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PION-NUCLEON Σ TERM IN THE QUARK MODEL OF A SUPERCONDUCTING TYPE

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It is shown that by using an intermediate scalar meson ϵ (700) naturally arising in the quark model of a superconductivity type one can obtain an experimental value for the $\Sigma_{\pi N}$ term considering only u and d quarks.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Пион-нуклонный Σ-член в кварковой модели сверхпроводящего типа

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Показано, что с учетом промежуточного скалярного мезона ϵ (700), естественным образом возникающего в кварковой модели сверхпроводящего типа, можно получить экспериментальное значение Σ-члена при рассмотрении только u и d кварков.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

The present paper is devoted to the calculation of the pion-nucleon σ term determined by the matrix element

$$\sigma = \frac{m_u^0 + m_d^0}{4M_P} <P\,|\,\bar{u}u + \bar{d}d\,|\,P>\,, \tag{1}$$

where $|\,P>$ is a one-proton physical state, M_P = 938 MeV is the proton mass and m_q^0 is the current q quark mass (q = u, d). Chiral symmetry allows one to connect the matrix element (1) with the even (with respect to isotopic transformations) amplitude of πN scattering $\bar{D}^+(\nu, t)$

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calculated at the Cheng-Dashen point [1]: $\nu = (s-u)/4M_P = 0,\ t = 2m_\pi^2$*

$$\Sigma = F_\pi^2\ \bar{D}^{(+)}(0, 2m_\pi^2) = \sigma + \Delta\,, \qquad (2)$$

where F_π = 93 MeV is the π meson decay constant. The first term in the right-hand side of (2) determined by the matrix element (I) can be treated as a contribution of the scattering amplitude of a massless pion on a physical nucleon, i.e. $\sigma = F_\pi^2 \bar{D}^{(+)}$ (0,0), whereas the second term Δ is a correction due to nonzero pion mass. The theoretical value of Δ does not exceed 5 MeV, i.e. we may assume that $\Delta \simeq 5$ MeV [2,3]. Extrapolation of the experimental values of the πN scattering amplitudes into Cheng-Dashen point enables one to get an experimental value for the Σ term (2):

$$\Sigma_{exp} = \begin{cases} 64 \pm 8 \text{ MeV } [4] \\ 56 \pm 8 \text{ MeV} [3,5]. \end{cases} \qquad (3)$$

Using (3) and relation (2) at Δ = 5 MeV [2,3] we get an experimental estimate for the matrix element (1)

$$\sigma_{exp} = \begin{cases} 59 \pm 8 \text{ MeV } [4], \\ 51 \pm 8 \text{ MeV } [3,5]. \end{cases} \qquad (4)$$

The experimental values for the σ term (4) turned out to be unexpectedly large. The theoretical estimate for the σ term from the mass formulae of baryons in approximation linear in the current quark masses leads to the value σ = 25 MeV. Allowance for higher orders in the current quark masses may increase this value only by 10 MeV [5], i.e. σ = 35 MeV.

Since a large experimental value of σ_{exp} (4) could not be described within usual approximations of current algebra and a standard baryon mode, it was suggested to increase the theoretical value of the σ term due to the contribution of the $\bar{s}s$ to the valence proton structure (see [2,3,6] and refs. therein)

$$\sigma = \sigma_0 / 1 - y\,, \qquad (5)$$

**Here* s, u *and* t *are the kinematic invariants of* πN *scattering:* $s + u + t = 2m_\pi^2 + 2M_P^2$.

where σ_0 = 35 MeV and y determines the portion of s quarks in the valence proton structure

$$y = \frac{2 \langle P \mid \bar{s}s \mid P \rangle}{\langle P \mid \bar{u}u + \bar{d}d \mid P \rangle} . \tag{6}$$

It can easily be calculated that the portion of s quarks in the valence proton structure, which is necessary to explain the experimental value of the σ term (4), should be of about 40%, i.e. $y \simeq 0.4$. Such a large number of strange quarks in a proton would mean that half of the observed proton mass should be determined by the s quark component in a proton [7]. This must cause drastical changes of the standard baryon model [8]*.

To conserve the standard baryon model, the experimental value of the σ term (4) should be explained without introducing the $\bar{s}s$ component in the valence proton structure. Since the current algebra cannot correctly describe the σ term (1), it is interesting to calculate the matrix element (1) within a phenomenological quark model with the chiral symmetry, based on the QCD and successfully describing strong low-energy hadron interactions. Interest in the model calculation of the σ term (1) stems also from the fact that the use of dispersion methods for calculating the values of the term (2) leads to the result Σ_{disp} = = 63 MeV [9] which is in good agreement with the experimental data (3)**.

