«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
1997, ТОМ 28, ВЫП.5

УДК 539.12.01

RENORMALIZATION OF BRS CHARGES

K.Nishijima, I.Demizu

Department of Physics, Chuo University,
Kasuga, Bunkyo-ku, Tokyo 112, Japan

Normalization of BRS charges and related renormalization group equations are discussed in connection with color confinement.

Обсуждаются нормировки БРС-зарядов и связанные с ними ренормгрупповые уравнения в связи с цветным конфайнментом.

1. INTRODUCTION

Quantum chromodynamics (QCD) is characterized, among other things, by BRS invariance and asymptotic freedom, and many of its fundamental properties are governed by renormalization group (RG). In a series of papers [1,2,3] and in a review article [4] the problem of color confinement has been discussed and we refer to these papers for mathematical details. It has been concluded that color confinement is an inevitable consequence of unbroken non-Abelian gauge symmetry and asymptotic freedom. A key feature of this theory consists in renormalization of BRS charges, and in this connection the renormalization constants of BRS charges and related RG equations are studied in this paper.

2. BRS INVARIANCE

The total Lagrangian density for QCD is given by

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}\cdot F_{\mu\nu} - \overline{\psi}(\gamma_\mu D_\mu + m)\psi + \partial_\mu B\cdot A_\mu + \frac{\alpha}{2} B\cdot B + i\partial_\mu \overline{c}\cdot D_\mu c, \qquad (2.1)$$

where both the color and flavor indices have been suppressed. B denotes the Nakanishi–Lautrup auxiliary fields, c or $\overline{c}$ are anticommuting Hermitian scalar fields called the Faddeev–Popov ghost fields, and the constant α stands for the gauge parameter. The covariant derivatives are defined by

$$D_\mu \psi = (\partial_\mu - igT\cdot A_\mu)\, \psi, \tag{2.2}$$

$$D_\mu c = \partial_\mu c + gA_\mu \times c, \tag{2.3}$$

where the antisymmetric cross product is defined in terms of the structure constant of the Lie algebra.

The BRS transformations of the gauge and quark fields denoted by δ and $\bar{\delta}$, respectively, are defined by replacing the infinitesimal gauge function by c or $\bar{c}$. For instance,

$$\delta A_\mu = D_\mu c, \quad \bar{\delta} A_\mu = D_\mu \bar{c}, \tag{2.4}$$

$$\delta\psi = ig(c\cdot T)\psi, \quad \bar{\delta}\psi = ig(\bar{c}\cdot T)\psi. \tag{2.5}$$

For the auxiliary fields, B, and c and $\bar{c}$, local gauge transformations are not defined, and their BRS transformations are so defined as to leave the total Lagrangian (2.1) invariant, namely,

$$\delta\mathcal{L} = \bar{\delta}\mathcal{L} = 0. \tag{2.6}$$

Then the BRS transformations of these auxiliary fields are given by [1,2,4]

$$\delta B = 0, \quad \delta\bar{c} = iB, \quad \delta c = -\frac{1}{2} g(c \times c), \tag{2.7}$$

$$\bar{\delta}\bar{B} = 0, \quad \bar{\delta} c = i\bar{B}, \quad \bar{\delta}\bar{c} = \frac{1}{2} g(\bar{c} \times \bar{c}), \tag{2.8}$$

where $\bar{B}$ is defined by

$$B + \bar{B} - ig(c \times \bar{c}) = 0. \tag{2.9}$$

The conserved BRS charges Q_B and $\bar{Q}_B$ are defined, respectively, as the generators of the corresponding BRS transformations by

$$\delta\phi = i[Q_B, \phi]_\pm, \quad \bar{\delta}\phi = i[\bar{Q}_B, \phi]_\pm, \tag{2.10}$$

where the $-(+)$ sign has to be chosen when the field ϕ is even (odd) in the ghost fields c and $\bar{c}$. Both charges are Hermitian and nilpotent. For instance, we have

$$Q_B^\dagger = Q_B, \quad Q_S^2 = 0. \tag{2.11}$$

The equation for the gauge field follows from the Lagrangian density (2.1) and can be expressed as

$$\partial_\mu F_{\mu\nu} + gJ_\nu = i\delta\bar{\delta}A_\nu, \tag{2.12}$$

where J_ν denotes the color current density.

It is then clear that we have

$$\partial_\lambda(\delta\bar{\delta}A_\lambda^\alpha) = 0. \tag{2.13}$$

3. RENORMALIZATION OF THE BRS CHARGE Q_B

The Lagrangian density (2.1) is apparently invariant under the scale transformation

$$c \to e^\lambda c, \quad \bar{c} \to e^{-\lambda}\bar{c}. \tag{3.1}$$

The corresponding Noether current is given by

$$j_\mu = i(\partial_\mu \bar{c}\cdot c - \bar{c}\cdot D_\mu c) \tag{3.2}$$

and satisfies the conservation law

$$\partial_\mu j_\mu = 0. \tag{3.3}$$

We define a conserved Hermitian operator Q_c by

$$Q_c = \int d^3x j_0(x). \tag{3.4}$$

This operator satisfies commutation relations of the form

$$i\,[Q_c, \phi] = N\phi, \tag{3.5}$$

where N denotes the number of the c fields minus that of the $\bar{c}$ fields involved in ϕ as factors. N is called the ghost number of the field ϕ and is clearly equal to an integer. The conservation of the ghost number implies that Green's functions or the vacuum expectation values of the chronological products of operators survive only when they involve equal numbers of c and $\bar{c}$ fields in the products.

