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NEXT-TO-NEXT-TO-LEADING ORDER QCD ANALYSIS OF COMBINED DATA FOR xF_3 STRUCTURE FUNCTION AND HIGHER-TWIST CONTRIBUTION

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The simultaneous QCD analysis of the xF_3 structure function measured in deep-inelastic scattering by several collaborations is done up to 3-loop order of QCD. The x dependence of the higher-twist contribution is evaluated and turns out to be in a qualitative agreement with the results of «old» CCFR data analysis and with renormalon approach predictions. The Gross–Llewellyn Smith sum rule and its higher-twist corrections are evaluated.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Совместный КХД анализ данных по xF_3 в третьем порядке теории возмущений и определение вклада высших твистов

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Совместный КХД анализ данных по структурной функции xF_3, измеренной различными коллаборациями, проведен в 1-, 2- и 3-петлевом приближении. Определена x-зависимость вклада высших твистов в структурную функцию. Определена экспериментальная величина вклада высших твистов в правило сумм Гросса – Льювеллина-Смита.

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1. The experimental data of the CCFR collaboration (we'll call them «old») obtained at Fermilab Tevatron [1] for the xF_3 structure functions of deep-inelastic scattering of neutrinos and antineutrinos on an iron target provide an important means of accurate comparison of QCD with experiment. However, in view of revision of «old» data announced by CCFR collaboration [2] the question arises: what can we say about the comparison of the QCD predictions on Q^2 dependence of the $xF_3(x, Q^2)$ structure function (SF) based on the data of neutrino DIS experiments different from those of CCFR?

In the present note, a combined fit of the experimental data of the CDHS [3], SCAT [4], BEBC-WA59 [5], BEBC-Gargamelle [6] and JINR-IHEP [7] collaborations for the xF_3 structure functions is done in order to determine the x dependence of the SF, higher twist (HT) contribution and the value of the scale parameter $\Lambda_{\overline{MS}}$.

2. We'll use, for the QCD analysis, the Jacobi polynomial expansion method proposed in [8]. It was developed in [8]—[14] and applied for the 3-loop order of peturbative QCD (pQCD) to fit F_2 [13] and xF_3 data [14,15].

The Q^2-evolution of the moments $M_3^{\mathrm{pQCD}}(N, Q^2)$ is given by the well-known perturbative QCD [16,17] formula:

$$M_3^{\mathrm{pQCD}}(N,Q^2) = \left[\frac{\alpha_S(Q_0^2)}{\alpha_S(Q^2)}\right]^{d_N} H_N(Q_0^2,Q^2) M_3^{\mathrm{pQCD}}(N,Q_0^2), \quad N = 2, 3, \ldots$$

$$d_N = \gamma^{(0),N}/2\beta_0. \tag{1}$$

The factor $H_N(Q_0^2,Q^2)$ contains next- and next-to-leading order QCD corrections* and is constructed in accordance with [14] based on theoretical results of [19].

The expression (1) provides an input for reconstruction of the SF by the Jacobi polynomial method. Following the method [10,11], we can write the leading twist contribution to the structure function xF_3 in the form:

$$xF_3^{\mathrm{pQCD}}(x,Q^2) = x^{\alpha}(1-x)^{\beta} \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha\ \beta}(x) \sum_{j=0}^{x} c_j^{(n)}(\alpha, \beta)(\beta) M_3^{\mathrm{QCD}}(j+2, Q^2), \tag{2}$$

where $\Theta_n^{\alpha\beta}(x)$ is a set of Jacobi polynomials and $c_j^{(n)}(\alpha, \beta)$ are coefficients of the series of $\Theta_n^{\alpha\beta}(x)$ in powers of x:

$$\Theta_n^{\alpha\beta}(x) = \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta)(\beta) x^j. \tag{3}$$

The unknown coefficients $M_3(N, Q_0^2)$ in (1) could be parametrized as Mellin moments of some function:

$$M_3^{\mathrm{pQCD}}(N, Q_0^2) = \int_0^1 dx x^{N-2} A x^b (1-x)^c (1+\gamma x), \quad N = 2, 3, \ldots \tag{4}$$

