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DESCRIPTION OF TRANSITIONAL NUCLEI IN THE FRAMEWORK OF «QUADRUPOLE PLUS PAIRING» COLLECTIVE MODEL

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The «quadrupole plus pairing» collective model is constructed and adopted to describe the quadrupole collective states in even-even transitional nuclei. An approximation scheme of solving the model is given. Exemplary results of microscopic calculations within the framework of the model are shown.

Since more than 30 years the General Bohr Hamiltonian (GBH) remains the main tool for description of collective states in even-even transitional nuclei (e.g., [1–5]). This is because of two important features of such a model. The former is that the collective Hamiltonian is a rotational scalar and its eigenfunctions are of a definite angular momentum. The latter is that a rotation-vibration coupling, so important for transitional nuclei, is taken into account. To be sure, algebraic collective models of the type of Interacting Boson Model (IBM) possess similar features (cf. [7]). However, such models act usually as purely phenomenological approaches whereas there exist efficient methods of microscopic constructing the GBH. Two standard microscopic approaches are: (i) the cranking model (see [6] for formulae and references quoted therein) which, however, gives a classical Hamiltonian and, therefore, requires a requantization procedure, (ii) the Gaussian Overlap Approximation (GOA) of the Generator Coordinate Method (GCM) [8]. But it should be admitted that the GBH treats collective excitations as an adiabatic phenomenon and does not take into account a coupling with other degrees of freedom, in particular, with two-quasiparticle excitations.

It is known for a long time that the GBH, when constructed microscopically, gives results which are not compatible with experimental data [3]. The calculated excitation energies of collective levels are not in a proper scale. This seems as if the inertial parameters are two to three times too small. These inertial parameters are sensitive functions of the pairing energy gaps and can easily be made bigger by an artificial weakening of pairing forces [3]. This observation has brought us to the conclusion that a coupling between the quadrupole and the pairing degrees of freedom should be taken into account [9,10]. It can be done by the construction of a collective Hamiltonian for both, the quadrupole and the pairing vibrations.

Below we present the «quadrupole plus pairing» collective model and discuss a way of its approximate solving. Next, we show some exemplary results of calculations and compare them with experimental data. At the end we draw some conclusions.

Apart from the five usual collective variables, namely, $\beta,\ \gamma$, the Bohr deformation parameters describing the nuclear shape or the quadrupole moment in the intrinsic frame, and $\phi,\ \theta,\ \psi$, the Euler angles describing the orientation of the intrinsic frame, we introduce a further four dynamical variables to the collective model, namely, $\Delta^{\rm p},\ \Delta^{\rm n}$, the proton and neutron energy gaps describing the proton and neutron pairing correlations, and $\Phi^{\rm p},\ \Phi^{\rm n}$, the proton and neutron gauge angles describing rotations in the proton and neutron gauge spaces or transfer of the proton and neutron pairs (cf. [11–13]). We assume the «quadrupole plus pairing» collective Hamiltonian (QPCH) of the following structure:

$$\begin{aligned}\hat{\mathcal{H}}_{\rm quad-pair} &= \hat{\mathcal{T}}_{\rm vib}(\beta,\gamma;\Delta^{\rm n},\Delta^{\rm p}) + V_{\rm def}(\beta,\gamma,\Delta^{\rm n},\Delta^{\rm p}) \\ &+\hat{\mathcal{H}}_{\rm rot}(\phi,\theta,\psi;\beta,\gamma,\Delta^{\rm n},\Delta^{\rm p}) \\ &+\sum_{\rm t=p,n}\Big[\hat{\mathcal{T}}^{(\rm t)}_{\rm pair}(\Delta^{\rm t},\Phi^{\rm t};\beta,\gamma) + V^{(\rm t)}_{\rm pair}(\beta,\gamma,\Delta^{\rm t}) \\ &+\hat{\mathcal{T}}^{(\rm t)}_{\rm quad-pair}(\beta,\gamma,\Delta^{\rm t})\Big]\,. \end{aligned} \tag{1}$$

The operators which enter into the Hamiltonian of Eq. (1) are differential operators of the second order in the arguments given in front of semicolon; $\hat{\mathcal{T}}^{(\rm t)}_{\rm quad-pair}$ is a differential operator in all of its arguments. We do not write down here exact forms of all terms of the Hamiltonian which are more or less obvious. We only mention that it is determined by the following functions of β, γ, $\Delta^{\rm p}$ and $\Delta^{\rm n}$

