«ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА»
2000, ТОМ 31, ВЫП. 7А

УДК 539.14

QUANTUM FIELD THEORY AND SYMMETRIES IN NUCLEAR PHYSICS

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Nuclear physics embraces a wide area of knowledge ranging from fundamental problems of matter structure up to the origin of the universe. Applied aspects of this science bear a direct relation to the most urgent problems of people's life — ecology and energetics. The present talk deals with one of these aspects, namely, a possible description of the properties of nuclear matter by means of the methods of modern mathematical physics which N.N.Bogoliubov has greatly contributed to.

The problem of describing nuclear processes, like all other physical processes, is solved on the basis of the construction of the space of defining parameters linking real physical objects. Nuclear physics originates from the discovery of the Mendeleev Periodic Law in which the parameters: atomic weight A and charge Z, play the fundamental role in the description of atomic properties. The proton-neutron structure of all the nuclei, including the synthesized ones, is given in the Figure, as a function of the parameters $A - Z$ and Z. The creation of quantum mechanics has resulted in the introduction of the quantum parameters of the ground and excited states of atomic nuclei. Later on, it was found that it was necessary to introduce the concept of non-nucleon degrees of freedom, as well as the concept of quark-gluon or colour degrees of freedom of nuclei. Then, the idea itself that matter consists of elementary particles has undergone essential changes. However the idea that the primary concept of physics is the concept of space has kept its fundamental importance. The comparison of the defining parameters' space with the mathematical one is the essential point of the construction of mathematical models.

Complicated real physical situations require simplified descriptions and determinations of the region of validity (measurability) of the introduced concepts. We have to define the region of applicability of the concept «elementary particle». By tradition, the elementary particles are taken to mean indecomposable structure constituents of matter. This concept has been formed in a close connection with the idea about the discrete structure of matter at the microscopic level. When constructing models the elementary particles are thought of as absolutely identical and their ensembles are described by the quantum fields which are just the basis of the mathematical space of a model. However, quantum field theory is suc-

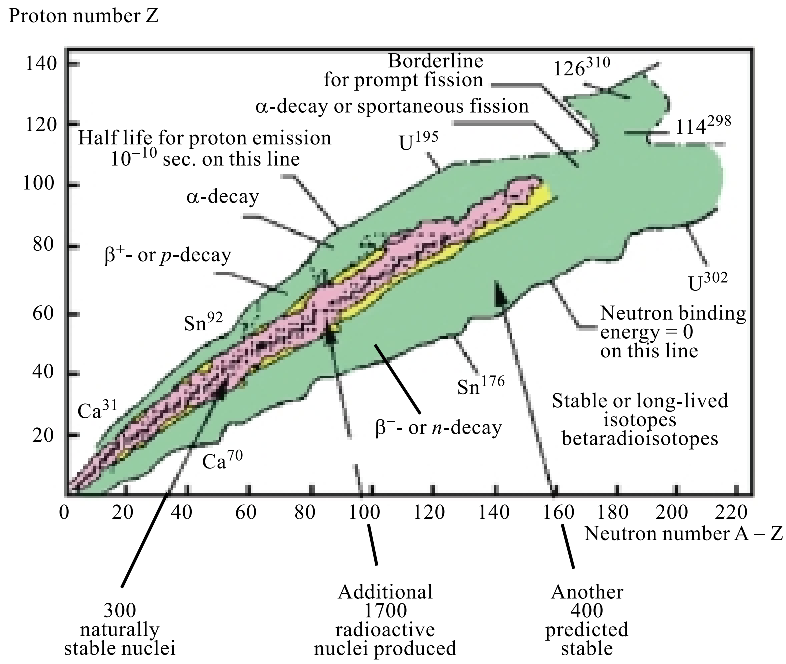

Fig. 1.

cessfully applied to both particles possessing inherent structure and decomposable objects, for example, helium atoms at low temperatures.

