ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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A kinetic theory of vacuum particle creation under the action of an inertial mechanism is constructed within a nonperturbative dynamical approach. At the semiphenomenological level, the inertial mechanism corresponds to quantum field theory with a time-dependent mass. At the microscopic level, such a dependence may be caused by different reasons: the nonstationary Higgs mechanism, the influence of a mean field or condensate, the presence of the conformal multiplier in the scalar-tensor gravitation theory, etc. In what follows, a kinetic theory in the collisionless approximation is developed for scalar, spinor and massive vector fields in the framework of the oscillator representation, which is an effective tool for transition to the quasiparticle description and for derivation of non-Markovian kinetic equations. Properties of these equations and relevant observables (particle number and energy densities, pressure) are studied. The developed theory is applied here to describe the vacuum matter creation in conformal cosmological models and explain the observed photons number density in the cosmic microwave background radiation. As other example, the self-consistent evolution of scalar fields with nonmonotonic self-interaction potentials (the W potential and Witten–Di Vecchia–Veneziano model) is considered. In particular, conditions for appearance of tachyonic modes and a problem of the relevant definition of a vacuum state are discussed.

В рамках непертурбативного полевого подхода построена кинетическая теория вакуумного рождения частиц под действием инерционного механизма. На полуфеноменологическом уровне инерционный механизм соответствует квантовой теории поля с массой, зависящей от времени. На микроскопическом уровне такая зависимость может быть обусловлена различными факторами: нестационарным механизмом Хиггса, влиянием среднего поля или конденсата, наличием конформного множителя в уравнениях скалярно-тензорной теории гравитации и т. п. В бесстолкновительном приближении в работе получены кинетические уравнения для скалярных, спинорных и массивных векторных полей на базе осцилляторного представления, которое является эффективным способом для перехода к квазичастичному описанию и для вывода кинетических уравнений немарковского типа. Изучены свойства этих уравнений, а также особенности поведения полученных на их основе макроскопических характеристик (плотности числа частиц и энергии, давления). Развитая теория применяется для описания вакуумного рождения материи в конформных космологических моделях и для обсуждения проблемы наблюдаемой плотности числа фотонов микроволнового реликтового излучения. В качестве другого примера рассмотрена самосогласованная эволюция скалярных квантово-полевых систем с немонотонными потенциалами самодействия (W-потенциал и модель Виттена–ди Веккиа–Венециано). В частности, рассмотрены условия возникновения тахионных мод и проблема адекватного выбора вакуумного состояния.

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INTRODUCTION

The present work is devoted to the construction of a kinetic theory of vacuum creation of particles with time-dependent masses. For brevity, this mechanism will be referred to as inertial one. Microscopic foundations of a mass change may be different. The Higgs mechanism leads to the most popular models of such a class, when the corresponding mean fields are time-dependent. General quantum field models with nonpolynomial interactions may also be considered, where the separation of nonstationary mean fields results in a time-dependent mass [1]. A well-known example of this kind is the Witten–Di Vecchia–Veneziano model [2,3] in the framework of which the mean-field concept was analyzed in [4]. The Nambu–Jona-Lasinio [5] and σ [6] models are other examples where the meson masses are defined by evolution of a quark condensate to be described at the hydrodynamic [7] or kinetic [8] level. The particle mass may depend on many-particle interactions in hot and dense nonstationary matter [9–11]. A general basis for a rather slowly-varying time dependence of the effective mass can be obtained within the Green function method [12, 13]. The field dependence of the mass is a general factor determining the time evolution in all these cases (F-class models). The conformal invariance of the scalar-tensor gravitational theory provides a time dependence of the particle mass by means of the conformal multiplier [14–17]. The mass can be changed also due to the parameterization stipulated by additional space dimensions [18]. Such theories should be referred to as the other class (C class). In the F-class theories, the vacuum particle creation admits a well-known interpretation based on the simplified vacuum tunnelling model in an external field [19–21]. A similar interpretation of the C-class models is difficult. At the phenomenological level, however, both classes have a uniform mathematical description as will be shown in Secs. 1, 2, and 3, respectively, for the scalar, fermion and massive vector boson in quantum field theories (QFTs).

