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## *g* FACTORS AS A PROBE FOR HIGH-SPIN STRUCTURE OF NEUTRON-RICH Dy ISOTOPES

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We have made theoretical investigations on the structure of high spin states of <sup>160–170</sup>Dy isotopes to supplement the current experimental progress being made on the production and spectroscopy of neutron-rich Dy nuclei with mass number  $A = 166$  and  $168$ . The calculations have been carried out within the framework of cranked Hartree–Fock–Bogoliubov (CHFb) theory employing a pairing + quadrupole + hexadecapole model interaction. Our results for the  $g$  factors show a remarkable variation as a function of angular momentum while moving from <sup>164</sup>Dy to neutron-rich isotopes <sup>166–170</sup>Dy. This is explained in terms of relative rate of change in proton and neutron single-particle contributions to the total aligned angular momentum as the system acquires higher spins. Amongst single-particle orbitals, the  $\pi h_{11/2}$  and  $\nu i_{13/2}$  states and their respective positions from the proton and neutron Fermi levels are found to play the most crucial role. A comparison with the available experimental data shows that our calculated spin dependence of  $g$ -factor values for yrast states in the stable isotopes <sup>160,162,164</sup>Dy are consistent with the recent measurements of Brandolini et al.

В статье представлены результаты расчета  $g$ -факторов для состояний ираст-полосы с большими моментами ряда изотопов диспрозия с  $A = 160 \div 170$ . Расчеты выполнены в рамках модели, развитой П. Рингом и П. Шуком для деформированных ядер в приближении Хартри–Фока–Боголюбова с учетом вращения. Учитываются остаточные силы парного и мультиполь-мультипольного ( $\lambda = 2, 4$ ) взаимодействий. Показано, что поведение  $g$ -фактора возбужденных состояний в зависимости от углового момента ядра различается для легких и тяжелых нейтроноизбыточных изотопов Dy. Причина этого связана с изменением вклада нейтронного  $i_{13/2}$  и протонного  $h_{11/2}$  состояний в полный выстроенный угловой момент. Результаты расчетов для легких изотопов находятся в соответствии с недавними экспериментальными результатами. В связи с этим ставится вопрос о необходимости проверки на эксперименте их предсказаний для тяжелых изотопов Dy.

### INTRODUCTION

The study of magnetic moment or  $g$  factor has been extensively employed in the past as a sensitive probe for a better understanding of the structure of ground state as well as excited states up to very high angular momentum in stable nuclei in different mass regions. With the advent of radioactive beam facilities, now the focus has shifted to the production and study of spectroscopic properties of neutron-rich nuclei. Such studies in the mass  $A > 160$  region have been limited though to the neighbourhood of stable nuclei because of difficulties in their production and observations. However, recently there have been made successful experimental attempts to study the properties of neutron-rich Dy isotopes: <sup>166</sup>Dy [1] and <sup>168</sup>Dy [2]. The change in the moment of inertia as a function of rotational frequency for the even–even <sup>160–166</sup>Dy isotopes as discussed in Ref. 1 reveals variation in the detailed structure of the excitation spectrum indicating separation in behaviour as a function of neutron number

$N$  beyond  $^{164}\text{Dy}$ . Similar neutron number dependence has been inferred from the systematics of measured [2] energy of the first  $2^+$  state  $E_2^+$ , and the excitation energy ratio  $R_4 = E_4/E_2$  for Dy and other rare-earth nuclei [3].

In yet another recent experiment Brandolini et al. [4] have measured the  $g$ -factor values for the yrast levels in  $^{160-164}\text{Dy}$  isotopes. These measurements comprise the results up to angular momentum states  $J = 12^+$  in  $^{160}\text{Dy}$  and up to  $J = 10^+$  in  $^{162-164}\text{Dy}$  nuclei. Earlier measurements by the Bonn group [5, 6] for the  $g$  factors in these nuclei were restricted up to the states with spin  $J = 6^+$ . The Bonn group data [5, 6] are not compatible with those of Brandolini et al. [4]. Apart from the overall differences in the  $g$ -factor values of the two measurements for all the three isotopes, one also finds that the values for the  $2^+$ ,  $4^+$ , and  $6^+$  states in  $^{160}\text{Dy}$  in Refs. 5, 6 show oscillatory behaviour whereas those of Ref. 4 are marked by a smooth reduction, albeit slow, right up to  $J = 12^+$  states. In this connection we should add that it is gratifying to note that the results of our earlier CHFB calculations [7] using the pairing-plus-quadrupole Hamiltonian [8] for the nuclei  $^{158,164}\text{Dy}$  and  $^{166}\text{Er}$  have proved to be consistent with experimental data [4, 9, 10] on the  $g$ -factor values providing a correct trend for their angular momentum dependence.