In atomic physics, the criterion that restricts the applicability of such an approach is the smallness of the kinetic energy of relative motion in comparison with the energy of the first excited level of the atom. In the opposite case, the interaction of atoms with one another results in a violation of the identity and it becomes necessary to enlarge the parameters' space. A relativistic generalization of the criterion of applicability of the concept «elementary particle» can be obtained by using the four-momentum conservation law $p_1 + p_2 = p_3 + p^*$:

$$(p_1 + p_2)^2 = (p_3 + p^*)^2.$$

From the definition of the threshold for the creation of an excited state of one of the colliding particles we have

$$(p_1 + p_2)^2 = (p_3 + p^*)^2 = (m + m^*)^2$$

from where

$$b_{12} = -(u_1 - u_2)^2 = \frac{m - m^*}{m} \left[4 + \frac{m - m^*}{m}\right] \ll 1. \qquad (1)$$

Here, m are the masses of the identical particles, and m^* is the mass of the excited state; p_1, p_2, p_3, and p^*, their momenta, respectively; $u_i = p_i/m_i$, the four-velocity vectors.

The four-velocity space is fundamental for describing relativistic multiparticle production processes. The criterion (1) is formulated in terms of invariant, dimensionless and measurable quantities, it does not involve parameters like the particle size, the degree of pointlikeness, the spacing, and so on.

On the basis of the criterion (1), we obtain the following classification of the nuclear systems:

— The region $0 \leq b_{ik} \leq 10^{-2}$ corresponds to nonrelativistic nuclear physics. Nucleons can accurately be treated as elementary particles.

— The region $b_{ik} \sim 1$ corresponds to excitation of the internal hadron (isobar, resonance) degrees of freedom. It is necessary to introduce non-nucleon degrees of freedom — the field quanta different from nucleon ones.

— The region $b_{ik} \gg 1$ corresponds to dominance of quark and gluon fields, that is, of quanta carrying color.

Values $b_{ik} \sim 10^{-9}$ characterize atomic physics. Here, for example, helium atoms lose electrons and are transformed from bosons into fermions. In relativistic nuclear physics, one collision process involves all relative velocities b_{ik} and, respectively, very different quanta.

Particle and nuclear physicists use the basic theoretical framework to describe the behavior of quantum system in quantum field theory. A basis for the Hilbert space of the system of arbitrary numbers of particles is composed of the following states:

$|\, O\rangle$ the «no particle» state,

$|\, p\rangle$ the «single particle» states,

$|\, p_1 p_2\rangle$ the «two-particle» states,

$\vdots$

$|\, p_1 \quad \ldots \quad p_N\rangle$ the «N-particle» states.

$\vdots$

The norm of this Hilbert space is

$$\langle p \mid p' \rangle = (2\pi)^3 \delta^3(\vec{p} - \vec{p}\,') 2E_p,$$

$$\langle p_1, p_2 \mid p_1{}', p_2{}' \rangle =$$
$$= (2\pi)^6 2E_{p_1} 2E_{p_2} \left\{ \delta^3(\vec{p_1} - \vec{p_1}{}') \delta^3(\vec{p_2} - \vec{p_2}{}') + \delta^3(\vec{p_1} - \vec{p_2}{}') \delta^3(\vec{p_2} - \vec{p_1}{}') \right\}$$

and the obvious generalization to the other states. This Hilbert space is called a Fock space. A general state in this space is

$$| \Psi\rangle = \Psi_0 \mid O\rangle + \int \frac{d^3p}{(2\pi)^3 2E_1} \cdot \Psi(\vec{p}) \mid p\rangle +$$
$$+ \frac{1}{2!} \int \frac{d\vec{p}_1 d\vec{p}_2}{(2\pi)^6 2E_1 \cdot 2E_2} \Psi_2(\vec{p}_1, \vec{p}_2) \mid p_1, p_2\rangle + \dots$$