To extract the HT contribution, the nonsinglet SF is parametrized as follows:

$$xF_3(x,Q^2) = xF_3^{\mathrm{pQCD}}(x, Q^2) + h(x)/Q^2, \tag{5}$$

where the Q^2 dependence of the first term in the r.h.s. is determined by perturbative QCD. Constants $h(x_i)$ (one per x-bin) parameterize the HT x dependence. We put x_i = 0.03, 0.05, 0.08, 0.15, 0.25, 0.35, 0.45, 0.50, 0.55, 0.65, 0.80 for i = 1, 2... 11. The HT contribution or F_2 was determined in [20]. The values of constants $h(x_i)$ as well as the parameters A, b, c, γ and scale parameter Λ are determined by fitting the combined set of data of 192 experimental points of xF_3 in a wide kinematic region: $0.5\ \mathrm{GeV}^2 \le Q^2 \le 196\ \mathrm{GeV}^2$ and $0.03 \le x \le 0.80$ and $Q_0^2 = 10\ \mathrm{GeV}^2$. We have put the number of flavors to equal 4. In accordance with the result of [3] concerning the disagreement of their data with perturbative QCD at small x, a cut $x \ge 0.35$

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*For reviews and references on higher order QCD results see [18].

Table. Results of the 1-, 2- ($N_{max} = 10$) and 3- order ($N_{max} = 8$) QCD fit (with TMC) of the combined xF_3 SF data for $f = 4$, $Q^2 > 0.5$ GeV^2 with the corresponding statistical errors, normalization coefficients and values of the HT contribution $h(x_i)$

	1-loop approx.	2-loop approx.	3-loop approx.
$\chi^2_{d.f.}$	312/176	316/176	312/176
A	6.68 ± 0.38	6.92 ± 1.43	7.11 ± 0.38
b	0.760 ± 0.027	0.768 ± 0.072	0.778 ± 0.027
c	4.03 ± 0.07	3.97 ± 0.17	3.82 ± 0.07
γ	0.675 ± 0.156	0.452 ± 0.624	0.189 ± 0.128
$\Lambda_{\overline{MS}}$, MeV	191 ± 46	159 ± 39	163 ± 31
x_i	$h(x_i)$, GeV^2		
0.03	0.086 ± 0.087	0.090 ± 0.091	0.067 ± 0.085
0.05	0.001 ± 0.028	0.022 ± 0.032	0.093 ± 0.047
0.08	-0.127 ± 0.123	-0.094 ± 0.126	-0.011 ± 0.131
0.15	-0.286 ± 0.046	-0.230 ± 0.050	-0.200 ± 0.050
0.25	-0.401 ± 0.058	-0.334 ± 0.056	-0.327 ± 0.054
0.35	-0.284 ± 0.073	-0.220 ± 0.068	-0.178 ± 0.062
0.45	-0.436 ± 0.093	-0.366 ± 0.090	-0.403 ± 0.083
0.50	0.005 ± 0.079	0.047 ± 0.077	0.036 ± 0.074
0.55	-0.243 ± 0.069	-0.200 ± 0.068	-0.242 ± 0.064
0.65	0.176 ± 0.063	0.202 ± 0.072	0.154 ± 0.060
0.80	0.020 ± 0.037	0.024 ± 0.039	-0.012 ± 0.039

was used for CDHS data. The target mass corrections (TMC) are taken into account to the order $o(M^4_{\rm nucl}/Q^4)$ [14].

The nuclear effect of the relativistic Fermi motion is estimated from below by the ratio $R_F^{D/N} = F_3^D/F_3^N$ obtained in the covariant approach in light-cone variables [21].

3. Results of the fit for distribution parameters, the shape of the next twist contribution $h(x)$ and parameter Λ are presented in the Table and in the Figure.

The experimental values of xF_3 for each collaboration were multiplied by the normalization factors C^{coll} which were considered as free parameters. Their values are not sensitive to the order of pQCD in use and were found to be equal to: $C^{\text{BEBC-WA59}} = 0.92 \pm 0.03$, $C^{\text{SCAT}} = 1.06 \pm 0.03$, $C^{\text{JINR-IHEP}} = 1.02 \pm 0.05$, and $C^{\text{BEBC-Gard}} = 0.97 \pm 0.04$. The value of $C^{\text{CDHS}} = 1$ was fixed.