In view of these interesting experimental developments, we have carried out a systematic study of the structure of ground state as well as high spin states of  $^{160-170}\text{Dy}$  isotopes within the framework of CHFB using the pairing + quadrupole + hexadecapole model interaction Hamiltonian. It should be mentioned that guided by the past experience we have not included at this stage of our investigations the particle number and angular momentum projections as we believe that such calculations, though desired, would not significantly change the gross features of the present results for the  $g$  factors. Further, since our main interest is in the high spin structure of neutron rich isotopes, we present here the results for  $^{164-170}\text{Dy}$  isotopes in greater detail. It may be emphasized that one of the main aims of the present work is to show that the low as well as high spin behaviour of  $g$  factors in  $^{166}\text{Dy}$  and in heavier isotopes is strikingly different from that in  $^{160,162,164}\text{Dy}$ . Thus a measurement of  $g$  factors even at low spins in  $^{166}\text{Dy}$  and in heavier isotopes would be extremely valuable to enrich our understanding of the neutron-rich rare-earth nuclei. For the purpose of our general study we have chosen the interaction strengths which provide a reasonably good description for the ground state shape parameters, the first  $2^+$  excitation energy, and the spin dependence of the  $g$  factors of  $^{164}\text{Dy}$ . The calculations are then performed with the same set of interaction parameters for all other isotopes  $^{166,168,170}\text{Dy}$  in order to illustrate the general trend for neutron number dependence of the variation in  $g$  factor as a function of increasing spin. The same set of parameters provides a consistent trend of  $g$  factor values for lighter isotopes  $^{160,162}\text{Dy}$  as well. A very small adjustment only of pairing interaction strengths, keeping other interaction parameters unchanged, even provides almost a quantitative agreement with experiments.

## 1. THE MODEL AND CALCULATIONAL DETAILS

The CHFB theory [11] has been successfully applied in the past especially for the study of high-spin structures and by now it has become a standard tool. We therefore describe here only the most relevant aspects of this approach. For a deformed nucleus, with  $z$  axis as its

symmetry axis, a cranked Hamiltonian is given by

$$H_\omega = H - \omega J_x, \quad (1)$$

where  $\omega$  is the cranking frequency for rotation about the  $x$  axis and  $\hat{H}$  represents the usual many-body Hamiltonian for the ground state with  $\omega = 0$ . The angular momentum and particle number are conserved on the average and satisfy the following constraints simultaneously,

$$\langle \Psi_{\text{CHFB}} | \hat{J}_x | \Psi_{\text{CHFB}} \rangle = \langle \hat{J}_x \rangle = \sqrt{J(J+1)}, \quad (2)$$

$$\langle \Psi_{\text{CHFB}} | \hat{N} | \Psi_{\text{CHFB}} \rangle = N. \quad (3)$$

Here  $N$  denotes the particle number.

For the purpose of our calculations we employ a quadrupole + hexadecapole + pairing model interaction Hamiltonian  $H$  written as,

$$H = H_0 - \frac{1}{2} \sum_{\lambda=2,4} \chi_\lambda \sum_{\mu} \hat{Q}_{\lambda\mu} (-1)^\mu \hat{Q}_{\lambda-\mu} - \frac{1}{4} \sum_{\tau=p,n} G_\tau \hat{P}_\tau^\dagger \hat{P}_\tau, \quad (4)$$

where  $H_0$  stands for the one-body spherical part;  $\chi_\lambda$  term represents the quadrupole and hexadecapole terms with  $\lambda = 2, 4$ ; and the  $G_\tau$  term represents the proton and neutron monopole pairing interaction. Explicitly we have

$$\hat{Q}_{\lambda\mu} = \left( \frac{r^2}{b^2} \right) Y_{\lambda\mu}(\theta, \phi), \quad (5)$$

$$\hat{P}_\tau^\dagger = \sum_{\alpha_\tau, \bar{\alpha}_\tau} c_{\alpha_\tau}^\dagger c_{\bar{\alpha}_\tau}^\dagger. \quad (6)$$

