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PHYSICS FROM EXTRA DIMENSIONS

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A brief review of the recent developments in the physics from extra dimensions is given with a focus on the effects of Kaluza–Klein excitations in the Standard Model sector. It is shown that the current accurate data on the Fermi constant and on other electro-weak parameters puts a lower bound on the scale of extra dimensions of ~ 3 TeV, and thus the observation of such dimensions lies beyond the reach of accelerators in the near future. The correction to the anomalous magnetic moment of the muon from extra dimensions is discussed and one finds that, with the current limit on the scale of extra dimensions from the Fermi constant, the correction to $g_\mu - 2$ does not compete with the potentially large contributions from the supersymmetric electro-weak correction. The possibility of generating Kaluza–Klein excitations associated with large-radius compactifications at the LHC is discussed. It is shown that if such excitations are indeed produced their resonance structure will encode information on the number of compactified dimensions as well as on the nature of the specific orbifold compactification. A brief discussion of difficulties, such as rapid proton decay, that one encounters in theories with large-radius compactifications is given.

Дается краткий обзор современного состояния в области физики дополнительных размерностей, особое внимание при этом уделяется возможным проявлениям возбуждений Калуцы–Кляйна в секторе стандартной модели. Показано, что современные прецизионные измерения фермиевской константы и других электрослабых параметров дают нижнюю границу для масштаба дополнительных размерностей на уровне 3 ТэВ и, таким образом, наблюдение такого сорта дополнительных размерностей лежит за пределами возможностей ускорителей ближайшего будущего. Обсуждается поправка к аномальному магнитному моменту мюона, обусловленная дополнительными размерностями, и показано, что на основе полученного из измерения фермиевской константы ограничения на масштаб дополнительных размерностей эта поправка к $g_\mu - 2$ значительно уступает потенциально большей электрослабой суперсимметричной поправке. Обсуждается возможность формирования на LHC возбуждений Калуцы–Кляйна, ассоциированных с большим радиусом компактификации. Показано, что если такие возбуждения действительно возникают, то их резонансная структура содержит информацию о числе компактифицируемых размерностей и характере компактифицируемого многообразия. Кратко обсуждаются и те трудности (например, быстрый распад протона), которые возникают в теориях с большим радиусом компактификации.

INTRODUCTION

The physics of extra dimensions has a long and interesting history [1–3] beginning with the work of Kaluza and Klein in the nineteen twenties [1,2]. Recent interest in Kaluza–Klein theories arises because such theories may occur in TeV scale strings [4–6]. Activity in these

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models is taking place along three directions: (a) effects of large extra dimensions in the Standard Model [3, 7, 8], (b) effects in the gravitational sector [9], and (c) non-factorizable geometries [10, 11]. The focus of this paper is on the constraints on extra dimensions from precision electro-weak data. Specifically we will discuss in detail the constraints arising from two of the most precisely determined quantities in all of particle physics, i.e., the Fermi constant and the anomalous magnetic moment of the muon. We will also discuss the possible signatures for extra dimensions in pp collisions at the Large Hadron Collider (LHC). Although the focus of this paper is on extra dimensions in the context of the Standard Model, we will make a brief detour to assess also the status of work on extra dimensions in low-scale quantum gravity. Finally, we will discuss some of the difficulties that surface in theories with extra dimensions. In section 1 we give a brief discussion of the conventional string phenomenology based on heterotic strings. In section 2 we discuss the more recent developments which lead to the possibility of a string scale in the TeV region. In section 3 we discuss compactifications of extra dimensions which lead to Kaluza–Klein excitations in the Standard Model. In section 4 we discuss the contributions from the Kaluza–Klein excitations on the Fermi constant and give an analysis of the constraints that the accurate determination of the Fermi constant places on the compactification scale M_R . In section 5 an analysis of the effects of Kaluza–Klein excitations on the anomalous magnetic moment of the muon is given. In section 6 we discuss the probe of extra dimensions at colliders. A brief discussion of low-scale quantum gravity is given in section 7. In Section 8 we discuss the difficulties encountered in models with large-radius compactifications.

