PERIODICAL SEQUENCES (TRAJECTORIES) OF OUTCOMES OF ATOMIC STATE MEASUREMENT ON EXIT FROM THE MICROMASER CAVITY

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A method of recurrence relations, developed in the paper [1], is applied for simulation of a random sequence (trajectory) of detector clicks that controls a state of atoms leaving the one-atom micromaser. The random sequence of relative frequencies of detector clicks for the fixed time interval was calculated. The change of the frequencies along the trajectory is chaotical and close to the mean level, which is independent of time. The character peculiarities of such a sequence were observed experimentally in the paper [2] where it was shown that the mean levels may alternate each other stepwise, but their value was reproduced. Hence, the mean levels is the observable quantity. A method of calculation of a stationary reduced density matrix of the field $\rho_{\rm st}$ of a subensemble, which is determined by the given mean relative frequencies of the detector clicks, is proposed here. The main idea is that the random sequence could be approximated by the simpler periodical one. The micromaser would approach $\rho_{\rm st}$ along the preset (imposed) periodical trajectory. A problem to find $\rho_{\rm st}$ is reduced to the eigenvalue problem for the evolution operator during the period. The latter is solved by means of the linearization procedure applied to the evolution operator of one period. Then the matrix $\rho_{\rm st}$ is obtained as the solution of the inverse problem that is a reconstruction of the field statistics from the statistics of the random trajectory. We notice an important property of the periodical trajectories: probabilities to observe leaving atom in the definite state at the end of every period are close to the relative frequencies calculated with help of statistical processing of the detector clicks.

1. A ONE-ATOM MICROMASER MODEL

At the present time, a lot of fundamental aspects of quantum mechanics are tested directly and applied in practice due to a high level of experimental achievements in atomic physics and quantum optics. Fast developing fields like quantum information processing, quantum computation and communication connected very close to a concept of entangled states of quantum-mechanical systems. An operation of quantum informational and communicational schemes is based on a realization of quantum measurement in informational elements which results in state reduction. A role of qubits is played by individual trapped atoms or ions and by single photons. Such a kind of experiments when a measurement is carried out under specially prepared individual quantum system solves general theoretic questions of interpretation of basis concepts in quantum theory [3].

In this connection, a so-called micromaser possesses interesting potentials [4]. Here, individual quantum system — a chosen mode of the microwave resonator interacts (and as a

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result entangles) during each period of operation with another quantum system — Rydberg atom which is initially prepared in the excited (maser) energy level. When the atom exits the cavity (and the systems stop interacting but stay in an entangled state), the atom undergoes quantum measurement of its energy. The measurement allows concluding indirectly about state of the quantum mode at the moment of detection. The measurement process is repeated for each atom flyby, resulting in peculiar dynamics of the field mode due to so-called *back-action of the detector on a detected quantum object*.

The one-atom micromaser model is based on the Hamiltonian of Jaynes–Cummings H [5] and developed in the paper [4]. Here,

$$H = \omega\hat{a}^\dagger\hat{a} + \omega_0\hat{S}_3 + g(\hat{a}^\dagger\hat{S}_- + \hat{a}\hat{S}_+).$$

In the expression, operators $\hat{S}_+, \hat{S}_-, \hat{S}_3$ belong to the group $SU(2)$, Bose operators $\hat{a}^\dagger, \hat{a}$, of creation and annihilation of quanta of the field mode.

Now let us consider the dynamics of the main diagonal — distribution law of the photons number probability in Fock's basis — in basis of the photon number operator. Denote by a symbol $\rho(l)$ a vector of the main diagonal of the reduced density matrix of the field (MDRDM) in the beginning of lth period satisfying the normalization condition

$$\mathrm{Tr}_f\,\rho(l) = \sum_{n=0}\rho_n(l) = 1. \tag{1}$$

Here, n is Fock's number; Tr_f is the field trace. Let us denote by $S(\xi, t)$ an operator of the evolution $\rho(l)$ over a micromaser operation cycle of time duration t. We introduced a notation of the random variable ξ taking a zero value $\xi = 0$, if the detector measured an atom in the lower state, and the value $\xi = 1$, if the atom was detected in the upper state, and $\xi = 2$, if the detector did not click because of nonideal efficiency. According to paper [4], this operator could be represented by the evolution operator $D(\xi)$ of the MDRDM, describing the atom–filed interaction in interval τ, and by an operator $W(T)$ of the MDRDM evolution in the relaxation process (interval T). Here,

