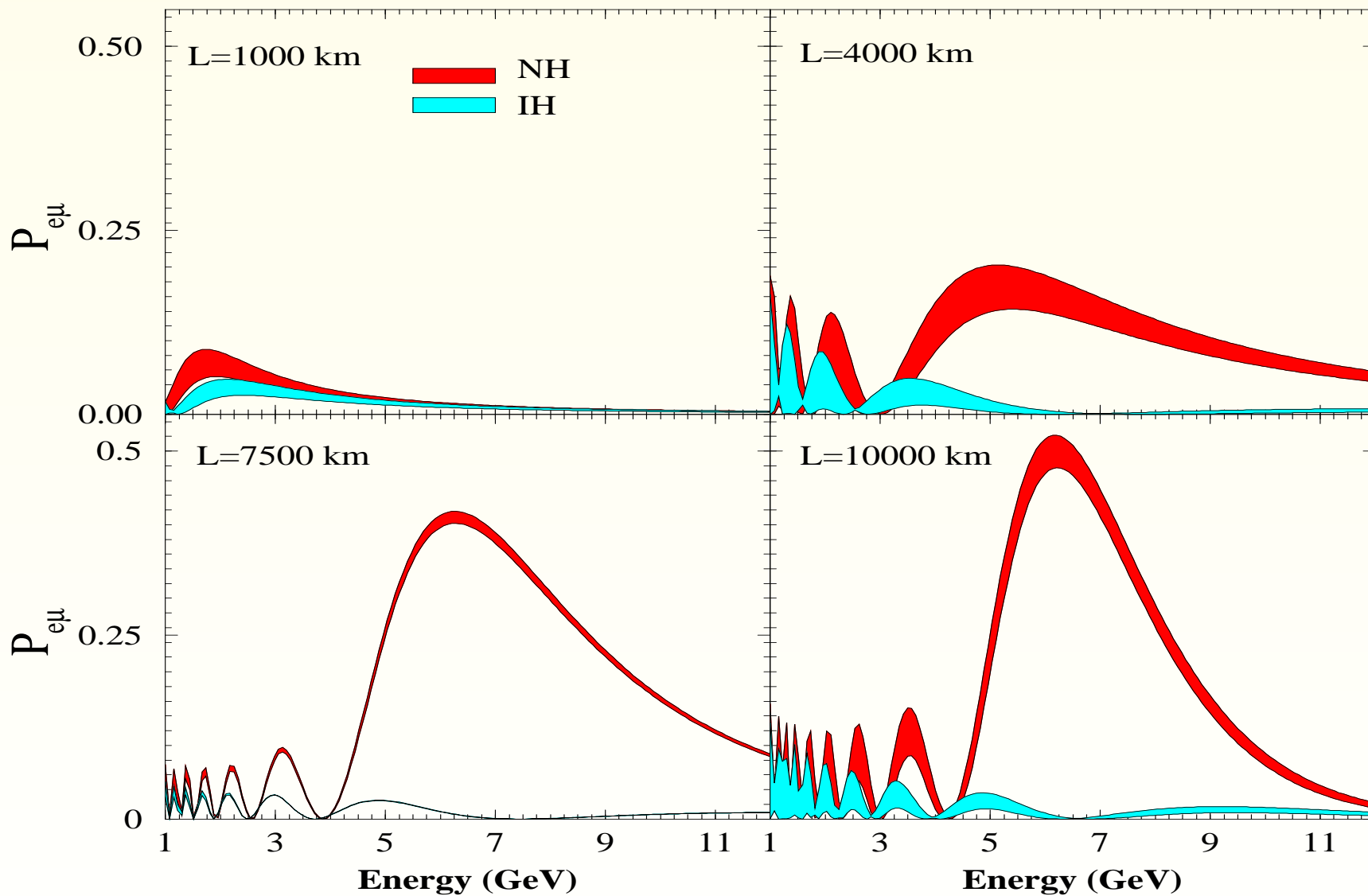
- For  $\Delta m_{31}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\sin^2 2\theta_{13} = 0.1$  and the PREM profile  $\rho_{av} = 4.13 \text{ gm/cc}$ ,  $E_{res} \simeq 7.5 \text{ GeV}$

- Maximal oscillations when  $\sin^2 2\theta_{13}^m \simeq 1$  and  $\sin^2\left(\frac{(\Delta m_{31}^2)^m L}{4E}\right) \simeq 1$  simultaneously

Gandhi et al, hep-ph/0408361

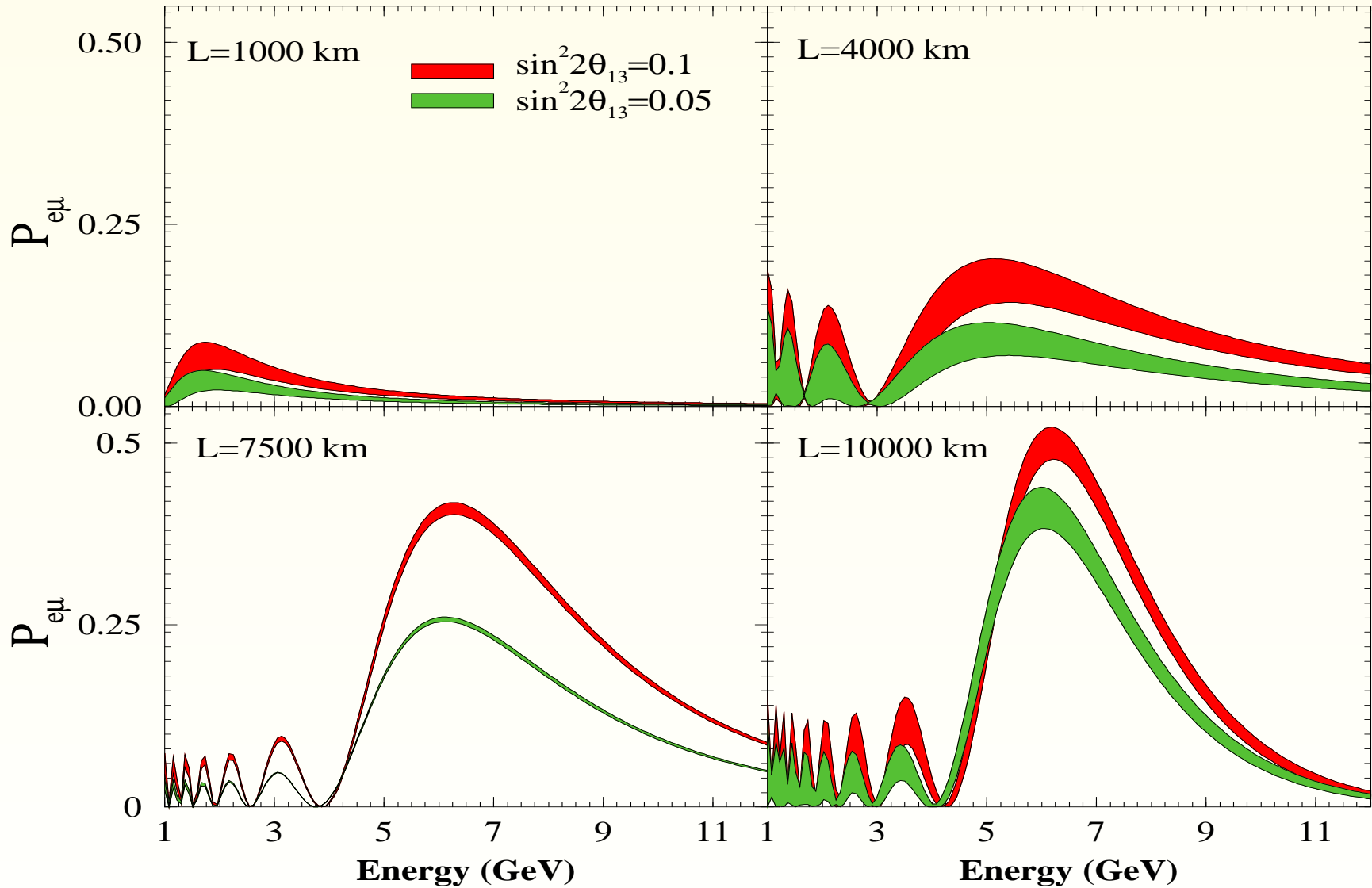
- At the magic baseline, largest oscillations come when  $E \simeq 6 \text{ GeV}$

# The Probability



Agarwalla, S.C., Raychaudhuri, hep-ph/0610333

# The Probability



Agarwalla, S.C., Raychaudhuri, hep-ph/0610333

# Conclusions (Comparison of different setups)

	$\gamma$	L(km)	Detector	$T_\nu/T_{\bar{\nu}}$	$\sin^2 2\theta_{13}$	$sgn(\Delta m_{31}^2)$	Max CPV
NF@3000		3000	50 (MI)	4/4	$2.5 \times 10^{-3}$	$(0.8 - 10) \times 10^{-3}$	$7 \times 10^{-5}$
NF@7500		7500	50 (MI)	4/4	$2 \times 10^{-4}$	$2 \times 10^{-4}$	No sens
CERN-	350	7152	50 (MI)	10	$2.1 \times 10^{-3}$	$1.1 \times 10^{-2}$	No sens
INO	500	7152	50 (MI)	10	$8.4 \times 10^{-4}$	$8.5 \times 10^{-3}$	No sens
hep-ph/ 0603172	100/100	130	440 (WC)	10/10	$5 \times 10^{-3}$ (W) $3 \times 10^{-4}$ (B)	$2.5 \times 10^{-3}$ +SPL+ATM	$2 \times 10^{-4}$
hep-ph/ 0506237	200/200	520	500 (WC)	8/8	$1.5 \times 10^{-3}$	$(0.7 - 2) \times 10^{-2}$	$2 \times 10^{-4}$
	500/500	650	50 (TASD)	8/8	$1.5 \times 10^{-3}$	$(0.6 - 4.5) \times 10^{-2}$	$1 \times 10^{-4}$
	1000/1000	1300	50 (TASD)	8/8	$4 \times 10^{-4}$	$(1 - 7) \times 10^{-3}$	$7 \times 10^{-5}$
hep-ph/ 0312068	100/60	130	400 (WC)	10(S)	Not	No Sens	$1 \times 10^{-3}$
	580/350	732	400 (WC)	10(S)	Given	$2 \times 10^{-2}$	$2 \times 10^{-4}$
	2500/1500	3000	40 (MI)	10(S)		$4 \times 10^{-3}$	$4 \times 10^{-4}$
hep-ph/ 0503021	120/120	130	440 (WC)	10(S)	$5 \times 10^{-3}$	Not	$1 \times 10^{-3}$
	150/150	300	440 (WC)	10(S)	$6 \times 10^{-4}$	Given	$2 \times 10^{-4}$
	350/350	730	440 (WC)	10(S)	$4 \times 10^{-4}$		$1 \times 10^{-4}$