The obtained value of $\Lambda_{\overline{MS}}$ is larger than that given by a similar analysis of CCFR data [15] $\Lambda_{\overline{MS}} = 134 \pm 57$ MeV but exhibits relatively small statistical errors. Results of the NLO and NNLO fit the constant of strong interaction $\alpha_S^{\text{NLO}}(M_Z^2) = 0.105 \pm 0.004$ and $\alpha_S^{\text{NNLO}}(M_Z^2) = 0.107 \pm 0.003$ in agreement, within the errors, with usual DIS results [22]. Additional uncertainties to the value of $\alpha_S(M_Z^2)$ due to extrapolation of the Q^2 dependence of the SF with four flavors ($f = 4$) in a wide kinematic interval $0.5\ \text{GeV}^2 \le Q^2 \le 196\ \text{GeV}^2$ were found to be about 0.001 in [23] and 0.5 should be taken into account, too.

The value of the perturbative part of the GLS sum rule [24] at $Q^2 = 10\ \text{GeV}^2$ estimated by using results of the Table is equal to $\int_0^{-1} \frac{xF_3^{\text{pQCD}}(x)}{x}\, dx = 2.60 \pm 0.23$ in agreement with results of the «old» CCFR data analysis [25,12].

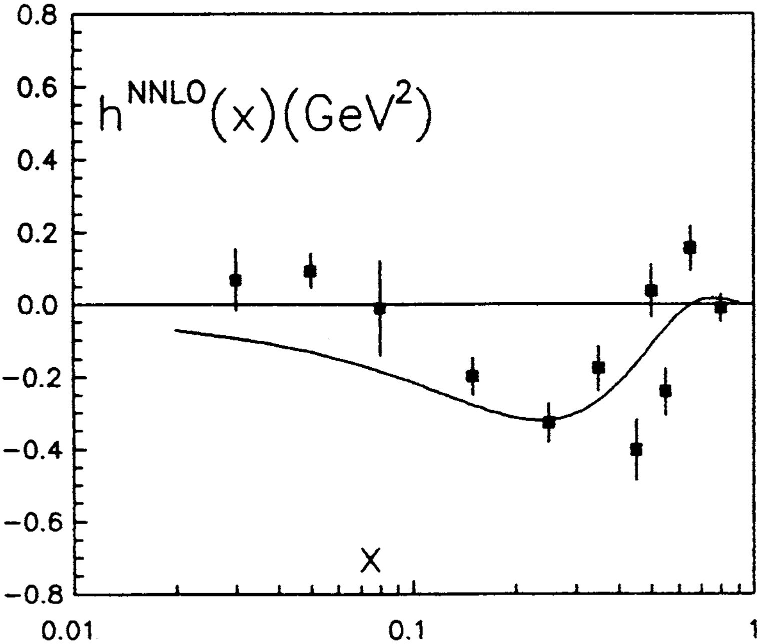

Figure. Higher-twist contributions from NNLO fit and the theoretical prediction for $h(x)$ from [26]

The shape of $h(x)$ is in qualitative agreement with theoretical predictions of the dispersion method of the renormalon approach [26] (for reviews and references see [27]) and with results of the QCD analysis of «old» CCFR data presented in [15]. The only difference is the positive value of measured $h(x)$ at small x. The obtained $h(x)$ obviously differs from the precise values of HT contribution for singlet SF F_2 presented in [20]. See also [28] for model dependent evolution of $h(x)$.

4. In conclusion it should be stressed that combined fit provides still a more precise determination of $\Lambda_{\overline{MS}}$ and $h(x_i)$ in comparison to the analysis of «old» CCFR data [15], while the shape of the SF ruled by parameters A, b, c and γ is determined less accurate. The most discrepancy with the «old» CCFR data analysis takes place for the HT contribution to the GLS sum rule and for the HT x dependence at large x.

Based on the results of the Table, one can estimate the value of the first moment of $h(x)$: $h_1 = \int_0^1 \frac{h(x)}{x}\,dx$. The obtained values: $h_1^{\rm LO} = -0.42 \pm 0.27$[*], $h_1^{\rm NLO} = -0.29 \pm 0.28$, and $h_1^{\rm NNLO} = -0.26 \pm 0.27$ are in agreement with theoretical predictions of [29] $h_1 = -0.29 \pm 0.14$ and [30] $h_1 = -0.47 \pm 0.04$ as well as with the recent result of [31].

For a more precise determination of the HT contribution to SF, the role of the nuclear effect should be clarified and a more realistic approximation for $R_F^{\rm Fe/N} = F_3^{\rm Fe}/F_3^{\rm N}$ is needed. We also did not take into account the threshold effects on Q^2 evolution of SF due to heavy quarks [32] which is necessary owing to a wide kinematic region of data under consideration and have been realized in [23].

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[*] Hereafter present the value of $h(x)$ in [GeV^2].

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