$$S(\xi, T) = W(T)D(\xi), \qquad W(T) = \exp(T\,\gamma\,\hat{L}), \tag{2}$$

$$\begin{gathered} D(0) = \varepsilon_0 Q0, \quad D(1) = \varepsilon_1 Q1, \quad D(2) = (1-\varepsilon_0)Q0 + (1-\varepsilon_1)Q1, \\ Q0 = \hat{a}^\dagger\left(\sin^2\left(g\tau\sqrt{\hat{a}^\dagger\hat{a}+1}\right)\Big/\sqrt{\hat{a}^\dagger\hat{a}+1}\right), \end{gathered} \tag{3}$$

$$Q1 = \cos^2(g\tau\sqrt{\hat{a}^\dagger\hat{a}+1}), \tag{4}$$

$$\hat{L} = -(2n_b+1)\,\hat{a}^\dagger\hat{a} - n_b + (n_b+1)\,\hat{a}\sqrt{\hat{a}^\dagger\hat{a}} + n_b\,\hat{a}^\dagger\sqrt{\hat{a}\,\hat{a}^\dagger}. \tag{5}$$

In the relations, γ is decay constant; n_b — average Plank's number of photons in the resonator; ε_1, ε_0 — efficiencies of detection of atoms in the upper (lower) states. We keep a notation of the photons creation and annihilation operators acting on the main diagonal in the case.

The MDRDM in the end of lth period (in the beginning of $(l+1)$th) is equal to

$$\rho(l+1) = \frac{S(\xi_l, T)\,\rho(l)}{\mathrm{Tr}_f S(\xi_l, T)\,\rho(l)}. \tag{6}$$

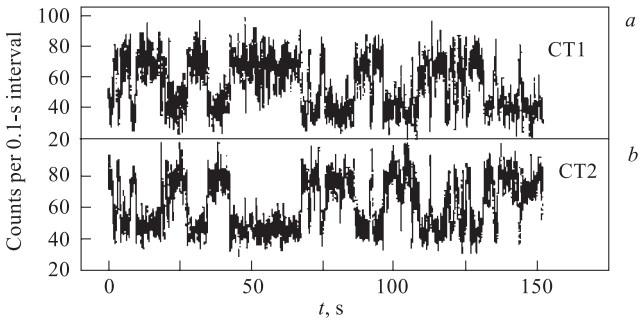

Fig. 1. The results of O. Benson, G. Raithel, and H. Walther experiment [2]

Here, ξ_l denotes the value of the random variable ξ found on lth cycle. Below the dependence of the random variable on the number of the micromaser, ξ_l, would be called the trajectory. Let us introduce a notation of a probability to detect the atom in the lower state immediately before measurement on $(l+1)$th cycle:

$$a_0 = \mathrm{Tr}_f D(0)\,\rho(l), \tag{7}$$

correspondingly, for the upper state $a_1 = \mathrm{Tr}_f D(1)\,\rho(l)$.

The relative frequencies of detector clicks for the fixed time interval Δt_{av} are defined as p_1 (for atoms in the upper state) and p_0 (in the lower):

$$p_0 = k\,/\,(\Delta t_{\mathrm{av}} R),\ p_1 = m\,/\,(\Delta t_{\mathrm{av}} R). \tag{8}$$

Here, R is injection rate; k — number of atoms detected in the lower state; m — number of atoms detected in the upper state from the whole number of atoms $\Delta t_{\mathrm{av}} R$ passed during averaging time Δt_{av}. In the experiment [6] (Fig. 1), these probabilities were determined from the sample of about 300 passed atoms ($\Delta t_{\mathrm{av}} \approx 0.1$ s) by direct accounting of favourable outcomes. The experiment was carried out with the following parameters: $g\tau \approx 0.92$, $N_{\mathrm{ex}} = R/\gamma \approx 200$, $R \approx 3300\ \mathrm{s}^{-1}$. The outcomes are obtained by detector with efficiency of order $\varepsilon_1 = \varepsilon_0 \approx 0.35$.