Therefore, in the present paper we undertake calculation of the σ term (1) in the quark model of a superconducting type (QMST). The latter is a generalization on the quark level of the known Nambu — Iona—Lasinio model [11] and describes strong low-energy interactions of four nonets of low-lying mesons (scalar, pseudoscalar, vector and axial) with the linear realisation of the chiral U(3)xU(3) symmetry. The vertices of low-energy interactions of mesons are approximated in the QMST by one loop quark diagrams with virtual constituents quarks. The rules of calculating one-loop quark diagrams in the QMST, which are in accordance with the requirements of the chiral symmetry and confinement, are expounded in ref. [12].

Note that the authors of ref. [8b] have shown that in a special regime of chiral symmetry breaking admissible in the Nambu–Iona–Lasinio model, one can obtain a large enough value for the Σ term with a small number of s quarks in a proton.

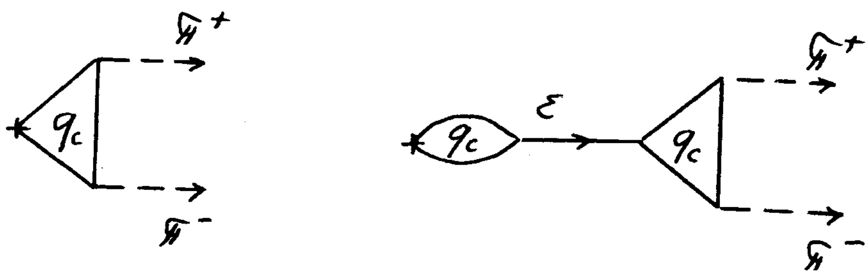
***The quantity $\Sigma_{disp.}$ = 63 MeV is mean over all the results obtained in refs. [10].*

First, let us demonstrate the calculational possibilities of the QMST by using the matrix element $<\pi^+|\bar{u}u+\bar{d}d|\pi^+>$ as an example. Then, we shall give the estimate for the matrix element $<\pi^+|uu+dd|\pi^+>$ with the help of the low-energy Gell—Mann—Oakes—Renner theorem (13) derived within the current algebra and PCAC hypothesis

$$<\pi^+(p)|\bar{u}u+\bar{d}d|\pi^+(p)> \simeq <\pi^+(0)|\bar{u}u+\bar{d}d|\pi^+(0)> = \\ = \frac{1}{F_\pi^2}<0|\bar{u}u+\bar{d}d|0> = \frac{2v}{F_\pi^2}, \quad (7)$$

where $v=-<0|\bar{u}u|0>=-<0|\bar{d}d|0>$ is the quark condensate.

Now let us calculate (7) in the QMST. In this model, the abovepointed matrix element is determined by two diagrams depicted in figs. 1 and 2*. The diagram in fig.2 describes the exchange by a virtual



Figs. 1 and 2. Quark diagrams describing the $<\pi^+|\bar{u}u+\bar{d}d|\pi^+>$ *matrix element in the QMST.*

scalar ϵ meson with the quark structure $(\bar{u}_c u_c+\bar{d}_c d_c)/\sqrt{2}$, where u_c and d_c are constituent u and d quarks. This diagram plays an important role in describing the matrix element in the QMST, as will be shown below.

The Lagrangian describing the diagram shown in figs.1 and 2 has the form

$$\mathcal{L} = g\bar{q}_c\,(\epsilon + iZ^{1/2}\gamma^5\vec{\tau}\vec{\pi})\,q_c\,, \quad (8)$$

**To obtain the dependence of the matrix element on p it is necessary to consider also the diagrams with* $\pi\to a_1$ *transitions on the pion-legs where* a_1 *is an axial-vector meson (see* /12/ *). Since we are interested only in the constant contribution to the term, we shall not consider these diagrams.*

where $\bar{q}_c = (\bar{u}_c, \bar{d}_c)$ are constituent u and d quarks, $g = m_u/(F_\pi Z^{1/2})$, $Z = (1 - 6m_u^2/m_{a_1}^2)$ is the constant of an additional renormalization of pion fields which arises after allowance for $\pi \to a_1$ transitions [12], $m_u = m_d = 280$ MeV is the mass of constituent quarks in the QMST, $m_{a_1} = 1260 \pm 30$ MeV [14], and $\vec{\tau}$ are the Pauli matrices.

In calculating the matrix element (7) in the QMST we should take into account only divergent parts of the loop quark diagrams depicted in figs.1 and 2. This requirement follows from the condition of conservation of chiral symmetry and fulfillment of quark confinement (see [12]). As a result, we get

$$<\pi^+ | \bar{u}u + \bar{d}d | \pi^+> = \frac{g_{\epsilon\pi\pi} C(m_\epsilon)}{g}, \tag{9}$$

$$C(m_\epsilon) = 1 + \frac{8g^2}{m_\epsilon^2}[I_1(m_u) - 2m_u^2 I_2(m_u)].$$

Here, the first term in $C(m_\epsilon)$ describes the contribution of diagram 1; whereas the remaining part, the contribution from diagram 2. $g_{\epsilon\pi\pi} = 4gm_u Z$ is the effective $\epsilon\pi\pi$ interaction constant (see [12]), and m_ϵ is the scalar ϵ meson mass. We consider two possible cases: 1) when $m_\epsilon^2 = 4m_u^2 + m_\pi^2/Z$ is the model value of m_ϵ following from QMST [12], and 2) when $m_\epsilon \simeq 700$ MeV is the physical value of the scalar resonance with a large decay width $\epsilon \to 2\pi$ [15]: $I_1(m)$ and $I_2(m)$ are the quadratically and logarithmically divergent integrals