# Backup Slides

---



# The CERN-INO Beta-Beam Experiment

---





# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at [INO](#)





# The CERN-INO Beta-Beam Experiment

---

- Proposal to build a large iron detector (ICAL) at **INO**
- A Beta-Beam facility might come up at **CERN**



# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at **INO**
- A Beta-Beam facility might come up at **CERN**
- CERN-INO distance is equal to **7152 km**



# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at **INO**
- A Beta-Beam facility might come up at **CERN**
- CERN-INO distance is equal to **7152 km**
- This is tantalizingly close to the **magic baseline**



# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at **INO**
- A Beta-Beam facility might come up at **CERN**
- CERN-INO distance is equal to **7152 km**
- This is tantalizingly close to the **magic baseline**
- Energy threshold of ICAL would be about **1 GeV**



# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at **INO**
- A Beta-Beam facility might come up at **CERN**
- CERN-INO distance is equal to **7152 km**
- This is tantalizingly close to the **magic baseline**
- Energy threshold of ICAL would be about **1 GeV**
- Beta beam spectrum depends on the end point energy of the beta unstable ion and Lorentz boost  $\gamma$



# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at INO
- A Beta-Beam facility might come up at CERN
- CERN-INO distance is equal to 7152 km
- This is tantalizingly close to the magic baseline
- Energy threshold of ICAL would be about 1 GeV
- Beta beam spectrum depends on the end point energy of the beta unstable ion and Lorentz boost  $\gamma$
- The standard Beta-Beam ions  $^{18}\text{Ne}$  and  $^6\text{He}$  would require very large gamma

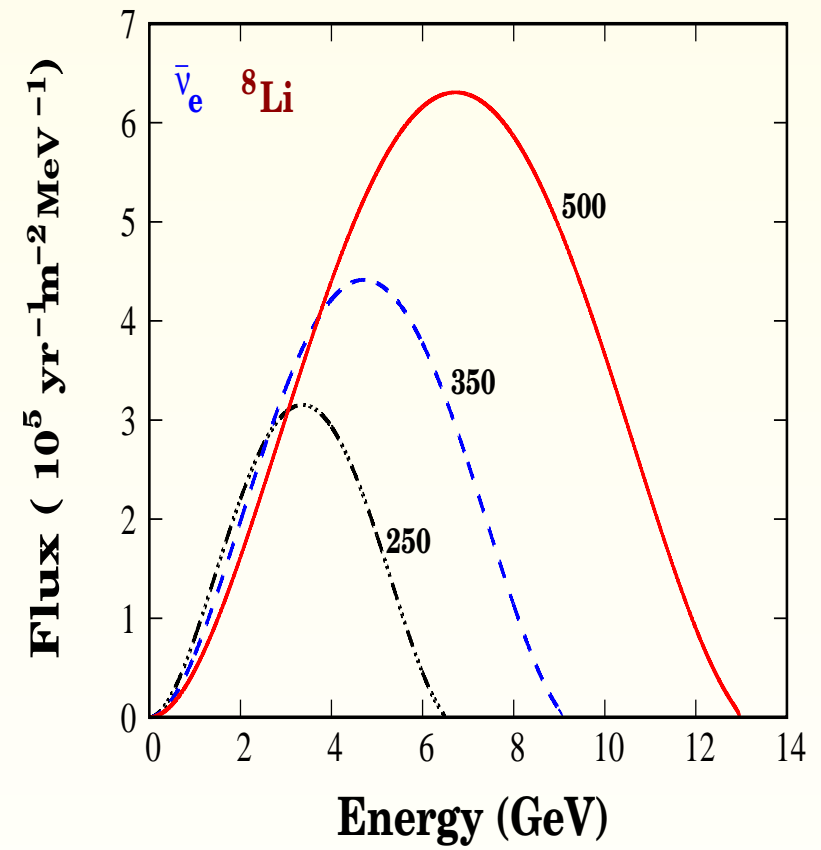
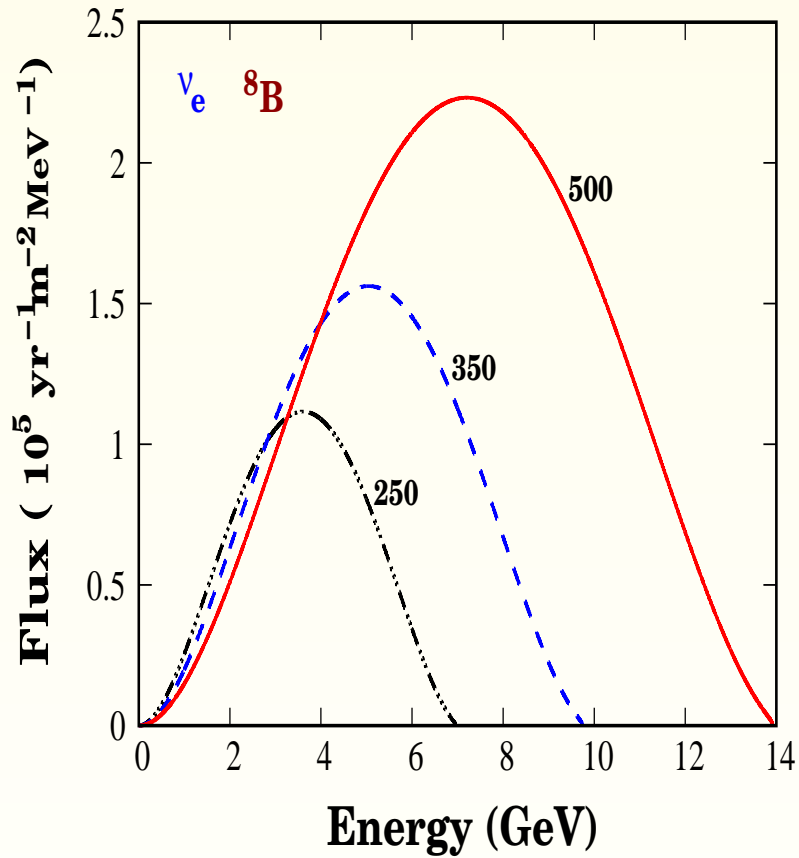


# The CERN-INO Beta-Beam Experiment

- Proposal to build a large iron detector (ICAL) at INO
- A Beta-Beam facility might come up at CERN
- CERN-INO distance is equal to 7152 km
- This is tantalizingly close to the magic baseline
- Energy threshold of ICAL would be about 1 GeV
- Beta beam spectrum depends on the end point energy of the beta unstable ion and Lorentz boost  $\gamma$
- The standard Beta-Beam ions  $^{18}\text{Ne}$  and  $^6\text{He}$  would require very large gamma
- Alternative ions  $^8\text{B}$  and  $^8\text{Li}$  have large end-point energy and hence “harder” spectra. Works!!



# The CERN-INO Beta-Beam Experiment

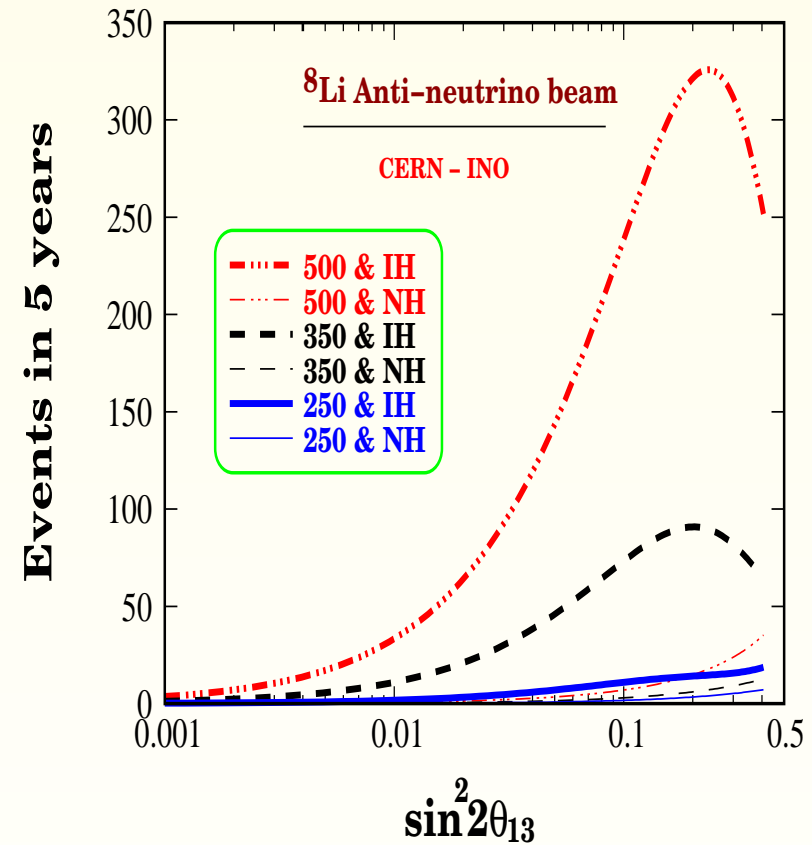
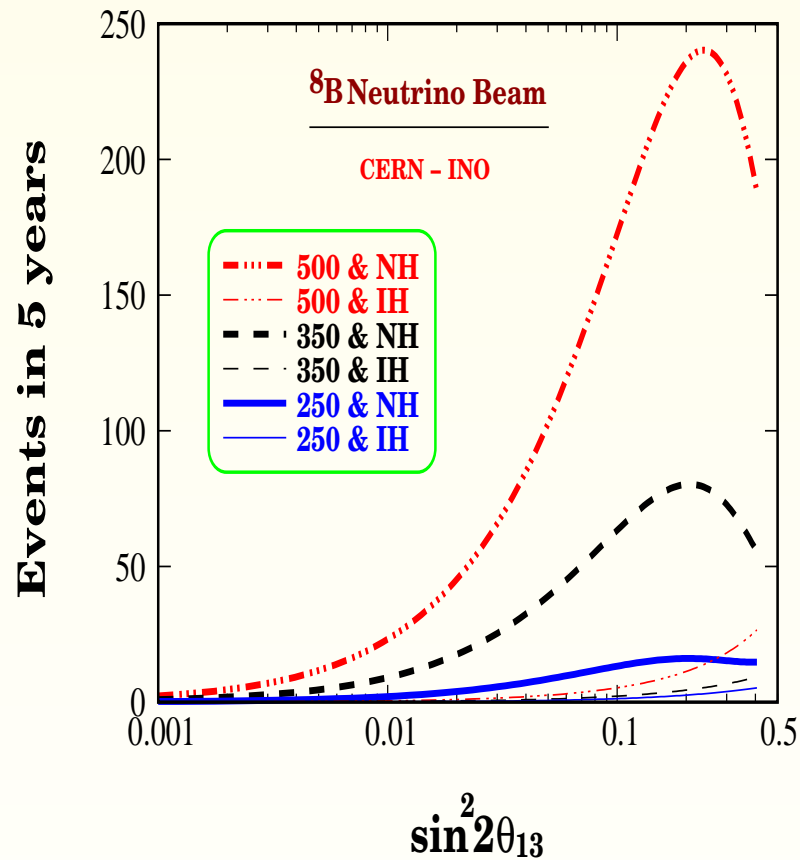