The first consideration of the vacuum creation of particles with the variable mass was proposed apparently in [22] as a possible variant for describing a quantum system response to the time variation of system parameters [23]. Using the Bogoliubov transformation method, residual momentum distributions for fermions, and pair correlators were found for the cases of step-like and smooth variations of the fermion mass (Subsec. 2.4).

In the present work, the kinetic theory will be based on the oscillator representation (OR) [24, 25], which is the most economical method for a nonpertrubative description (as compared to the Bogoliubov method of canonical transformation [26] or other accurate approaches to the problem [15, 27, 28]) of the vacuum particle creation under the action of time-dependent strong fields. This approach leads directly to the quasiparticle representation (QPR) with diagonal operator forms in the momentum space for the set of dynamical variables. It allows one to get easily the Heisenberg-type equations of motion for creation and annihilation operators. An important feature of the time-dependent Fock representation is the necessary consistency of commutation (anticommutation) relations with the equations of motion. Otherwise, this circumstance can bring to the noncanonical quantization rules (an example will be considered in Subsec. 3.1).

In terms of the OR it is possible to immediately derive the corresponding kinetic equations (KE) by the well-known method [29]. Some particular results are published in [1, 30]. The kinetic theory for scalar, spinor, and massive vector fields is constructed in Secs. 1, 2, and 3, respectively. The main attention is paid to the particle creation in conformal cosmological models [31–33] (Sec. 4). It is shown that the choice of the equation of state (EoS) of the Universe allows one to obtain, in principle, the observed number density of matter participants and photons and, possibly, dark matter. The basic problem here is the description of vacuum particle creation which should be consistent with EoS but it is beyond the present article.

Finally, in Sec. 5 the other class of scalar QFT systems is considered with nonmonotonic self-interaction potentials to apply the decomposition of the field amplitude into the quasi-classical space-homogeneous time-dependent background field and the fluctuation part. In this case, the particle mass is defined by intensity of a quasi-classical field. As an example, self-interaction potentials of the simplest polynomial type and those for a nontrivial case (the pseudoscalar sector of the Witten–Di Vecchia–Veneziano model) are analyzed. It is shown, that the relevant definition of vacuum states allows one to avoid the tachyonic mode beginning. The main purpose of this review is to summarize all known relevant results on the inertial mechanism of the vacuum particle production and to call attention to unsolved problems which are shortly listed in Sec. 5.3.3.

We use the metric $g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$ and natural units $\hbar = c = 1$.

1. SCALAR FIELD

1.1. Oscillator and Quasiparticle Representations. Let us start our consideration with the simplest case of the real scalar field with the time-dependent mass $m(t)$, whose equation of motion is

$$[\partial_\mu \partial^\mu + m^2(t)]\varphi(x) = 0. \tag{1}$$

The corresponding Lagrange function is given as

$$L = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2(t)\varphi^2. \tag{2}$$

In the considered case, the system is space-homogeneous and nonstationary. Therefore, the transition to the Fock space can be realized on the basis functions $\varphi(\mathbf{x}) \sim \exp(\pm i\mathbf{px})$, and creation and annihilation operators become time-dependent. The assumption about the space homogeneity allows one to look for solution of Eq. (1) in discrete momentum space in the following form:

$$\varphi(\mathbf{x},t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \mathrm{e}^{i\mathbf{px}} \varphi(\mathbf{p},t), \tag{3}$$

where $V = L^3$ and $p_i = (2\pi/L)n_i$, at that the integers n_i $(i = 1,2,3)$ run from $-\infty$ to $+\infty$. The thermodynamic limit can be covered in the resulting equations. Then the oscillator-type equation of motion follows from Eq. (1) and decomposition (3) as