1. CONVENTIONAL STRING PHENOMENOLOGY

Extra space-time dimensions are an integral part of string theory. However, in conventional string phenomenology compactifications of extra dimensions occur at a high scale close to the 4-dimensional Planck scale $M_{\text{Pl}} = 1.2 \cdot 10^{19}$ GeV. The effects of extra dimensions in this case are at the level of threshold effects of heavy states with masses of order 10^{17} GeV. Such models, based on compactifications on Calabi–Yau manifolds, orbifolds, and free fermionic constructions possess many desirable features [12]. Thus they contain the Standard Model gauge group, $N = 1$ supersymmetry and can accommodate the spectrum of the Minimal Supersymmetric Standard Model (MSSM). In these models there is an automatic unification of the gauge coupling constants at the scale M_{str} . Unfortunately, compatibility with the LEP data is not guaranteed. In fact, one has to invoke the presence of additional phenomena in the form of either an additional set of states over and above the MSSM spectrum with intermediate scale masses or large threshold corrections to get agreement with the LEP data. Further, in some models there is the problem of extra light Higgs doublets and the problem of proton stability in some others. Nonetheless it is quite remarkable that several string models come close to becoming realistic. Since supersymmetric grand unification is very successful in accommodating the unification of gauge couplings given by the LEP data, recent efforts have focussed on deducing grand unification from higher-level Kac Moody levels [13]. However, some phenomenological problems still remain to be resolved in these constructions.

2. RECENT STRING MODEL BUILDING

Recent developments in string model building has proceeded along two main directions: (i) M theory compactification, and (ii) Type I [Type IIB] string compactifications. The generic feature of such models is that there is no longer a rigid relation between M_{str} and M_{Pl} [4]. In fact in the context of Type I [Type IIB] compactifications, the scale M_{str} can be as low as a TeV scale. We shall discuss models where the string scale and the compactification scale are indeed quite low, i.e., in the TeV range. In addition to M_{str} being low the fundamental scale of gravity in higher dimensions may be low [9]. This is possible because the observed Planck scale M_{Pl} in four dimensions and the fundamental gravity scale in higher dimensions are related by extra dimensions [9]. We shall discuss the implications of a low-scale quantum gravity in further detail in section 7.

3. TEV SCALE STRINGS AND KALUZA–KLEIN MODES

In this section we discuss the effects of Kaluza–Klein modes in the Standard Model sector in models with large-radius compactification. The simplest phenomenologically viable example of a higher dimensional theory is the case with one extra dimension which we assume is compactified on S^1/Z_2 with a compactification radius of $R = M_R^{-1}$. After compactification the resulting spectrum contains massless modes with $N = 1$ supersymmetry in four dimensions (4D), which precisely form the spectrum of MSSM in 4D. The massive Kaluza–Klein modes form $N = 2$ multiplets in 4D with masses $(m_i^2 + n^2 M_R^2)$, $n = 1, 2, 3, \dots, \infty$, where m_i^2 is the electro-weak mass and $n^2 M_R^2$ is the compactification mass. For simplicity we consider a direct 5-dimensional extension of the MSSM with the matter fields (quarks, leptons and Higgs) confined to the orbifold points which constitutes the 4-dimensional wall of the physical space-time, while the $SU(3)_C \times SU(2)_L \times U(1)$ gauge bosons propagate in the bulk. In this model after compactification of the fifth dimension we will have only zero modes for the matter fields, while the gauge bosons will contain both the zero modes and the Kaluza–Klein modes. In the effective theory in 4D a rescaling of the five-dimensional gauge coupling constant is necessary so that $g_i^{(5)}/\sqrt{\pi R} = g_i$. After rescaling the low-energy effective lagrangian in four dimensions is of the form

$$L_{\text{int}} = g_i j^\mu (A_{\mu i} + \sqrt{2} \sum_{n=1}^{\infty} A_{\mu i}^n), \quad (1)$$

where $A_{\mu i}$ are the gauge fields and are massless, $A_{\mu i}^n$ are their massive Kaluza–Klein excitations, and j_μ are the matter sources which contain the massless quark, lepton and Higgs fields. It is interesting to note that the coupling of the vector Kaluza–Klein modes to matter is by a factor of $\sqrt{2}$ larger than the coupling of the zero mode to matter. The above compactification scheme can be generalized to the case with more than one extra dimension. However, as the number of extra dimensions becomes larger the number of possible compactifications also grows. Thus, for example, for the case of two extra dimensions one may compactify on $Z_2 \times Z_2$, Z_3 and Z_6 orbifolds. In general, different compactifications will lead to different low-energy effective 4D theories and to different signatures in low-energy physics.