– $V_{\rm def},\ V^{(\rm p)}_{\rm pair},\ V^{(\rm n)}_{\rm pair}$, the deformation and pairing potentials,

– $B_{\beta\beta},\ B_{\beta\gamma},\ B_{\gamma\gamma}$, the quadrupole vibrational inertial functions (mass parameters),

– $\mathcal{J}_1,\ \mathcal{J}_2,\ \mathcal{J}_3$, the quadrupole moments of inertia,

– $B_{\Delta^{\rm t}\Delta^{\rm t}}$ for t = p, n, the pairing vibrational inertial functions,

– $B_{\beta\Delta^{\rm t}},\ B_{\gamma\Delta^{\rm t}}$ for t = p, n, the quadrupole–pairing mixed vibrational inertial functions,

– $\mathcal{J}_{\Phi^{\rm t}}$ for t = p, n, the pairing moments of inertia,

– $\lambda^{(\rm t)}$ for t = p, n, the chemical potentials.

To calculate electromagnetic characteristics of nuclei, like reduced probabilities of γ-transitions, electric and magnetic multipole moments, collective multipole operators, which are determined again by some functions of β, γ, $\Delta^{\rm p}$ and $\Delta^{\rm n}$, should be constructed. All these functions determining the collective Hamiltonian and multipole operators can be calculated from a microscopic theory. This problem is discussed elsewhere [6].

Solving the collective model formulated above may consist in the numerical diagonalizing the set of operators: $\hat{\mathcal{H}}_{\rm quad-pair}$, the collective Hamiltonian, $\hat{I}^2$, $\hat{I}_z$, the total angular momentum and its projection onto a lab axis z, $\hat{N}_{\rm t} = -i\partial/\partial\Phi_{\rm t}$ for t = p, n, the particle number excess operators. We do not solve this eigenvalue problem exactly as yet. Instead, we have adopted an approximation scheme, which proceeds in the following steps:

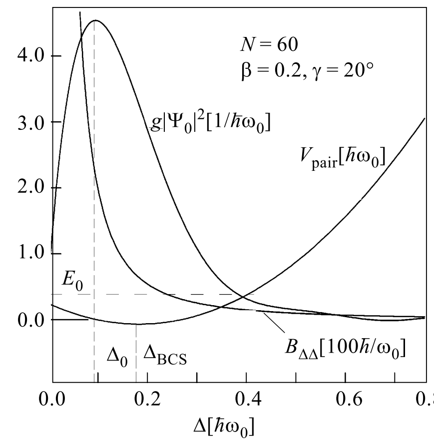

Fig. 1. Zero-point pairing vibration of neutrons in ^{104}Ru at deformation $\beta = 0.2$, $\gamma = 20^\circ$. The equilibrium value of the energy gap is $\Delta_{\rm BCS} \approx 0.14\hbar\omega_0$ whereas the most probable value is $\Delta_0 \approx 0.09\hbar\omega_0$. The oscillator frequency is $\hbar\omega_0 \approx 41/A^{1/3}$ MeV

1. Neglect the quadrupole-pairing coupling in the kinetic energy, i.e., put $\hat{\mathcal{T}}^{(\rm t)}_{\rm quad-pair}(\beta,\gamma,\Delta^{\rm t}) = 0$ for t = p, n in Eq. (1).

2. Find the zero-point pairing vibration of neutrons and protons for given β and γ solving the eigenvalue problem

$$\hat{\mathcal{H}}^{(\rm t)}_{\rm pair}(\Delta^{\rm t};\beta,\gamma)\Psi_0(\Delta^{\rm t};\beta,\gamma) = E_0^{(\rm t)}(\beta,\gamma)\Psi_0(\Delta^{\rm t};\beta,\gamma), \qquad (2)$$

$$\hat{N}_{\rm t}(\Phi^{\rm t})\Psi_0(\Delta^{\rm t};\beta,\gamma) = 0, \qquad (3)$$

where

$$\hat{\mathcal{H}}^{(\rm t)}_{\rm pair}(\Delta^{\rm t};\beta,\gamma) = \hat{\mathcal{T}}^{(\rm t)}_{\rm pair}(\Delta^{\rm t};\beta,\gamma) + V^{(\rm t)}_{\rm pair}(\Delta^{\rm t},\beta,\gamma) \qquad (4)$$

and the collective pairing kinetic energy (excluding the pair transfer effect) reads [12, 13]

$$\hat{\mathcal{T}}^{(\mathrm{t})}_{\mathrm{pair}} = -\frac{\hbar^2}{2\sqrt{g(\Delta^{\mathrm{t}})}}\frac{\partial}{\partial\Delta^{\mathrm{t}}}\frac{\sqrt{g(\Delta^{\mathrm{t}})}}{B_{\Delta^{\mathrm{t}}\Delta^{\mathrm{t}}}(\Delta^{\mathrm{t}})}\frac{\partial}{\partial\Delta^{\mathrm{t}}}; \tag{5}$$

here $g(\Delta^{\mathrm{t}},\beta,\gamma)$ is a normalization weight.