Our results of simulation are presented in Fig. 2. Here, the abscissa axis corresponds to the moment of time following one by one with interval Δt taken as equal to 1. We assumed that the interaction time τ fulfills a condition $\tau \ll \Delta t$. Number of intervals Δt between successive atoms is random and is generated in our model according to Poisson distribution

$$P(j) = R\,\Delta t\,(1 - R\,\Delta t)^{j-1}, \quad j = 1, 2\ldots \tag{9}$$

Here, $P(j)$ is the waiting probability of j elementary intervals between successive atoms or of the time $T = j\Delta t$. Particularly, Fig. 2 is plotted for $R\,\Delta t = 1/4$. The axis of ordinates corresponds to the relative frequency (8) to detect unexcited atom on the time interval $\Delta t_{\mathrm{av}} = N_{\mathrm{ex}}/R$ (Fig. 2). The solid line in Fig. 2 represents average steady (quasistationary) level of the relative frequency $\overline{p}_0$, obtained by averaging of the histogram over observation time. Specifically for plotting Fig. 2, we used the following parameters:

$$N_{\mathrm{ex}} = 71, \quad \Delta t_{\mathrm{av}} = 284, \quad n_b = 0.1, \quad g\tau = 0.92, \quad \varepsilon_0 = \varepsilon_1 = 1/3. \tag{10}$$

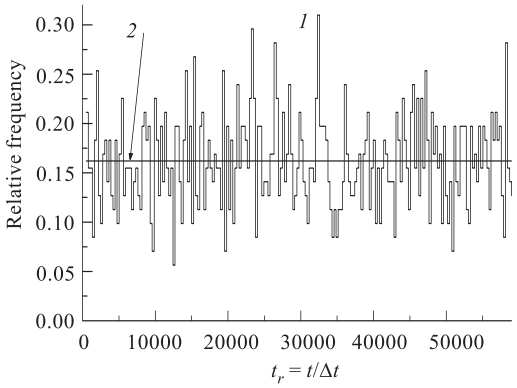

Fig. 2. *1* — the relative frequency to detect the atom exiting the micromaser in the ground state p_0, waiting times are random and distributed according to Poisson law; *2* (solid horizontal line) — time averaged relative frequency $\overline{p}_0$. The abscissa axis corresponds to time measured in units of elementary interval Δt. Parameters of the simulation are given in the present paper

As we mentioned above, experimentally measured observable is an average over the time $\Delta t_{\rm av}$, relative frequencies p_1 and p_0. It is natural to average over the number of passed atoms which is larger (or equal) than number $N_{\rm ex}(\Delta t_{\rm av} R \geqslant N_{\rm ex})$. In such a case, the analysis could be simplified replacing operators (2) by their average over the distribution (9). Average formulas have a form

$$W = \frac{R\Delta t \, \exp\left(R\Delta t \hat{L}/N_{\rm ex}\right)}{I - (1 - R\Delta t) \, \exp\left(R\Delta t \hat{L}/N_{\rm ex}\right)}, \quad S(\xi) = W D(\xi), \quad \xi = 0, 1, 2, \tag{11}$$

$$\rho(l+1) = \frac{S(\xi_l)\,\rho(l)}{{\rm Tr}_f S(\xi_l)\,\rho(l)}, \tag{12}$$

$$\rho(l+1) = W(Q0 + Q1)\,\rho(l). \tag{13}$$

Here, I is identical operator. The equation (13) describes the evolution of MDRDM without measurement. The steady state of this equation denotes $\rho^{(ss)}$. Comparing recurrence relations (12), (13), one can conclude that the evolution process of MDRDM in the nonmeasurement case is linear (13), whereas the atomic state measurement result is unpredictable and the stochastic recurrence relation (12) is nonlinear. In Fig. 3 results of simulation with the same parameters (10) are presented, but the calculations are based on (11)–(13). As follows from

