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GENERATING FUNCTIONALS METHOD OF N.N.BOGOLIUBOV AND MULTIPLE PRODUCTION PHYSICS

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The generating functionals (GF) method in Bogoliubov's formulation and its application for particle physics is considered. Effectiveness of the method is illustrated by two examples. So, GF method can be used as the technical trick solving the infinite sequence of algebraic equations. We will consider the example, where GF allows one to express the multiplicity distributions (topological cross sections) through the particles correlation functions (inclusive cross sections) to «predict» the so-called Koba–Nielsen–Olesen scaling. We will use the GF to define validity of the thermal description of the multiple production phenomena also. It will be seen that this will lead to the «correlations relaxation condition» of N.N.Bogoliubov. This will allow one to offer the experimentally measurable criteria of applicability of thermodynamical description of multiple production processes. As a result, we will find the closed form of perturbation theory applicable for kinetic phase of nonequilibrium processes. A way is shown how the approach may be adapted to the definite external conditions.

1. INTRODUCTION

It is hard to imagine modern particle physics without such fundamental notions as, for instance, the phase transitions, topological defects, taken from statistical physics. This extremely fruitful connection among two branches of physics is based on the Euclidean postulate [1]: the formulae of particle physics coincide with corresponding formulae of statistical physics if the transformation $t \to it$ is applied. But this coincidence exists if the media is equilibrium only, since the time order of physical process is lost after the transition to imaginary time it. So, the particles static properties only can be considered by Euclidean field theories.

The Euclidean postulate does not «work» for arbitrary element of S matrix and, by this reason, there is no, at first glance, general connection between particles and statistical physics. Our aim is to demonstrate this connection considering the multiple production example, staying in the real-time theory frame.

The multiple production is a typical dissipative process of the incident kinetic energies transition into the energies (masses) of produced particles. This is the

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*Permanent address: Inst. of Physics, Tbilisi, Georgia nonequilibrium process and the fluctuations, generally speaking, may be high in it. Experimental data confirms this general expectation at the mean multiplicities region, when $n \sim \bar{n}$ [2].

Considering multiple production we would like to note firstly that the mean multiplicity $\bar{n}$ of hadrons for modern accelerator energies ($\sim$10 Tev) is large $\bar{n}(s)$ $\simeq$100. So, it is practically impossible to describe the system with $N = 3\bar{n} - 10 \simeq 300$ degrees of freedom using ordinary methods.

Secondly, it is natural to assume that the entropy $\mathcal{S}$ tends to maximum with rising multiplicity n and reaches the maximum at $n \sim n_{\max} \sim \sqrt{s}$, since the dissipation takes place in the vacuum (presumably with zero energy density)*. But the experiment shows that at high energies $n \sim \bar{n}(s) \sim \ln^2 s << n_{\max}(s)$ are essential. This means that there is no total dissipation of incident energy in the considered thermalization process [3]. Absence of thermalization may be a consequence of hidden conservation laws [4].

We would like to adopt the following fundamental principle of nonequilibrium statistics introduced by N.N.Bogoliubov [5]. It is natural to assume that the system evaluates to the equilibrium in such a way that the «nonequilibrium» fluctuations in it should tend to zero. In the frame of Bogoliubov's principle the quantitative measure of «nonequilibrium» fluctuation is the mean value of correlation functions and, therefore, these quantities should tend to zero when the media tends to equilibrium.

In our interpretation the Bogoliubov's correlations relaxation principle means the following. So, for nonequilibrium state the presence of «nonequilibrium» fluctuations in the form of the macroscopic flow of, for instance, energy ε is natural. Then the mean value of m-point correlation functions K_m cannot be small as the consequence of macroscopic flow. But in the vicinity of equilibrium the macroscopic flows should relax and, accordingly, the $mean$ value of correlation functions should be small, $K_m \approx 0$. To characterize the equilibrium one may consider also the particles, charge, spin, etc., densities macroscopic flows and their relaxation.

We would like to show in result that the correlations relaxation principle leads to the quantitative connection with real time thermodynamics of Schwinger–Keldysh type** [6]. Just for this purpose the generating functionals (GF) method of Bogoliubov will be used since it allows one to find the quantitative connections, where the Euclidean postulate is not applicable.