3. Find the most probable neutron or proton energy gap $\Delta^{\mathrm{t}}_0(\beta,\gamma)$, i.e., the value of Δ^{t}_0 for which $g|\Psi_0|^2$ takes its maximum (see Fig. 1).

4. Solve the eigenvalue problem for the following general Bohr Hamiltonian

$$\begin{aligned}\hat{\mathcal{H}}_{\mathrm{coll}} &= \hat{\mathcal{T}}_{\mathrm{vib}}(\beta,\gamma;\Delta^{\mathrm{n}}_0(\beta,\gamma),\Delta^{\mathrm{p}}_0(\beta,\gamma)) + V_{\mathrm{coll}}(\beta,\gamma,\Delta^{\mathrm{n}}_0(\beta,\gamma),\Delta^{\mathrm{p}}_0(\beta,\gamma))\\ &+ \hat{\mathcal{H}}_{\mathrm{rot}}(\phi,\theta,\psi;\beta,\gamma,\Delta^{\mathrm{n}}_0(\beta,\gamma),\Delta^{\mathrm{p}}_0(\beta,\gamma)),\end{aligned} \tag{6}$$

where the quadrupole collective potential is

$$V_{\mathrm{coll}} = V_{\mathrm{def}} + E^{(\mathrm{n})}_0 + E^{(\mathrm{p})}_0 \tag{7}$$

and the quadrupole kinetic energy reads

$$\begin{aligned}\hat{\mathcal{T}}_{\mathrm{vib}} = -\frac{\hbar^2}{2\sqrt{wr}}\Bigg\{&\frac{1}{\beta^4}\left[\partial_\beta\left(\beta^4\sqrt{\frac{r}{w}}B_{\gamma\gamma}\partial_\beta\right) - \partial_\beta\left(\beta^3\sqrt{\frac{r}{w}}B_{\beta\gamma}\partial_\gamma\right)\right]\\ +&\frac{1}{\beta\sin 3\gamma}\left[\frac{1}{\beta}\partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\beta}\right)\partial_\gamma - \partial_\gamma\left(\sqrt{\frac{r}{w}}\sin 3\gamma B_{\beta\gamma}\partial_\beta\right)\right]\Bigg\},\end{aligned} \tag{8}$$

$$\hat{\mathcal{H}}_{\mathrm{rot}} = \frac{1}{2}\sum_{k=1}^{3}\hat{I}^2_k/\mathcal{J}_k; \tag{9}$$

here $\hat{I}_1,\ \hat{I}_2,\ \hat{I}_3$ are the operators of intrinsic angular momenta (differential operators in the Euler angles). The GBH of Eq. (6) is Hermitian with the volume element

$$\mathrm{d}\tau = \beta^4\sqrt{wr}|\sin 3\gamma|\mathrm{d}\beta\mathrm{d}\gamma\sin\theta\mathrm{d}\theta\mathrm{d}\phi\mathrm{d}\psi, \tag{10}$$

where $w = B_{\beta\beta}B_{\gamma\gamma} - B^2_{\beta\gamma}$ and $r = \mathcal{J}_1\mathcal{J}_2\mathcal{J}_3/(4\beta^6\sin^2 3\gamma)$.

The physical meaning of the above approximation consists in taking into account an effect of the zero-point pairing vibration on the quadrupole collective excitations.