4. KALUZA–KLEIN EFFECTS ON THE FERMİ CONSTANT

The Fermi constant is very accurately known from the muon lifetime. From the complete two-loop corrections one has [14]

$$G_F = 1.16639(1) \cdot 10^{-5} \text{ GeV}^{-2}. \quad (2)$$

A comparison of the Standard Model value with the experimental value of (2) shows an excellent agreement between theory and experiment. However, the error in the theoretical determination of G_F^{SM} is much larger, by a factor of around two orders of magnitude, than the error in the experimental determination given by (2). It is this theoretical error that allows for the possibility of a correction from the Kaluza–Klein states. Specifically, the Kaluza–Klein correction to the Fermi constant must lie in the error corridor of the experimental value and the Standard Model prediction, i.e., $\Delta G_F^{\text{KK}}/G_F^{\text{SM}} = G_F/G_F^{\text{SM}} - 1$. Thus for d extra dimensions the effective Fermi constant including the Kaluza–Klein corrections is given by [8]

$$G_F^{\text{eff}} = G_F^{\text{SM}} \int_0^\infty dt e^{-t} \left[\theta_3 \left(\frac{itM_R^2}{M_W^2\pi} \right) \right]^d, \quad (3)$$

where $\theta_3(z)$ is the Jacobi function defined by $\theta_3(z) = \sum_{k=-\infty}^\infty e^{(i\pi k^2 z)}$. For the case of one extra dimension one finds to leading order in M_W/M_R the result [8]

$$G_F^{\text{eff}} \simeq G_F^{\text{SM}} \left(1 + \frac{\pi^2}{3} \frac{M_W^2}{M_R^2} \right). \quad (4)$$

Thus in the case of one extra dimension $\Delta G_F^{\text{KK}}/G_F^{\text{SM}} = \frac{\pi^2}{3} \frac{M_W^2}{M_R^2}$ and a direct determination of M_R is possible from the error corridor between G_F and G_F^{SM} . However, the error corridor which is essentially determined by the error in G_F^{SM} is very sensitively dependent on the scheme in which radiative corrections are computed as well as on the process used to extract it. We illustrate this with two parametrizations of G_F^{SM} . First in the on-shell scheme one has

$$G_F^{\text{SM}} = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W (1 - \Delta r)}, \quad (5)$$

where $\sin^2 \theta_W = (1 - M_W^2/M_Z^2)$ and Δr is the radiative correction in this scheme. Alternately one may parametrize G_F^{SM} by

$$G_F^{\text{SM}} = \frac{\pi\alpha}{\sqrt{2}M_Z^2 \hat{s}^2 \hat{c}^2 (1 - \hat{\Delta r})}, \quad (6)$$

where $\hat{\Delta r}$ is the radiative correction and $\hat{s} = \sin \theta_W(\overline{\text{MS}})$ and $\hat{c} = \cos \theta_W(\overline{\text{MS}})$. Several other determinations of G_F^{SM} exist as well, e.g., from the leptonic partial decay widths of the Z boson (see, e.g., Marciano in [8]). With these parametrizations and the current errors in the electro-weak parameters one finds $M_R \geq 3$ TeV with a ± 1 TeV fluctuation depending on the parametrization used. With the above limit on M_R none of the Kaluza–Klein excitations of γ , W or Z boson will become visible at the Tevatron. However, the current limit on M_R still allows for the possibility that these excitations may become visible at the LHC.