$$\ddot{\varphi}^{(\pm)}(\mathbf{p},t) + \omega^2(\mathbf{p},t)\varphi^{(\pm)}(\mathbf{p},t) = 0 \tag{4}$$

with

$$\omega^2(\mathbf{p},t) = m^2(t) + \mathbf{p}^2. \tag{5}$$

The symbols $(\pm)$ correspond to the positive and negative frequency solutions of Eq. (4) defined by its free asymptotics in the infinite past (future) [26],

$$\varphi^{(\pm)}(\mathbf{p}, t \to \mp\infty) \sim \mathrm{e}^{\pm i\omega_{\mp} t}, \tag{6}$$

where $\omega_{\mp} = \sqrt{m_{\mp}^2 + \mathbf{p}^2}$ are defined by asymptotics of the mass

$$m_{\mp} = \lim_{t \to \mp\infty} m(t). \tag{7}$$

The asymptotics (6) corresponds to the in(out)-states and is necessary for definition of in(out)-vacuum. This requirement, however, can be broken in cosmology [34]. We suppose here that such asymptotics exists and the relevant vacuum states will be denoted by $|0\rangle$ without indices «in» or «out», that is evident from the context. In the considered class of problems, the classification of states in the frequency sign turns out to be impossible for an arbitrary time moment. According to a general analysis [35], this leads to instability of a vacuum state during the action period of the external fields and to the vacuum particle creation. In this case, it is possible to consider quasiparticle excitations during the system evolution ($\dot{m}(t) \neq 0$) and describe their creation and annihilation in the vacuum state $|0\rangle$. When the external field action is completed, residual particles of some finite density remain in the out-state. However, it is necessary to emphasize that these particles are defined in respect of the in-vacuum state [26].

In the general case, S matrix does not exist in the considered formalism. Its role in description of the system with unstable vacuum is performed by other mathematical objects: the operator of the canonical Bogoliubov transformation [26], distribution functions in the kinetic approach [24, 29, 30, 36–38], or some set of correlation functions [39, 40], etc.

The conception of «quasiparticle» plays the central role in the QFT with strong time-dependent quasi-classical external fields [26, 35]. Under the considered conditions, this approach is the shortest realization of the quasiparticle concept within the standard Fock representation of the QFT, where the external field can be taken into account nonperturbatively. Thus, the QPR corresponds to a possibility of writing down the set of commutative operators of physical (observable) quantities (the complete QPR) in the diagonal form in an arbitrary time. It is naturally related to the question whether operators have the quadratic form in the Fock representation. Hence, the interaction between the field constituents and self-interaction is not taken into account. It corresponds to a nondissipative approximation in the kinetic theory [41]. An alternative definition of the quasiparticle was given in [25] for constrained systems.

The transition to the QPR can be realized in different ways. The traditional method is based on the time-dependent canonical Bogoliubov transformation [26]. The alternative approach uses the oscillator («holomorphic») representation, which leads directly to the QRP [24]. In the considered case the transition to the OR is made by substituting $m_\pm \to m(t)$ into the dispersion law for the free field and postulating the following decompositions:

$$\begin{aligned} \varphi(x) &= \frac{1}{\sqrt{2V}} \sum_{\mathbf{p}} \frac{1}{\sqrt{\omega(\mathbf{p},t)}} \Big\{ a(\mathbf{p},t)\, \mathrm{e}^{i\mathbf{px}} + a^\dagger(\mathbf{p},t)\, \mathrm{e}^{-i\mathbf{px}} \Big\}, \\ \pi(x) &= -\frac{i}{\sqrt{2V}} \sum_{\mathbf{p}} \sqrt{\omega(\mathbf{p},t)} \left\{ a(\mathbf{p},t)\, \mathrm{e}^{i\mathbf{px}} - a^\dagger(\mathbf{p},t)\, \mathrm{e}^{-i\mathbf{px}} \right\}, \end{aligned} \tag{8}$$

where $\pi(x)$ is the generalized momentum; $a^\dagger(\mathbf{p},t)$ and $a(\mathbf{p},t)$ are the creation and annihilation operators of particles with the momentum $\mathbf{p}$ at the time moment t. The in-vacuum state is defined as

$$a(\mathbf{p}, t \to -\infty)|0\rangle = 0, \quad \langle 0|0\rangle = 1. \tag{9}$$