A more convenient notation is to label the states by $N(p)$ which «counts» particles with momentum p

$$\hat{P}^\mu = \int \frac{d^3p}{(2\pi)^3 2E} \hat{N}(p) p^\mu$$

that can be expressed by introducing the creation and annihilation operators as basic operators from which we shall construct all observables

$$\left[\hat{a}_p, \hat{a}^+_{p'}\right] = (2\pi)^3 2E \delta^3(\vec{p} - \vec{p}\,'),$$
$$\hat{N}(p) \equiv \hat{a}^+_p \cdot \hat{a}_p,$$

$\hat{a}^+_p$ creates an extra particle and $\hat{a}_p$ annihilates particle with momentum p,

$$\left[\hat{N}(p), \hat{a}^+_p\right] = (2\pi)^3 \delta^3(\vec{p} - \vec{p}\,') \cdot 2E_p \cdot \hat{a}^+_p.$$

From these definitions it follows that an ensemble consisting of massless particles possesses a mass. For example, the eigenvalue of the operator of the four-momentum of a system consisting of two photons with momenta k_1 and k_2 is

$$\hat{P}^\mu \mid k_1, k_2\rangle = (k_1^\mu + k_2^\mu) \mid k_1, k_2\rangle = P^\mu \mid k_1, k_2\rangle$$

and

$$P^\mu \cdot P_\mu = (k_1 + k_2)^2 = 2(k_1 \cdot k_2).$$

Experimentalists measuring the mass of a neutral π meson decaying into two photons have been knowing this fact for a long time. Another example is the discovery of the electron-positron pair production which has made it possible to define the «positive electron» mass and has signified the necessity of expanding the Fock space to electron field quanta, that is, the creation of the Maxwell–Dirac electrodynamics (QED).

In nuclear physics, the introduction of color fields*, the quanta of which have a negligibly small mass, has made the Fock space as a basis for constructing quark models (quark-parton model, models on the light cone, and so on).

*See the talk of N.N.Bogoliubov at a general meeting of the Academy of Sciences of the USSR on March 1985, JINR Communications D2-85-206, Dubna, 1985.

The obtaining of the color field Lagrangian on the basis of gauge symmetry has resulted in the formulation of quantum chromodynamics (QCD). Hovewer, the analogy between QED and QCD is far from being total. The auxiliary conditions which are to be imposed on the solutions of the Eiler–Lagrange equations are cardinally different in QCD and QED. Most of all, this concerns the definition of the quark and gluon asymptotic states. Without additional hypotheses on the boundary and initial conditions, on the quark-gluon structure of hadrons, and on the transformation of quarks and gluons into hadronic jets it is impossible to connect QCD with observable processes.

In solving differential equations it is necessary to take into account the properties of the space as a whole. In nonlinear models, to which QCD is attributed, there arise extended localized structures: solitons, vortices, instantons, skyrmeons, and so on.

In the 1930's and 1940's L.S.Pontrjagin and other mathematicians have discovered, without undergoing the influence of physical models, interesting topological invariants playing an ever-growing role in modern physics.

Merging of the newest areas of mathematics and theoretical physics enables us to hope that, along this way, one will succeed in finding an approach to nonperturbative solutions of QCD.

The perturbative QCD solutions are based on a specific dependence of the «invariant charge», discovered by N.N.Bogoliubov and D.V.Shirkov [1] and unhappily named «running coupling constant», on the momentum transfer. The decrease of the running coupling constant at large momentum transfers predicted theoretically and confirmed by experiment has given rise to a very important concept — «asymptotic freedom». Unlike the invariant charge, topological invariants are not the invariants of the Lee group. However both are additional conditions on the solutions.

The topological integral of motion is a particle number N in dynamics where the processes of production and annihilation of new particles are eliminated. This law of conservation is important in nonrelativistic nuclear physics.