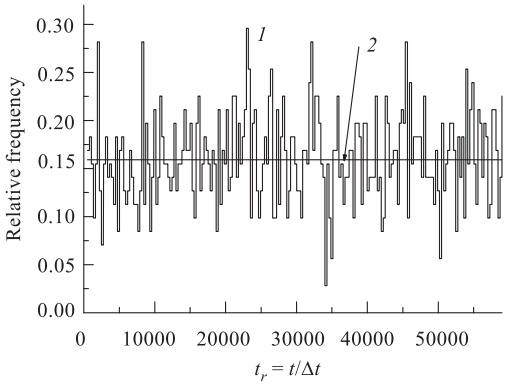

Fig. 3. The relative frequencies p_0 and $\overline{p}_0$ to detect the atom exiting the micromaser in the ground state. For simulation we have used the evolution operator W. The average time interval between successive atoms is $4\Delta t$. The notations and simulation parameters are the same as for Fig. 2

the comparison of the figures, the statistical characteristics of both figures p_0 are close to each other. The presented comparison gives a possibility to study the model with the help of formulas (11)–(13), without generation of the Poisson variable T, which significantly simplifies our analysis.

2. PECULIARITIES OF THE EVOLUTION OPERATORS $S(\xi)$ AND THEIR PRODUCTS. PERIODICAL TRAJECTORIES

Let us consider some features of the system dynamics evolving to the stationary state by choosing the period operator as an evolution operator

$$SL = \prod_{\{\xi_j,\, 1\le j\le L\}} S(\xi_j).$$

Here, $\prod$ is a symbol denoting product of operators. The vector of MDRDM is calculated in the end of each period; the periods are enumerated by integer number p. The equation describing nonlinear dynamics has an analogous form to equation (12), but in the present case this is not a stochastic relation:

$$\rho(p+1) = \frac{SL\,\rho(p)}{\mathrm{Tr}_f SL\,\rho(p)}. \tag{14}$$

In contrast to (6), (12), equation (14) describes the micromaser evolution according to preset (imposed) trajectory. All operators SL have eigenvalues less than 1, and as a consequence, they do not preserve the trace (in contrast to the operator in recurrence relation (13)). Nevertheless, due to nonlinearity the relation (14) has a stationary solution. We write the general solution of (14) by applying Fourier method. For this purpose, we expand the initial vector $\rho(0)$ by eigenvectors of SL:

$$\rho(0) = \sum_{\lambda} c_\lambda \rho_\lambda. \tag{15}$$

Here, c_λ are the coefficients of the expansion, ρ_λ and λ are eigenvector and eigenvalue of SL, respectively:

$$SL\,\rho_\lambda = \lambda\,\rho_\lambda. \tag{16}$$

We denote the maximal eigenvalue in the expansion (15) by $\widetilde{\lambda}$ and get the solution of (14) on period p in the form

$$\rho(p) = \left(\rho_{\widetilde{\lambda}} + \sum_{\lambda\neq\widetilde{\lambda}} \frac{c_\lambda}{c_{\widetilde{\lambda}}} \left(\frac{\lambda}{\widetilde{\lambda}}\right)^p \rho_\lambda\right) \Bigg/ \left(1 + \sum_{\lambda\neq\widetilde{\lambda}} \frac{c_\lambda}{c_{\widetilde{\lambda}}} \left(\frac{\lambda}{\widetilde{\lambda}}\right)^p\right), \quad \rho(p) \underset{p\longrightarrow\infty}{\longrightarrow} \rho_{\widetilde{\lambda}}. \tag{17}$$

