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MULTIBARYON AND MESON-LIKE STATES IN THE SU(2)-SKYRME MODEL

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The effective quantum mechanical Hamiltonian for topologically trivial and nontrivial sectors of the Skyrme model is obtained in the framework of the collective coordinate method. The collective variables correspond to the vibrations and rotations in the space and isospace. Some numerical results for the observable values for the lowest states are presented.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Мультибарионы и мезоноподобные состояния в SU(2)-модели Скирма

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В методе коллективных переменных, соответствующих вибрациям, а также вращениям во внешнем и внутреннем пространствах, получен эффективный квантовый гамильтониан для топологически тривиального и нетривиальных секторов модели Скирма. Приводятся некоторые результаты численных расчетов, наблюдаемых для нижайших состояний.

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The Skyrme model /1/, as a chiral soliton model of baryons, has some phenomenological success in describing the static properties of nucleons and their interactions. Recently, a new ansatz for the solutions of the stationary Euler-Lagrange equations has been proposed in /2,3/ for the multibaryon states. As it was shown in /2/ and /3/, some of the solitons with baryon number $B > 1$ were classically stable objects. In /2/ the new series of meson-like solitons with $B = 0$ are also obtained, and the classical properties are investigated. The purpose of this paper is to obtain the quantum mechanical effective Hamiltonian in the framework of the collective coordinate method. We use the vibrational and rotational degrees of freedom as collective coordinates and calculate the masses and binding energies of the lowest states in this method.

Let us describe the important steps needed to obtain effective Hamiltonian. We start with the Skyrme Lagrangian density $\mathfrak{L}$:

$$\mathfrak{L} = \frac{F_\pi^2}{16} \mathrm{Tr}(L_\mu \cdot L_\mu) + \frac{1}{32 e^2} \mathrm{Tr}[L_\mu , L_\nu]^2 , \tag{1}$$

where $L_\mu = U^+ \partial_\mu U$ are the left currents and assume, as in [2,3] that the chiral field U(r) has the following structure:

$$U(\vec{r}) = \cos(F(r)) + i(\vec{\tau}\vec{N}) \sin(F(r)) . \tag{2}$$

Here $\vec{N}$ determines a direction in the isotopic space. It is expressed by the components in the spherical coordinates system

$$\vec{N} = (\cos(\Phi(\phi)) \sin(T(\theta)), \sin(\Phi(\phi)) \sin(T(\theta)), \cos(T(\theta))) . \tag{3}$$

As it was shown in [2,3] the mass of the Skyrmion is given by the functional of the $T(\theta)$, $\Phi(\phi)$, $F(r)$:

$$M = M_2 + M_4 , \tag{4}$$

$$M_2 = \frac{\gamma}{4} \int_0^\infty dx\, x^2 \int_0^\pi d\theta \sin\theta \{(F')^2 + [\frac{\sin^2 T}{\sin^2\theta} k^2 + (T')^2] \frac{\sin^2 F}{x^2} \} , \tag{5}$$

$$M_4 = \gamma \int_0^\infty dx\, x^2 \int_0^\pi d\theta \sin\theta \{[\frac{\sin^2 T}{\sin^2\theta} k^2 + (T')^2] (F')^2 + $$

$$+ \frac{\sin^2 F}{x^2} \frac{\sin^2 T}{\sin^2\theta} k^2 (T')^2 \} \frac{\sin^2 F}{x^2} , \tag{6}$$

where $\gamma = \pi \cdot F_\pi / e$ and $x = F_\pi \cdot e \cdot r$.

We do not give here the system of equations [2] for the solution $T(\theta)$, $\Phi(\phi)$ and $F(x)$, but we have to note the requirements to be imposed on solutions. Thus, we consider only the configurations with finite masses, that is why we have $F(0) = \pi n$ and $T(0) = 0$, $T(\pi) = \pi \ell$ with integer n and ℓ. Some integer k determines $\Phi(\phi) = k \cdot \phi$, and leads to a single-valued solution. Now all the solutions $U_{nk\ell}(x)$ are classified by the set of integer numbers n, k and ℓ.

