

УДК 621.384.634.5

## CHARGE EXCHANGE INJECTION FOR NUCLOTRON AND NUCLOTRON BOOSTER

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The acceleration of polarized beams is between the major items in the JINR LHE's heavy ion superconducting synchrotron Nuclotron research programme. One effective way to increase the intensities of polarized deuteron beams is the application of the charge exchange injection into the Nuclotron.

The paper represents the results of a new analytical description of the heavy ion stripping injection based on the Boltzmann kinetic equation. Expressions for the ion density evolution in the transverse phase plane, for the emittance growth due to the elastic scattering and to energy losses in the stripping foil and for the number of successfully stored particles have been derived. These results have been applied to the stripping injection of polarized deuterons into the Nuclotron as well as to the stripping injection of heavy ions into the now under consideration Nuclotron rapid cycling booster. It has been shown that an estimated 40-fold intensity gain could be achieved for the stripping injection of polarized  $D^-$  into the Nuclotron and that an effective stripping injection of light and medium ions into the booster could be realized.

The investigation has been performed at the Laboratory of High Energies, JINR.

## Перезарядная инжекция для нуклотрона и бустера нуклотрона

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Ускорение поляризованных пучков — важный пункт научной программы на сверхпроводящем синхротроне тяжелых ионов ЛВЭ ОИЯИ — нуклотроне. Одним из эффективных путей повышения интенсивности тяжелых ионов является применение перезарядной инжекции в нуклотрон.

Данная статья представляет результаты нового аналитического описания инжекции с обдиркой тяжелых ионов, основанные на решении кинетического уравнения Больцмана. Получены выражения для эволюции плотности ионов на фазовой плоскости, роста эмиттанса из-за упругого рассеяния и энергетических потерь при прохождении обдирочной фольги, а также формулы для количества накопленных частиц. Эти результаты применимы как для инжекции с перезарядкой поляризованных дейтронов в нуклотрон, так и для инжекции с перезарядкой тяжелых ионов в

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проектируемый в настоящее время быстроциклический бустер нуклотрона. Показано, что увеличение интенсивности примерно в 40 раз может быть достигнуто с помощью перезарядной инжекции поляризованных  $D^-$  в нуклотрон и что эффективная инжекция с перезарядкой легких и средних ионов в бустер может быть реализована.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

## 1. INTRODUCTION

The superconducting heavy ion synchrotron Nuclotron uses an Alvarez type linac capable to accelerate ions with  $0.28 < Z/A < 0.5$  up to 5 MeV/u and protons up to 20 MeV as an injector [1]. The available ion sources are: duoplasmatron for producing protons, deuterons and  $\alpha$  particles; a laser ion source for producing heavy ions; an EBIS ion source for producing high charge state ions, and a cryogenic polarized deuteron source «Polaris». The laser and EBIS ion sources generate short beam pulses with duration comparable to the revolution time. Hence the single turn injection is the appropriate injection method and it has been used at the Nuclotron since 1992.

The acceleration of polarized particles is one of the major items in the Nuclotron research programme. The first test runs of polarized deuteron injection and acceleration in the Nuclotron have already been carried out.

It is important to increase the intensity of polarized beams. As the emittance of the injected beam is comparable with the Nuclotron acceptance no multiturn injection can be applied. In this paper we study the possibility to use the charge exchange injection to store polarized deuterons in the Nuclotron.

A more fundamental way to increase the intensity of Nuclotron beams is the use of a booster synchrotron injector [2]. The booster lattice can have large enough acceptance which will allow a multiturn injection to be applied. The high vacuum in the booster ring will reduce the beam losses due to interactions with the residual gas to a great extent. The booster will also raise the final energy of ions applying ion stripping before the injection into the main ring.

In this paper we study the injection of heavy ions into the booster synchrotron by means of ion stripping.

Proposed by G.I.Dimov in Novosibirsk in 1969 [3], nowadays the charge exchange, or stripping, injection is a preferred injection method for proton machines due to its relative simplicity and a very high intensity of stored beams. Recently this injection method has been successfully applied for light ion storage in CELSIUS [4].

In this paper we develop an analytical approach to description of the stripping injection. It is based on a kinetic treatment of the injection process. Analytical expressions for the particle density evolution in the transverse phase space, for the emittance growth due to the elastic scattering and to energy losses in the stripper and for the number of successfully stored particles have been derived.

## 2. PROCESSES IN CHARGE EXCHANGE INJECTION

A comprehensive review of the processes taking place during the heavy ion charge exchange injection could be found in [5]. Here we will summarize only those results which are of importance for the following description.

As the beam travels through the stripping foil the relative content of ions in different charge states changes due to the processes of electron loss and of electron capture. For thick enough foils the charge state distribution reaches an equilibrium [6]. This equilibrium distribution is independent of the charge state distribution in the incident beam and is determined only by the relations between different charge-exchange cross sections and by the ion velocity.