$$I_1(m_u) = -i\frac{3}{(2\pi)^4}\int \frac{d^4k\,\theta(\Lambda^2 - k^2)}{m_u^2 - k^2} = \frac{3}{(4\pi)^2}[\Lambda^2 - m_u^2 \ln(1 + \frac{\Lambda^2}{m_u^2})],$$

$$I_2(m_u) = -i\frac{3}{(2\pi)^4}\int \frac{d^4k\,\theta(\Lambda^2 - k^2)}{(m_u^2 - k^2)^2} = \frac{3}{(4\pi)^2}[\ln(1 + \frac{\Lambda^2}{m_u^2}) - \frac{\Lambda^2}{\Lambda^2 + m^2}], \tag{10}$$

where $\Lambda = 1.25$ GeV is the cutoff parameter limiting the region where a spontaneous breaking of chiral symmetry takes place [12].

Assuming $m_\epsilon^2 \simeq 4m_u^2$ and using the condition $4g^2 I_2(m_4) = 1$ following from the definition of g [12] we get

$$<\pi^+ | \bar{u}u + \bar{d}d | \pi^+> = \frac{2v_{QMST}}{F_\pi^2}, \tag{11}$$

i.e., the result of the low-energy theorem (7). Here, the relation $v_{QMST} = 4m_u I_1(m_u)$ has been used (see [12]).

Thus, we have shown how to calculate the matrix element (11) in the QMST. The matrix element $\langle P|\bar{u}u + \bar{d}d|P\rangle$ is calculated in an analogous way: therefore we give the final result

$$\langle P|\bar{u}u + \bar{d}d|P\rangle = \frac{g_{\epsilon PP}}{g_{\epsilon\pi\pi}} \frac{2v_{QMST}}{F_\pi^2} \bar{U}(P)U(P) = \frac{g_{\epsilon PP}}{g_{\epsilon\pi\pi}} \frac{2v_{QMST}}{F_\pi^2} 2M_P. \quad (12)$$

Here $g_{\epsilon PP}$ is the interaction ϵPP constant on the proton mass shell ($p^2 = M_P^2$), and $U(P)$ is the bispinor normalized by the condition $\bar{U}(P)U(P) = 2M_P$. Using (12) we find the σ term (1) in the QMST

$$\sigma_{\pi N}^{QMST} = \frac{g_{\epsilon PP}}{g_{\epsilon\pi\pi}} m_\pi^2 = \frac{1}{Z^{1/2}} \left(\frac{m_\pi}{2m_u}\right)^2 M_P . \quad (13)$$

In deriving this formula we have used the Goldberger — Treiman relation

$$g_{\epsilon PP} = \frac{g_{\pi PP}}{g_A} = \frac{M_P}{F_\pi} \quad (14)$$

following from the requirements of chiral symmetry (see [16]), where $g_A = 1.259 + 0.004$ is the axial current renormalization constant of β decay [14], and

$$m_\pi^2 = \frac{v}{F_\pi^2}(m_u^0 + m_d^0) . \quad (15)$$

is the Gell—Mann—Oakes—Renner formula that holds valid in the QMST [12]. Using the numerical values of the parameters of our model and experimental values of other quantities, for the σ term we get

$$\sigma_{\pi N}(0) = 50 \text{ MeV}. \quad (16)$$

This quantity is in good agreement with experimental data.

Now let us see how this estimate changes if instead of the theoretical model value $m_\epsilon \simeq 2m_u$ one uses the physical value $m_\epsilon \simeq 700$ MeV. For the $\epsilon\pi\pi$ vertex one should take into account the form factor, as it has been done, for instance, in calculating the $\pi\pi$ scattering lengths and in describing other processes (see /12b/)

$$f_{\epsilon\pi\pi}(q) = 1 + \frac{m_\epsilon^2 - q^2}{Z(4\pi F_\pi)^2}. \tag{17}$$

Then, for the σ term we get ($\frac{C_f(m_\epsilon = 700\ \mathrm{MeV})}{C(m_\epsilon = 2m_u)} = 0.85$)

$$\sigma_{\pi N}^{(\epsilon(700))} = 43\ \mathrm{MeV}. \tag{16'}$$

Without taking the form factor into account $\sigma_{\pi N}^{(\epsilon(700))} = 38$ MeV. The value of (16') is rather close to the experimental data. One should remember that the obtained estimates may somewhat increase if other scalar resonances are taken into account.

Thus, it is shown that in our model one can obtain a satisfactory enough estimate for the experimentally observed σ term with allowance made only for u and d quarks.

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