In the above  $c^\dagger$  are the creation operators with  $\alpha \equiv (n_\alpha l_\alpha j_\alpha m_\alpha)$  as the spherical basis-states quantum numbers with  $\bar{\alpha}$  denoting the conjugate time-reversed orbital. The standard mean field CHFB equations for which an excellent account is available in Ref. 11, are solved self-consistently for the quadrupole, hexadecapole and pairing gap parameters. The deformation parameters are defined in terms of the following expectation values:

$$D_{2\mu} = \chi_2 \langle \hat{Q}_{2\mu} \rangle, \quad D_{4\mu} = \chi_4 \langle \hat{Q}_{4\mu} \rangle, \quad (7)$$

$$\hbar\omega\beta \cos \gamma = D_{20}, \quad \hbar\omega\beta \sin \gamma = \sqrt{2}D_{22}, \quad \hbar\omega\beta_{40} = D_{40}, \quad (8)$$

$$\Delta_\tau = \frac{1}{2} G_\tau \langle \hat{P}_\tau \rangle. \quad (9)$$

The oscillator frequency  $\hbar\omega = 41.0A^{-1/3}$  (MeV), whereas  $\beta, \gamma$ , and  $\beta_{40}$  are the usual deformation parameters, while  $\Delta_p$  and  $\Delta_n$  are the pairing gap parameters for protons and neutrons, respectively. For the *g*-factor calculations we use the relation

$$g = \langle \Psi_{\text{CHFB}} | \mu_x | \Psi_{\text{CHFB}} \rangle / \langle \hat{J}_x \rangle, \quad (10)$$

where the *x* component of the magnetic moment operator is given by

$$\mu_x = g_l \sum_i j_x(i) + (g_s - g_l) \sum_i s_x(i) \quad (11)$$

with  $g_l = 1$  and  $g_s = 5.586$  for proton, and  $g_l = 0$  and  $g_s = -3.826$  for neutrons. From the form of the expression for *g* it is evident that the contribution of intrinsic spin  $s_x$  of neutron single particle states is negative whereas that from protons is positive. Thus in the case of predominant neutron alignments the *g* factor values may become close to zero (or even negative) as is observed in many rare earth nuclei at relatively higher angular momenta.

Present calculations have been performed within the basis space consisting of  $N = 4, 5$  harmonic oscillator major shells  $+0i_{13/2}$  orbitals for protons, and  $N = 5, 6$  major shells  $+0j_{15/2}$  orbitals for neutrons with the assumption of an inert core  $Z = 40$  and  $N = 70$ . For the model Hamiltonian with multipole separable forces having  $r^2$  radial dependence a basis space constituted of many shells is not required as has been elaborated in Ref. 8. Nevertheless, we have considered the additional subshells  $0i_{13/2}$  for protons, and  $0j_{15/2}$  for neutrons in our calculations. It is observed that these additional orbitals do not play any significant role even at high spins. The spherical single particle energies are taken as the spherical Nilsson model single particle energies with *A*-dependent Nilsson parameters [12]. The upper shell radial matrix elements are reduced by factors,  $(N_0 + 3/2)/(N + 3/2)$ , as discussed in Ref. 8, where  $N_0$  takes the value 4 for protons and 5 for neutrons. Finally the interaction strengths are chosen such that reasonable values of the ground-state shape parameters, the first  $2^+$  excitation energy ( $\sim 100$  keV), and the spin-dependent (up to  $J = 10$ ) *g* factors of  $^{164}\text{Dy}$  are obtained. On the basis of this consideration we have taken the following values of interaction strengths (all in MeV)

$$\chi_2 = 60/A^{1.4}, \quad \chi_4 = 55/A^{1.4}, \quad G_p = 25.3/A, \quad G_n = 21.5/A. \quad (12)$$

For our general qualitative study of spin dependence of *g*-factor values we use only this set of global interaction strengths for all the isotopes  $^{164-170}\text{Dy}$  throughout. However, it should be emphasized that for a quantitative agreement with experimental data one has to slightly fine tune the pairing interaction strengths, as the proton and neutron contribution to the total alignment  $\langle j_x \rangle$  is very sensitive to the single particle energies and the position of the proton and neutron Fermi levels as has been demonstrated in the case of  $^{160,162}\text{Dy}$  isotopes.