Agarwalla, SC, Raychaudhuri, hep-ph/0610333

- Flux peaks at  $E \simeq 6 \text{ GeV}$  for  $\gamma = 350 - 500$



# The CERN-INO Beta-Beam Experiment

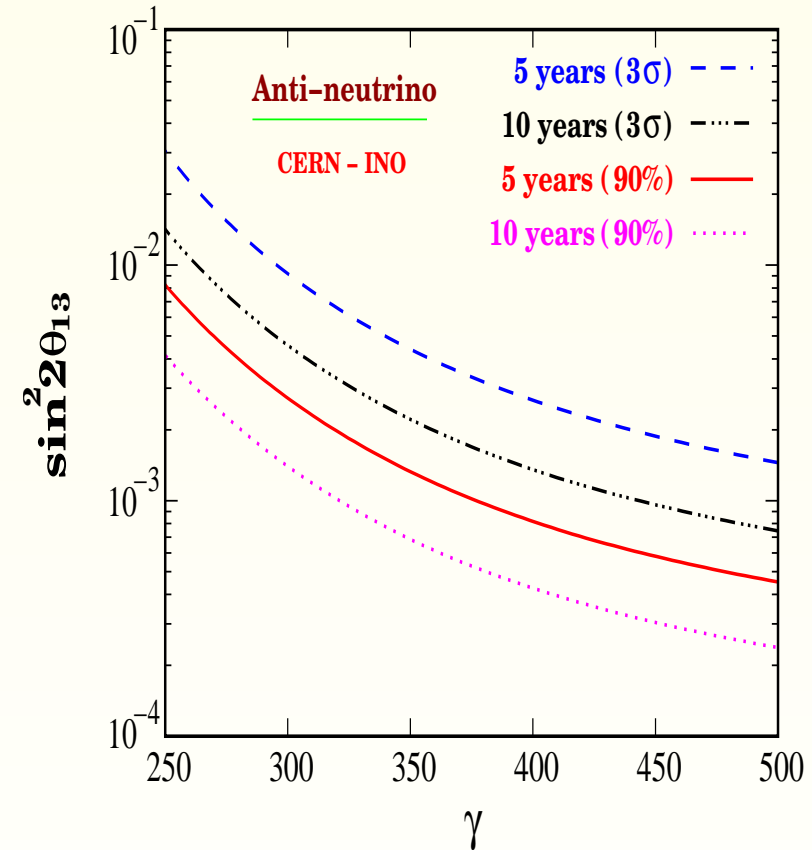
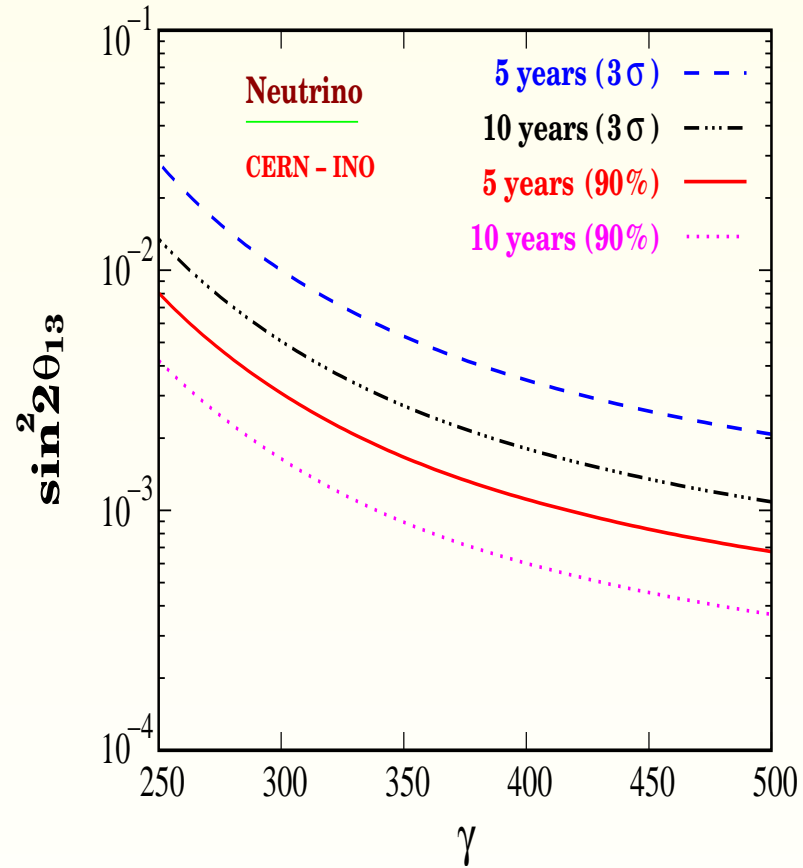


Agarwalla, SC, Raychaudhuri, hep-ph/0610333

- The rate shows a sharp dependence on the **hierarchy** and  $\theta_{13}$

# The CERN-INO Beta-Beam Experiment

## ● Sensitivity to $\theta_{13}$

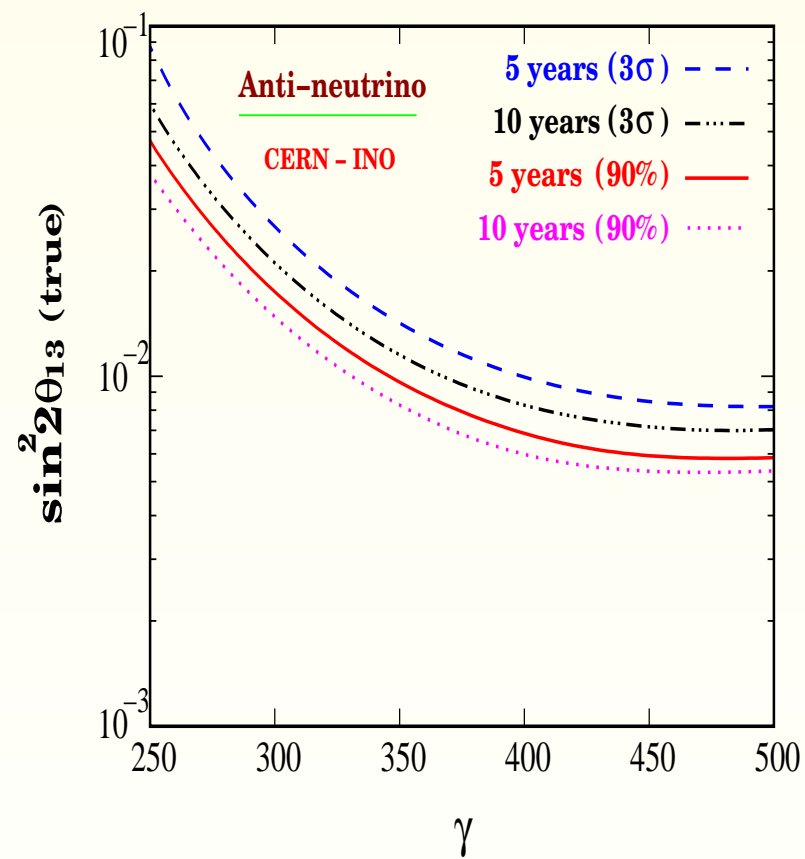
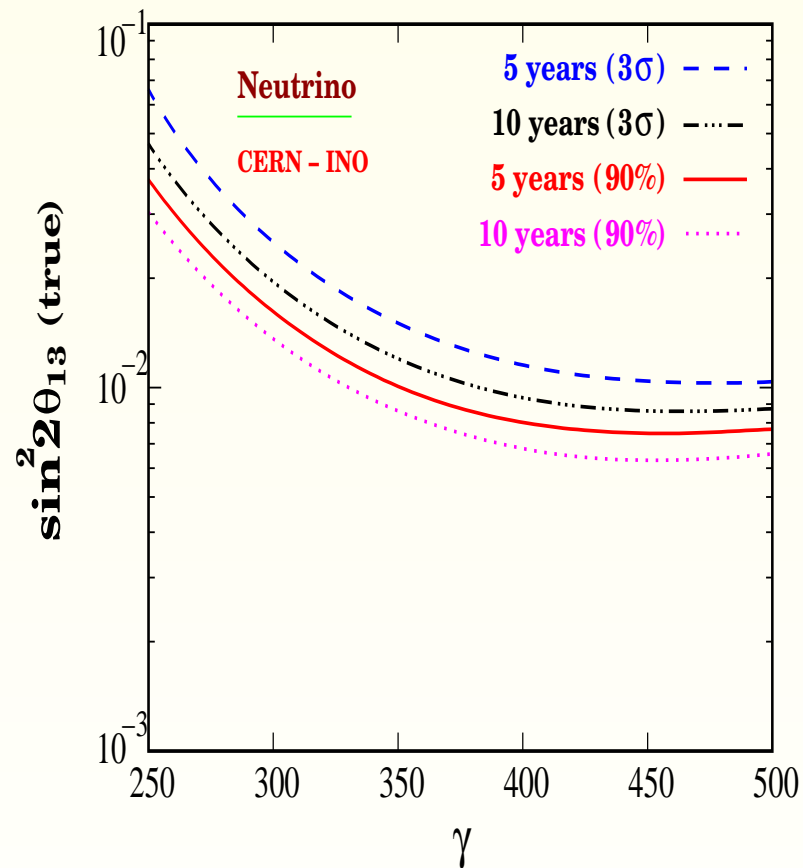