We will use more the natural, for particles physics, microcanonical formalism. In this formalism the thermodynamical «rough» variables are introduced as the

*This consideration lies in the basis of the earliest Fermi–Landau «statistical» model of hadrons multiple production.

**Last one includes the nonequilibrium thermodynamics also.

Lagrange multipliers of corresponding conservation laws. Their physical meaning is defined by corresponding equations of state. So, if the fluctuations in the vicinity of solutions of corresponding equations are Gaussian, then one can use these variables for description of the system. Corresponding condition is the Bogoliubov's correlations relaxation condition.

Formally, the generating functions method presents the integral transformation to new variables. One can choose them as the «rough» thermodynamical variables. To describe the far from equilibrium system we will introduce the «local equilibrium hypothesis». In its frame the preequilibrium state consist of equilibrium domains. In this case new variables should depend on the coordinates of domain and, in result, we are forced to use the generating functionals (GF) formalism.

We will consider two examples to illustrate effectiveness of the GF method. In Sec. 2 we will consider the transformation (multiplicity $n \to$ activity z) to show the origin of the Koba–Nielsen–Olesen scaling (KNO-scaling)*.

In Sec. 3 we will investigate a possibility of temperature description of the multiple production processes. We will consider for this purpose the transformation (particles energies $\varepsilon \to$ temperature $1/\beta$) to find the S-matrix interpretation of thermodynamics. It will be shown that this interpretation would be rightful if the correlations are relax.

In Sec. 4 we will use this interpretation to formulate the perturbation theory in the case when β and z are local coordinates of temperature (x, t) [7]. One can use this closed form of perturbation theory for description of nonequilibrium media (in kinetic phase) and for description of the multiple production process as well.

2. KNO-SCALING

We would like to start from the note that the generating functions method allows one to connect inclusive spectra f_k [8] and exclusive cross sections $\sigma_n(s)$. One can use for this purpose the normalization condition:

$$\bar{f}_k \sigma_{\rm tot} \equiv \int d\omega_k(q) f_k(q_1, q_2, ..., q_k) = \sum_{n=k} \frac{n!}{(n-k)!}\sigma_n, \;\; \bar{f}_k \equiv 0 \quad k > n_{\max}, \tag{2.1}$$

*In private discussion with one of the authors (A.S.) in summer of 1973 Z.Koba noted that the main reason of investigation leading to the KNO-scaling was just the GF method of N.N.Bogoliubov.

where, as usual,

$$d\omega_k(q) = \prod_{i=1}^{k} d^3q_i/(2\pi)^3 2\varepsilon(q_i),\ \varepsilon(q) = \sqrt{q^2+m^2}$$

is the Lorentz-covariant element of phase space.

Eq. (2.1) can be considered as the set of coupled equations for σ_n. One may multiply both sides of (2.1) on $(z-1)^k/k!$ and sum over k to solve them. We will see that this is equivalent of introduction of «big partition function» $\Xi(z)$, where z is the «activity»: the chemical potential $\mu \sim \ln z$.

We will find in result of summation over k that

$$\Xi(z) \equiv \sum_k \frac{(z-1)^k}{k!}\bar{f}_k = \sum_n z^n \frac{\sigma_n}{\sigma_{\text{tot}}}. \tag{2.2}$$

Then, assuming that $\Xi(z)$ is known,

$$\sigma_n = \sigma_{\text{tot}} \frac{1}{2\pi i} \oint_C \frac{dz}{z^{n+1}} \Xi(z), \tag{2.3}$$

where the closed contour C includes point $z=0$. Here $\Xi(z)$ is defined by left-hand side of (2.2) and is the generating function of σ_n.

The coefficients C_m in decomposition:

$$\ln \Xi(z) = \sum_m \frac{(z-1)^m}{m!} C_m \tag{2.4}$$

are the (binomial) correlators. Indeed,

$$C_1 = \bar{f}_1 = \bar{n},\ C_2 = \bar{f}_2 - \left\{\bar{f}_1\right\}^2,\ C_3 = \bar{f}_3 - 3\bar{f}_2\left\{\bar{f}_1\right\}^2 + 2\left\{\bar{f}_1\right\}^3 \tag{2.5}$$

an so on. If $C_m = 0$, $m > 1$, then σ_n is described by Poisson formulae:

$$\sigma_n = \sigma_{\text{tot}} e^{-\bar{n}} \frac{(\bar{n})^n}{n!}. \tag{2.6}$$

It corresponds to the case of absence of correlations.