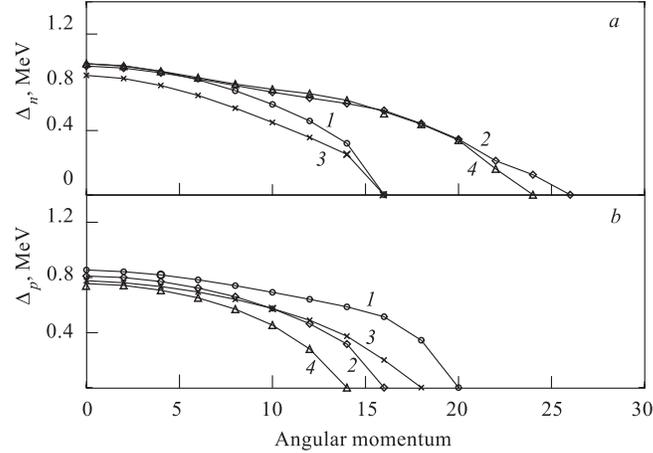
## 2. RESULTS AND DISCUSSIONS

First we make some remarks about the ground state properties of these nuclei summarized in Table 1. Our results show that in their ground state all these isotopes, excepting  $^{164}\text{Dy}$  which shows a tendency to have small prolate deformation  $\gamma$ , are symmetrically deformed

with almost similar values of  $\beta$  ranging from 0.3444 for  $^{170}\text{Dy}$  to 0.3529 for  $^{166}\text{Dy}$ . This  $\gamma$  asymmetry of  $^{164}\text{Dy}$  becomes more pronounced at higher spins as compared to that in other isotopes. For example, it becomes  $8^\circ$  at  $J = 20^+$  for  $^{164}\text{Dy}$  but remains less than  $4^\circ$  for others. Further, the systematics of energy of  $2^+$  state and the ratio  $R_4 = E_4/E_2$  obtained in our calculations are found to be consistent with the experimental results [2]. The proton and neutron pairing gaps  $\Delta_p$  and  $\Delta_n$  for this isotope are found to be largest, though the  $\Delta_n$  values for  $^{166}\text{Dy}$  and  $^{170}\text{Dy}$  are similar to that of  $^{164}\text{Dy}$ . These attributes are finally seen to be responsible for  $^{164}\text{Dy}$  to show marked differences in its angular momentum dependence of the  $g$  factors as compared to neutron-rich isotopes studied here.

**Table 1. CHFB results for the ground-state intrinsic-shape parameters quadrupole deformation  $\beta$ , asymmetric deformation  $\gamma$ , hexa-decapole deformation  $\beta_{40}$ . The table also shows the proton and neutron pairing gaps  $\Delta_p$  and  $\Delta_n$ , and the Fermi energies  $\lambda_p$  and  $\lambda_n$  for the ground state along with the excitation energy  $E_2$  and the ratio  $R_4 = E_4/E_2$  for the  $^{164-170}\text{Dy}$  isotopes**

Nucleus	$\beta$	$\gamma$ , deg	$\beta_{40}$	$\Delta_p$ , MeV	$\Delta_n$ , MeV	$\lambda_p$ , MeV	$\lambda_n$ , MeV	$E_2^+$ , KeV	$R_4$
$^{164}\text{Dy}$	0.3528	0.0299	0.0293	0.854	0.816	-2.095	-4.386	96.5	3.219
$^{166}\text{Dy}$	0.3529	0.0080	0.0160	0.811	0.801	-2.147	-3.994	97.3	3.258
$^{168}\text{Dy}$	0.3524	0.0003	0.0025	0.774	0.743	-2.204	-3.520	91.7	3.226
$^{170}\text{Dy}$	0.3444	0.0130	-0.0109	0.756	0.815	-2.257	-3.013	98.1	3.233



**Fig. 1.** The angular momentum dependence of neutron (a) and proton (b) pairing gaps  $\Delta_n$  and  $\Delta_p$  for the yrast levels in  $^{164-170}\text{Dy}$  isotopes: 1,  $\circ$  —  $A = 164$ ; 2,  $\diamond$  —  $A = 166$ ; 3,  $\times$  —  $A = 168$ ; 4,  $\triangle$  —  $A = 170$

In order to look into the mass dependence of the high-spin structure of these isotopes, we have shown in Fig. 1 the variation of proton and neutron pairing gaps,  $\Delta_p$  and  $\Delta_n$ , for the