Agarwalla, SC, Raychaudhuri, hep-ph/0610333

● At  $3\sigma$ ,  $\sin^2 2\theta_{13} < 8.5 \times 10^{-4} (1.5 \times 10^{-3})$  with 80% efficiency and 10(5) years data

# The CERN-INO Beta-Beam Experiment

## ● Sensitivity to $sgn(\Delta m_{31}^2)$



Agarwalla, SC, Raychaudhuri, hep-ph/0610333

● At  $3\sigma$ ,  $\sin^2 2\theta_{13} < 8.5 \times 10^{-3}$  ( $9.8 \times 10^{-3}$ ) with 80% efficiency and 10(5) years data

# Large Matter Effects in $P_{ee}$

---



# Large Matter Effects in $P_{ee}$

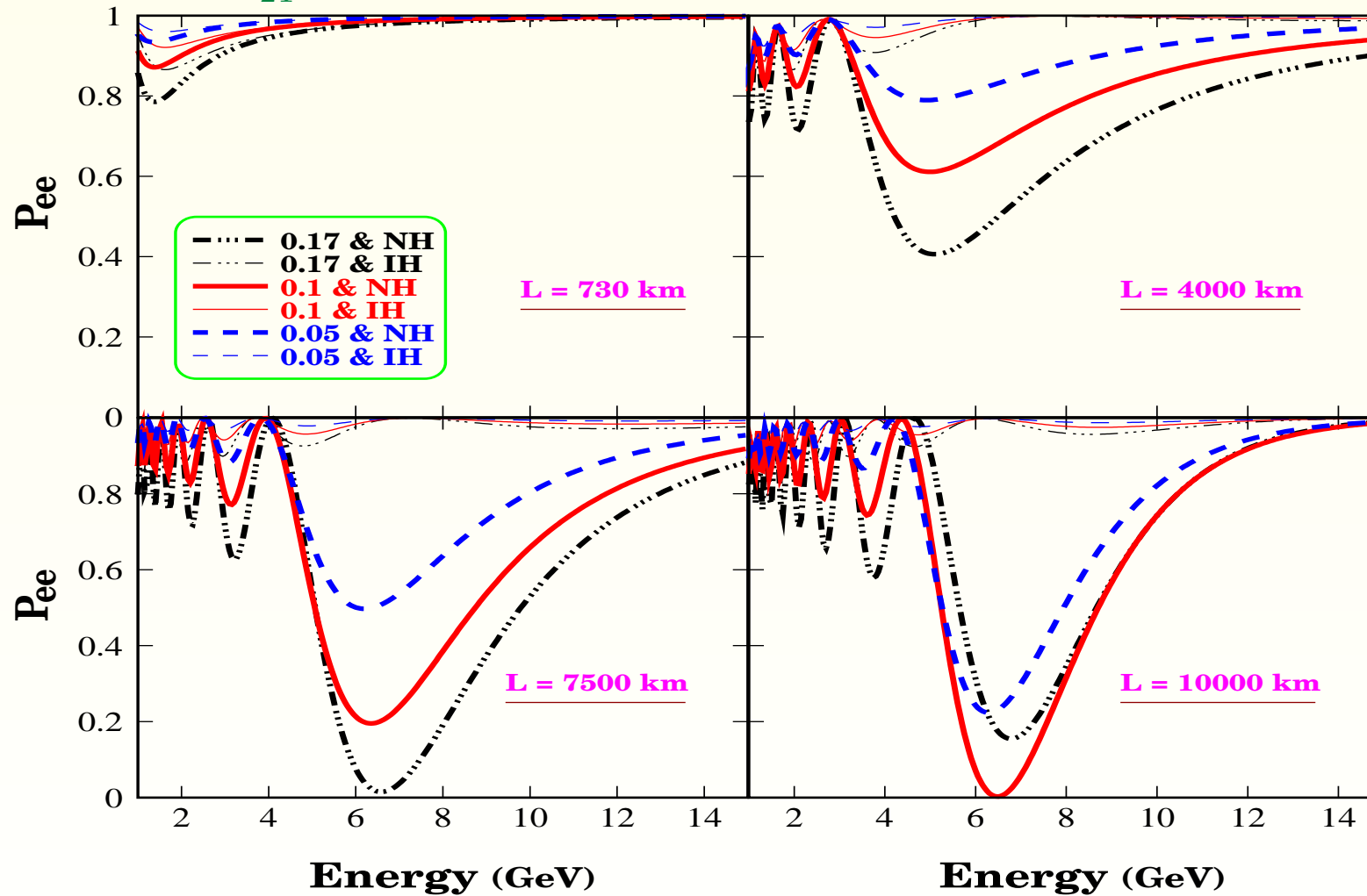
$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{e\mu} = \sin^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$

# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{e\tau} = \cos^2 \theta_{23} \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$

# Large Matter Effects in $P_{ee}$

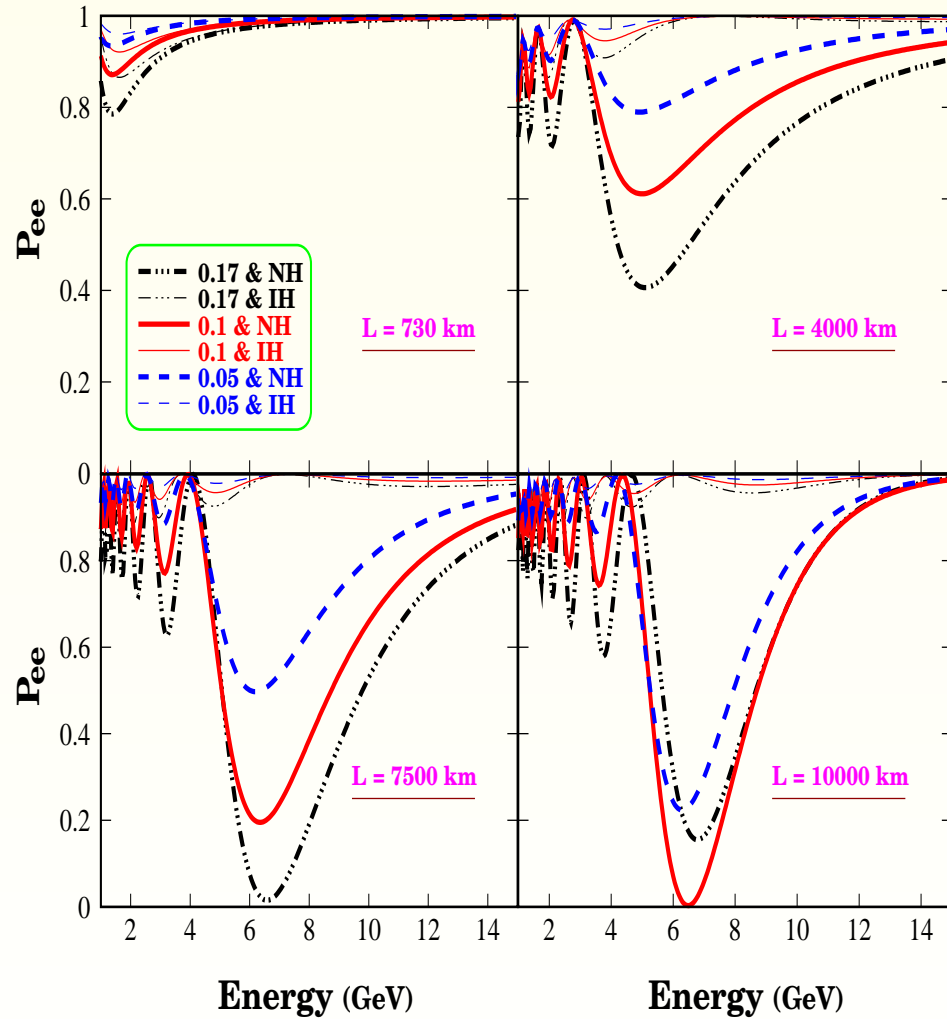
$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$



Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233

# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$

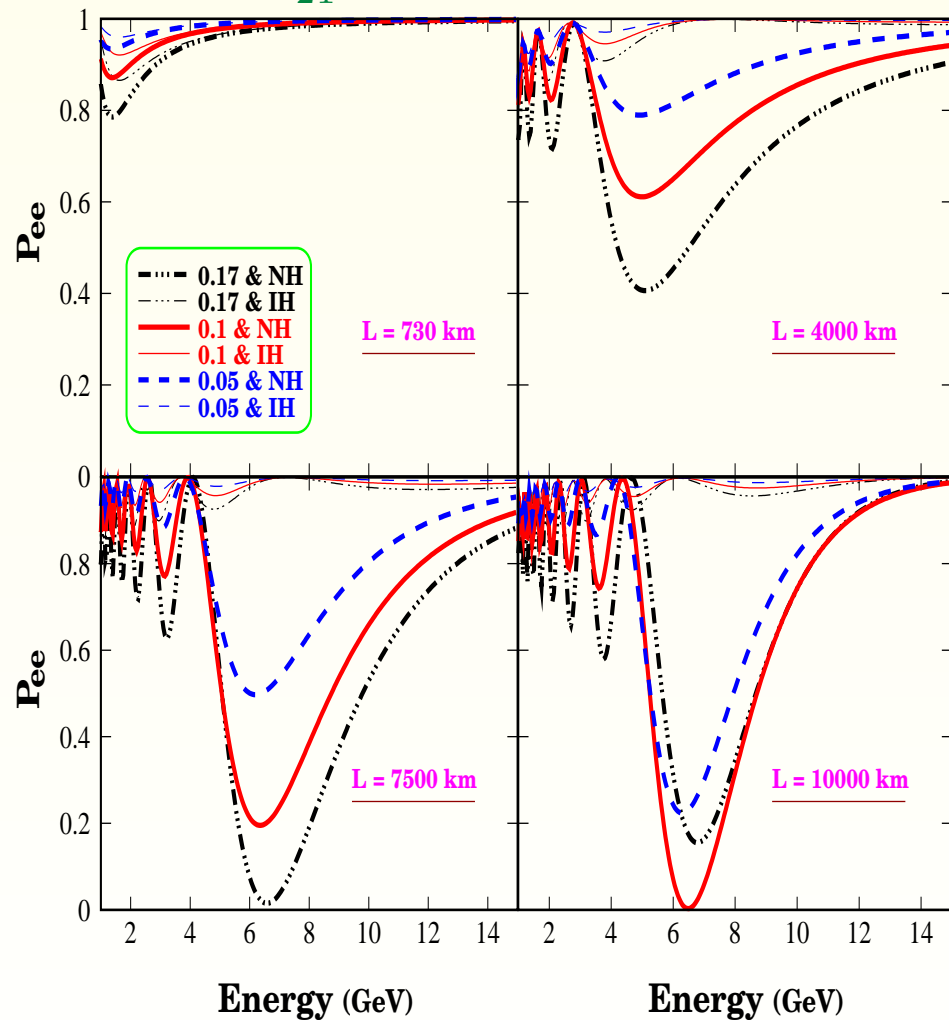


Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233



# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$



- Matter effects large: almost 2 times

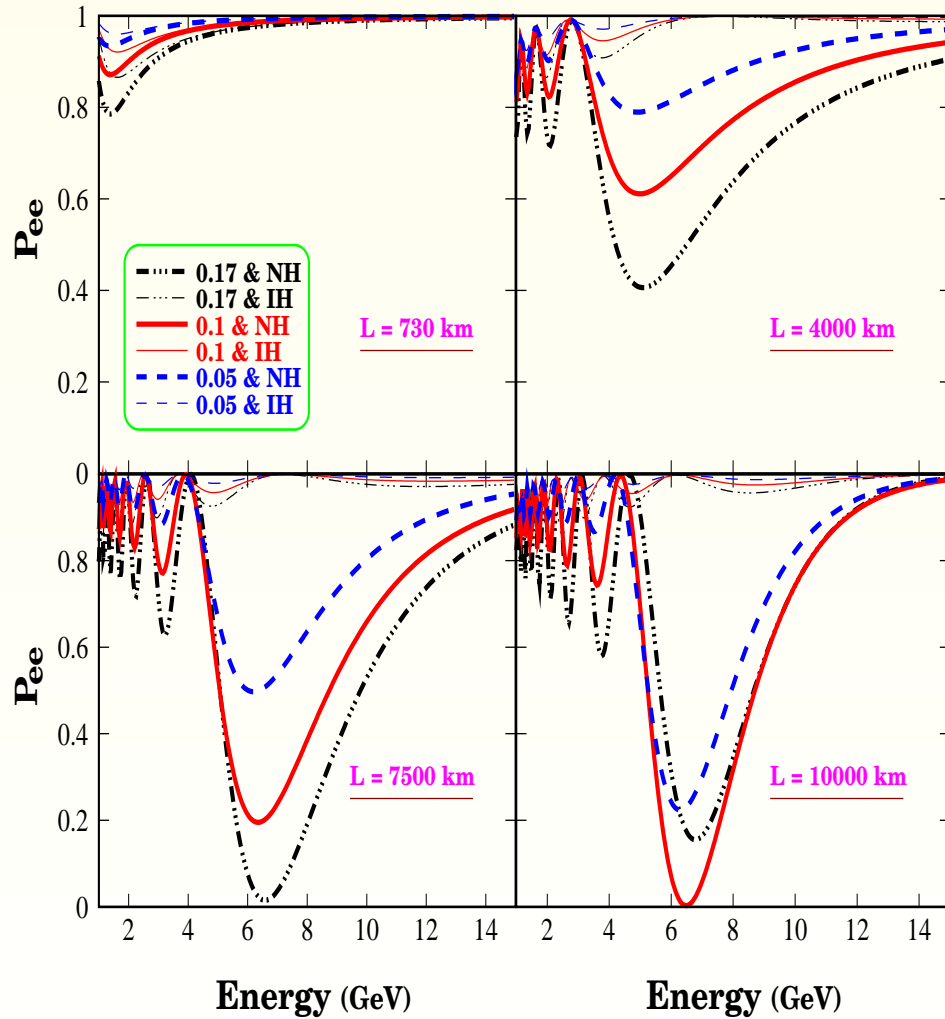
- No dependence on  $\delta_{CP}$  and  $\theta_{23}$

- No parameter degeneracies!!

Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233

# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$

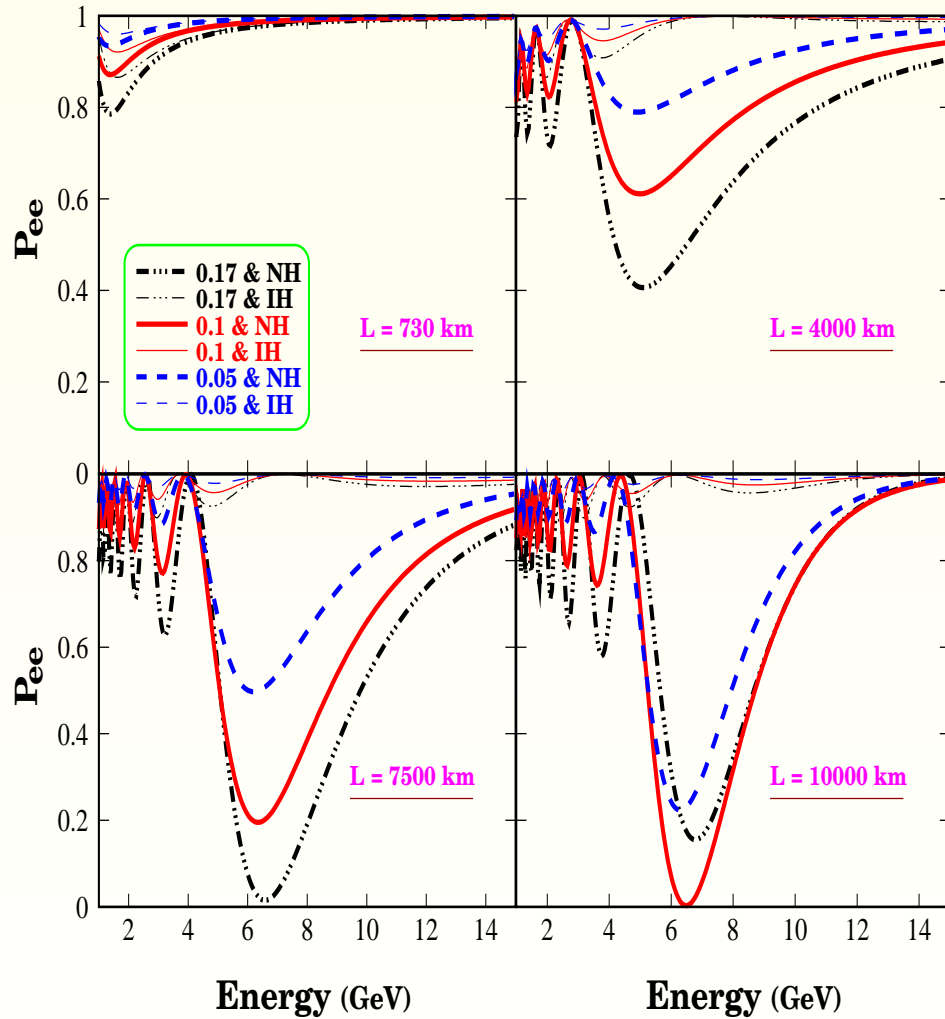


● Pure  $\nu_e/\bar{\nu}_e$ : Beta Beams

Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233

# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$

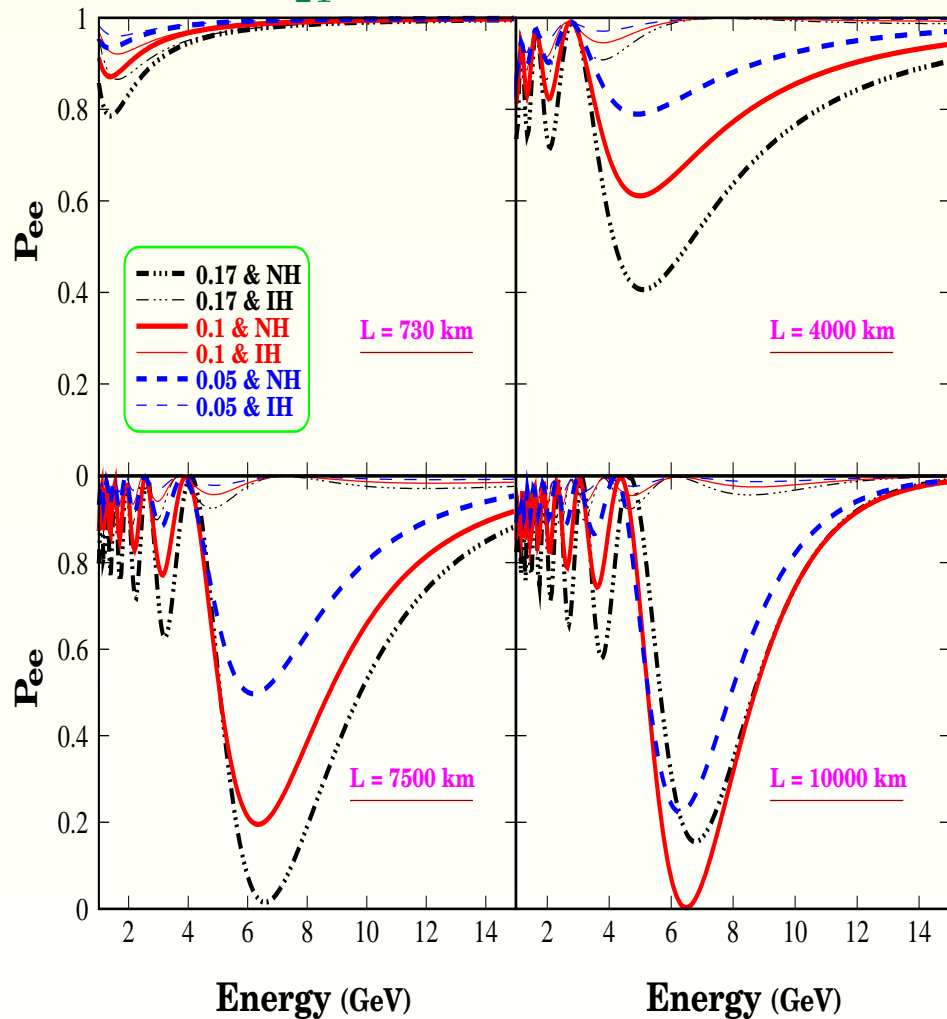


- Pure  $\nu_e/\bar{\nu}_e$ : Beta Beams
- $E \sim 6$  GeV:  ${}^8B$  and  ${}^8Li$

Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233

# Large Matter Effects in $P_{ee}$

$$\lim_{\Delta m_{21}^2 \rightarrow 0} P_{ee} = 1 - \sin^2 2\theta_{13}^m \sin^2 \frac{(\Delta m_{31}^2)^m L}{4E}$$



● Pure  $\nu_e/\bar{\nu}_e$ : Beta Beams

●  $E \sim 6$  GeV:  ${}^8B$  and  ${}^8Li$

● Very long baselines:

CERN-UNO:  $L = 7000 - 8600$  km

FNAL-MEMPHYS:  $L = 7313$  km

FNAL-HK:  $L = 10184$  km

CERN-HK:  $L = 9647$  km

Agarwalla, S.C., Goswami, Raychaudhuri, hep-ph/0611233

# CP Discovery Reach with Beta-Beams ( $P_{e\mu}$ )

- At magic baseline CP sensitivity is smothered

# CP Discovery Reach with Beta-Beams ( $P_{e\mu}$ )

- At magic baseline CP sensitivity is smothered
- Must move away from the magic baseline

# CP Discovery Reach with Beta-Beams ( $P_{e\mu}$ )

- At magic baseline CP sensitivity is smothered
- Must move away from the magic baseline
- Which set-up could be best?

# CP Discovery Reach with Beta-Beams ( $P_{e\mu}$ )

- At magic baseline CP sensitivity is smothered
- Must move away from the magic baseline
- Which set-up could be best?

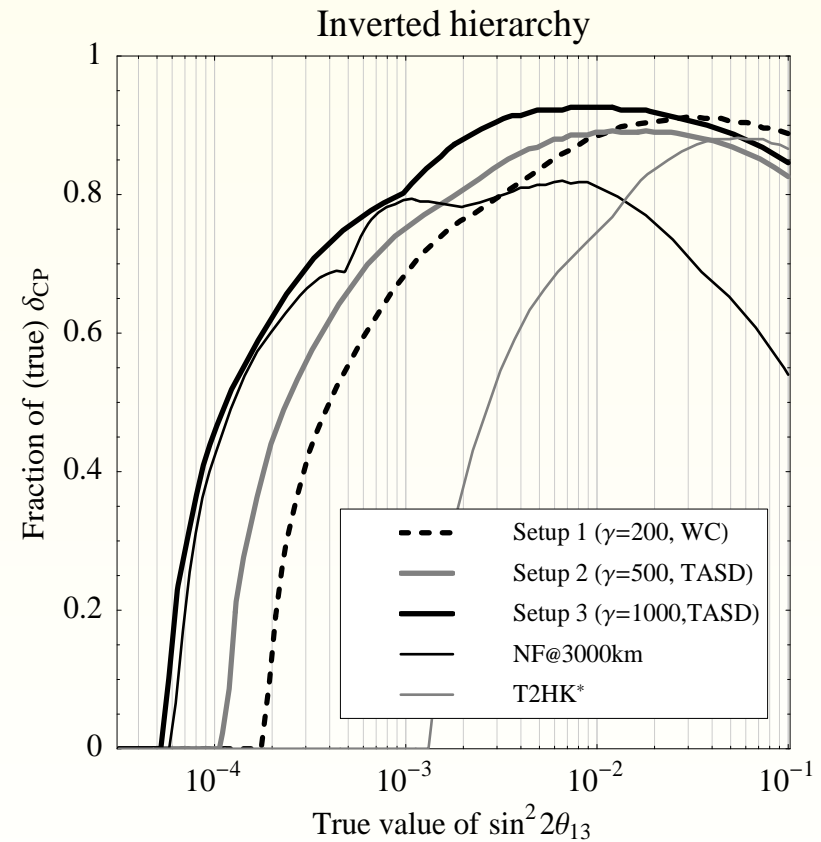
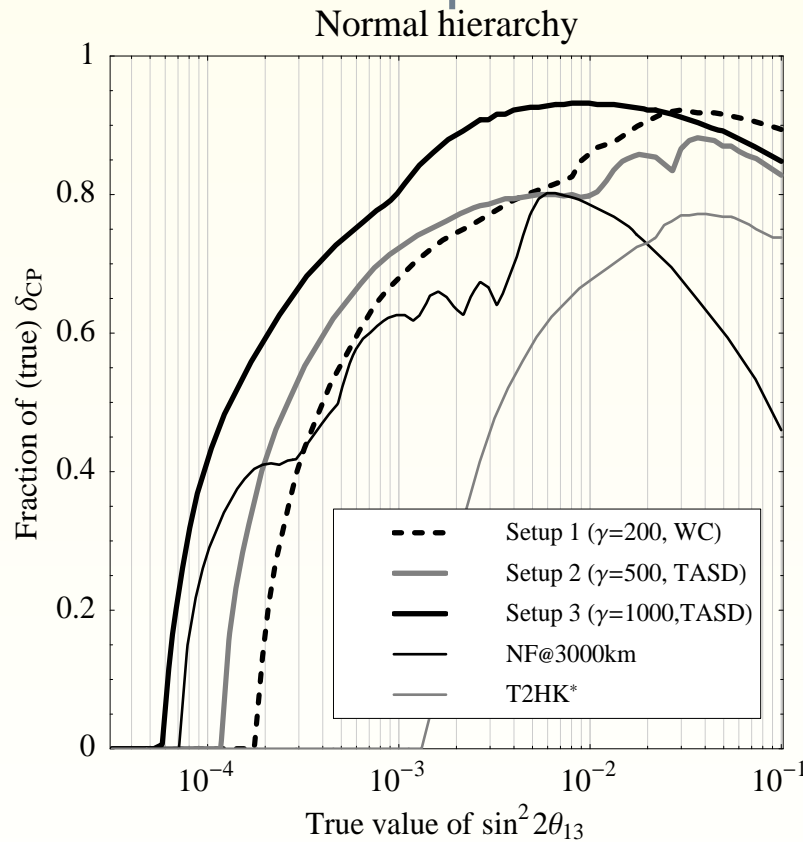
#	1	2	3
type	WC	TASD	TASD
$m$ [kt]	500	50	50
$\gamma$	200	500	1000
$L$ [km]	520	650	1000
$\nu$ signal	1983	2807	7416
$\nu$ background	105	31	95

The following results are taken from  
PH, M. Lindner, M. Rolinec, W. Winter,  
[hep-ph/0506237](https://arxiv.org/abs/hep-ph/0506237).



# CP Discovery Reach with Beta-Beams ( $P_{e\mu}$ )

- At magic baseline CP sensitivity is smothered
- Must move away from the magic baseline
- Which set-up could be best?

