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ANOTHER POSSIBILITY OF CONSERVED-VECTOR-CURRENT HYPOTHESIS VERIFICATION

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Based on the conserved-vector-current (CVC) hypothesis and a four-ρ-resonance unitary and analytic VMD model of the pion electromagnetic form factor, the behaviour of a total cross section and energy distribution of the final state pions of $\bar{\nu}_e e^- \to \pi^- \pi^0$ process are predicted theoretically for the first time. An experimental confirmation of the latter could provide another reliable method of CVC-hypothesis verification for all energies above the two-pion threshold.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Другая возможность проверки гипотезы сохранения векторного тока

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На основе гипотезы сохранения векторного тока (СВТ) и унитарной и аналитической ВМД-модели с четырьмя ρ-резонансами для описания электромагнитного формфактора пиона впервые теоретически предсказывается поведение полного сечения и энергетическое распределение пионов в конечном состоянии и процессе $\bar{\nu}_e e^- \to \pi^- \pi^0$. Экспериментальная проверка предсказанного представляет другой метод проверки СВТ-гипотезы для всех энергий выше двухпионного порога.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

In the V-A theory of weak interactions [1] the hadronic charged weak current J_μ^W is a compound of vector V_μ and axial-vector A_μ currents. In the second half of the fifties the conserved-vector-current (CVC) hypothesis

$$\partial_\mu V_\mu = 0 \qquad (1)$$

was postulated [2,3] in order to explain an approximate numerical equality of the muon decay constant G^μ and the neutron decay vector constant $G^{(V)}$. Later on the CVC-hypothesis manifested to be very powerful. Here we

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notice only the relation between a matrix element $\langle\pi^0|V_\mu|\pi^+\rangle$ and a matrix element $\langle\pi^+|J_\mu^E|\pi^+\rangle$ of the electromagnetic (e.m.) current J_μ^E following directly from (1), which finally leads to the relation

$$F_\pi^W(s) = \sqrt{2}\, F_\pi^{E,I=1}(s) \tag{2}$$

between the weak pion form factor (ff) $F_\pi^W(s)$ of a virtual W^--boson transition $(W^-)^* \to \pi^-\pi^0$ and the pure isovector e.m. pion form factor $F_\pi^{E,I=1}(s)$ of a virtual photon $\gamma^* \to \pi^+\pi^-$ transition, where $\sqrt{2}$ is a Clebsch-Gordan coefficient of the SU(2) isotopic group. The relation (2) has shown to be very useful. There are neither data on $F_\pi^W(s)$ nor an accomplished theory, nor a phenomenology giving a reliable behaviour of the weak pion ff. On the other hand, the behaviour of the e.m. pion ff is understood from the experimental (for a compilation see [4]) and phenomenological points of view [5,6] quite well. So, the relation (2) already in an early stage allowed [2,3] to predict a probability of the pion beta-decay $\pi^+ \to \pi^0 e^+\nu_e$. Its experimental confirmation [7] is now presented as one of the brilliant demonstrations of a general validity of the CVC-hypothesis in the weak interaction theory. However, there is a release of a negligible amount of energy in the pion beta-decay and in fact one is authorized to speak about a CVC-hypothesis verification only in surroundings of $s \approx 0$.

To validate experimentally the CVC-hypothesis outside this restricted region, we propose here to investigate the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ process. Its threshold energy is $E_\nu^{(0)} \approx 2m_\pi^2/m_e \approx 76.7$ GeV and so, it is already attainable experimentally on existing accelerators.

The differential cross-section of the weak $\bar{\nu}_e e^- \to \pi^-\pi^0$ reaction in the c.m. system is given by the expression

$$\frac{d\sigma}{d\Omega} = \frac{1}{(2s_{\bar{\nu}}+1)(2s_e+1)} \frac{1}{64\pi^2 s} \frac{k}{p} \sum_{s_{\bar{\nu}},s_e} |M|^2, \tag{3}$$

where $s \geq 4m_\pi^2$ is the c.m. energy squared, $k = ((s-4m_\pi^2)/4)^{1/2}$ is the length of a 3-dimensional momentum of produced pions, $p = (s/4)^{1/2}$ is the length of a 3-dimensional neutrino-momentum and $s_{\bar{\nu}}$ and s_e are spins of the antineutrino and electron, respectively. The matrix element M in the lowest order of a perturbation expansion can be calculated from the Feynman diagram presented in Fig.1a, that for $s << m_{W^-}^2$ is reduced to a contact diagram presented in Fig.1b.