It follows from (17) that the stationary state ρ of the equation (14) could be any eigenvector of the initial state expansion if its eigenvalue is the largest. So the relation (14) has infinitely many stationary states. The vector with the maximum possible eigenvalue $\lambda_{\max}$ is the main stationary state ρ_{st}:

$$\rho_{\mathrm{st}} = \rho_{\lambda_{\max}}. \tag{18}$$

As follows from (16), the eigenvector of SL is normalized according to (1) and corresponds to eigenvalue $\lambda = \mathrm{Tr}_f\, SL\, \rho_\lambda$. But a value $\mathrm{Tr}_f\, SL\, \rho_\lambda$ is equal to conditioned probability that L atoms are found by the detector in states described by the random variable ξ taking successive values from a set $\{\xi_j,\, 1 \le j \le L\}$ (on condition that the initial state of the field was ρ_λ). So the eigenvalue problem (16) for a product of the operators is of special importance. The main stationary state (18) determines (average) probability levels to detect the atom in the lower (upper) state in the end of each period — $\bar{a}_0$, $\bar{a}_1$:

$$\bar{a}_0 = \mathrm{Tr}_f\, D(0)\, \rho_{\mathrm{st}}, \quad \bar{a}_1 = \mathrm{Tr}_f\, D(1)\, \rho_{\mathrm{st}}. \tag{19}$$

These average a priori probabilities satisfy an exact relation

$$\bar{a}_0/\varepsilon_0 + \bar{a}_1/\varepsilon_1 = 1. \tag{20}$$

The parameters of a periodical trajectory are: a period $L = k + m + r$; a number of atoms in the upper and in the lower states during the period k and m, correspondingly; r atoms are left undetected. Then the average relative frequencies of detector clicks $\overline{p}_1$ and $\overline{p}_0$ can be evaluated:

$$\bar{p}_1 = m/L, \quad \bar{p}_0 = k/L. \tag{21}$$

Let us find the main stationary state (18) solving an eigenvalue problem (16) for the operator of the period

$$SL = S(0)^k S(1)^m S(2)^r. \tag{22}$$

Here, $S(\xi)$ is defined in (11). Now we want to study a connection between a relative frequencies (21) and their a priori analog (19). In further analysis, we assume the following connection between parameters of the operator (22):

$$\overline{p}_0/\varepsilon_0 + \overline{p}_1/\varepsilon_1 = k/(L\varepsilon_0) + m/(L\varepsilon_1) = 1. \tag{23}$$

For this purpose, we perform a shift of operators $\hat{a}^\dagger$, $\hat{a}$ in formulas (3)–(5):

$$\hat{a}^\dagger \longrightarrow \hat{a}^\dagger + \mu^\dagger, \quad \hat{a} \longrightarrow \hat{a} + \mu,$$

where $\mu^\dagger, \mu$ are numbers, and linearization of the operator (22), keeping its linear and quadratic term with respect to $\hat{a}^\dagger$, $\hat{a}$. After that we find $\mu^\dagger, \mu$ from a condition that the linear terms with respect to $\hat{a}^\dagger$, $\hat{a}$ become zero in SL. The method of linearization is restricted by the condition

$$g\,\tau < 1, \tag{24}$$

but Fillipowicz parameter $\Theta = g\,\tau\sqrt{N_{\mathrm{ex}}}$ can have arbitrary value.

Let us consider another simpler case of equal detector efficiencies $\varepsilon_0 = \varepsilon_0 = \varepsilon$. We have

$$\varepsilon = (k+m)/(k+m+r), \quad \chi = \overline{p}_0/\varepsilon = k/(k+m), \quad \nu = \mu^\dagger \mu\, /\, N_{\mathrm{ex}}, \tag{25}$$

$$\begin{aligned} F(\nu) = {} & (\sin^2(\Theta\sqrt{\nu}) - \chi)^2 + \\ & + (\sin^2(\Theta\sqrt{\nu}) - \chi)(\chi\,(2-\varepsilon) - 1 - \nu\,(\varepsilon+1)) + (1-\varepsilon)(1-\chi)(\nu-\chi), \end{aligned} \tag{26}$$

$$G(\nu)=\left(\frac{\nu(1+\sin^2(\Theta\sqrt{\nu}))^2}{(1+\nu)^2}\right)^k\left(\frac{1-\sin^4(\Theta\sqrt{\nu})}{1+\nu}\right)^m\times$$
$$\times\left(\frac{2\nu+\cos^2(\Theta\sqrt{\nu})}{(1+\nu)^2}(1+\sin^2(\Theta\sqrt{\nu}))\right)^r\varepsilon^{k+m}(1-\varepsilon)^r. \quad (27)$$