As a hypothesis about the properties of the solutions of statistical physics N.N.Bogoliubov has formulated the correlation depletion principle [2]. The principle is based on the intuitive idea that the correlation between spatially separated groups of particles of a microscopic system practically vanishes. The correlation depletion principle was successfully applied to the development of the theory of ferromagnetism, superfluidity and superconductivity. Also it is possible to formulate the notion of quasi-averages and the properties of the solutions that afterwards were given the name of spontaneous symmetry breaking. It is interesting that the well-known attempt of Dirac to formulate a relativistic theory of dynamical systems [3] led him to the realization that it was possible to state only the necessary but not the sufficient conditions for this theory to exist. At the end of his remarkable article, Dirac writes, «Some further condition is needed to ensure that the interaction between two physical objects becomes small when the objects become far apart. It is not clear how this condition can be formulated mathematically». Bogoliubov's correlation depletion principle is formulated as an asymptotic form of the Green functions as universal (independent of the specific features of the system) linear form from averages of the product of field functions. This principle gives mathematical formulation for the additional condition of the relativistic theory (Poisson's brackets) developed by Dirac.

In Refs. 4, 5 the correlation depletion principle is formulated in both the relative four-velocity space and the Lobachevsky space. The application of this principle to quantum chromodynamics of large distances (or, more precisely, of small relative velocities), to the description of multiple particle production processes, and, particularly, to relativistic nuclear physics was found to be especially productive. In these areas, the perturbative approach does not work, thus hypotheses of a fundamental character, i.e., auxiliary conditions, are needed. A collision of relativistic nuclei results in the production of many particles, and the interaction picture is very complicated. Both nucleon and quark-gluon degrees of freedom participate in the same collision. The number of the parameters of the problem is extremely large, and it is particularly important to discover the invariants.

Relativistic nuclear physics that was born at the beginning of the '70s at Dubna became one of the most intensively developed areas of high energy physics in many laboratories of the world. The discovery of the laws of relativistic nuclear physics is a part of the general search for the laws describing relativistic multiparticle systems. These problems were studied by outstanding scientists of the 20th century. The first studies were devoted to the transport equations which allowed the formulation of the thermodynamic properties of dilute relativistic multiple systems. The great success of quantum field theory in describing multiparticle systems on the basis of the Hamiltonian method has not resulted however in great progress in the development of the problems of relativistic nuclear physics.

In Refs. 5, 6 it is shown that the approach to relativistic nuclear physics based on the geometry of velocity space and the hypotheses about the asymptotic nature of the laws in this space allows us to put in order an enormous amount of experimental data and make quantitative predictions. Some of these predictions make many experiments on huge accelerators unnecessary and even condemned to failure. The methods of symmetry of the solutions utilized in these papers are analogous to the methods of the mechanics of continuous media.

In the case of relativistic nuclear physics, the defining parameters are the cross sections, quantities derived from them, and the invariant dimensionless intervals in relative four-velocity space $\vec{u}_i = \vec{p}_i/m_i$; $u_i{}^0 = E_i/m_i$:

$$b_{ik} = -(u_i - u_k)^2 = 2\left[(u_i \cdot u_k) - 1\right] = 2\left[\frac{E_i \cdot E_k - \vec{p}_i \cdot \vec{p}_k}{m_i \cdot m_k} - 1\right].$$

As far as the energies E_i and the momenta $\vec{p}_i$ are linked by the known relation ${E_i}^2 - {\vec{p}_i}^{\ 2} = {m_i}^2$, then $(u_i)^2 = (u_o)^2 - (\vec{u}_i)^2 = 1$. Instead of the four-dimensional space it is possible to introduce a three-dimensional one with a fourth coordinate expressed in terms of the other three:

$$u_i^0 = \pm\sqrt{1 + u_x^2 + u_y^2 + u_z^2}. \tag{2}$$

This equation is a two-sheeted hyperboloid. The geometry on the surface of the hyperboloid is the geometry of the three-dimensional Lobachevsky space, analogous to the geometry on the surface of a sphere. The interval between the points on the surface of a sphere is given by the cosine of the angle of the great circle, and the interval on the surface of the hyperboloid is given by the hyperbolic cosine of the rapidity

$$\rho = \frac{1}{2}\ln\frac{E + \mid \vec{p} \mid}{E - \mid \vec{p} \mid}.$$

The relation between the intervals b_{ik} and ρ_{ik} is of the form:

$$b_{ik} = 2\left[(u_i \cdot u_k) - 1\right] = 2\left[ch\rho_{ik} - 1\right].$$

The number of the parameters of b_{ik} is $n(n-1)/2$. The most complete description of the final states of nuclear collisions is connected with the use of triangulation and the construction of polyhedra in velocity space.