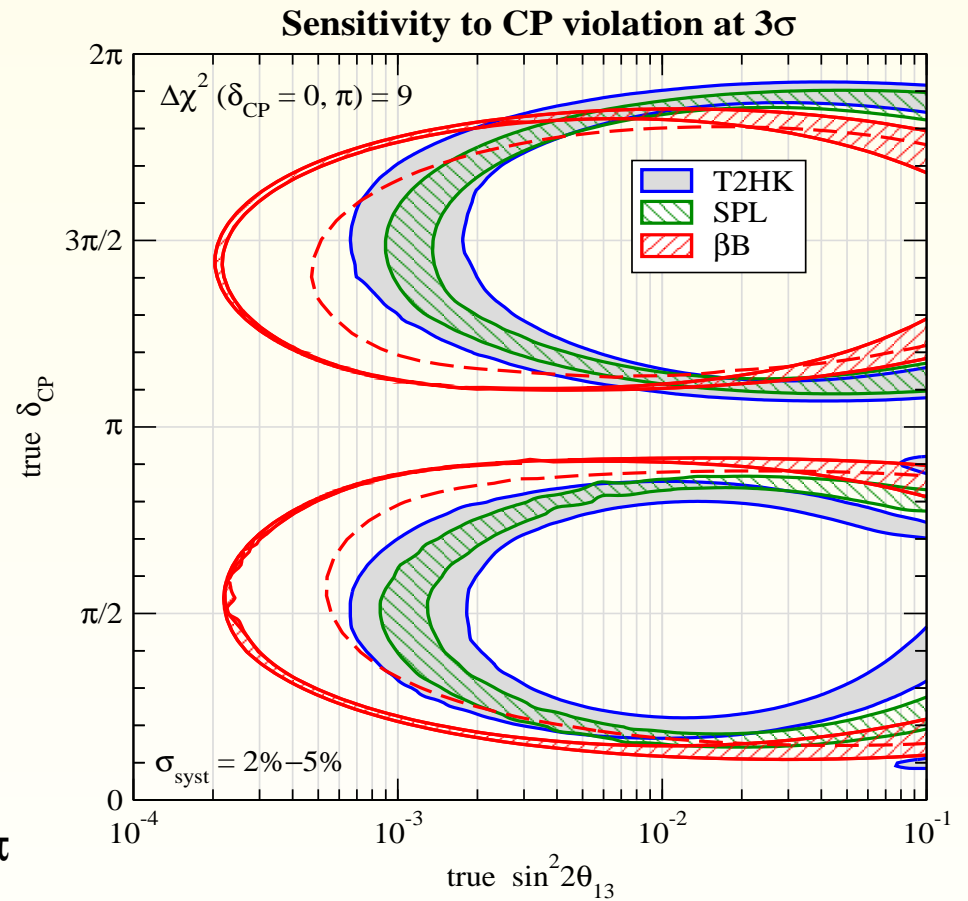
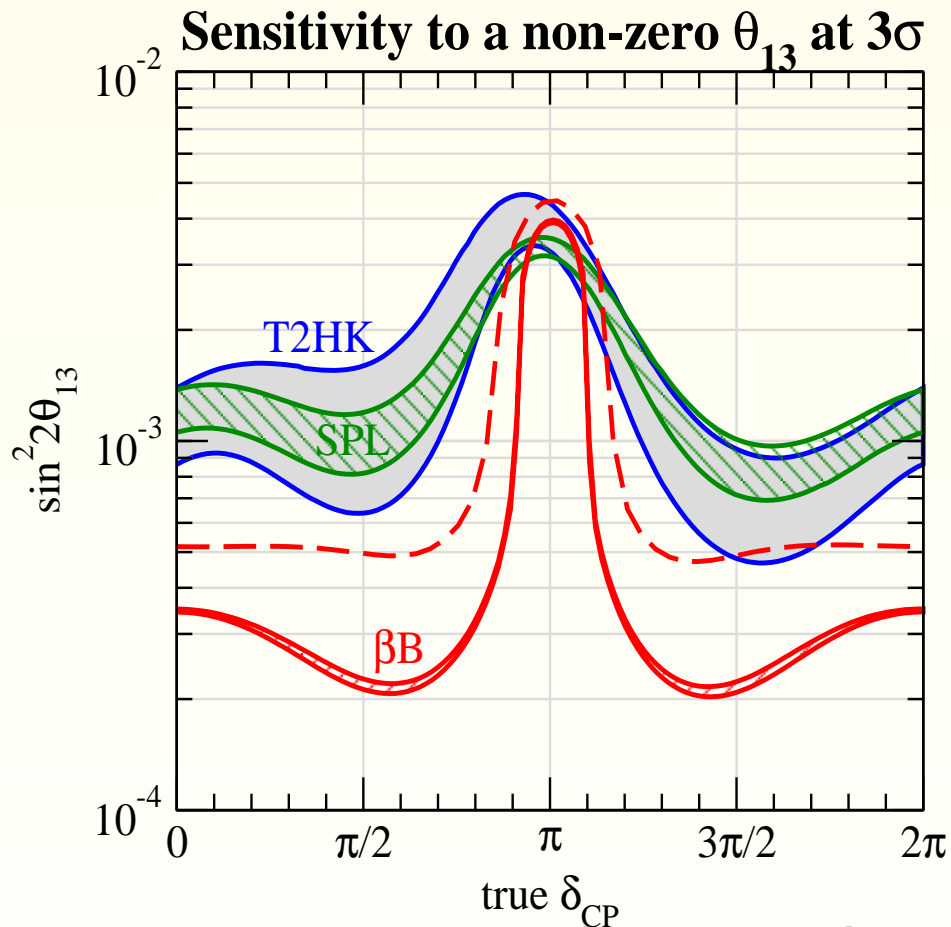


Huber *et al.*, hep-ph/0506237

- Beta-Beams CP sensitivity is similar to NuFacts

# Physics Potential of CERN-MEMPHYS

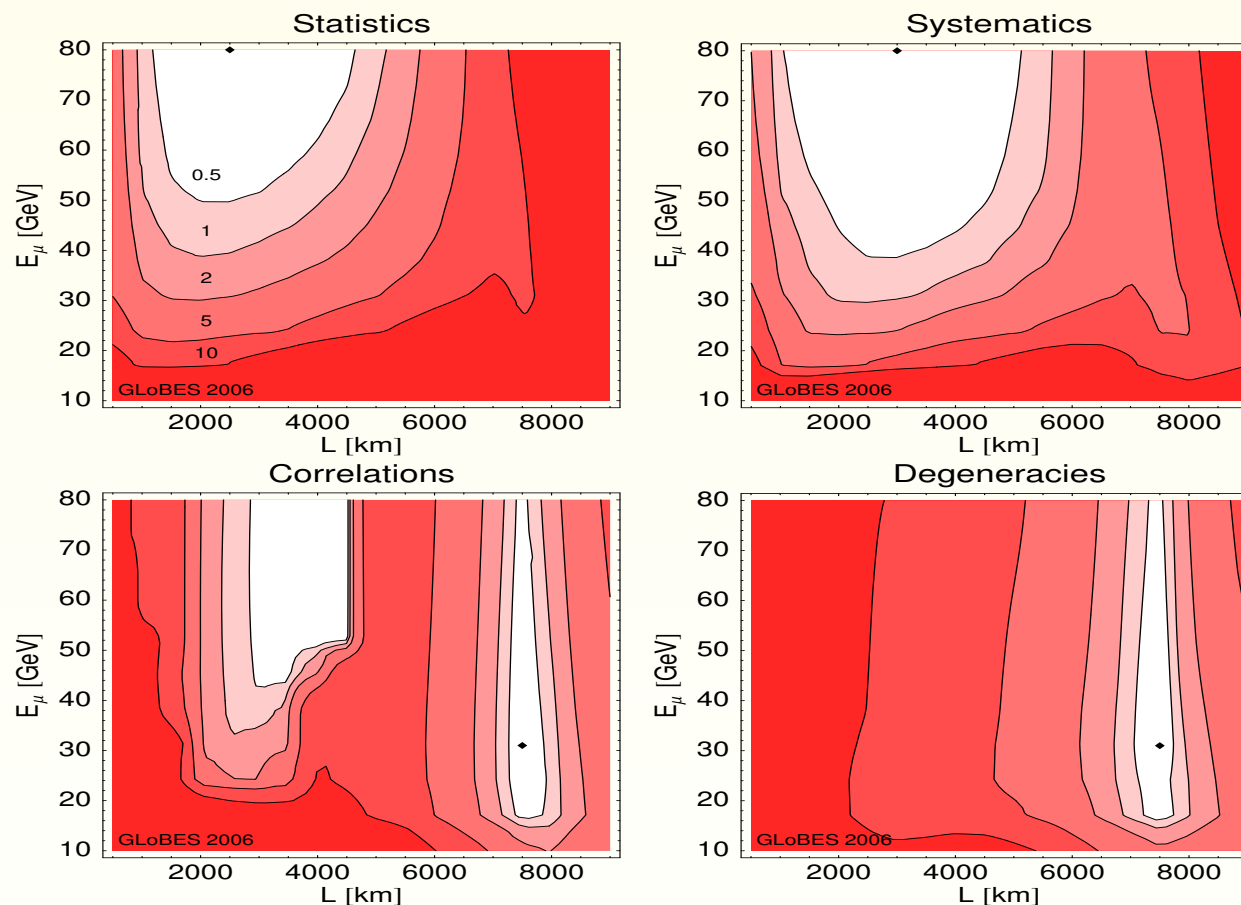
Campagne *et al.*, hep-ph/0603172



- $\gamma = 100$ ,  $L = 130$  km, 440 kton WC detector
  - Combined  $\beta B$  and SPL in only  $\nu$  mode takes 5 yrs only
  - The megaton water detector will also collect atm data
- helps in determining  $sgn(\Delta m_{31}^2)$  and octant of  $\theta_{23}$

# Physics Reach of Neutrino Factory

- Sensitivity to  $\sin^2 2\theta_{13} \lesssim 5.0 \times 10^{-4}$  ( $5\sigma$ )