The equilibrium charge state distribution is well described by a Gaussian [7]:

$$\Phi_q = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(q-\bar{q})^2}{2\sigma^2}}. \quad (1)$$

In (1)  $\Phi_q$  denotes the relative content of ions in the charge state  $q$ ,  $\bar{q}$  is the average charge state (generally not integer) and  $\sigma$  is the standard deviation of the charge state distribution. Formula (1) is valid for  $1 < \bar{q} < Z_{\text{pr}} - 1$ .

Several semiempirical formulae have been proposed for the average charge state  $\bar{q}$ , for example, the Nikolaev–Dmitriev formula [8]:

$$\frac{\bar{q}}{Z_{\text{pr}}} = (1 + X^{-\frac{1}{0.6}})^{-0.6}, \quad (2)$$

where

$$X = \frac{v}{v' Z_{\text{pr}}^{0.45}}, \quad v' = 3.6 \cdot 10^8 \text{ cm/s}. \quad (3)$$

In (2) and (3)  $v$  is the ion velocity and  $Z_{\text{pr}}$  is the projectile atomic number.

For the standard deviation of the distribution Nikolaev and Dmitriev propose the following expression [8]:

$$\sigma = 0.5 \sqrt{\bar{q} \left( 1 - \left( \frac{\bar{q}}{Z_{\text{pr}}} \right)^{1.67} \right)}. \quad (4)$$

Two processes are of big importance for the charge exchange injection: Coulomb elastic scattering and ionization losses of ion energy.

The Coulomb elastic scattering causes a change in the ion trajectory slope. In [9] the following experimental formula for the heavy ion multiple scattering mean square angle in solid foils is given:

$$\langle \Delta\theta^2 \rangle = 0.250 \frac{Z_t(Z_t + 1)}{A_t} \frac{Z_{\text{pr}}^2}{E_{\text{pr}}^2} t, \quad (5)$$

where the scattering angle  $\theta$  is in mrad, the stripper thickness  $t$  is in  $\mu\text{g}/\text{cm}^2$  and the projectile energy  $E_{\text{pr}}$  is in MeV.

The distribution of the multiple scattering angle could be approximated with good accuracy by a Gaussian.

The losses of the ion energy in the stripping foil are due to the excitation and ionization of the foil atoms. Mean losses are described by the well-known Bethe–Bloch formula [10].

The ionization losses straggling is distributed according to Landau's, Vavilov's or normal distributions depending on the ion velocity. In practice the straggling is small and could be neglected.

### 3. EQUATION OF MOTION

The Floquet normalized coordinates are the most convenient for the description of the kinetics of charge exchange injection. In this paper we will use

i) generalized azimuth  $\phi$  as an independent («time») variable

$$\phi = \int_0^s \frac{ds}{Q\beta(s)}, \quad (6)$$

ii) normalized transverse displacement

$$u = \frac{x}{\sqrt{\beta}}, \quad (7)$$

iii) conjugate momentum

$$p_u = \frac{du}{d\phi} = Q \left( \frac{\alpha x + \beta x'}{\sqrt{\beta}} \right). \quad (8)$$

The coordinates  $u$  and  $p_u$  have dimensions  $\sqrt{m}$ .

In the above formulae  $x$  denotes the horizontal coordinate,  $s$  — the longitudinal coordinate along the reference orbit and  $\alpha$ ,  $\beta$ ,  $\gamma$  are the well-known Twiss functions. We will use «/» for the differentiation with respect to  $\phi$ .

Taking into account the process of elastic Coulomb scattering and of ionization losses of energy in the stripper, we can write the following equation of motion describing the behaviour of heavy ions passing through the stripper:

$$\frac{d\mathbf{u}}{d\phi} = A\mathbf{u} + \mathbf{a}(\phi) + \mathbf{b}(\phi)\delta + \mathbf{c}V(\phi), \quad (9)$$

where

$$\mathbf{u} = \begin{pmatrix} u \\ p_u \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 \\ -Q^2 & 0 \end{pmatrix}, \quad (10)$$

$$\mathbf{a} = \begin{pmatrix} 0 \\ Q^2 f(\phi) \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ -Q^2 g(\phi) \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (11)$$

The function  $f(\phi)$  describes linear perturbations causing the closed orbit distortion:

$$f(\phi) = \beta^{3/2} \frac{\Delta B}{B\rho}. \quad (12)$$

The function  $g(\phi)$  is connected with the dispersion:

$$g(\phi) = -\frac{\beta^{3/2}}{\rho}. \quad (13)$$

In the above formulae  $\Delta B$  denotes field errors,  $\delta$  — the relative momentum spread and  $\rho$  — the radius of curvature.