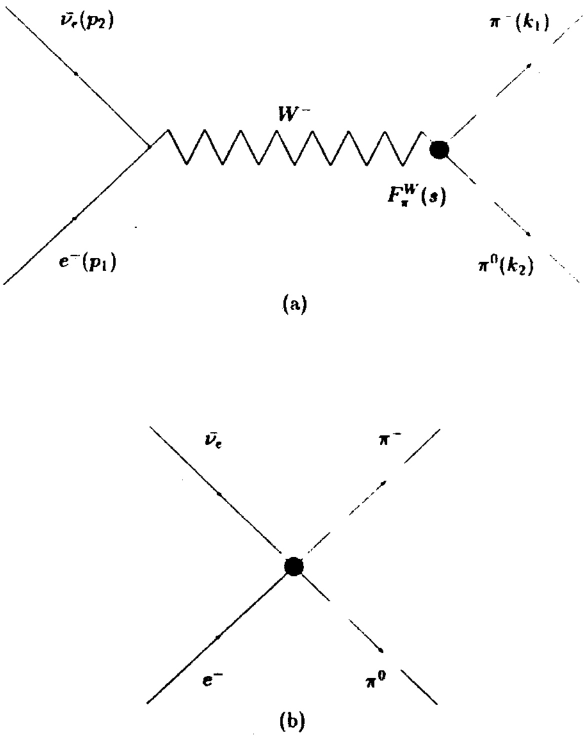

Fig. 1. The lowest order perturbation expansion Feynman diagram giving a dominant contribution to the weak $\bar{\nu}_e e^- \to \pi^- \pi^0$ process

It takes the form

$$M = \frac{G}{\sqrt{2}} \bar{\nu}_e(p_2)\gamma_\mu(1 + \gamma_5)e(p_1)\,(k_1 - k_2)^\mu F_\pi^{\ W}(s), \tag{4}$$

where $G = 1.1663 \cdot 10^{-5}$ GeV^{-2} is the weak interaction Fermi constant. The expression (4) leads to

$$\frac{d\sigma}{d\Omega} = \frac{G^2}{128\pi^2} s \beta_\pi^3 \,|F_\pi^{\ W}(s)|^2 \sin^2\theta, \tag{5}$$

where $\beta_\pi = (1 - 4m_\pi^2/s)^{1/2}$ is the velocity of produced pions and θ is the scattering angle in the c.m. system. To predict a behavior of $\sigma_{tot}(E_\nu^{lab})$, first

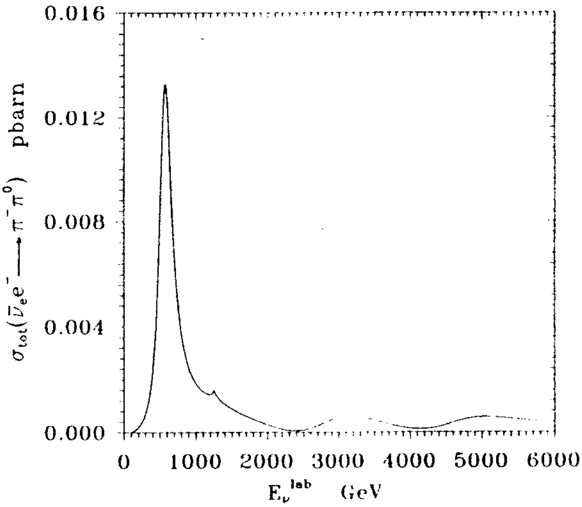

Fig.2. The predicted by (6) behaviour of the total cross-section of the $\bar{\nu}_e e^- \to \pi^- \pi^0$ process at the laboratory system

