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Language: Fortran

The function ZHYPG2 computes the hypergeometric functions ${}_2F_1(a, 2; c, z)$ with complex parameters a, c and an argument z. For calculation of the hypergeometric function ${}_2F_1(a, 2; c, z)$ it used the beforehand calculated values of the hypergeometric functions and their derivatives by means of the subroutine HYPGEO [2] at points

$$z_1 = 1/2 + 2\imath,\ z_2 = 3/2\imath,\ z_3 = 1 + 3/2\imath,\ z_4 = -1/2 + \imath,\ z_5 = 1/2 + \imath,$$
$$z_6 = 3/2 + \imath,\ z_7 = -1 + 1/2\imath,\ z_8 = 1/2\imath,\ z_9 = 1 + 1/2\imath,\ z_{10} = 2 + 1/2\imath, \qquad (1)$$
$$z_{21-i} = \overline{z}_i,\ i = 1, \ldots, 10.$$

Methods:

The considered procedure was constructed on the basis of the algorithms published in [3]. The described algorithm allows one to save calculation time of the multidimensional integrals, whose kernel contains hypergeometric functions ${}_2F_1(a, 2; c, z)$, approximately 10–60 times depending on accuracy of the calculation $(10^{-4} - 10^{-14})$ in comparison with a direct use of the subroutine HYPGEO at each value z. This type of calculations occurs in investigation of the single and double ionizations by electron impact of two-nuclear molecules (see papers [4,5]).

References:

[1] Abramowitz M. and Stegun I. *Handbook of mathematical functions.* National Bureau of Standards Applied Mathematics series. 55. 1964.

[2] Press W.H., Teukolsky S.A., Vetterling W.T. and Flannery B.P. *Numerical recipes: The art of scientific computing.* Cambridge University Press, Cambridge, 1986.

[3] Chuluunbaatar O. Bulletin of Tver State University: Ser. Applied Mathematics. 2008, №. 26(86), pp. 47–64.

[4] Chuluunbaatar O., Joulakian B.B., Tsookhuu Kh. and Vinitsky S.I. J. Phys. B, 2004, v. 37, pp. 2607–2616.

[5] Chuluunbaatar O., Joulakian B.B., Puzynin I.V., Tsookhuu Kh. and Vinitsky S.I. J. Phys. B, 2008, v. 41, pp. 015204–1–6.


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Structure:

     FUNCTION
     Name: ZHYPG2
     Internal subroutines: ZSUM, ZGAMMA, DRHYP, CGAMA, HYPGEO, HYPDRV,
                           HYPSER, ODEINT, BSSTEP, MMID, PZEXTR

Usage:

     ZFUNC = ZHYPG2(ZA,ZC,ZZ,NMAX,EPS)
     INPUT: ZA, ZC, ZZ, NMAX, EPS:

 ZA      -- double complex number, contains value of parameter A.
 ZC      -- double complex number, contains value of parameter C.
 ZZ      -- double complex number, contains value of argument Z.
 NMAX    -- integer number, the maximum number of summation of truncated Gaussian
            series.
 EPS     -- double precision number, given accuracy.
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Test:

$$\int_0^3 dx \int_0^3 dy \exp(-x-y)\,{}_2F_1(-\imath, 2; 3 + 0.5\imath, x + \imath y) \qquad (2)$$