In (9)

$$V(\phi) = \sum_{0 \leq k \leq \phi/2\pi} Q \sqrt{\beta_t} \Delta \theta_k \delta(\phi - 2\pi k). \quad (14)$$

The random kicks  $\Delta \theta_k$  which particles undergo in each passing the stripper are due to the Coulomb scattering. They are uncorrelated, with zero mean values and equal dispersions  $\langle \Delta \theta^2 \rangle$ . The function  $V(\phi)$  is consequently a white noise with intensity

$$v(\phi) = \sum_{k=1}^n Q^2 \beta_t \langle \Delta \theta^2 \rangle \delta(\phi - 2\pi k), \quad (15)$$

where  $n = [\phi/2\pi]$  is the number of realized turns and  $[ ]$  denotes the integer part of the number.

The above equation of motion is a linear stochastic differential equation. Its solution is

$$\mathbf{u} = \mathbf{u}_\beta + \mathbf{u}_{co} + \mathbf{D}_u \delta + \mathbf{u}_{sc}. \quad (16)$$

In (16)

i)  $\mathbf{u}_\beta$  is the general solution of the uniform equation and describes the betatron oscillations:

$$\mathbf{u}_\beta = K(\phi, \phi_0) \mathbf{u}_0, \quad (17)$$

where

$$K(\phi, \phi_0) = \begin{pmatrix} \cos Q(\phi - \phi_0) & \frac{1}{Q} \sin Q(\phi - \phi_0) \\ -Q \sin Q(\phi - \phi_0) & \cos Q(\phi - \phi_0) \end{pmatrix} \quad (18)$$

is the Cauchy matrix for the differential equation normalized to the unity matrix at  $\phi = \phi_0$  and  $\mathbf{u}_0$  is the vector of initial conditions;

ii)  $u_{co}$ , the normalized closed orbit, is a private, periodic (with period  $2\pi$ ) solution of the nonuniform equation  $u'' + Q^2 u = Q^2 f(\phi)$ :

$$u_{co} = \frac{Q}{2 \sin \pi Q} \int_{\phi}^{\phi+2\pi} f(\phi) \cos Q(\phi + \pi - \varphi) d\varphi; \quad (19)$$

iii)  $D_u$ , the normalized dispersion, is a private, periodic (with period  $2\pi$ ) solution of the nonuniform equation  $u'' + Q^2 u = Q^2 g(\phi)$ :

$$D_u = -\frac{Q}{2 \sin \pi Q} \int_{\phi}^{\phi+2\pi} \frac{\beta^{3/2}}{\rho} \cos Q(\phi + \pi - \varphi) d\varphi. \quad (20)$$

The ionization energy losses in the stripper increase the relative momentum spread according to

$$\delta = \delta_0 + n \frac{\Delta p}{p}, \quad (21)$$

where

$$\frac{\Delta p}{p} = \frac{\gamma}{\gamma + 1} \frac{\Delta T}{T}, \quad (22)$$

$\Delta T$  being the change in the ion kinetic energy which is well described by the Bethe–Bloch formula for ionization losses;

iv)  $u_{sc}$  is a private solution of the nonuniform equation

$$u'' + Q^2 u = \sum_{0 \leq k \leq \phi/2\pi} Q \sqrt{\beta_t} \Delta \theta_k \delta(\phi - 2\pi k)$$

and describes the elastic scattering in the stripper:

$$\mathbf{u}_{sc} = \int_0^\phi K(\phi, \varphi) \mathbf{c} V(\varphi) d\varphi, \quad (23)$$

which after substituting for the Cauchy matrix  $K$  and the function  $V(\phi)$  gives

$$u_{sc} = \sum_{0 \leq k \leq \phi/2\pi} \sqrt{\beta_t} \Delta \theta_k \sin Q(\phi - 2\pi k), \quad (24)$$

i.e.  $u_{sc}$  is a sum of elementary random functions.

It is the mean square value of  $u_{sc}$  which is of interest. From (24) one could calculate

$$\langle u_{sc}^2 \rangle = \beta_t \langle \Delta \theta^2 \rangle \sum_{0 \leq k \leq \phi/2\pi} \sin^2 Q(\phi - 2\pi k) \approx \frac{1}{2} n \beta_t \langle \Delta \theta^2 \rangle. \quad (25)$$

#### 4. THE BOLTZMANN EQUATION

Let us consider the evolution of the particle density in a thin slice of the beam.

In the consecutive moments  $\phi = 0, 2\pi, 4\pi, \dots$  the slice will pass through the stripper and the particles in the slice will undergo elastic Coulomb scattering. The elastic scattering results in kicks in the slope of the particle trajectories —  $\Delta \theta$ .

A bit earlier new portions of particles are injected into the accelerator. As we will see, not all of the particles which have passed the stripper will be accepted by the accelerator;  $(1 - \Phi_2)f(u, u', \phi)$  of them will be lost.