The introduction of the variables N_I and N_{II} characterizing the effective numbers of particles participating in the collisions of nuclei I and II has proved to be very productive. In a wide interval of relative velocities, the additional variables N_I and N_{II} turned out to be continuous and smooth.

The invariant that is employed to express a large number of the laws of relativistic nuclear physics has the meaning of the minimal mass

$$min\ \left[m_0^2(u_I N_I + u_{II} N_{II})^2\right]^{1/2} = 2m_0 \prod$$

under the condition of conservation of four-momentum:

$$m_0 u_I N_I + m_0 u_{II} N_{II} = \sum_i p_i.$$

Here, u_I and u_{II} are the four-velocities of the nucleus as a whole, m_0 is the mass of one nucleon. The introduction of the single self-similarity parameter (invariant)

$$\prod = \frac{1}{2}\sqrt{(u_I N_I + u_I N_{II})^2} \tag{3}$$

allowed a quantitative description of the cumulative effect, deep subthreshold, near-threshold phenomena, and antimatter production in nucleus-nucleus collisions. Of special interest is the prediction, on this basis, of the results of future experiments on nuclear colliders that are presently being designed.

Building of nuclear colliders and huge detectors is motivated by the possibility of obtaining at $b_{I,II} \gg 1$ of an extremely excited nuclear matter — quark-gluon plasma. The quantitative predictions based on the dependence of the cross sections upon the invariants (3) make it possible to conclude that the hopes for obtaining dense and hot matter in heavy ultrarelativistic nuclear collisions will not be realized (see the talk by A.I.Malakhov at a Parallel Session of the present Conference).

The best studied domain of nuclear physics that corresponds to nucleon relative motion, characterized by the criterion $b_{ik} \gtrsim< 10^{-2}$, contains a large amount of theoretical approaches (models), results and research goals. In a nonrelativistic approach, the nucleus is thought of as a system, consisting of a definite number of nucleons.

The Hamiltonian of a nonrelativistic nuclear physics is of the form:

$$H = \sum_{f_1 f'} \{T(f_1 f') - \lambda \delta_{f_1 f'}\} \hat{a}^+_{f_1} \hat{a}_{f'} - \frac{1}{4} \sum_{f_1 f_2 f_2' f_1'} V(f_1 f_2; f_2' f_1') \hat{a}^+_{f_1} \hat{a}^+_{f_2} \hat{a}_{f_2'} \hat{a}_{f_1'},$$

where $\hat{a}^+_f$ and $\hat{a}_f$ are the nucleon creation and annihilation operators, f is the set of quantum numbers describing the nucleon state, and λ stands for the chemical potential.

Using geometric, kinematic and dynamic symmetries one succeeds, to a large extent, in putting in order the nuclear level system and essentially simplifying the finding of the solutions describing a broad spectrum of phenomena of nuclear physics. The Hartree–Fock variational method is one of the fundamental approaches to the study of the many-body problem. This method is used to find the energy minimum with the aid of a class of one-particle wave functions. In this case, pairing and more complicated correlations are not taken into account.

N.N.Bogoliubov has suggested a new variational principle, a natural generalization, of the Hartree–Fock method. According to Bogoliubov's method, the energy minimum is found with the aid of a wider class of functions: in addition to the one-particle wave functions, the wave functions of pairs of particles are taken into account. The method, named the Hartree–Fock–Bogoliubov method, was discussed at a Parallel Session devoted to nuclear physics.