The next expression for topological (baryon) charge with such an ansatz in [2] has been obtained

$$B = \frac{k \cdot n}{2} (1 - \cos(\pi \cdot \ell)). \tag{7}$$

This expression points out that between the solutions there are ones (with even ℓ) that will be quantized as bosons with an integer spin and isospin quantum number. On the other hand, odd ℓ will correspond to the multibaryon state with $B = n \cdot k$.

To obtain quantum mechanical effective Hamiltonian we employ the collective coordinate method. Now the chiral fields are considered to be time-dependent:

$$U(r, t) = \exp(i \cdot \tau^i \cdot I^{ij}(t) \cdot N^j (R^{-1}_{nk}(t) x_k) \cdot F(x \cdot e^{\lambda})), \tag{8}$$

where R and I are the spatial and isospin rotation 3 x 3 matrices, and $\lambda(t)$ is the time-dependent parameter of the dilatation vibrations. Inserting Eq.(8) into the Lagrangian in which the time components L_0 of the currents now play their important role we have

$$L = -M - \frac{F_\pi^2}{16} \cdot \int dV \, Tr(L_0 L_0) - \frac{1}{16e^2} \cdot \int dV \, Tr[L_0, L_k]^2 . \tag{9}$$

Performing the canonical transformation and determining canonically conjugate variables

$$p = \frac{\partial L}{\partial \dot{\lambda}}, \quad T = \frac{\partial L}{\partial \omega_i}, \quad S = \frac{\partial L}{\partial \Omega_i}, \tag{10}$$

where the angular velocities Ω_i and ω_i for the rotation and isorotation are given by

$$(R^{-1})_{ik} \dot{R}_{kj} = \epsilon_{ijk} \Omega_k , \quad \dot{I}_{ik} (I^{-1})_{kj} = \epsilon_{ijk} \omega_k , \tag{11}$$

we obatin the Hamiltonian for $k^2 \neq 1$

$$H = M(\lambda) + \frac{\hat{p}^2}{2 \cdot m(\lambda)} + \frac{\hat{T}^2}{2 \cdot \mathcal{G}_T} + \frac{\hat{S}^2}{2 \cdot \mathcal{G}_s} - \frac{1}{2} \cdot \left(\frac{1}{\mathcal{G}_T} + \frac{k^2}{\mathcal{G}_s} - \frac{1}{\mathcal{G}}\right) \cdot \hat{T}_3^2 . \tag{12}$$

Here symbols p, T and S are interpreted now as follows: impulse p corresponds to the vibrational operator, T and S are the isospin and spin operators. The vibrational potential $M(\lambda)$ is given by the next expression

$$M(\lambda) = M_2 \cdot \exp(-\lambda) + M_4 \cdot \exp(\lambda). \tag{13}$$

For the interval values $m(\lambda)$, $Q_T(\lambda)$, $Q_S(\lambda)$, $Q(\lambda)$ we have:

$$m(\lambda) = \frac{2\pi}{F_\pi e^3} \int_0^\infty (F')^2 \{\frac{e^{-3\lambda}}{2} + e^{-\lambda}\frac{\sin^2 F}{x^2} \int_0^\pi (k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} + (T')^2) \sin\theta\, d\theta\, \} x^4 dx, \tag{14}$$

$$Q_T(\lambda) = \frac{\pi}{F_\pi e^3} \cdot \int_0^\infty x^2 dx \int_0^\pi \sin\theta\, d\theta\, \{\sin^2 F \cdot [\frac{e^{-3\lambda}}{4} +$$

$$+ e^{-\lambda} \cdot [(F')^2 + \left(k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} + (T')^2\right) \cdot \frac{\sin^2 F}{x^2}]] \cdot (1 + \cos^2 T) - \tag{15}$$

$$- e^{-\lambda} \cdot \frac{\sin^4 F}{x^2} \cdot (k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} \cdot \cos^2 T + (T')^2) \},$$