The canonical commutation relation

$$[\varphi(x), \pi(x')]_{t=t'} = i\delta(\mathbf{x} - \mathbf{x}') \tag{10}$$

together with the decomposition (8) provides the standard commutation relation for time-dependent creation and annihilation operators

$$\left[a(\mathbf{p},t), a^\dagger(\mathbf{p}',t)\right] = \delta_{\mathbf{pp}'}. \tag{11}$$

The substitution of the decompositions (8) into the Hamiltonian

$$H(t) = \frac{1}{2} \int d^3x \left\{ \pi^2(x) + [\nabla\varphi(x)]^2 + m^2(t)\varphi^2(x) \right\} \tag{12}$$

leads immediately to a diagonal form which corresponds to the QPR

$$H(t) = \sum_{\mathbf{p}} \omega(\mathbf{p},t) \left\{ a^{\dagger}(\mathbf{p},t) a(\mathbf{p},t) + \frac{1}{2} \right\}. \tag{13}$$

In the considered case the vacuum energy of zero oscillations («Zitterbewegung») depends on time.

Equations of motion for the operators $a, a^{\dagger}$ can be obtained now from the minimal action principle [24] or from the Hamiltonian equations

$$\dot{\varphi} = \frac{\delta H}{\delta \pi} = \pi, \quad \dot{\pi} = -\frac{\delta H}{\delta \varphi} = \triangle\varphi - m^2(t)\varphi. \tag{14}$$

Here and below we use the notation $(\dot{\ }) = d/dt(\ )$. Combining Eqs. (8) and (14), we get

$$\begin{aligned} \dot{a}(\mathbf{p},t) &= \frac{1}{2}\Delta(\mathbf{p},t)\, a^{\dagger}(-\mathbf{p},t) - i\omega(\mathbf{p},t)\, a(\mathbf{p},t), \\ \dot{a}^{\dagger}(\mathbf{p},t) &= \frac{1}{2}\Delta(\mathbf{p},t)\, a(-\mathbf{p},t) + i\omega(\mathbf{p},t)\, a^{\dagger}(\mathbf{p},t), \end{aligned} \tag{15}$$

where

$$\Delta(\mathbf{p},t) = \frac{\dot{\omega}(\mathbf{p},t)}{\omega(\mathbf{p},t)} = \frac{m(t)\,\dot{m}(t)}{\omega^2(\mathbf{p},t)} \tag{16}$$

is the factor defining the mixing of states with positive and negative energies. This equation obviously is consistent with the commutation relations (11).

Equations of motion (15) can be rewritten as the Heisenberg-type equation, e.g.,

$$\dot{a}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t) a^{\dagger}(-\mathbf{p},t) + i\left[H(t), a(\mathbf{p},t)\right]. \tag{17}$$

In the instantaneous QPR these equations serve as a basis for a nonperturbative derivation of the KE describing scalar particle creation and annihilation processes within the inertial mechanism.