states with angular momentum up to  $J = 26^+$ . While moving towards neutron-rich isotopes from  $A = 164$  to  $A = 170$ , it is found that the proton-pairing gap collapses for these isotopes, respectively at spin value  $J = 20^+$ ,  $J = 16^+$ ,  $J = 18^+$ ,  $J = 14^+$ ; whereas the neutron pairing collapses at  $J = 16^+$ ,  $J = 26^+$ ,  $J = 16^+$ , and  $J = 24^+$ , respectively. It is seen from the figure that the rate of decrease of  $\Delta_p$  with increasing angular momentum is much slower for  $^{164}\text{Dy}$ . In contrast to  $^{170}\text{Dy}$  it is quite fast as compared to other isotopes. An almost opposite trend is observed for the neutron-pairing gap  $\Delta_n$ . It may be emphasized that in a number projected calculation the finer details of the variations in  $\Delta_p$  and  $\Delta_n$  could be changed, but guided by our experience we believe that the gross features of the results discussed above will not be washed out to change our conclusions. The change of angular frequency  $\omega$  with spin  $J$  has been shown in Fig. 2. It is seen that these curves do not exhibit any backbending feature, though in case of  $^{168}\text{Dy}$  and  $^{170}\text{Dy}$  the rate of change in  $\omega$  increases sharply at around  $J = 14$  in contrast to  $^{164}\text{Dy}$  which shows moderate change around  $J = 20$ .

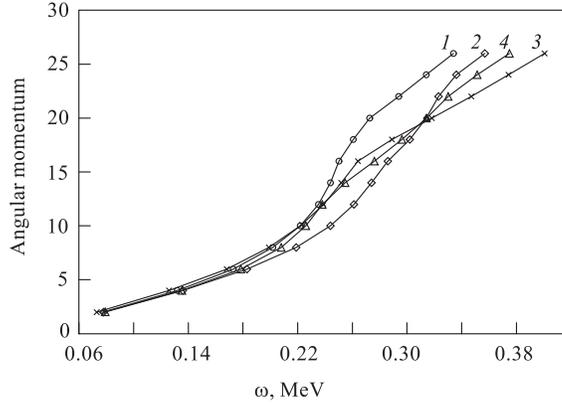


Fig. 2. Variation of angular frequency  $\omega$  with angular momentum  $J$  for the yrast levels in  $^{164-170}\text{Dy}$ : 1,  $\circ$  —  $^{164}\text{Dy}$ ; 2,  $\diamond$  —  $^{166}\text{Dy}$ ; 3,  $\times$  —  $^{168}\text{Dy}$ ; 4,  $\triangle$  —  $^{170}\text{Dy}$

In Fig. 3 we have shown the variation of calculated  $g$  factors for the four Dy isotopes. The experimental data for the  $^{164}\text{Dy}$  isotope taken from the work of Brandolini et al. [4] and that of Bonn group [5,6] are also displayed. The experimental data of Ref. 4 and Refs. 5, 6 are not consistent with each other. We have not shown the Canberra data [13] as these are quite old and have large uncertainty. Our results for the isotope dependence show that the enhancement in  $g$ -factor values starting already at low spin becomes increasingly sharp while moving from  $^{164}\text{Dy}$  to higher mass number. In contrast, for the  $^{164}\text{Dy}$  isotope the trend is much different. The  $g$  factor continues to decrease slowly until  $J = 14^+$  and then goes up suddenly to level off at  $J = 20^+$ . For the isotopes  $^{166}\text{Dy}$  and  $^{170}\text{Dy}$  a very sharp increase is followed by a rapid reduction in  $g$  factors beyond  $J = 14^+$  and  $J = 16^+$ , respectively. This variation with increasing spin for  $^{168}\text{Dy}$  characteristically shows intermediate behaviour between the two extremes described above for  $^{164}\text{Dy}$  and  $^{170}\text{Dy}$  and underlines the interplay between the single particle and collective degrees of freedom. In this respect the positions of the single-particle levels with respect to the Fermi surface play the most crucial role as

has been illustrated in Fig. 4 which displays the predominant contributions coming from the single-particle states  $\pi h_{11/2}$  and  $\nu i_{13/2}$  to the total angular momentum of the system through alignments due to the coriolis coupling. It may be mentioned that the effect of variation in Fermi energy with increasing mass number on the spectra and electromagnetic properties of nuclei has been demonstrated earlier in the context of our studies [14] of coriolis antipairing and alignment effects in transitional nuclei. From the form of the magnetic moment operator given by Eq.(11), it is evident that the proton states contribute positively and increase the  $g$  factor whereas in contrast the neutron single particles reduce it as these contribute only through the anomalous  $g_s$  term which has a negative value.