Let us consider more week assumption:

$$C_m(s) = \gamma_m \left(C_1(s)\right)^m, \tag{2.7}$$

where γ_m is the energy independent constant. Then

$$\ln \Xi(z,s) = \sum_{m=1} \frac{\gamma_m}{m!}\{(z-1)\bar{n}(s)\}^m. \tag{2.8}$$

To find consequences of this assumption let us find the most probable values of z. The equation:

$$n = z\frac{\partial}{\partial z}\ln \Xi(z, s) \tag{2.9}$$

is increasing with n solutions $\bar{z}(n, s)$ since $\Xi(z, s)$ is the increasing function of z, if $\Xi(z, s)$ is the nonsingular at finite z function. Last condition has deep physical meaning and practically assumes the absence of the first order phase transition [9].

Let us introduce new variable:

$$\lambda = (z - 1)\bar{n}(s). \tag{2.10}$$

Corresponding Eq. (2.9) looks as follows:

$$\frac{n}{\bar{n}(s)} = \left(1 + \frac{\lambda}{\bar{n}(s)}\right)\frac{\partial}{\partial \lambda}\ln \Xi(\lambda). \tag{2.11}$$

So, with $O(\lambda/\bar{n}(s))$ accuracy, one can assume that

$$\lambda \simeq \lambda_c(n/\bar{n}(s)) \tag{2.12}$$

are essential. It follows from this estimation that such scaling dependence is rightful at least in the neighborhood of $z = 1$, i.e., in the vicinity of main contributions to σ_{tot}. This gives:

$$\bar{n}(s)\sigma_n(s) = \sigma_{\mathrm{tot}}(s)\psi(n/\bar{n}(s)), \tag{2.13}$$

where

$$\psi(n/\bar{n}(s)) \simeq \Xi(\lambda_c(n/\bar{n}(s)))\exp\{n/\bar{n}(s)\lambda_c(n/\bar{n}(s))\} \leq O(e^{-n}) \tag{2.14}$$

is the unknown function. The asymptotic estimation follows from the fact that $\lambda_c = \lambda_c(n/\bar{n}(s))$ should be, as follows from nonsingularity of $\Xi(z)$, nondecreasing function of n.

The estimation (2.12) is rightful at least at $s \to \infty$. The range validity of n, where solution of (2.12) is acceptable, depends on the exact form of $\Xi(z)$. Indeed, if $\ln \Xi(z) \sim \exp\{\gamma\lambda(z)\}$, $\gamma = \mathrm{const} > 0$, then (2.12) is rightful at all values of n and it is enough to have the condition $s \to \infty$. But if $\ln \Xi(z) \sim (1 + a\lambda(z))^\gamma$, $\gamma = \mathrm{const} > 0$, then (2.12) is acceptable if $n << \bar{n}^2(s)$.

Representation (2.13) shows that just $\bar{n}(s)$ is the natural scale of multiplicity n [10]. This representation was offered firstly as a reaction on the so-called Feynman scaling for inclusive cross section:

$$f_k(q_1, q_2, ..., q_k) \sim \prod_{i=1}^{k}\frac{1}{\varepsilon(q_i)}. \tag{2.15}$$

As follows from estimation (2.14), the limiting KNO prediction assumes that $\sigma_n = O(e^{-n})$. In this regime $\Xi(z,s)$ should be singular at $z = z_c(s) > 1$. The normalization condition

$$\frac{\partial \Xi(z,s)}{\partial z}|_{z=1} = \bar{n}(s)$$

gives: $z_c(s) = 1 + \gamma/\bar{n}(s)$, where $\gamma > 0$ is the constant. Note, such behavior of big partition function $\Xi(z,s)$ is natural for stationar Markovian processes described by logistic equations [11]. In the field theory such equation describes the QCD jets [12].

It is known that at $Tevatron$ energies the mean hadrons multiplicity rises with transverse momentum. The associated mean multiplicity is

$$C_1(q_{tr}) = \bar{n}(q_{tr}) = \frac{\sum_n n d\sigma_n/dq_{tr}}{\sum_n d\sigma_n/dq_{tr}}.$$

So, if

$$C_m(q_{tr}) = \gamma_m \left(C_1(q_{tr})\right)^m : \quad f_k(q_1, q_2, ..., q_k) \sim \prod_{i=1}^{k} \frac{1}{\varepsilon(q_i)} \Omega(q_{tr}),$$

then:

$$\bar{n}(q_{tr}) \frac{d\sigma_n/dq_{tr}}{\sum_n d\sigma_n/dq_{tr}} = \Psi(n/\bar{n}(q_{tr})).$$

This prediction is in good agreement with the experiment [13].