Thus we may choose a convention in which c and $\bar{c}$ share the same renormalization constant

$$c^{(0)} = \tilde{Z}_3^{1/2} c, \quad \bar{c}^{(0)} = \tilde{Z}_3^{1/2} \bar{c}. \tag{3.6}$$

The gauge field is renormalized as

$$A_\mu^{(0)} = Z_3^{1/2} A_\mu. \tag{3.7}$$

In these equations the superscript (0) is attached to «unrenormalized» operators. On the basis of field equations derived from (2.1) we can derive the following identity:

$$\langle A_\mu^a(x), B^b(y) \rangle = -\delta_{ab} \partial_\mu D_F(x-y), \tag{3.8}$$

where D_F denotes the free massless propagator and a and b are color indices. This identity holds in both renormalized and unrenormalized versions of the operators A_μ and B, and we can derive from Eqs.(3.7) and (3.8) the relation

$$B^{(0)} = Z_3^{1/2} B. \tag{3.9}$$

Now we shall compare the following two identities in the renormalized and unrenormalized versions that follow from Eqs.(2.7):

$$\{Q_B, \bar{c}\} = B, \tag{3.10}$$

$$\{Q_B^{(0)}, \bar{c}^{(0)}\} = B^{(0)}. \tag{3.11}$$

By inserting Eqs.(3.6) and (3.9) in the above equations we find

$$Q_B^{(0)} = (Z_3 \tilde{Z}_3)^{-1/2} Q_B. \tag{3.12}$$

Thus we have obtained the renormalization constant for Q_B. It is necessary, however, to pay some attention to fix the renormalization constant for $\bar{Q}_B$.

4. RENORMALIZATION OF THE BRS CHARGE Q_B

So far we have introduced three conserved charges Q_B, $\overline{Q}_B$ and Q_c satisfying the following commutation relations:

$$i\,[Q_c, Q_B] = Q_B\,, \quad i\,[Q_c, \overline{Q}_B] = -\overline{Q}_B\,, \tag{4.1}$$

$$Q_B^2 = \overline{Q}_B^2 = Q_B\overline{Q}_B + \overline{Q}_B Q_B = 0. \tag{4.2}$$

They form a graded algebra called BRS algebra.

In the Landau gauge ($\alpha = 0$), however, Nakanishi and Ojima [5] have found two more conserved charges denoted by $Q(c, c)$ and $Q(\overline{c}, \overline{c})$, which are obtained by replacing $\overline{c}$ by c and $\overline{c}$ by c in Q_c, respectively. Here we shall give their commutation relations:

$$i\,[Q_c, Q(c, c)] = 2Q(c, c), \tag{4.3}$$

$$i\,[Q_c, Q(\overline{c}, \overline{c})] = -\,2Q(\overline{c}, \overline{c}), \tag{4.4}$$

$$[Q(c, c), Q(\overline{c}, \overline{c})] = 4iQ_c, \tag{4.5}$$

$$[Q(c, c), \overline{Q}_B] = 2iQ_B, \tag{4.6}$$

$$[Q(\overline{c}, \overline{c}), Q_B] = -\,2i\overline{Q}_B, \tag{4.7}$$

$$[Q(c, c), Q_B] = [Q(\overline{c}, \overline{c}), \overline{Q}_B] = 0. \tag{4.8}$$

In other gauges ($\alpha \neq 0$) these two extra charges are no longer conserved, but the above commutation relations are still valid. Thus the role played by the extended algebra is similar to that of current algebra.

Furthermore, they also satisfy the following commutation relations:

$$[Q(c, c), \overline{c}\,] = 2ic, \tag{4.9}$$

$$[Q(\overline{c}, \overline{c}), c] = -\,2i\,\overline{c}. \tag{4.10}$$

In the convention given by Eq.(3.6) we find non-renormalization of $Q(c, c)$ and $Q(\bar{c}, \bar{c})$ on the basis of these commutation relations. Thus we may avail ourselves of Eqs.(4.6) and (4.7) to conclude that the renormalization constants for Q_B and $\bar{Q}_B$ should be the same, namely, we have

$$\bar{Q}_B^{(0)} = (Z_3\tilde{Z}_3)^{-1/2}\bar{Q}_B \tag{4.11}$$

in conformity with Eq.(3.12)

5. RENORMALIZATION GROUP

Having fixed the renormalization constants for the BRS charges we can write down the RG equations for various Green's functions. We shall give some examples related to color confinement.