Summarizing these three effects, we can write the following kinetic equation for the distribution function  $f(u, p_u, \phi)$  in the transverse phase plane:

$$\frac{\partial f}{\partial \phi} + \frac{\partial f}{\partial u} u' + \frac{\partial f}{\partial u'} (-Q^2 u + \sum_{0 \leq k \leq \phi/2\pi} Q \sqrt{\beta_t} \frac{\Delta \theta_k}{\Delta \phi} h_k(\phi)) = q_{\text{source}} - q_{\text{drain}}, \quad (26)$$

where  $h_k(\phi)$  is a unity pulse from  $2\pi k$  to  $(2\pi k + \Delta\phi)$ , with  $\Delta\phi$  being the phase thickness of the stripping foil ( $\Delta\phi \ll 2\pi$ ):

$$h_k(\phi) = e(\phi - 2\pi k) - e(\phi - (2\pi k + \Delta\phi)), \quad (27)$$

where  $e(\phi)$  is the unity step function (Heaviside's function).

The second and the third members in (26) describe betatron oscillations while the fourth member reflects the Coulomb scattering.

In equation (26)  $q_{\text{source}}$  denotes the power of sources of particles. If  $f_0(u, u')$  is the distribution function in the injected beam we could write for the power of sources

$$q_{\text{source}} = \sum_{0 \leq k \leq \phi/2\pi} f_0(u, u') \delta(\phi - 2\pi k). \quad (28)$$

In equation (26)  $q_{\text{drain}}$  denotes the power of drains of particles. In order to obtain an expression for  $q_{\text{drain}}$  we will use the following assumption.

First of all we consider here the machine as being able to accelerate only ions in the equilibrium charge state  $\langle q \rangle$  ( $\langle q \rangle$  is an integer closest to the parameter  $\bar{q}$  in (1)). Ions in charge states different from the equilibrium one will finally be lost on the vacuum chamber walls.

When the injected beam, which contains ions in the charge state  $q_0$ , crosses the stripper a whole chain of electron losses and captures occurs. As a result, the beam will embrace a spectrum of charge states behind the stripper. Only the part  $\Phi_1$  of injected ions will go to the charge state  $\langle q \rangle$ .

On the other hand, the circulating beam contains ions in the charge state  $\langle q \rangle$  and with the same energy as the injected ions. When it crosses the stripper the part  $\Phi_2$  of ions go back to the charge state  $\langle q \rangle$ .

Generally speaking,  $\Phi_1 \neq \Phi_2$ , but if the foil has equilibrium thickness  $\Phi_1 = \Phi_2$ . In this case the charge state distribution behind the foil will not depend on the charge state distribution in the incident beam.

Under all these assumptions we can write for the power of particle drains

$$q_{\text{drain}} = (1 - \Phi_1) \sum_{0 \leq k \leq \phi/2\pi} f_0(u, u') \delta(\phi - (2\pi k + \Delta\phi)) + \\ + (1 - \Phi_2) \sum_{1 \leq k \leq \phi/2\pi} (f_0(u, u', 2\pi k + \Delta\phi - 0) - f_0(u, u')) \delta(\phi - (2\pi k + \Delta\phi)). \quad (29)$$

The Boltzmann equation (26) contains the random function

$$\xi(\phi) = \sum_{0 \leq k \leq \phi/2\pi} Q \sqrt{\beta_t} \frac{\Delta\theta_k}{\Delta\phi} h_k(\phi) \quad (30)$$

and is in essence a stochastic PDE. Our first step will be to cope with this stochasticity. In order to do this we will try to find an equation for the particle density  $\langle f \rangle$  averaged over the realization of  $\xi(\phi)$ . The PDE for the averaged particle density was derived in [11]:

$$\frac{\partial \langle f \rangle}{\partial \phi} \phi + u' \frac{\partial \langle f \rangle}{\partial u} - Q^2 u \frac{\partial \langle f \rangle}{\partial u'} - \sum_{0 < k < \phi/2\pi} \frac{1}{2} Q^2 \beta_t \frac{\langle \Delta\theta^2 \rangle}{\Delta\phi} \frac{\partial^2 \langle f \rangle}{\partial u'^2} h_k(\phi) = q_{\text{source}} - \langle q_{\text{drain}} \rangle. \quad (31)$$

This PDE is already free of any stochasticity. From this point further we will omit the ugly brackets, writing  $f$  instead of  $\langle f \rangle$ .

We must solve equation (31) under zero initial and boundary conditions.