1.2. Kinetic Equation. The key object of the kinetic theory is the quasiparticle distribution function which for the space-homogeneous case is

$$f(\mathbf{p},t) = \langle 0 | a^{\dagger}(\mathbf{p},t) a(\mathbf{p},t) | 0 \rangle, \tag{18}$$

where $|0\rangle = |0\rangle_{\mathrm{in}}$ is the initial ($t \to -\infty$) vacuum state. Differentiating the distribution function (18) with respect to time and using (15) we get

$$\dot{f}(\mathbf{p},t) = \Delta(\mathbf{p},t)\,\mathrm{Re}\,\{\mathrm{f}^{(+)}(\mathbf{p},\mathrm{t})\}. \tag{19}$$

Here the auxiliary correlation function is introduced

$$f^{(+)}(\mathbf{p},t) = \langle 0|a^\dagger(\mathbf{p},t)a^\dagger(-\mathbf{p},t)|0\rangle. \tag{20}$$

This function provides a coherent connection between the states with positive and negative energies (the so-called entangled states [28]).

The equation of motion for $f^{(+)}(\mathbf{p},t)$ can be obtained by analogy with equation (19). We present it here in the integral form as

$$f^{(+)}(\mathbf{p},t) = \frac{1}{2}\int\limits_{t_0}^{t} dt'\Delta(\mathbf{p},t')\left[1+2f(\mathbf{p},t')\right]\mathrm{e}^{2i\theta(\mathbf{p};t,t')}, \tag{21}$$

where the initial condition $f^{(+)}(\mathbf{p},t_0)=0$ was used. This condition corresponds to the initial condition for the distribution function $f(\mathbf{p},t_0)=0$ and is a direct consequence of the definition (9). Eventually, the dynamical phase in Eq. (21) is equal to

$$\theta(\mathbf{p};t,t') = \int\limits_{t'}^{t} d\tau\omega(\mathbf{p},\tau). \tag{22}$$

The substitution of Eq. (21) into Eq. (19) leads us to the resulting KE written in the thermodynamical limit $V\to\infty$ at the fixed particle density

$$\dot{f}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\int\limits_{t_0}^{t} dt'\Delta(\mathbf{p}',t)\left[1+2f(\mathbf{p},t')\right]\cos\left[2\theta(\mathbf{p};t,t')\right]. \tag{23}$$

The source term in the r.h.s. of Eq. (23) describes a variation of the particle number with the given momentum due to vacuum creation and annihilation processes for the inertial mechanism, $\Delta(\mathbf{p},t)$ is defined by Eq. (16). The non-Markovian KE (23) has the structure as that for the Schwinger mechanism of pair creation in an electric field [29]. This equation was investigated in detail for description of the pre-equilibrium evolution of quark–gluon plasma created in collisions of ultrarelativistic heavy ions [36, 37]. The case of the scalar QED was considered in [24] for an electric field of arbitrary polarization.

As follows from Eq. (16), in the framework of the inertial mechanism the particle production rate is defined by the rate of the mass change

$$\xi(t) = \frac{1}{m(t)}\frac{dm(t)}{d(m_0t)}, \tag{24}$$

where m_0 is the characteristic mass to fix the time scale (e.g., $m_0=m_-$).

Let us note that the KE (23) is valid under two basic assumptions: a) there are no particles (or antiparticles) in the in-state; b) a collisionless approximation is applicable (i.e., the corresponding dissipative processes are not taken into consideration).

In the low-density approximation $f(\mathbf{p},t) \ll 1$, the KE (23) results in the following solution [36]:

$$f(\mathbf{p},t) = \frac{1}{4}\left|\int\limits_{t_0}^{t} dt'\Delta(\mathbf{p},t')\exp\left[2i\theta(\mathbf{p};t,t')\right]\right|^2 \geqslant 0. \tag{25}$$

The KE (23) can be transformed to linear equations of the non-Hamiltonian dynamical system with zero initial conditions

$$\dot{f} = \frac{1}{2}\Delta u, \qquad \dot{u} = \Delta(1+2f) - 2\omega v, \qquad \dot{v} = 2\omega u, \tag{26}$$

which is convenient for numerical analysis. This equation system has the first integral

$$(1+2f)^2 - u^2 - v^2 = 1, \tag{27}$$

according to which the phase trajectories are located on the two-cavity hyperboloid with top coordinates $f = u = v = 0$ (physical branch) and $f = -1$, $u = v = 0$ (nonphysical one). If the function f is excluded from Eqs. (26), we obtain the nonlinear two-dimensional dynamical system with

$$\begin{aligned} \dot{u} &= \Delta\sqrt{1+u^2+v^2} - 2\omega v, \\ \dot{v} &= 2\omega u. \end{aligned} \tag{28}$$