3. TEMPERATURE DESCRIPTION

By definition,

$$\sigma_n^{ab}(s) = \int d\omega_n(q) \delta(q_a + q_b - \sum_{i=1}^{n} q_i) |A_n^{ab}|^2, \tag{3.1}$$

where A_n^{ab} is the amplitude of n creation at interaction of particles a and b.

Considering Fourier transform of energy-momentum conservation δ function one can introduce the generating function ρ_n [14]. We may find in result that σ_n is defined by equality:

$$\sigma_n(s) = \int_{-i\infty}^{+i\infty} \frac{d\beta}{2\pi} e^{\beta\sqrt{s}} \rho_n(\beta), \tag{3.2}$$

where

$$\rho_n(\beta) = \int \left\{ \prod_{i=1}^{n} \frac{d^3 q_i e^{-\beta\varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} |A_n^{ab}|^2. \tag{3.3}$$

The most probable value of β is defined by equation of state:

$$\sqrt{s} = -\frac{\partial}{\partial\beta} \ln \rho_n(\beta). \tag{3.4}$$

Let us consider the simplest example of noninteracting particles:

$$\rho_n(\beta) = \{2\pi m K_1(\beta m)/\beta\}^n ,$$

where K_1 is the Bessel function. Inserting this expression into (3.4) we can find that in the nonrelativistic case ($n \simeq n_{\max}$)

$$\beta_c = \frac{3}{2} \frac{(n-1)}{(\sqrt{s} - nm)}.$$

That is, $E_{\text{kin}} = \frac{3}{2}T$, where $E_{\text{kin}} = (\sqrt{s} - nm)$ is the kinetic energy.

It is important to note that the equation (3.4) has unique real rising with n and decreasing with s solution $\beta_c(s,n)$ [15].

The expansion of integral (3.2) near $\beta_c(s,n)$ unavoidably gives asymptotic series with zero convergence radii since $\rho_n(\beta)$ is the essentially nonlinear function of β. From physical point of view this means that, generally speaking, fluctuations in the vicinity of $\beta_c(s,n)$ may be arbitrarily high and in this case $\beta_c(s,n)$ has not any physical sense. But if fluctuations are small (strictly speaking, they may be arbitrarily high, but distribution in the vicinity of $\beta_c(s,n)$ should be Gaussian), then $\rho_n(\beta)$ should coincide with partition function of n particles and $\beta_c(s,n)$ may be interpreted as the inverse temperature.

Let us define the conditions when the fluctuations are small [7]. Firstly, we should expand $\ln \rho_n(\beta + \beta_c)$ over β:

$$\ln \rho_n(\beta + \beta_c) = \ln \rho_n(\beta_c) - \sqrt{s}\beta + \frac{1}{2!}\beta^2 \frac{\partial^2}{\partial\beta_c^2} \ln \rho_n(\beta_c) -$$

$$-\frac{1}{3!}\beta^3 \frac{\partial^3}{\partial\beta_c^3} \ln \rho_n(\beta_c) + \ldots \tag{3.5}$$

and, secondly, expand the exponent in the integral, for instance, over $\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3$ neglecting higher decomposition terms in (3.5). As a result, kth term of the perturbation series

$$\rho_{n,k} \sim \left\{ \frac{\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3}{(\partial^2 \ln \rho_n(\beta_c)/\partial\beta_c^2)^{3/2}} \right\}^k \Gamma\left(\frac{3k+1}{2}\right). \tag{3.6}$$

Therefore, one should assume that

$$\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3 \ll (\partial^2 \ln \rho_n(\beta_c)/\partial\beta_c^2)^{3/2} \tag{3.7}$$

to neglect this term. One of the possible solutions of this condition is

$$\partial^3 \ln \rho_n(\beta_c)/\partial\beta_c^3 \approx 0. \tag{3.8}$$

If this condition is hold, then the fluctuations are Gaussian, but arbitrary since their value is defined by $\{\partial^2 \ln \rho_n(\beta_c)/\partial\beta_c^2\}^{1/2}$, see (3.5).