We give simplified expressions of functions $F(\nu)$ and $G(\nu)$ that are correct for small $n_b \ll 1$ and large $\Theta \gg 1$. Zeros of function $F(\nu)$ determine values of parameter ν for which linear terms of $\hat{a}^\dagger$, $\hat{a}$ in SL become zero. Function $F(\nu)$ has many zeros. Each one is connected to a series of close to equidistant eigenvalues, and distances between those eigenvalues depend on the series number. In each series there is an eigenvector of the ground state. The vector is very localized in Fock's space, its maximum lies close to the corresponding zero of function $F(\nu)$, and the eigenvalue is determined by value of the function $G(\nu)$ taken in the chosen zero. Functions $F(\nu)$ and $G(\nu)$ are connected by important relation: locations of maxima of the function $G(\nu)$ are determined by zeros of the function $F(\nu)$. The analysis of quadratic terms shows that the stable states are localized in the so-called trapping regions [6], whose location is defined by inequality

$$(\pi\,(0.5+j\,)/\Theta\,)^2 \leqslant \nu \leqslant (\pi\,(1+j\,)/\Theta\,)^2, \quad j=1,2\ldots \quad (28)$$

Here, integer j is a number of the trapping region. The trapping regions have an important property: their location (28) does not depend on the measurements result (of numbers k, m, r) and on the detector efficiencies. We want to point out that functions $F(\nu)$ and $G(\nu)$ do not depend on the order of efficients in the period operator (22), and therefore they determine properties of the beam $(m+n+r)!/(m!n!r!)$ of periodical trajectories. There is dependence of coefficients of the quadratic form of operators $\hat{a}^\dagger$, $\hat{a}$ on permutation of the efficients.

To justify formulas (23), (25), we note that for such a chosen connection the equation $F(\nu)=0$ has a root $\nu=\widetilde{\nu}$ satisfying two approximate conditions: $\sin^2(\Theta\sqrt{\widetilde{\nu}}) \approx \chi$, $\widetilde{\nu} \approx \chi$. It is a root defining the main steady state $\rho_{\rm st}$ corresponding to the maximal eigenvalue of SL that we were looking for. Hence the ground state is a very localized vector, according to formulas (19), (3) in the approximation of linearization $\overline{a}_0 = \mathrm{Tr}_f\, D(0)\rho_{\rm st} \approx \varepsilon \sin^2\left(\Theta\sqrt{\widetilde{\nu}}\right)$, then we have a connection of (average) probability levels to detect the atom in the lower state in the end of each period and relative frequencies of detector clicks $\overline{p}_0 \approx \overline{a}_0$.

3. DISCUSSION

The method developed in our paper is based on assumption that the random trajectory ξ_l could be approximated in quasistationary conditions by the most simple periodical trajectory whose parameters k, m, r should be taken from the experiment. Then our search of the distribution law of photon numbers in the resonator MDRDM takes a form of calculating eigenvector of the period operator (22) corresponding to the maximal eigenvalue. Our estimate under condition (24) shows that equation of motion (14) in definite periodical trajectory ξ_l is self-consistent that is, it leads the quantum system to evolve into the state when, in order to detect the atom in the lower (upper) state in the end of each period $\overline{a}_0$ and $\overline{a}_1$, (average) probability levels become close to mean relative frequencies of