5. FERMIONIC MOMENTS

The extra dimensions will also have an effect on the fermionic moments, and specially on the muon anomalous magnetic moment. This moment is one of the most accurately determined quantities in physics. The most recent measurement of $a_\mu = (g_\mu - 2)/2$ from the Brookhaven experiment E821 combined with the previous CERN measurement [15] gives [16, 17]

$$a_\mu^{\text{exp}} = 11659235.7(46) \cdot 10^{-10}. \quad (7)$$

The result of the Standard Model for this quantity computed to α^5 QED corrections [18] and up to α^2 hadronic [19] and two-loop electro-weak corrections gives the value [17]

$$a_\mu^{\text{SM}} = 11659159.7(6.7) \cdot 10^{-10}. \quad (8)$$

The two-loop electro-weak correction by itself is $a_\mu^{\text{EW}}(2\text{-loop}) = 15.2(0.4) \cdot 10^{-10}$ [20]. It is expected that in the next round of analysis the BNL experiment will reach a sensitivity of about $\pm 15 \cdot 10^{-10}$ and eventually it will measure a_μ to a sensitivity of $\pm 4 \cdot 10^{-10}$. The term a_μ is generally regarded as a sensitive probe of new physics and we wish to determine if this is also the case for extra dimensions. Thus it is already known that a_μ is a sensitive probe of supersymmetry [21] specifically for the case of large $\tan\beta$ since in SUSY $\Delta a_\mu \sim \tan\beta$ and for large $\tan\beta$ the BNL experiment in fact can favorably compete with the Tevatron in discovering new physics [21].

Next we discuss the effects on a_μ due to corrections from the Kaluza–Klein excitations of the photon and of the W and Z bosons [22]. In the analysis we have to take into account the effects of the Kaluza–Klein W states on G_F , which we have already discussed. Including these effects we find that for the case $d = 1$ the correction to a_μ due to the Kaluza–Klein excitations of γ , W , and Z is given by [22]

$$(\Delta a)_\mu^{\gamma-W-Z, \text{KK}} = \frac{G_F m_\mu^2}{\pi^2 2\sqrt{2}} \left(-\frac{5}{12} + \frac{4}{3} \left(\sin^2 \theta_W - \frac{1}{4} \right)^2 \right) \left(\frac{M_Z^2 - M_W^2}{M_R^2} \right) + \alpha \frac{\pi}{9} \frac{m_\mu^2}{M_R^2}. \quad (9)$$

From (9) one finds that there is a partial cancellation between the W and Z Kaluza–Klein exchange contribution and the photonic Kaluza–Klein exchange contribution. The net result is that with $M_R \geq 1$ TeV the contribution of the Kaluza–Klein modes to a_μ falls more than 1–2 orders of magnitude below the sensitivity that will be achievable in the new a_μ experiment. Thus extra dimensions even as low as 1 TeV provide no serious background for SUSY effects and if a deviation in a_μ from the Standard Model prediction is seen at BNL it could not be attributed to effects of extra dimensions.

The basic reason why there is a very large suppression of the Kaluza–Klein contributions to a_μ is the redefinition of the Fermi constant which absorbs most of the correction in a_μ from Kaluza–Klein modes. As discussed in the first paper [8], there is, however, a variant model where muon decay and consequently G_F receive no contribution from the Kaluza–Klein excitations. This is a model where the first quark lepton generation lies in the bulk, while the second generation lies on the 4D wall. In this model, while the correction to G_F from the Kaluza–Klein states are suppressed, there is no such suppression for the Kaluza–Klein correction to a_μ . The detailed analysis here shows that the new BNL experiment will be able to probe extra dimensions for this model as follows [22]: $M_R \sim 0.65$ TeV ($d = 2$),

$M_R \sim 1$ TeV ($d = 3$), and $M_R \sim 1.4$ TeV ($d = 4$). The effects of extra dimensions in the context of quantum gravity are discussed in [23]. With the current estimates on the scale of low-scale quantum gravity (section 7) the quantum gravity effects on a_μ are again expected to be rather small. Thus, aside from the special case of the variant model discussed above, one finds that the effect of extra dimensions on a_μ will in general be too small to provide any serious background to the supersymmetric electro-weak correction.