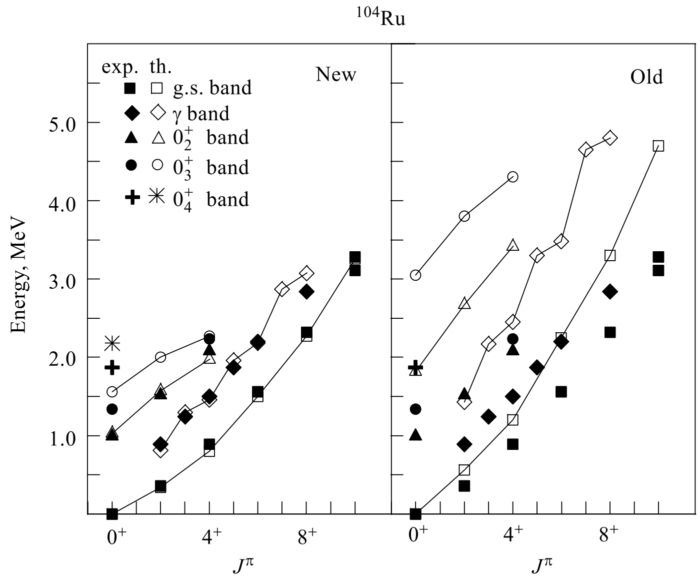

Fig. 2. Experimental (black figures) and theoretical (open figures connected by straight lines) energy levels in ^{104}Ru versus angular momentum J^π. The theoretical levels are calculated with («new») and without («old») the effect of zero-point pairing vibration taken into account

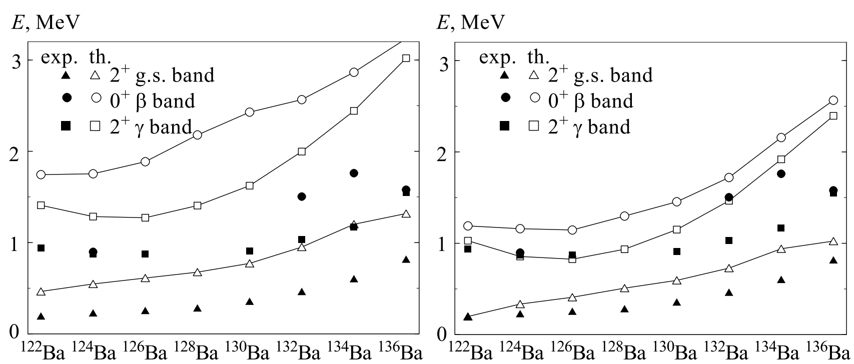

Fig. 3. Experimental (black figures) and theoretical (open figures connected by straight lines) values of the lowest excited 2^+ levels and band-heads of the β- and the γ-band in the Ba isotopes. The theoretical levels are calculated without (left part) and with (right part) the effect of zero-point pairing vibration taken into account

Results of calculations show that the zero-point-pairing-vibration effect is really essential for the quadrupole excitations and improves considerably an agreement with experimental data. It is easily seen in Fig. 2, where the collective energy levels in ^{104}Ru calculated without («old») and with («new») this effect taken into account, are compared with experimental levels [14]. The «new» calculation reproduces the data almost perfectly. The ground-state rotational bands in isotopes of Erbium, $^{152-166}$Er are reproduced equally well and the γ-bands only a little bit worse (Fig. 4). From Fig. 3 we see that the effect of pairing vibration improves the results also for isotopes of Barium. However, it does not make the job in this case. The qudrupole-pairing coupling in the kinetic energy and also a coupling with the octupole degrees of freedom may probably play a role in the quadrupole excitations of the neutron-deficient nuclei of $50 < Z, N < 82$.

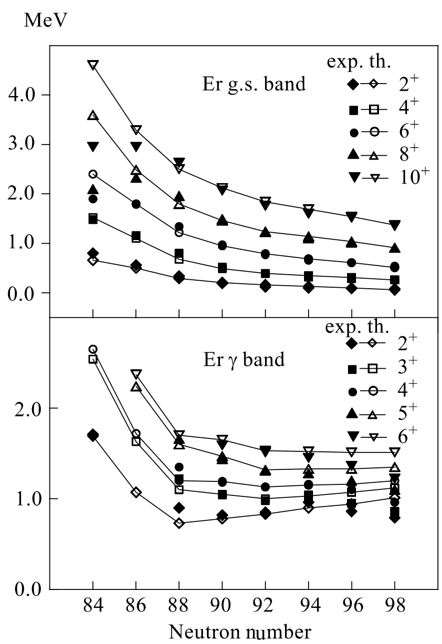

Fig. 4. Experimental (black figures) and theoretical (open figures connected by straight lines) rotational bands in the Er isotopes. The upper part shows the ground-state band, the lower — γ-band

In conclusion, the «quadrupole plus pairing» collective model can successfully be applied to the description of collective states in even-even transitional nuclei. An essential role of the zero-point pairing vibration in the behaviour of quadrupole excitations is observed. When the pairing vibration is taken into account, microscopic calculations with no free parameters yield results which are in good agreement with experimental data. However, it is still an open question whether the description of low-lying collective states by the «quadrupole plus pairing» model takes all main and/or proper effects into account.

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