Let us consider now (3.8) carefully. We will find computing derivatives that this condition means the following approximate equality:

$$\frac{\rho_n^{(3)}}{\rho_n} - 3\frac{\rho_n^{(2)}\rho_n^{(1)}}{\rho_n^2} + 2\frac{(\rho_n^{(1)})^3}{\rho_n^3} \approx 0, \tag{3.9}$$

where $\rho_n^{(k)}$ means the kth derivative. For identical particles (see definition (3.3)),

$$\rho_n^{(k)}(\beta_c) = n^k(-1)^k \int \left\{ \prod_{i=1}^{n} \varepsilon(q_i) \frac{d^3q_i e^{-\beta\varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} |A_n^{ab}|^2 =$$
$$= \sigma_{\mathrm{tot}} n^k \int \left\{ \prod_{i=1}^{k} \varepsilon(q_i) \frac{d^3q_i e^{-\beta\varepsilon(q_i)}}{(2\pi)^3 2\varepsilon(q_i)} \right\} \bar{f}_k(q_1, q_2, ..., q_k), \tag{3.10}$$

where $\bar{f}_k$ is the $(n-k) \geq 0$-point inclusive cross section. It coincides with k-particle distribution function in the n-particle system. Therefore, l.h.s. of (3.9) is the 3-point correlator K_3:

$$K_3 \equiv \int d\omega_3(q) \times$$

$$\times \left(< \prod_{i=1}^{3} \varepsilon(q_i) >_{\beta_c} -3 < \prod_{i=1}^{2} \varepsilon(q_i) >_{\beta_c} < \varepsilon(q_3) >_{\beta_c} +2 \prod_{i=1}^{3} < \varepsilon(q_i) >_{\beta_c} \right), \tag{3.11}$$

where the index means averaging with the Boltzmann factor $\exp\{-\beta_c\varepsilon(q)\}$.

As a result, to have all fluctuations in vicinity of β_c Gaussian, we should have $K_m \approx 0$, $m \geq 3$. But, as follows from (3.7), the set of minimal conditions looks as follows:

$$K_m \ll K_2, \; m \geq 3. \tag{3.12}$$

If experiment confirms this conditions, then, independently of the number of particles, the final state may be described by one parameter β_c with high enough accuracy β_c.

Considering β_c as physical (measurable) quantity, we are forced to assume that both the total energy of the system $\sqrt{s} = E$ and the conjugate to it variable β_c may be measured with high accuracy*.

4. REAL-TIME FINITE TEMPERATURE GENERATING FUNCTIONALS

We would like to show now why and in a what conditions our S-matrix interpretation of statistics is rightful.

In modern formulations, see, e.g., the textbook [16], the temperature is introduced by the so-called periodic Kubo–Martin–Schwinger (KMS) boundary condition [17]. Namely, in the Feynman–Kac functional integral representation of the partition function

$$\Xi(\beta) = \int D\varphi e^{-S_\beta(\varphi)} \tag{4.1}$$

the action $S_\beta(\varphi; z)$ is defined on the Matsubara imaginary time contour C_M: $(t_i, t_i - i\beta)$, but fields should obey KMS boundary condition:

$$\varphi(t_i) = \varphi(t_i - i\beta). \tag{4.2}$$

This is a natural consequence of definition: $\Xi(\beta) = \mathrm{Sp} e^{-\beta \mathbf{H}}$.

It was offered to deform Matsubara contour in the following way:

$$C_M \to C_{SK}: \ (t_i, t_f) + (t_f, t_i + i\beta), \tag{4.3}$$

where C_{SK} is the Mills time contour [18] and $t_f > t_i$ belongs to real axis [19]. Including the real-time parts we obtain a possibility of describing the time evolution of the system

But this attempt was not successful. First of all, we have not an evident interpretation of t_i and t_f [20]. Secondly, in spite of the real-time parts, this formulation is unable to describe the time evolution [21].

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*Note, the uncertainty principle $\sim \hbar$ did not restrict ΔE and $\Delta\beta$.

4.1. Equilibrium Media. It was shown above that if σ_n is defined by (3.1), then one may introduce ρ_n using definition (3.3). The Fourier transform (3.2) connects $\sigma_n < \rho_n$. On the other hand, ρ_n reminds the partition function.