## 5. SOLUTION OF THE KINETIC EQUATION

We will reason that particles in the injected beam have normal distribution in the transverse phase plane  $(u, p_u)$ :

$$f_0(u, u') = \frac{\Delta N}{2\pi Q \sigma_0^2} e^{-\frac{Q^2 u^2 + u'^2}{2Q^2 \sigma_0^2}}, \quad (32)$$

where  $\Delta N$  is the number of particles injected into the considered beam slice ( $\Delta N = I_0 \Delta t = I_0 \Delta s / v_s = I_0 \beta \Delta \phi / v_s$ ;  $I_0$  being the injected current,  $v_s$  being the longitudinal velocity of the synchronous particle).

We will work with the accuracy  $O\left(\frac{\beta \langle \Delta \theta^2 \rangle}{2\sigma_0^2}\right)$ . As we will see later, this means that we consider the emittance growth  $\Delta \varepsilon$  in single stripper crossing much smaller than the initial emittance  $\Delta \varepsilon / \varepsilon \ll 1$ .

It can be shown that the solution of equation (31) consists of the main stationary part, which represents a sum of Gaussians, and a small (of order  $\beta \langle \Delta \theta^2 \rangle / 2\sigma_0^2$ )  $\phi$ -dependent part with more complicated structure [11].

For the stationary part of the solution we can obtain

$$\bar{f}(u, u') = \Phi_1 \bar{f}_1(u, u') + \Phi_1 \Phi_2 \bar{f}_2(u, u') + \dots + \Phi_1 \Phi_2^{n-1} \bar{f}_n(u, u')$$

with

$$\bar{f}_k(u, u') = \frac{\Delta N}{2\pi Q \sigma_k^2} e^{-\frac{Q^2 u^2 + u'^2}{2Q^2 \sigma_k^2}},$$

$$\sigma_k^2 = \sigma_0^2 + \frac{1}{2} k \beta_t \langle \Delta \theta^2 \rangle. \quad (33)$$

In equation (33)  $n$  denotes the number of realized injection turns. The meaning of this formula is as follows: a portion of  $\Delta N$  particles with normal distribution  $f_0(u, p_u) \sim N(0, \sigma_0)$  is injected into the accelerator; passing through the stripper the distribution gets wider in  $u'$  direction, i.e. the RMS is replaced by  $\sigma \rightarrow \sigma + a/\sigma$ ; the following betatron oscillations spread this widening also to the coordinate  $u$ ; a part  $(1 - \Phi_2)f$  of these particles goes to charge states different from the equilibrium charge state  $\langle q \rangle$  after the stripping foil and is cut later by the accelerator, so only  $\Phi_2 f$  of the slice survives. This process repeats  $n$  times.

For the variable with  $\phi$  part of the solution we can obtain

$$\begin{aligned} \tilde{f}(u, u', \phi) = & \sum_{k=0}^n \sum_{i=0}^{n-k} \Phi_1 \Phi_2^{n-k} \frac{a}{2Q^2 \sigma_i^4} \bar{f}_i(u, u') \times \\ & \times \left[ \left( \frac{u'^2 - Q^2 u^2}{2} \right) \cos 2Q(\phi - (i+k)2\pi) + Quu' \sin 2Q(\phi - (i+k)2\pi) \right]. \end{aligned} \quad (34)$$

The stationary part of the distribution function is normalized, while the integral of the variable part over  $u$  and  $u'$  from  $-\infty$  to  $+\infty$  is equal to zero.

## 6. PARTICLE STORAGE

Integrating (33) over  $u$  and  $u'$  from  $(-\infty)$  to  $(+\infty)$  and over the azimuth from 0 to  $2\pi$ , we receive the number of particles successfully injected in the accelerator

$$N = N_\infty(1 - \Phi_2^{n-1}), \quad (35)$$

where

$$N_\infty = I_0 T \frac{\Phi_1}{1 - \Phi_2}, \quad (36)$$

$T$  being the period of the synchronous particle.

The above formula is valid for relatively large acceptance of the machine, when the new emittance still remains smaller than the acceptance.

In the case of aperture limitations on the beam we could modify it in the following way. It is known that for a beam with Gaussian distribution in the phase plane the part  $p$  of the beam that lies outside an ellipse with surface  $\varepsilon_p$  is given by

$$p = e^{-\frac{2\varepsilon_p}{\varepsilon_{\text{RMS}}}}. \quad (37)$$

Applying this we reach the following expression for the stored beam:

$$N = \Phi_1 I_0 T (\Phi_{A,1}^X \Phi_{A,1}^Z + \Phi_{A,2}^X \Phi_{A,2}^Z \Phi_2 + \dots + \Phi_{A,n}^X \Phi_{A,n}^Z \Phi_2^{n-1}), \quad (38)$$

where the new «aperture» factors  $\Phi_{A,k}$  are given by

$$\Phi_{A,k} = \begin{cases} 1, & \text{if } 4\sigma_k^2 < A \\ \frac{1 - e^{-\frac{2A}{\varepsilon_{\text{RMS}}}}}{1 - e^{-2}} = 1.16 \left( 1 - e^{-\frac{A}{2\sigma_k^2}} \right), & \text{if } 4\sigma_k^2 \geq A, \end{cases} \quad (39)$$

$A$  being the acceptance.