The functions $f(\mathbf{p},t)$ and $u(\mathbf{p},t)$ have a certain physical meaning (the last function describes vacuum polarization effects, see Subsec. 1.3) below and are invariants with respect to the time inversion $t \to -t$ while the auxiliary function $v(\mathbf{p},t)$ and factor (16) change their signs. Thus, the KE (23) is invariant at the time inversion.

The presented formalism of vacuum particle creation is specific for kinetic theory and allows natural generalization to the case of interacting fields that leads to introduction of corresponding collision integral of a non-Markovian type [42]. This approach is close to the modern method expounded in the book [26] where the time-dependent Bogoliubov transformation is used. The same method was used in pioneer works [43–45] (see also [34]). Some modification of the formalism [26] (the $r, \bar{\theta}$ representation) was developed in [25] and then used widely (e.g., in [32] and references cited there). The correspondence between the $r, \bar{\theta}$ representation and our approach (as well as that used in the book [26]) may be easily established.

1.3. Observables and Regularization. The KE (23) describes the vacuum quasiparticle excitations rising at an external force ($\dot{m}(t) \neq 0$, in the considered case). When this action is switched off, there is still some remaining density of real (residual) particles and antiparticles. In the absence of any interaction between the system constituents, the real particles are «on-shell» ones and have the free-particle dispersion law $\omega_\pm$ with the mass $m_\pm$ (7), while quasiparticles are «off-shell» with the dispersion law (5). Within the Green function method [13], one can say that the time-dependent dispersion law like (5) corresponds to the t-parametric mass shell surface of slowly time-dependent $m(t)$, i.e., $m(T \pm \tau/2) \approx m(T)$, where T and τ correspond to slow and fast time scales. This case is not of interest for the considered problem. Thus, the dispersion law (5) does not belong to the mass shell surface. In the general case, the on-shell condition

$$\left| \frac{m(t) - m_0}{m_0} \right| \ll 1 \tag{29}$$

($m_0 = m_\pm$) is not connected directly with the condition of efficiency of the vacuum particle creation, $\xi(t) \lesssim 1$, where $\xi(t)$ is defined by Eq. (24). On the contrary, the presence of high frequencies in the function $m(t)$ is necessary for vacuum creation and does not contradict the on-shell condition (29). In principle, the KEs of such a type are designed for the description of evolution of both real particles and quasiparticles. In particular, the distribution function of residual particles is $f_{\rm out}(\mathbf{p}) = \lim\limits_{t \to \infty} f(\mathbf{p}, t)$. This simple formula for $f_{\rm out}(\mathbf{p})$ follows from Eq. (25) in the low-density approximation. However, the presence of the fast oscillated multiplier in the source term in the r.h.s. of the KE (23) leads to a large amount of numerical calculations which make impossible the study of the system evolution for rather large times after the switching on external forces. The corresponding large scaling methods of calculations based on the KE (23) have not been worked out at present. Some properties of a residual particle–antiparticle plasma due to a limited pulse of the external field action can be estimated by the imaginary time method [46].

The distribution function is the key quantity of the system. The density of observable variables is some integral in the momentum space containing the distribution function and auxiliary functions $u(\mathbf{p}, t)$, which describe the effects of vacuum polarization. The simplest variable of such a type is the density of quasiparticles. In the thermodynamic limit $L \to \infty$ we have

$$n_{\rm tot}(t) = \int [dp] f(\mathbf{p}, t), \tag{30}$$

where $[dp] = (2\pi)^{-3} d^3p$. To proceed to the thermodynamical limit, the rule

$$\frac{1}{L^3} \sum_{\mathbf{p}} \to \int [dp] \tag{31}$$

is used here and below.