First of all we have

$$\delta\bar{\delta}A_\mu^{(0)} = (Z_3\tilde{Z}_3)^{-1}Z_3^{1/2}(\delta\bar{\delta}A_\mu). \tag{5.1}$$

Thus we have, for instance,

$$\langle (\delta\bar{\delta}A_\mu^a(x))^{(0)}, A_\mu^b(y)^{(0)} \rangle = \tilde{Z}_3^{-1} \langle \delta\bar{\delta}A_\mu^a(x), A_\nu^b(y) \rangle . \tag{5.2}$$

Because of Eq.(2.13) we have a Ward-Takahashi (WT) identity of the form

$$\partial_\mu\langle i\delta\bar{\delta}A_\mu^a(x), A_\nu^b(y) \rangle = i\delta_{ab}C\partial_\nu\delta^4(x-y). \tag{5.3}$$

The above two-point function and consequently the constant C should satisfy the same RG equation, and we have

$$(\mathcal{D} - 2\gamma_{FP})C = 0, \tag{5.4}$$

where γ_{FP} denote the anomalous dimensions of the Faddeev–Popov ghost fields, and $\mathcal{D}$ is given by

$$\mathcal{D} = \mu\frac{\partial}{\partial\mu} + \beta\frac{\partial}{\partial g} - 2\alpha\gamma_V\frac{\partial}{\partial\alpha}, \tag{5.5}$$

in the well-known notation, and γ_V denotes the anomalous dimension of the gauge field.

In the unrenormalized version of Eq.(5.3) we have $C^{(0)} = 1$, but in the renormalized version we have

$$C = Z_3^{-1} + (\text{GIS term}), \tag{5.6}$$

where «GIS term» denotes the Goto–Imamura–Schwinger term [6,7]. Since the anomalous dimension of Z_3^{-1} is given by $2\gamma_V$ different from $-2\gamma_{FP}$ in Eq.(5.4) emergence of the GIS term is unavoidable. In Refs.2,3,4 it has been shown that C vanishes in gauges in which Z_3^{-1} vanishes, and this is a sufficient condition for confinement as has been discussed in detail [1,2,3,4]. It should be stressed here that color confinement is a sort of renormalization effect since unrenormalized C can never vanish.

In QED the ghost fields are free so that $\gamma_{FP} = 0$, and $C = 1$ also in the renormalized version indicating absence of confinement in QED. In QCD the constant C is gauge-dependent and vanishes in certain gauges [2,3,4].

Now we turn to three-point functions. In Refs.1,2,3,4 we have given a set of WT identities.

$$\partial_\lambda \langle \delta\bar{\delta}A_\lambda^a(x), \psi^\alpha(y), \overline{\psi}^\beta(z) \rangle = ig_R T^a_{\alpha\beta} [\delta^4(x-y) - \delta^4(x-z)] S_F(y-z), \tag{5.7}$$

$$\partial_\lambda \langle \delta\bar{\delta}A_\lambda^a(x), A_\mu^b(y), A_\nu^c(z) \rangle = g_R f_{abc} [\delta^4(x-y) - \delta^4(x-z)] D_{F_{\mu\nu}}(y-z), \tag{5.8}$$

where g_R denotes the RG improved coupling constant. These WT identities served to conclude that color confinement is realized when we can find gauges in which Z_3^{-1} and consequently C vanish.

In the unrenormalized version the constant g_R agrees with the unrenormalized coupling constant, but in the renormalized version g_R deviates from g, since it satisfies an RG equation

$$(\mathcal{D} - 2\gamma_{FP} - \gamma_V) g_R = 0. \tag{5.9}$$

Let us put

$$g_R = g + \Delta, \tag{5.10}$$

where Δ denotes a kind of GIS term, and we find

$$(\mathcal{D} - 2\gamma_{FP} - \gamma_V)\Delta = 2g\gamma_{FP} + (g\gamma_V - \beta). \tag{5.11}$$

In this case the precise value of g_R is irrelevant in the discussion of color confinement provided that g_R does not vanish identically, but it is generally different from g.

In QED, however, we have

$$\gamma_{FP} = 0, \quad \beta = e\gamma_V. \tag{5.12}$$

So that we have

$$e_R = e. \tag{5.13}$$

To conclude this paper we would like to emphasize that renormalization plays an essential role in realizing color confinement.

REFERENCES

1. **Nishijima K.** — Int. J. Mod. Phys., 1994, v.A9, p.3799.
2. **Nishijima K.** — Int. J. Mod. Phys., 1995, v.A10, p.3155.
3. **Nishijima K., Takase N.** — Int. J. Mod. Phys., 1996, v.A11, p.2281.
4. **Nishijima K.** — Czech. J. Phys., 1996, v.46, p.1.
5. **Nishijima K., Ojima I.** — Z. Phys., 1980, v.C6, p.155.
6. **Goto T., Imamura T.** — Prog. Theor. Phys., 1955, v.14, p.396.
7. **Schwinger J.** — Phys. Rev. Lett., 1959, v.3, p.296.