6. PROBE OF EXTRA DIMENSIONS AT COLLIDERS

If the compactification radius of extra dimensions is large enough, the Kaluza–Klein excitations of the γ , W and Z could be produced at the LHC [24–26]. In this case one can show that quite remarkably the experimental data on the Kaluza–Klein excitations encodes information on the nature of compactification [26]. Thus the resonance structure in the production cross section associated with Kaluza–Klein excitations will provide information on the number of compactified dimensions as well as on the nature of the specific orbifold compactification. The most dramatic signals arise from the interference pattern involving the exchange of the Standard Model spin-1 bosons (γ and Z) and their Kaluza–Klein modes. Additional signals arise from the Kaluza–Klein excitations of the W boson and of the gluon.

The main signal of the Kaluza–Klein modes is the Drell–Yan process $pp \rightarrow l^+l^- + X$ via the Kaluza–Klein excitations of the Z and γ . The detailed analysis of the above process yields the cross section for the Kaluza–Klein case which is ~ 10 times larger than that for the case of the SSM Z' boson. The reason for this enhancement is twofold. First, one has the extra factor of $\sqrt{2}$ in the couplings of the Kaluza–Klein states to matter as discussed in section 3 (see (1)). Second, there is also an enhancement from a constructive interference between Kaluza–Klein modes of the photon and of the Z boson, which essentially overlap. There are also other remarkable features associated with the production of the dilepton pair via Kaluza–Klein states. An interesting quantity to plot is the cross section $d\sigma/dm_{ll}$ as a function of the dilepton invariant mass m_{ll} (Fig. 1). This cross section exhibits clear resonance peaks corresponding to the masses of the Kaluza–Klein states. An interesting feature is that Breit–Wigner resonances arising from the Kaluza–Klein excitation of the photon and from the Kaluza–Klein excitation of the Z boson superpose and lead to a net distorted Breit–Wigner resonance. Another interesting phenomenon is that there are sharp dips below the resonance peaks. The origin of these dips is due to a destructive interference between the contributions arising from the exchange of the γ and Z gauge bosons and of their Kaluza–Klein excited states in the region below the peaks. This phenomenon is unique to the Kaluza–Klein excitations. The analysis shows that Kaluza–Klein excitations with M_R up to 6 TeV can be explored with a luminosity of 100 fb^{-1} [26].

Next we consider the case of more than one extra dimension. Here for $d > 1$ there are in general several orbifold compactifications possible and thus the compactifications are more model-dependent in this case. For example, for the case $d = 2$ one can get a $Z_2 \times Z_2$ orbifold model where the compactified space is $S^1/Z_2 \times S^1/Z_2$, where the two S^1 have a common radius R . Another possibility is Z_3 or Z_6 compactification with a 2D torus of periodicity $2\pi R$. We note in passing that the mass spectra of the Kaluza–Klein excitations for the Z_3 and Z_6 orbifold cases are related. Thus for the Z_3 orbifold compactification, masses for the Kaluza–Klein excitations are given by $M_{Z_3}^2 = \frac{4}{3R^2}(m_1^2 + m_1m_2 + m_2^2)$, where m_1, m_2 are positive or

negative integers. Analogously for the Z_6 orbifold compactification the mass formula for the Kaluza–Klein excitations is $M_{Z_6}^2 = \frac{4}{3R^2}(m_1^2 - m_1 m_2 + m_2^2)$. We note that the mass formulae for the Z_3 and Z_6 cases are related by $(m_1, m_2) \rightarrow (m_1, -m_2)$. In general the masses of the Kaluza–Klein excitations, their multiplicities and the strength of their couplings to the boundary fermions depend on the nature of the compactification and these should manifest themselves in the production cross section and in the resonance structure of these states at the LHC. A detailed analysis of the above bears this out and one finds that the $d = 1$ and the $d = 2$ compactifications can be distinguished by a detailed study of the dileptonic cross section $d\sigma_U/dm_{ll}$ as a function of the dilepton invariant mass m_{ll} (Fig. 2). Further, as the analysis of Fig. 2 shows, one can even distinguish between the $Z_2 \times Z_2$ and Z_3 compactifications for the $d = 2$ case. Thus a study of the resonance structure of the Kaluza–Klein states will allow one to determine the dimensionality of the compactified space as well as the detailed nature of specific orbifold compactification. In general the compactification radii for different compact dimensions could be different, leading to a richer resonance structure. However, the general observations made above should still hold. A similar analysis can be carried out for the study of Kaluza–Klein excitations at future lepton colliders which also present an interesting possibility for the study of extra dimensions [27].