$$Q_S(\lambda) = \frac{\pi}{F_\pi e^3} \cdot \int_0^\infty x^2 dx \int_0^\pi \sin\theta\, d\theta\, \{ \sin^2 F \cdot [\frac{e^{-3\lambda}}{4} +$$

$$+ e^{-\lambda} \cdot [(F')^2 + (k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} + (T')^2) \cdot \frac{\sin^2 F}{x^2}]] \cdot \left(k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} \cdot \cos^2\theta + (T')^2\right) -$$

$$- e^{-\lambda} \cdot \frac{\sin^4 F}{x^2} \cdot (k^4 \cdot \frac{\sin^4 T}{\sin^4\theta} \cdot \cos^2\theta + (T')^4) \}, \tag{16}$$

$$Q(\lambda) = \frac{2\pi}{F_\pi e^3} \cdot \int_0^\infty x^2 dx \int_0^\pi \sin\theta\, d\theta\, \{ \sin^2 F \cdot [\frac{e^{-3\lambda}}{4} +$$

$$+ e^{-\lambda} \cdot [(F')^2 + (k^2 \cdot \frac{\sin^2 T}{\sin^2\theta} + (T')^2) \cdot \frac{\sin^2 F}{x^2}]] \cdot \sin^2 T - \qquad (17)$$

$$- e^{-\lambda} \cdot \frac{\sin^4 F}{x^2} \cdot k^2 \cdot \frac{\sin^4 T}{\sin^2\theta} \} .$$

It should be noted that we have $S_3^{b.f.} - k \cdot T_3^{b.f.} = 0$. It is a constraint for the wave function of the quantized Skyrmion. More strictly speaking the wave function is given by

$$<I, R \mid TK,\ CM,\ L> = \frac{\sqrt{(2T+1)(2S+1)}}{8\pi^2} D^T_{KL}(I) \cdot D^{S}_{M-KL}(R) \qquad (18)$$

as in[4] and its parity is given as $P = (-1)^L$. If we exclude the vibrational degrees of freedom from our consideration we obtain the expression for the mass spectra for B = 2 (k = 2, ℓ = 1):

$$E_{S,T,T_3} = \frac{F_\pi}{e} \{70.55 + \frac{e^4}{2} \cdot [\frac{S(S+1)}{272.4} + \frac{T(T+1)}{183.0} - \frac{1}{83.2} T_3^2]\} \qquad (19)$$

(for an arbitrary value of F_π and e).

Now we present here some numerical results for the calculated soliton states with B = 2 (see Table 1) and lowest multibaryon states with B = 3,4 (Table 2). The calculations were performed in the harmonic approximation with the next values of the constant: e = 4.84 and F_π = 108 MeV (M_{nucl} = 931 MeV).

Table 1. The calculated energies for the B=2 (k=2, =1) soliton states with isospin T, spin parity S^P and quantum number n_λ=0 corresponding to the vibration mode

T	S^P	$E-2M_{nucl}$
0	0^+	−214 MeV
0	1^+	−172 MeV
1	0^+	−154 MeV
1	1^+	−118 MeV
1	2^-	− 53 MeV

Table 2. The lowest multibaryon states (k = 3,4, ℓ = 1)

B	T	S	T_3	$E-B \cdot M_{nucl}$
3	1/2	3/2	1/2	−268.0 MeV
3	3/2	3/2	1/2	−210.5 MeV
4	0	0	0	−324.0 MeV
4	0	1	0	−312.7 MeV
4	1	0	0	−294.5 MeV

The calculation shows that the classically nonstable state $k = 4$, $\ell = 1$ has the binding energy [2] +220.7 MeV, and becomes stable when the quantum correction is taken into account (see Table 2). It may have some more general sense.

The effective quantum mechanical Hamiltonian for baryon and heavy meson-like states of the Skyrme model is obtained in the framework of the collective coordinate method. The collective variables correspond to vibrations and rotations.

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