For the case of nonzero dispersion in the stripper, (33) is generalized to

$$\bar{f}(u, u') = \Phi_1 \bar{f}_1(u, u') + \Phi_1 \Phi_2 \bar{f}_2(u, u') + \dots + \Phi_1 \Phi_2^{n-1} \bar{f}_n(u, u')$$

with

$$\bar{f}_k(u, u') = \frac{\Delta N}{2\pi Q \sigma_k^2} e^{-\frac{Q^2(u - D\delta_k)^2 + (u' - D'\delta_k)^2}{2Q^2\sigma_k^2}},$$

$$\sigma_k^2 = \sigma_0^2 + \frac{1}{2} k \beta_t \langle \Delta\theta^2 \rangle, \quad \delta_k = \delta_0 + k \Delta\delta. \quad (40)$$

For the number of stored particles we can use again formula (38), but now in (39) stands the reduced acceptance  $A_k$  instead of  $A$ . The reduced acceptance  $A_k$  is given by

$$\sqrt{\beta_t A_k} = \sqrt{\beta_t A} - \sqrt{D_t^2 + \beta_t^2 D_t'^2} k \Delta\delta. \quad (41)$$

## 7. EMITTANCE GROWTH

The diffusion in the stripper due to the Coulomb scattering leads to transverse emittance growth.

There are different definitions of the beam emittance [12]. Here we will use the RMS emittance which for a beam with two-dimensional Gaussian distribution is given by  $\varepsilon_{\text{RMS}} = 4\sigma_x\sigma_{x'} = 4\sigma_u^2$ . In the charge exchange injection we have on the orbit simultaneously particles which have passed through the foil  $n$  times,  $(n-1)$  times and so on up to one time.

If we consider the maximum (not the averaged) emittance we must take the standard deviation of the particles having crossed the foil  $n$  times.

It follows from (33) that

$$\Delta\varepsilon_{\text{RMS}} = 2n\beta_t\langle\Delta\theta^2\rangle. \quad (42)$$

## 8. CHARGE EXCHANGE INJECTION OF DEUTERONS INTO THE NUCLOTRON

A natural development of the JINR LHE spin physics programme will be the acceleration of polarized beams of deuterons in the Nuclotron. The scheme of acceleration covers a cryogenic source of polarized deuterons «Polaris», a 5 MeV/u linac, charge exchange  $D^- \uparrow \rightarrow D^+ \uparrow$  injection into the Nuclotron and acceleration in it up to 6 GeV/u.

Using the above derived formulas, we have studied the possibility to apply the method of stripping injection to the storage of polarized deuterons in the Nuclotron.

Figs. 1 and 2 show the emittance growth due to the elastic Coulomb scattering in the stripping foil.

Fig. 3 shows the process of deuteron storage.

An estimated 40-fold intensity gain could be achieved for a 100-turn stripping injection.

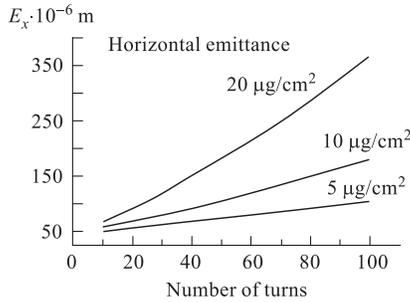


Fig. 1. Horizontal emittance growth for charge exchange injection of deuterons into the Nuclotron

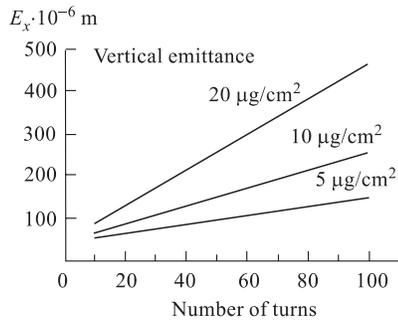


Fig. 2. Vertical emittance growth for charge exchange injection of deuterons into the Nuclotron

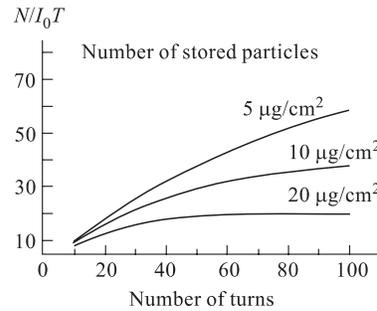


Fig. 3. Storage of deuterons in the Nuclotron by means of stripping injection

## 9. CHARGE EXCHANGE INJECTION OF HEAVY IONS INTO THE NUCLOTRON BOOSTER

As a second example we will describe in brief the stripping injection of heavy ions into the Nuclotron booster. This will be a superconducting synchrotron with circumference of 84 m capable to accelerate ions with  $Z/A = 0.5$  up to 250 MeV/u [2]. The now in operation linac LU-20, which accelerates protons up to 20 MeV and ions with  $Z/A = 0.5$  up to 5 MeV/u will be used as an injector into the booster. The booster will increase the beam intensities in the Nuclotron more than ten times, will raise the final energy of ions applying ion stripping and will improve beams quality by electron cooling.