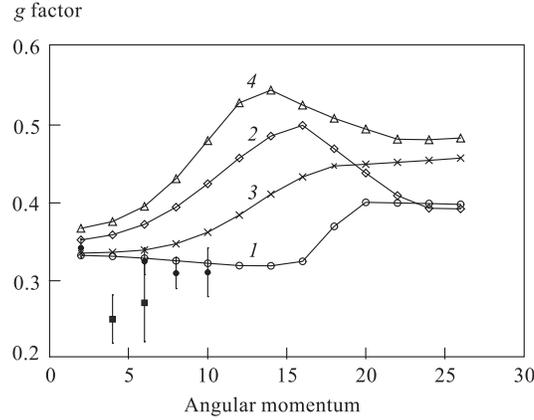


Fig. 3. Angular momentum dependence of  $g$ -factor values for the yrast levels in  $^{164-170}\text{Dy}$  isotopes: 1,  $\circ$  —  $^{164}\text{Dy}$ ; 2,  $\diamond$  —  $^{166}\text{Dy}$ ; 3,  $\times$  —  $^{168}\text{Dy}$ ; 4,  $\triangle$  —  $^{170}\text{Dy}$ . The full circles and full squares show the experimental data of Brandolini et al. [4] and that of Bonn group [5,6], respectively

We first consider the  $g$ -factor variation for the yrast states in  $^{164}\text{Dy}$ . From Fig. 5 it is seen that the contribution of  $\nu i_{13/2}$  single-particle orbital to the total angular momentum of  $^{164}\text{Dy}$  is larger than that coming from the alignment of the  $\pi h_{11/2}$  state through out for all spins starting from  $J = 2^+$ . This is due to closer proximity of the  $\nu i_{13/2}$  state to the neutron Fermi energy  $\lambda_n$  as compared to that of  $\pi h_{11/2}$  state to the proton Fermi energy  $\lambda_p$ . Remembering that the neutron states have negative contribution, this accounts for a rather small value of the  $g$  factor for  $^{164}\text{Dy}$  as shown in Fig. 3. The  $\langle j_x \rangle$  for the  $\nu i_{13/2}$  increases linearly up to spin  $J = 16^+$  where the neutron-pairing gap  $\Delta_n$  becomes zero as is seen from Fig. 2. Similar increase is observed for the  $\langle j_x \rangle$  contribution from the  $\pi h_{11/2}$  state until spin value  $J = 20^+$  where the proton-pairing gap  $\Delta_p$  almost vanishes. This relative variation of the  $\langle j_x \rangle$  values from the two states with increasing spin is reflected in the variation of the  $g$  factor of  $^{164}\text{Dy}$  observed experimentally as depicted in Fig. 3. Thus one observes that due to relatively large increase of neutron contribution to  $\langle j_x \rangle$  the net result is that  $g$  factor continuously decreases until the total angular momentum of the system attains a value  $J = 14^+$  and then increases sharply around  $J = 16^+$  beyond which the rate of increase for the proton contribution to the  $\langle j_x \rangle$  value is enhanced and that of neutron contribution is diminished due to the collapse of the neutron pairing. This increase in the  $g$  factor for  $^{164}\text{Dy}$  is sustained until it acquires the

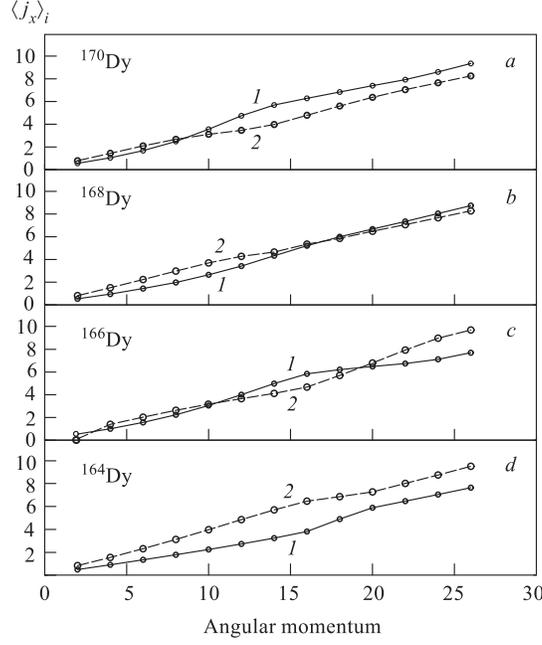


Fig. 4. Contribution of single-particle orbitals  $\pi h_{11/2}$  (1, small circles), and  $\nu i_{13/2}$  (2, large circles) to the total angular momentum  $J$  in  $^{164}\text{Dy}$  (d),  $^{166}\text{Dy}$  (c),  $^{168}\text{Dy}$  (b) and  $^{170}\text{Dy}$  (a). The contributions from other proton and neutron single-particle states are rather small and have not been depicted in this plot

total spin  $J = 20^+$ , when the proton pairing also collapses to zero. Beyond spin  $J = 20^+$  the contributions of proton and neutron states to  $\langle j_x \rangle$  exhibit almost similar rate of increase and therefore the  $g$  factor remains almost constant. A comparison with the experimental data in Fig.3 shows that the new measurements of Brandolini et al. [4] carried out up to spin  $J = 10^+$  show a similar trend of slowly decreasing  $g$  factor value with increasing spin as obtained in our CHFB calculations. As mentioned earlier, if particle number projection is performed, a small pairing gap will continue to persist even at very high spins. However, our experience with such calculations [15] is that the rotation alignment is not much affected to change the  $g$  factor values appreciably.