Special attention should be given to the influence of Bogoliubov's ideas and methods on nuclear physics. In Ref. 7 Bogoliubov has suggested that the mathematical methods, developed in constructing the superconductivity theory are very general and may be applied to the description of nuclear matter. This idea has initiated the study of the effect of superfluidity on the description of the ground and excited nuclear states. Later on, it was shown that the superconducting pairing correlations are of great importance in medium and heavy nuclei. At the Conference devoted to the centenary of the discovery of Mendeleev's Table (Tokino–Roma 15–21 September 1969), in his talk, Bogoliubov has described the main results obtained by V.G.Soloviev and his colleagues concerning the development of the nuclear superfluid model. There are also given concrete physical results based on experimental data. (For review see [8]).

Bogoliubov's idea about the existence of bosons in the nucleus and about a possible consideration of the ground states of even-even nuclei as boson condensates was further developed as applied to supersymmetry in nuclear physics [9, 10].

The supergroups are relevant to mixed systems of bosons and fermions. The bosons are the low-lying collective degrees of freedom of a heavy nucleus. Six dynamical bosons, namely scalar, I=0 , (called s) and quadrupole, I=2 , (called d) are assigned to the six-dimensional representation of $U(6)$. The boson creation and annihilation operators are:

$$\hat{b}^+_\alpha\ (\hat{b}_\alpha), \qquad \alpha = 1, \ldots, 6 \qquad\qquad \text{(Bogoliubov's bosons).}$$

The 36 generators of $U(6)$ are:

$$G^{(B)}_{\alpha\alpha'} = \hat{b}^+_\alpha\ \hat{b}_{\alpha'}.$$

The dimension of the fermionic degrees of freedom is $m = \sum_i (2j_i + 1)$.

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$$j\ =\ 5/2,\ 7/2,\ 11/2,\ 3/2,\ 1/2.$$

The creation and annihilation operators for fermions are denoted as

$$\hat{a}^+_i\ (\hat{a}_i), \qquad i = 1, \ldots, m.$$

The m^2 generators of $U(m)$ are:

$$G^{(F)}_{ii'} =\ \hat{a}^+_i\ \hat{a}_{i'}.$$

The mixed problem of bosons and fermions is described by the Hamiltonian

$$H = H_B + H_F + V_{BF}, \quad H_B = H_0 + \sum_{\alpha\alpha'} \in_{\alpha\alpha'} G^{(B)}_{\alpha\alpha'} + \sum_{\alpha\alpha'\beta\beta'} U_{\alpha\alpha'\beta\beta'} G^{(B)}_{\alpha\alpha'} G^{(B)}_{\mu\beta'},$$

$$H_F = H'_0 + \sum_{ii'} \eta_{ii'} G^{(F)}_{ii'} + \sum_{ii',kk'} \nu_{ii'kk'} G^{(F)}_{ii'}, \quad V_{BF} = \sum_{\alpha\alpha' ii'} \omega_{\alpha\alpha' ii'} G^{(B)}_{\alpha\alpha'} G^{(F)}_{ii'}.$$

The supergroup appropriate to nuclear problems appears to be $U(6/m)$ in the matrix form

$$\begin{pmatrix} b^+b & b^+a \\ a+b & a+a \end{pmatrix}.$$

The Bose sector of the algebra is $U^{(B)}(n) \times U^{(F)}(m)$.

If the supersymmetry scheme applies, all states in the supermultiplet should be described by the same energy formula corresponding to the chain of subgroups:

$$U(6/4) \supset U^{(B)}(6) \oplus U^{(F)}(4) \supset SO^{(B)}(6) \oplus SU^{(F)}(4)$$
$$\supset spin(6) \supset spin(5) \supset spin(3) \supset spin(2).$$

In quantum field theory and particle physics, supersymmetry implies somewhat different mathematical constructions. Search for supermultiplets uniting bosons and fermions brings these concepts together. Nevertheless, Yu.A.Gol'fand, one of the discoverers of supersymmetry, in his paper «Supersymmetry» published in Physical Encyclopaedia, remarks that the prefix «super» in this word bears no semantic load at all. In nuclear physics, supermultiplets are found among the low-lying levels of complex nuclei. Search for supermultiplets in the elementary particle physics is the most difficult and extremely expensive problem of the high-energy experimental physics.

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