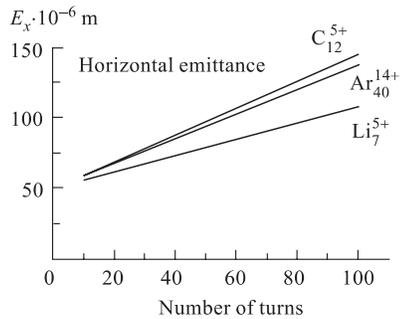


Fig. 4. Horizontal emittance growth due to the elastic Coulomb scattering for heavy ion stripping injection into the Nuclotron booster

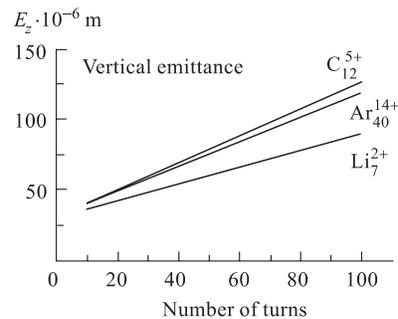


Fig. 5. Vertical emittance growth due to the elastic Coulomb scattering for heavy ion stripping injection into the Nuclotron booster

Figs. 4 and 5 show the emittance growth due to the elastic Coulomb scattering in the foil. Fig. 6 shows the growth of the relative momentum spread due to energy losses in the foil. Fig. 7 shows the process of ion storage in the booster.

It is seen from Fig.7 that only light ions could be successfully stored in the booster.

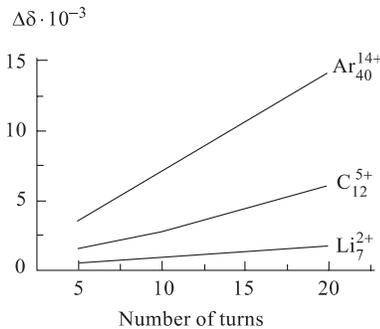


Fig. 6. Relative momentum spread growth due to energy losses in the foil for heavy ion stripping injection into the Nuclotron booster

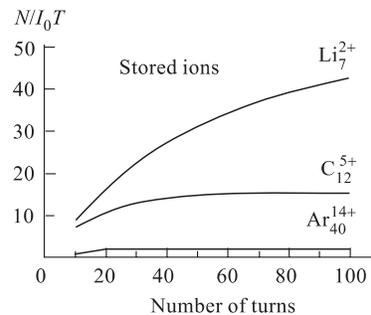


Fig. 7. Ion storage during charge exchange injection of heavy ions into the Nuclotron booster

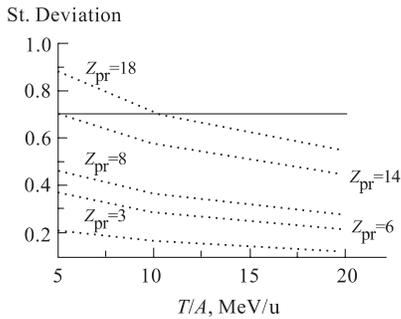


Fig. 8. Standard deviation vs. ion energy; approximately all the ions below the horizontal straight line could be successfully stored

The following general considerations concerning the ion storage can be expressed. In order to have successive ion storage,  $\Phi_1$  and  $\Phi_2$  should be large enough. This means that the spectrum of charge states should be narrow, i.e.  $\sigma$  should be small. It follows from (2) and (4) that small  $\sigma$  requires large  $\bar{q}$ , i.e.  $\bar{q}$  close to  $Z_{pr}$ . For a given ion ( $Z_{pr}$ ) this is achieved for large enough energy. The figure of merit is the parameter  $X$  (3). On the contrary, for a given ion energy ( $T/A$ ) only light enough ions could be stored.

This is illustrated in Fig. 8, in which standard deviation vs. energy is plotted for several ion atomic numbers. Approximately all the ions below the marked horizontal straight line can be stored in the accelerator, i.e. heavy ions with  $Z$  up to 14.