The variation of  $g$  factor for other isotopes shown in Fig. 3 can be understood in a similar manner. While moving from  $^{164}\text{Dy}$  to neutron rich isotopes, the proton Fermi energy remains almost constant while that for the neutron moves up as shown in Table 1. Consequently contribution of the neutron single-particle state  $\nu i_{13/2}$  to the  $\langle j_x \rangle$  value decreases due to somewhat reduced coriolis coupling and already for  $^{168}\text{Dy}$  the proton contribution becomes larger to that from neutron at spin values beyond  $J = 18^+$ . This take-over in the case of  $^{170}\text{Dy}$  occurs at much lower angular momentum  $J = 10^+$ . In case of  $^{166}\text{Dy}$  it is seen from Fig. 4 that the rate of enhancement for proton contribution at lower spins is larger as compared to that of neutrons. Consequently total contribution of proton states becomes larger than that from neutron states beyond spin  $J = 10^+$  and remains so until  $J = 18^+$ . This causes the

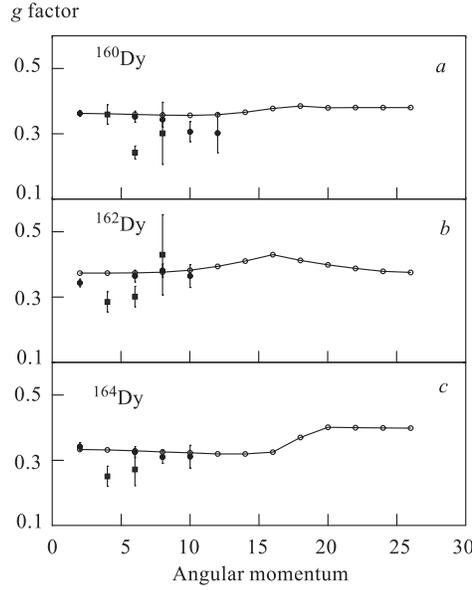


Fig. 5. Angular momentum dependence of  $g$ -factor values for the yrast levels in  $^{160}\text{Dy}$  (a),  $^{162}\text{Dy}$  (b) and  $^{164}\text{Dy}$  (c). The experimental data of Brandolini et al. [4] and that of Bonn group [5,6] have been shown by full circles and full squares, respectively; empty circles and curves — theory

$g$  factor to rise continuously up to  $J = 18^+$  as seen in Fig. 3. The proton-pairing gap for  $^{166}\text{Dy}$  tends to become negligible at  $J = 16^+$ , whereas the neutron pairing collapses only at higher spin around  $J = 26^+$ . This is the reason that the rate of rise in the proton alignment is drastically reduced beyond spin value  $J = 16^+$  whereas it continues to increase for the neutron right up to spin value  $J = 26^+$ . This variation causes the  $g$  factor to have much reduced rate of increase beyond spin  $J = 18^+$  as is seen from Fig. 3. The slope of the curve for  $\pi h_{11/2}$  at lower spins up to  $J = 14^+$ , as seen in Fig. 4, is maximum in the case of  $^{170}\text{Dy}$  and thereafter it is reduced. Consequently one observes in Fig. 3 that growth in the  $g$  factor for  $^{170}\text{Dy}$  within this region of spin values is very sharp. The pairing gap  $\Delta_p$  for this isotope collapses around  $J = 14^+$  whereas  $\Delta_n$  becomes zero at  $J = 24^+$ . This in turn results in the sharp reduction of  $g$  factor after  $J = 14^+$ . The  $g$  factor keeps decreasing up to spin value  $J = 24^+$  due to neutron contribution to  $\langle j_x \rangle$ . After both  $\Delta_p$  and  $\Delta_n$  become zero, that is for  $J \geq 24^+$ , the  $g$  factor attains almost a constant value as can be seen in Fig. 3. A similar explanation holds for the case of  $^{166}\text{Dy}$  isotope.