we multiply (5) by $d\Omega = \sin\theta \, d\theta \, d\varphi$, then integrate over angles θ and φ and finally substitute the expression (2) (generally valid for $-\infty < s < +\infty$) and the relation $s = m_e^2 + 2m_e E_\nu^{lab}$. As a result, one gets

$$\sigma_{tot}(E_\nu^{lab}) = \frac{G^2}{24\pi}(m_e^2 + 2m_e E_\nu^{lab})\beta_\pi^3 \, |F_\pi^{E,I=1}(E_\nu^{lab})|^2 \tag{6}$$

from which, by using four-ρ-resonance unitary and analytic VMD model [6] of the pion e.m. ff, the behaviour of $\sigma_{tot}(E_\nu^{lab})$ in the laboratory system as shown in Fig.2 is predicted.

Besides the $\sigma_{tot}(E_\nu^{lab})$ it is interesting also to predict an energy distribution of the pions created in the weak $\bar{\nu}_e e^- \to \pi^- \pi^0$ process to be given by $d\sigma / dE_\pi^{lab}$. The latter is obtained from (5), first by integration over the φ angle and then by substitution of the relations

$$d\cos\theta = -\frac{m_e}{k^{c.m.} \, p^{c.m.}} dE_\pi^{lab}; \quad s \approx 2m_e E_\nu^{lab};$$
$$\sin^2\theta = 1 - \frac{(E_\pi^{c.m.} E_e^{c.m.} - m_e E_\pi^{lab})^2}{(k^{c.m.})^2 (p^{c.m.})^2} dE_\pi^{lab}, \tag{7}$$

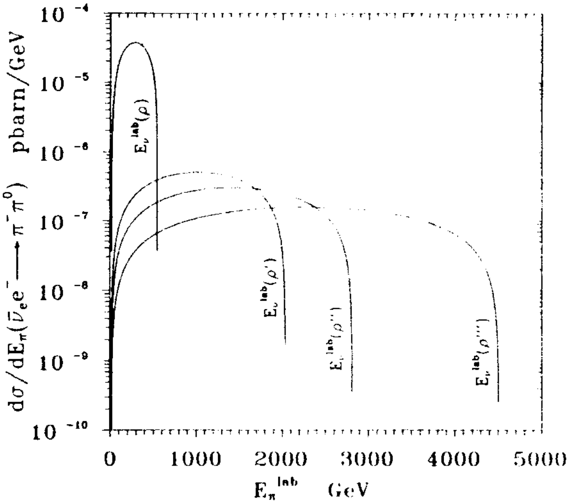

Fig.3. The predicted behaviour of the energy distribution of pions as given by (8)

where

$$E_\pi^{\text{c.m.}} \approx E_e^{\text{c.m.}} \approx p^{\text{c.m.}} \approx \sqrt{\frac{m_e}{2} E_\nu^{\text{lab}}} \quad \text{and} \quad k^{\text{c.m.}} \approx \sqrt{\frac{m_e}{2} (E_\nu^{\text{lab}} - E_\nu^{(0)})}.$$

Consequently, one gets the expression

$$\frac{d\sigma}{dE_\pi^{\text{lab}}} = \frac{m_e G^2}{8\pi} \left(\frac{E_\nu^{\text{lab}} - E_\nu^{(0)}}{E_\nu^{\text{lab}}} \right) \left\{ 1 - \frac{(E_\nu^{\text{lab}} - 2E_\pi^{\text{lab}})^2}{E_\nu^{\text{lab}}(E_\nu^{\text{lab}} - E_\nu^{(0)})} \right\} |F_\pi^{E,I=1}(E_\nu^{\text{lab}})|^2 \tag{8}$$

from which the energy distribution of the final state pions of the reaction $\bar{\nu}_e e^- \to \pi^- \pi^0$ at four different energies corresponding just to $\rho(770)$, $\rho'(1450)$, $\rho''(1700)$, $\rho'''(2150)$ resonances is calculated. The results are graphically presented in Fig.3.

The experimental approval of our predictions for $\sigma_{\text{tot}}(E_\nu^{\text{lab}})$ and $d\sigma/dE_\pi^{\text{lab}}$ could validate the CVC-hypothesis for all investigated energies above the two-pion threshold.

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