To find complete analogy with statistical physics we should consider transition $m \to n$ particles with amplitude $A_{nm} =<\mathrm{out}; n|\mathrm{in}; m>$. Summation over n and m is assumed. The corresponding δ function of energy-momentum conservation law should be written in the form:

$$\delta(\sum_{i=1}^{n} q_i - \sum_{i=1}^{m} p_i) = \int d^4P\delta(P - \sum_{i=1}^{n} q_i)\delta(P - \sum_{i=1}^{m} p_i), \quad P = (E, \vec{P}). \tag{4.4}$$

This will lead to necessity to introduce independently the temperature of initial $(1/\beta_i)$ and final $(1/\beta_f)$ states. In particle physics we can consider the final state temperature only.

As a result we get to the Fourier–Mellin transform $\rho(\beta, z) = \rho(\beta_i, z_i; \beta_f, z_f)$. Direct calculations give important factorized form:

$$\rho(\beta, z) = e^{\hat{N}(\beta,z;\phi)}\rho_0(\phi),$$

where the operator

$$\hat{N}(\beta, z; \phi) = \int dxdx'(\hat{\phi}_+(x)D_{+-}(x - x', \beta_f, z_f)\hat{\phi}_-(x') - \\ -\hat{\phi}_-(x)D_{-+}(x - x', \beta_i, z_i)\hat{\phi}_+(x')), \ \hat{\phi} = \frac{\delta}{\delta\phi} \tag{4.5}$$

acts on the functional:

$$\rho_0(\phi_\pm) = \int D\Phi_+ D\Phi_- e^{iS(\Phi_+)-iS(\Phi_-)-iV(\Phi_++\phi_+)+iV(\Phi_-+\phi_-)}. \tag{4.6}$$

At the very end of calculations one should take auxiliary variables $\phi_\pm$ equal to zero.

Here $D_{+-} < D_{-+}$ are the frequency correlation functions:

$$D_{\pm\mp}(x - x', \beta) = \mp i \int d\omega(q)e^{\pm iq(x-x'+i\mp\beta)}z(q).$$

They obey the equations:

$$(\partial^2 + m^2)_x G_{+-} = (\partial^2 + m^2)_x G_{-+} = 0.$$

So, all «thermodynamical» information is contained in the operator $\hat{N}(\beta, z; \phi)$, but interactions are described by $\rho_0(\phi)$. One can say that the operator $\hat{N}$ (adiabatically) maps the interacting filed system on the observable states. This important property allows one to consider only «mechanical» processes and exclude from consideration the «thermal» ones.

Calculating $\rho_0(\phi)$ perturbatively one can find:

$$\rho(\beta, z) = e^{-iV(-i\hat{j}_+)+iV(-i\hat{j}_-)} e^{\frac{i}{2}\int dx dx' j_a(x) D_{ab}(x-x',\beta,z) j_b(x')}, \tag{4.7}$$

where D_{++} is the Feynman (causal) Green function and

$$D_{--} = (D_{++})^*$$

is the anticausal one and, as usual, $\hat{j} = \delta/\delta j$. At the very end one should take $j = 0$.

Let us assume now that our system is a subsystem of bigger system. This would lead to transformation of Boltzmann factor $\exp\{-\beta\varepsilon\}$ on corresponding statistics occupation number $\bar{N}(-\beta\varepsilon)$. This means that our interacting fields system is surrounded by black body radiation. This is mechanical model of the thermostat (heat bath of thermodynamics).

In result the matrix D_{ab} takes the form (we put for simplicity $z_i = z_f = 1$):

$$iG(q;\beta) = \begin{pmatrix} \frac{i}{q^2-m^2+i\epsilon} & 0 \\ 0 & -\frac{i}{q^2-m^2-i\epsilon} \end{pmatrix} +$$

$$+2\pi\delta(q^2-m^2) \begin{pmatrix} \tilde{n}(\frac{\beta_f+\beta_i}{2}|q_0|) & \tilde{n}(\beta_i|q_0|)a_+(\beta_i) \\ \tilde{n}(\beta_f|q_0|)a_-(\beta_f) & \tilde{n}(\frac{\beta_f+\beta_i}{2}|q_0|) \end{pmatrix}, \tag{4.8}$$