Brandoni et al. [4] have carried out measurements of the  $g$  factors also in the yrast band of  $^{160}\text{Dy}$ ,  $^{162}\text{Dy}$  nuclei and the data are understood to be greatly improved over earlier results obtained by the Bonn group [5,6]. With this in view, we have also studied the spin dependence of  $g$  factor values for these isotopes as well. It is found that using the same strengths for the quadrupole and hexa-decapole interactions as used for  $^{164-170}\text{Dy}$  and with a slight change in the pairing interaction strength, the experimental results are well accounted for by the theory as shown in Fig. 5. The pairing strengths used for  $^{160}\text{Dy}$  are  $AG_p = 23.02$

and  $AG_n = 21.50$ , and those for  $^{162}\text{Dy}$  are  $AG_p = 24.00$  and  $AG_n = 22.00$ . Such a small variation in pairing strengths from one nucleus to another is quite expected in order to obtain a quantitative description of the data.

The experimental *g*-factor values in  $^{160}\text{Dy}$  and  $^{164}\text{Dy}$  exhibit a slight decrease with increasing spin values whereas in the case of  $^{162}\text{Dy}$  it tends to increase a bit up to  $J = 6^+$  and then decreases again. As discussed above for the  $^{164-170}\text{Dy}$  isotopes such a behaviour reflects the relative increase in contributions to total alignment coming from proton and neutron single particle orbitals as the system acquires higher and higher spin. This in turn depends sensitively on the position of the Fermi energy with respect to single particle levels, especially the  $\pi h_{11/2}$  and  $\nu i_{13/2}$  as explained for heavier isotopes above.

## CONCLUSIONS

In view of the new experiments being successfully performed for the neutron rich  $^{166}\text{Dy}$  and  $^{168}\text{Dy}$  nuclei, we have carried out the standard *x*-axis CHFB calculations using a separable pairing + quadrupole + hexadecapole model Hamiltonian for the  $^{164-170}\text{Dy}$  isotopes to study the spin dependence of *g*-factor values for the yrast states, especially in the neutron rich isotopes.

The systematics of CHFB results for the ground and excited  $2^+$  states, and that of the ratio  $R_4 = E_4/E_2$  for these isotopes indicate that in the ground state the maximum deformation occurs at neutron number  $N = 102$  corresponding to  $^{168}\text{Dy}$ , instead of the expected  $N = 104$  which signifies the midshell for the magic numbers 82 and 126. Our results also suggest that  $^{164}\text{Dy}$  is asymmetrically deformed in the ground state having quadrupole deformation  $\beta$  close to that for  $^{168}\text{Dy}$ . This implies a second maximum deformation occurs at neutron number  $N = 98$ . These findings are consistent with the recent experimental results [2, 3]. The axial component of hexadecapole deformation in the ground state of these isotopes varies from  $\beta_{40} = 0.029$  in  $^{164}\text{Dy}$  to  $\beta_{40} = -0.01$  in neutron rich  $^{170}\text{Dy}$ . Our  $\beta_{40} = 0.029$  in  $^{164}\text{Dy}$  is slightly larger than the value  $\beta_{40} = 0.019$  reported by Stuchbery et al. [16].

The spin dependence of the *g*-factor values in  $^{166,168,170}\text{Dy}$  are found to be characteristically different from that observed in stable  $^{164}\text{Dy}$ . The *g*-factor values in neutron rich  $^{170}\text{Dy}$  exhibit a sharp rise with increasing spin attaining a maximum at  $J = 14^+$ . This sharp increase in  $^{164}\text{Dy}$  is found to set in at much higher spin beyond  $J = 16^+$ . This marked separation of behaviour is explained in terms of the relative rate of increase in contributions to the total aligned angular momentum coming from proton and neutron single particle orbitals. Our calculations indicate that for  $^{166,168,170}\text{Dy}$   $g_J > g_2$  for  $J = 4$  to 26 contrary to  $g_J < g_2$  for  $J$  values up to 14. At much higher spins  $J \geq 24^+$  the *g* factor values in all these isotopes are seen to level off. In the Dy nuclei studied here it is found that the  $\pi h_{11/2}$  and  $\nu i_{13/2}$  single-particle states and their positions with respect to the Fermi levels play important role in influencing the main trend of spin dependence of the *g* factors. Finally it is shown that the recent experimental data [4] for the spin dependence of *g* factor values for the yrast states in  $^{160,162,164}\text{Dy}$  can be satisfactorily accounted for within the cranking approach with multipole interactions. In order to verify the new trend in spin dependence of *g*-factor values for the neutron rich nuclei it would be of great interest to have measurements of *g*-factor values even at low spins for the yrast states in  $^{166,168,170}\text{Dy}$ .

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