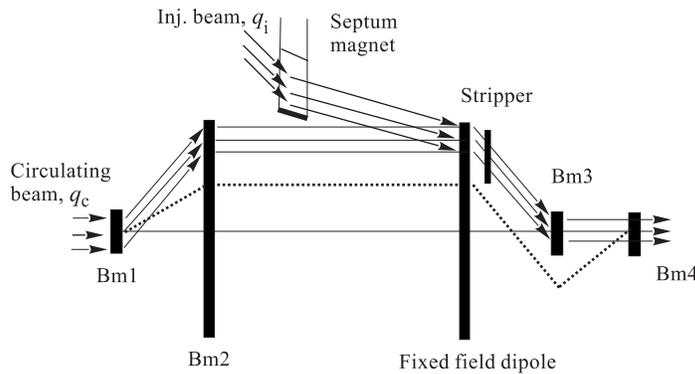


Fig. 9. Stripping injection of heavy ions into the Nuclotron booster

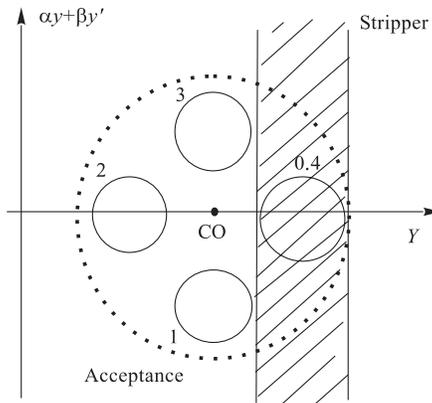


Fig. 10. The use of betatron oscillations to reduce the number of foil crossings

Heavy ions which change their charge from 1.3 to 1.7 times in stripping foil crossings could be injected into the booster through a four-magnet closed orbit bump (Fig. 9).

As the ratio of the booster acceptance to the beam emittance is rather small,  $A_x/\varepsilon_x = 8$  and  $A_z/\varepsilon_z = 7$ , a simplest painting scheme making use of the fact that the fractional part of the betatron tune is equal to 0.75 is proposed (Fig. 10). The number of foil crossings will be reduced due to betatron oscillations and correspondingly the intensity will increase by a factor of two (taking into account the emittance growth).

A promising way to increase the intensity multiplication factor is to combine charge exchange injection with electron cooling. This kind of injection

tion was successfully applied to TSR [13] and CELSIUS [14]. The principle of the method is explained in Fig. 11. The phase space already filled with particles is shrunk by electron cooling thus making available space for injection of a new portion of particles. A new orbit bump with appropriate amplitude reduction follows so that the already stored beam is not displaced into the stripper foil.

With this injection method the intensity multiplication factor could be increased at least by one order of magnitude.

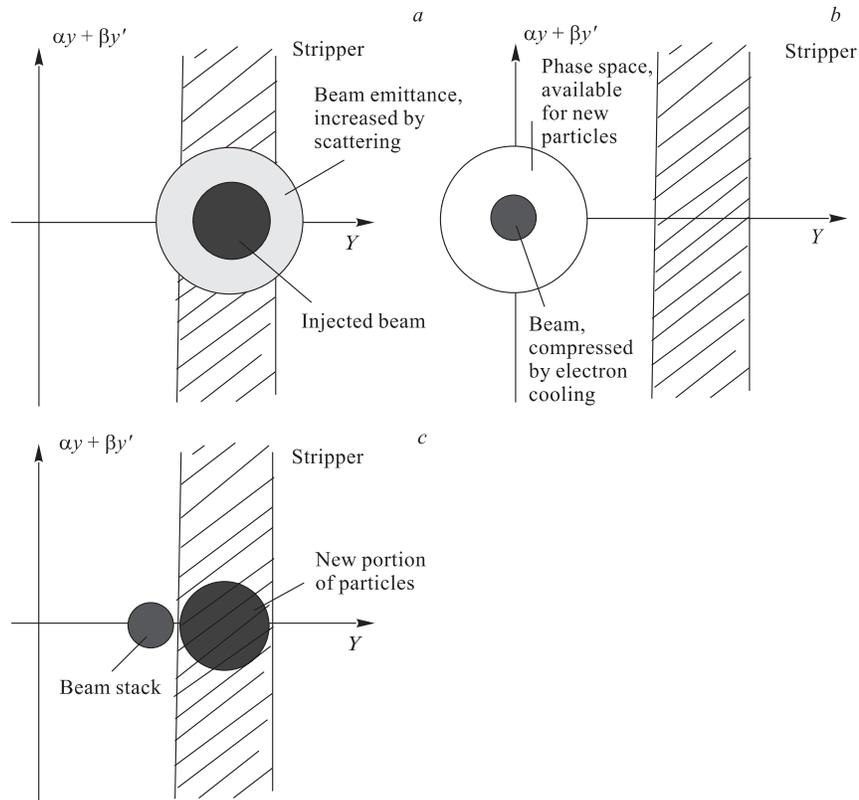


Fig. 11. Combination of stripping injection with electron cooling

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Received on January 17, 2001.