where

$$a_\pm(\beta) = -e^{\frac{\beta}{2}(|q_0|\pm q_0)}.$$

Following Green functions:

$$D_{ab}(x-x',\beta) = \int \frac{d^4q}{(2\pi)^4} e^{iq(x-x')} G_{ab}(q,\beta)$$

was introduced and the occupation number

$$n_{++}(q_0) = n_{--}(q_0) = \left\{ e^{|q_0|(\beta_f+\beta_i)/2} - 1 \right\}^{-1} \equiv \tilde{n}(|q_0|\frac{\beta_i+\beta_f}{2}), \tag{4.9}$$

and

$$n_{+-}(q_0) == \Theta(q_0)(1+\tilde{n}(q_0\beta_f)) + \Theta(-q_0)\tilde{n}(-q_0\beta_i), \tag{4.10}$$

$$n_{-+}(q_0) = \Theta(q_0)\tilde{n}(q_0\beta_i) + \Theta(-q_0)(1+\tilde{n}(-q_0\beta_f)). \tag{4.11}$$

Assuming that $\beta_i = \beta_f = \beta_c$, it is easy to find:

$$G_{+-}(t-t') = G_{-+}(t-t'-i\beta), \quad G_{-+}(t-t') = G_{+-}(t-t'+i\beta), \quad (4.12)$$

i.e., our Green function obeys KMS boundary condition.

So, representation (4.7) with Green functions (4.8) coincides identically with (4.1), calculated perturbatively, see also [19].

4.2. Nonequilibrium Media. Our attempt to introduce the temperature as the quantitative characteristic of the $whole$ system is based on the assumption that mean value of correlators is small. We can «localize» this condition assuming that this rough description may be extended only on subdomains of the system. For definiteness the subdomains may be marked by space-time coordinate r.

It should be underlined that we divide on the subdomains not the system under consideration but the device, where external particles are measured. Noting that external flow consist of noninteracting particles (including the flow of black body radiation) the division on subdomains cannot affect the fields interaction.

In result we introduce the «local» temperature $1/\beta(r)$ for rth group of interacting particles assuming that fluctuations in the vicinity of $\beta(r)$ are Gaussian. This means that the mean value of correlation in the group is small, but the correlation between groups may be high. Nevertheless, last one is not important since the external particles are on the mass shell. At the same time dimension of group may be arbitrary, but larger than some r_0 to have possibility to introduce the temperature as the collective variable.

We can distinguish the following scales. Let L_q be the characteristic 4-scale of quantum fluctuations, L_s be the scale thermodynamical fluctuations and L be the scale of subdomain. It is natural to assume that $L_s \gg L \gg L_q$.

Corresponding generating $functional$ has the form:

$$\rho_{cp}(\alpha_1, \alpha_2) = e^{\hat{N}(\phi_a^* \phi_b)} \rho_0(\phi_\pm).$$

One may note that the «localization» gives influence on the operator only:

$$\hat{N}(\phi_a^* \phi_b) = \int dY dy \hat{\phi}_a(Y + y/2) \tilde{n}_{ab}(Y, y) \hat{\phi}_b(Y - y/2).$$

The occupation numbers $n_{ab}(Y, q)$ have the same form, $\beta \to \beta(Y)$ and

$$\tilde{n}_{ij}(Y, y) = \int d\omega(q) e^{iqy} n_{ij}(Y, q).$$

We find calculating ρ_0 perturbatively that:

$$\rho_{cp}(\beta) = \exp\{-iV(-i\hat{j}_+) + iV(-i\hat{j}_-)\} \times$$
$$\exp\{i \int dY dy [j_a(Y + y/2) G_{ab}(y, (\beta(Y)) j_b(Y - y/2)\}, \quad (4.13)$$

where the matrix Green function $G(q, (\beta(Y)))$ was defined in (4.8).

5. CONCLUSION

One more detail. Our consideration has shown the uniqueness of Bogoliubov's solution of the nonequilibrium thermodynamics problem. Indeed, without vanishing of the correlations, perturbation series in the β_c vicinity, being asymptotic, is divergent.

We would like to stress in conclusion that Bogoliubov's creative works naturally unite particle and statistical physics. In result, using Bogoliubov's mathematical basis, we have the united scientific space in which both branches of physics, thermodynamics and quantum field theory, supplement each other.

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