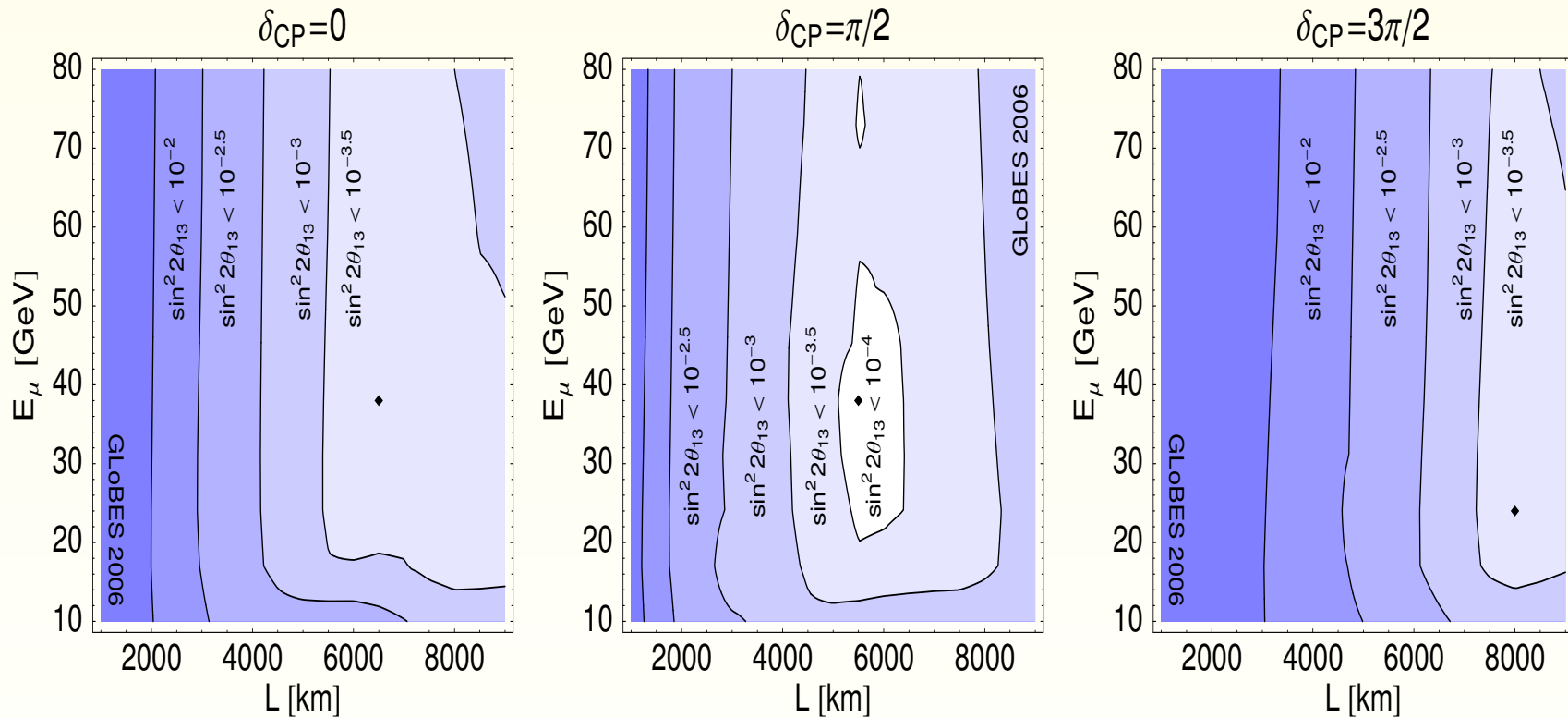


Huber *et al.*, hep-ph/0606199

- Best around  $E_\mu = 30$  GeV and close to magic baseline

# Physics Reach of Neutrino Factory

- Sensitivity to  $sgn(\Delta m_{31}^2) \gtrsim 1.8 \times 10^{-4} (3\sigma)$

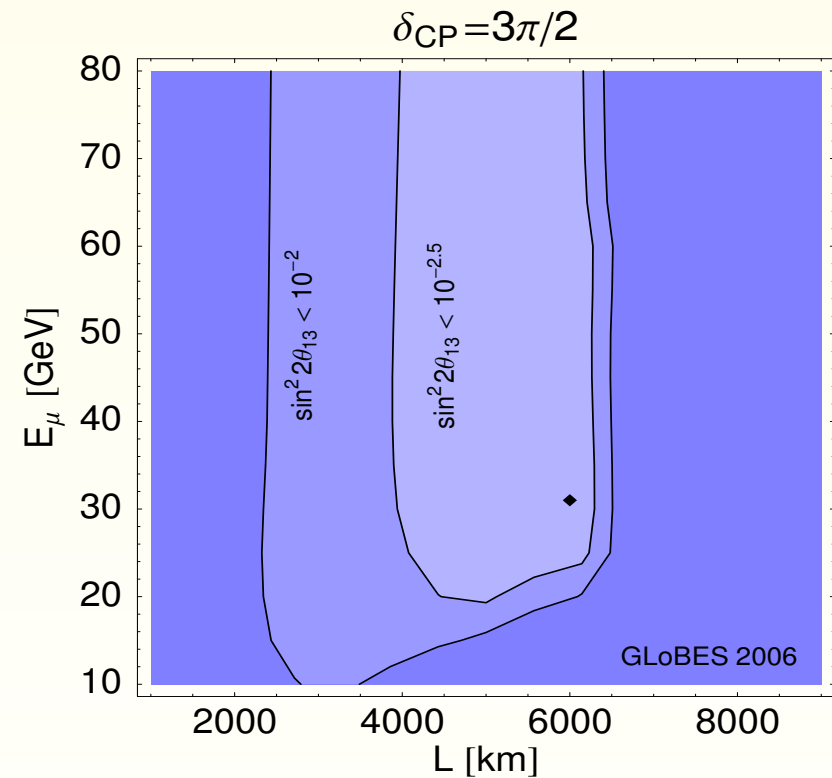
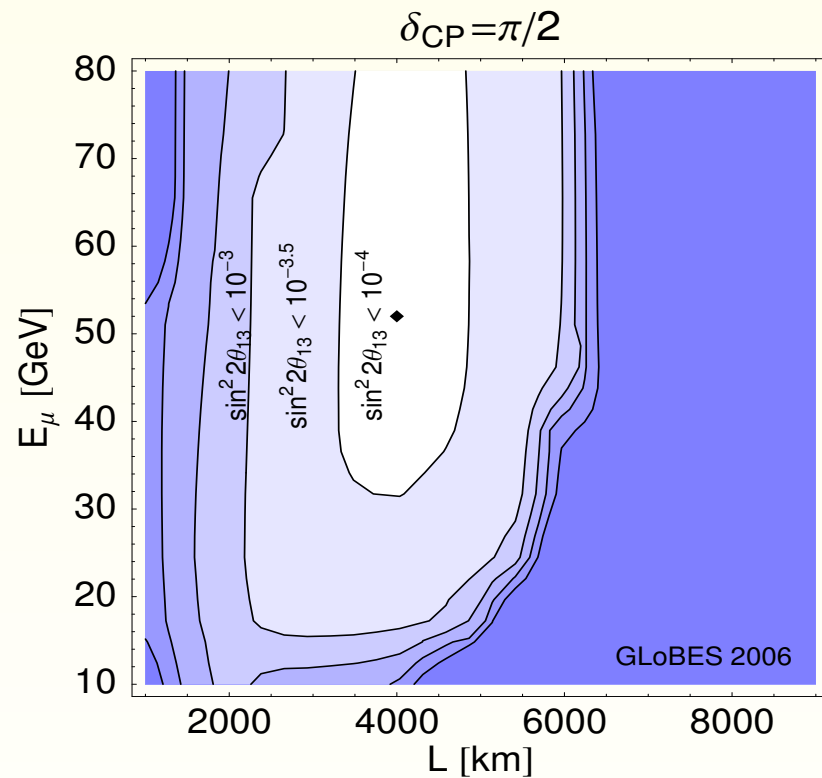


Huber *et al.*, hep-ph/0606199

- Best around  $E_\mu = 20 - 40$  GeV and close to magic baseline

# Physics Reach of Neutrino Factory

## ● CP Discovery Reach ( $3\sigma$ )

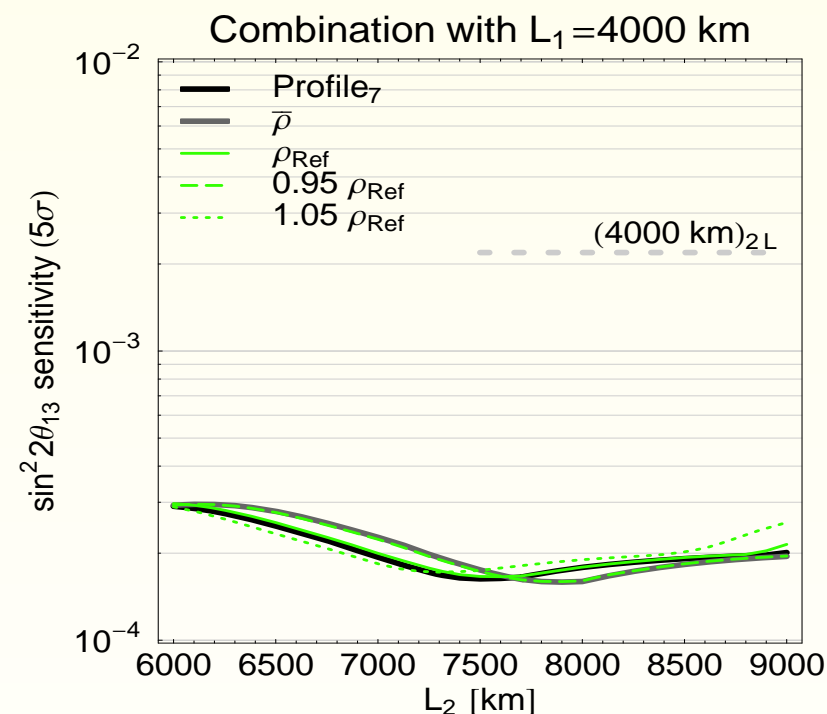
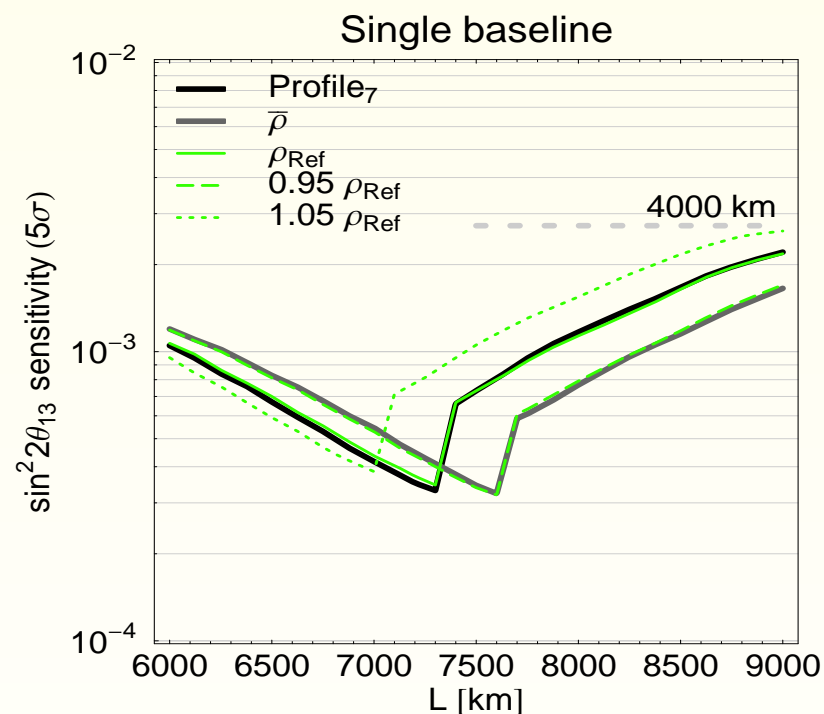


Huber *et al.*, hep-ph/0606199

## ● Best at around $L = 4000$ km

# Physics Reach of Neutrino Factory

- Combining baselines for best  $\theta_{13}$  sensitivity



Gandhi, Winter, hep-ph/0612158

- With two detectors, one at  $L = 4000$  km and another farther away, the range of “optimal” baselines widens