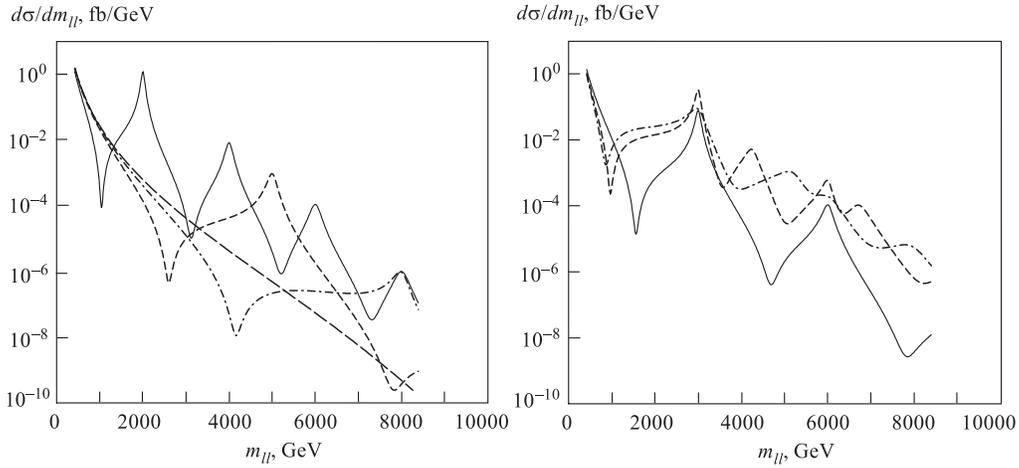


Fig. 1. A plot of the differential cross section $d\sigma/dm_{ll}$ as a function of the dilepton invariant mass m_{ll} for the process $pp \rightarrow l^+l^- + X$ including the effects of Kaluza–Klein excitations for the case when the compactification scale is $M_R = 2$ TeV (solid), $M_R = 5$ TeV (dashed), and $M_R = 8$ TeV (dot-dashed). The resonance structure exhibits the existence of Kaluza–Klein modes. For comparison the result of SM case is also shown (long-dashed). (Taken from [26])

Fig. 2. A plot of the differential cross section $d\sigma/dm_{ll}$ as a function of the dilepton invariant mass m_{ll} for the process $pp \rightarrow l^+l^- + X$ including the effects of Kaluza–Klein excitations for the case $d = 1$ (solid) and for the case $d = 2$ for two orbifold compactifications, $Z_2 \times Z_2$ (dashed) and Z_3 (dot-dashed), when the mass of the first Kaluza–Klein excitation is taken to be 3 TeV. The features of the resonance structure distinguish cases with different number of compactified dimensions as well as cases with different orbifold compactifications. (Taken from [26])

7. LOW-SCALE QUANTUM GRAVITY

As discussed in section 2, in addition to the Planck scale being low, the fundamental scale of gravity may also be low because the relation between the Planck scale and the fundamental scale of gravity depends on the number of extra dimensions. Thus from Gauss's law the relation between the volume R^n of n new dimensions, the fundamental scale M and the observed Planck scale in four dimensions is $M_{\text{Pl}}^2 = R^n M^{n+2}$ [9], where $M_{\text{Pl}} = G_N^{-1/2}$. One might investigate what happens if the fundamental scale M of quantum gravity is 1 TeV. The case $n = 1$ is then excluded since it would modify Newtonian law of gravitation at planetary distances. The case $n = 2$ gives $R \simeq 1$ mm and represents an interesting possibility for exploration. The low-energy effective lagrangian for theories of this type is discussed in [28] and the implications at accelerators for this class of theories have been discussed by several authors [29]. However, astrophysical considerations seem to indicate a limit on the fundamental scale which is rather large and would seem to exclude the possibility of observation of quantum gravity phenomenon at accelerator energies. Thus for the interesting case of two extra dimensions one finds that the analysis of graviton decay to the cosmic diffuse gamma radiation [30] and studies of graviton emission into large compact dimensions from a hot supernova core using the SN1987A data [31] put bounds on the fundamental scale which are very stringent, in the range of 50–100 TeV, and place the exploration of extra dimensions beyond the reach of the laboratory experiment. Extra compact dimensions can also be probed directly in gravity experiments and there are experiments proposed to probe distances at the submillimeter scale to look for possible deviations from the inverse square law [32]. These experiments look for modifications of the type