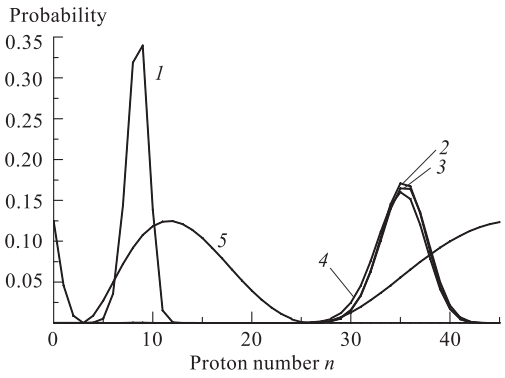

Fig. 4. Eigenvectors for the largest eigenvalue of the period operator SL. *1* — $\rho_{\rm st}$ for $k = 1$, $m = 4$, $r = 0$; *2* — $\rho_{\rm st}$ for $k = 1$, $m = 1$, $r = 0$; *3* — $\rho_{\rm st}$ for $k = 1$, $m = 1$, $r = 20$; *4* — $\rho^{(ss)}$; *5* — $\cos^2(g\,\tau\sqrt{n})/8$. The simulation parameters are the same as for Fig. 2

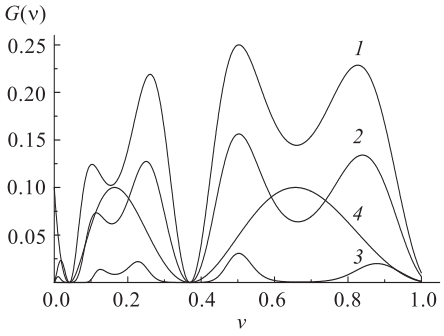

Fig. 5. Function $G(\nu)$ for parameters $k = 1$, $m = 1$, $\chi = 0.5$. *1* — function $G(\nu)$ for $r = 1$, $\varepsilon = 1$; *2* — function $G(\nu) \cdot 10$ for $r = 2$, $\varepsilon = 0.5$; *3* — function $G(\nu) \cdot 100$ for $r = 20$, $\varepsilon = 0.091$; *4* — $\cos^2(\Theta\sqrt{\nu})/10$. The simulation parameters are the same as for Fig. 2

detector clicks $\overline{p_0}$ and $\overline{p_1}$ which determine the definite trajectory. A statistical treatment of the random sequence shown in Figs. 2, 3 gives the following results: $\bar{p}_0 = k/L \approx 0.162$, $\bar{p}_1 = m/L \approx 0.163$. The result agrees with (23): $\bar{p}_0 + \bar{p}_1 \approx \varepsilon = 1/3$. To get the minimal period, one can take $k = 1$, $m = 1$, $\chi = k/(k+m) = 1/2$. Then the period $L = 1/\bar{p}_0 \approx 6$. We determine parameters of the periodical trajectory found by detector with efficiency $\varepsilon = 1/3$: $k = 1$, $m = 1$, $r = 4$. In Fig. 4 we plotted MDRDM $\rho^{(ss)}$ (13) and $\rho_{\rm st}$ (18) for $k = 1$, $m = 1, 4$; $r = 0, 20$. As one can see from Fig. 4, $\rho^{(ss)}$ almost coincide with $\rho_{\rm st}$ that have been predicted since for chosen parameters of simulation $\rho^{(ss)}$ have only one peak.

As follows from the obtained results, a problem of experimenter is to determine parameter of the trajectory: its minimal period and number of clicks for the upper and lower states of atoms. From Fig. 5 we see that it is not important what type of detector — with small or large efficiency — is used to find the parameters. Indeed, location of the maxima of function $G(\nu)$ in Fig. 5 depends weakly on efficiency. Obviously, high-efficiency detectors are more preferable, since they allow observation of trajectories with small time of stay in quasistationary state. In conclusion we want to note once more that the results presented in the paper are correct in the approximation of the linearization method. The main states of eigenvalue problem (16) localized in the regions do not overlap and weakly interact. It is the case when a probability of any part of the trajectory weakly depends on order of events and is determined by number of happened events. The analysis becomes significantly complicated when condition (24) breaks down. The system has unstable behaviour; mean relative frequencies may have two (or more) competitive values. In case the trajectory also becomes unstable, and abrupt jumps from one state to another are possible. The jumps happen in the time interval much smaller than the time of living in quasistationary state. Such frustration results from events with very small prob-

ability — critical fluctuation in the sequence of clicks. The peculiarities of the critical fluctuation and the statistics of cavity mode photons are planned to be studied in succeeding works.

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