$$V(r) = -G_N \frac{m_1 m_2}{r} \left(1 + \alpha e^{-r/\lambda} \right), \quad (10)$$

a form which is valid for $r \gg \lambda$ and $\alpha = n + 1$ for n -sphere and $\alpha = 2n$ for n -torus compactification [33]. For the case of interest of two extra dimensions $\lambda = R \simeq 1$ mm and $\alpha = 3(4)$ for the sphere (torus) case. The very recent result from the Seattle Group probes distances well below the 1 mm level and finds no deviation from the inverse square law [34]. The experiment places a limit on M of $M \geq 3.5$ TeV [34].

Next we discuss the generation of neutrino masses in models of this type. In grand unified theories and in string theories a small neutrino mass is generated by a see-saw mechanism which gives $m_\nu \sim m_f^2/M_X$, where m_f is the fermion mass and M_X is a heavy mass scale. In this mechanism the neutrino mass is small because M_X is heavy, where M_X is taken to lie between the intermediate scale and the GUT scale. In a model with large-radius compactification such a large mass scale does not exist and one needs to rethink how a small neutrino mass will arise in such a scenario. One mechanism used is to assume that aside from gravity some matter fields could also propagate in the bulk. Specifically it is possible to generate a small Dirac neutrino mass by assuming that the right-handed component of this neutrino is a Standard Model singlet which resides in the bulk [35, 36]. Here the couplings between the singlet and the Standard Model particles arise at the wall and the Dirac neutrino mass is thus suppressed because of the volume factor from extra dimensions [35]. However, generation of a Majorana mass for the neutrino is more difficult as one needs to violate lepton number on a distant brane (or in the bulk) and communicate this breaking to the physical brane by a bulk field.

A more recent work in a similar direction is on non-factorizable geometries [10,11]. An example of this is a 5D model with gravity in the bulk and the 5th dimension compactified on S^1/Z_2 : $\{x^M\} = \{x^\mu, \phi\}$; $\mu = 0, \dots, 3$, $-\pi \leq \phi \leq \pi$. One assumes the existence of two 3-branes, one at $\phi = 0$ and the other at $\phi = \pi$. One of these could be viewed as the brane for the hidden sector and the other as the brane for the visible sector. The total action is $S = S_{\text{grav}} + S_{\text{vis}} + S_{\text{hid}}$. One looks for solutions with Lorentz invariance with the form $ds^2 = e^{-2\sigma(\phi)} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 d\phi^2$. A fine tuning among the cosmological constants in the bulk and on the boundary is necessary, generating a sort of ADS_5 geometry, to achieve a 4D Poincare invariance. Solutions require $\sigma(\phi) = kr_c|\phi|$ generating a warp factor of $e^{-2kr_c|\phi|}$ which decays exponentially as one moves away from the wall at $\phi = 0$. Some of the phenomenological consequences of this model are discussed in [37].

8. DIFFICULTIES IN MODELS WITH LARGE-RADIUS COMPACTIFICATIONS

There are several phenomena other than those discussed above which are affected in scenario with large-radius compactifications creating in some cases extra challenges or problems depending on one's point of view. We begin with a discussion of the problem of gauge coupling unification. In MSSM, unification of the gauge couplings takes place naturally with a unification scale of $\sim 2 \cdot 10^{16}$ GeV. In models with large-radius compactifications the evolution of the gauge couplings above the compactification scale obeys a power law behavior as a function of the scale factor [7,38]. This power law behavior arises as a consequence of the contributions from the Kaluza–Klein excitations. Thus in models with large-radius compactification the meeting of two of the gauge couplings constants, say α_1 and α_2 , can occur at a low scale. However, with the MSSM spectrum the low-scale unification leads to a value of α_3 which in general is larger than the one given by the LEP data. Thus one of the successes of MSSM, i.e., a natural unification of the gauge coupling constants, is lost. Suggestions on how to recover unification with additional contributions are discussed in some of the papers in [39,40]. However, in this case the unification of the gauge couplings becomes more of an accident rather than a prediction of the model.

Perhaps the most serious problem in models with large-radius compactification is that of proton stability. Since in theories of large-radius compactifications the unification mass is typically in the TeV range compared to the unification scale of $\sim 10^{16 \div 17}$ GeV in unified theories of the normal sort which includes grand unified theories [41] and old-fashioned string models [42], one has to suppress baryon and lepton number violating operators to a very high order. One suggestion made to overcome this difficulty is to assume that the baryon number is gauged in the bulk and that this symmetry is then broken on a brane different from the physical brane [43]. In this case one can arrange proton decay to be suppressed by a huge exponential factor. However, it is not clear how one may naturally arrange the breaking of baryon number symmetry on the distant brane. Another set of suggestions to suppress proton decay require imposition of a discrete symmetry [5,7,40,44]. A detailed analysis of such discrete gauge symmetries is given in [40], where a generalized matter parity of the type $Z_3 \times Z_3$ is proposed in an extended MSSM model which suppresses dangerous operators to high orders. However, it has been argued that unless a theory has an exact or an almost exact baryon number conservation one may have rapid proton decay induced by quantum gravity effects [45]. To suppress this type of proton decay one would need a scale of quantum gravity

which is similar to the scale one needs to stabilize the proton in grand unified theories and in ordinary string unified theories [41,42].

CONCLUSIONS

Type I [Type II] strings allow for the possibility of models with large-radius compactifications. In this paper we have considered the physical implications of models where the mass scale $M_R = R^{-1}$ associated with the extra compactified dimensions is in the TeV region. We showed that in this case if the accelerator energies are large enough to produce Kaluza–Klein excitations, then the experimental data can provide information on the number of compactified dimensions as well as on the nature of orbifold compactification. Such information can be gleaned from a detailed study of the differential cross section $d\sigma/dm_{ll}$ as a function of the dilepton invariant mass m_{ll} from the Drell–Yan process $pp \rightarrow l^+l^- + X$. Specifically this process is an important channel for the discovery of such states up to $M_R \approx 6$ TeV for an integrated luminosity of 100 fb^{-1} . Additional processes such as $pp \rightarrow l^\pm \nu_l + X$ and $pp \rightarrow jj + X$ also provide further signals for the discovery of Kaluza–Klein modes. An important unknown in these analyses is the compactification scale M_R . Currently the strongest constraint on M_R arises from the closeness of the Standard Model prediction of the Fermi constant G_F and its precision determination from the muon lifetime. The Standard Model prediction depends on the accuracy of the experimental determinations of electro-weak parameters, and with their current errors one finds $M_R \geq 3$ TeV. This limit will increase as the precision of the electro-weak parameters increases. Effects of Kaluza–Klein excitations on a_μ are found to be small when the constraint from G_F is imposed. Thus if a deviation in a_μ from the Standard Model value is observed in the BNL experiment it will most likely be an effect other than from contributions from the Kaluza–Klein excitations.

A similar situation holds in the quantum gravity sector where the recent experiment on the submillimeter tests of gravity explores distances well below the 1 mm level and finds no deviations from the inverse square law. Interestingly the lower limit on M of $M \geq 3.5$ TeV deduced in this experiment is similar to the limit on M_R of $M_R \geq 3$ TeV gotten from the G_F constraint. Finally, we note that problems regarding the consistency of theories with large-radius compactifications persist, the most serious being that of rapid proton decay in such theories.

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