Other important characteristics of the system are the energy density ε and pressure P, which can be obtained as the average value of the energy-momentum tensor corresponding to the Lagrangian density (2),

$$T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} L. \tag{32}$$

As a result, we have [26]

$$\varepsilon = \langle 0|T_{00}|0\rangle = \int [dp]\, \omega f, \tag{33}$$

$$3P = \varepsilon - \int [dp] \left[\frac{m^2}{\omega} \left(f + \frac{1}{2} u \right) + \omega u \right]. \tag{34}$$

The last two terms in integrand (34) represent the contribution of vacuum polarization.

Finally, the entropy density can be introduced

$$S(t) = -\int [dp] \big[f \ln f - (1+f) \ln (1+f) \big]. \tag{35}$$

It is not conserved ($\dot{S}(t) \neq 0$) even in the considered nondissipative approximation because the system is open (the mass change is defined by external causes).

A direct proof of the convergence of integrals (30), (33)–(35) is complicated because of the absence of an explicit form for functions $f(\mathbf{p}, \mathbf{t})$ and $u(\mathbf{p}, \mathbf{t})$. Therefore, one usually uses the method of asymptotic expansions in power series of the inverse momentum p^{-N} (N-wave regularization technique) [47] (another approach rests on the WKB approximation [48]). Our present consideration is based on the explicit asymptotic solutions of the system (26) for $|\mathbf{p}| \gg m$. The integral (30) is assumed to be convergent at any time moment. Then the function $f(\mathbf{p}, t)$ should decrease at $\mathbf{p} \to \infty$ and hence $f(\mathbf{p}, t) \ll 1$ in this region. This inequality corresponds to the low-density approximation (25), where the KE solution can be written in the explicit form as

$$f^{\infty}(\mathbf{p}, t) = \frac{1}{4p^4} \left| \int\limits_{t_0}^{t} dt' m(t') \dot{m}(t') \exp\left[2ip\,(t - t')\right] \right|^2, \tag{36}$$

$p = |\mathbf{p}| \to \infty$ because $\Delta^{\infty}(\mathbf{p}, t) = m(t)\dot{m}(t)/p^2$ in accordance with Eq. (16). These solutions are consistent with the integral of motion (27). Thus, indeed asymptotic solutions are some quickly oscillating functions (this fact was first noted in [30] for the case of massive vector bosons, see Subsec. 3.3). Such behavior matches with the quasiparticle interpretation of vacuum excitations by the inertial mechanism. The real (observed) particles are the result of the evolution by the moment when $\dot{m} = 0$ and the out-vacuum state is realized. The asymptotics (36) may be influenced by other (noninertial) mechanisms of vacuum particle creation (e.g., in the case of harmonic «laser» electric field [49–51]).

The asymptotics of the integral (36) can be obtained by the stationary phase method [52]

$$\int_{t_0}^{t} dt'\; m(t')\dot{m}(t')\, \mathrm{e}^{2ip(t-t')} = \frac{m(t)\dot{m}(t)}{ip} + O(p^{-2}), \tag{37}$$

if $\dot{m}(t_0) = 0$. Using Eqs. (36) and (26) we get the leading contributions

$$\begin{aligned} f^{(6)}(\mathbf{p}, t) &= \left[\frac{m(t)\dot{m}(t)}{2p^3}\right]^2, \\ u^{(4)}(\mathbf{p}, t) &= \frac{1}{p^4}\left[\dot{m}^2(t) + m(t)\ddot{m}(t)\right], \end{aligned} \tag{38}$$

where the upper indices show the inverse momentum degree for the corresponding leading terms (we are not interested in the asymptote of the function $v(\mathbf{p}, t)$, which plays some auxiliary role only). Relations (38) are identical to the results of application of the N-wave regularization method to Eqs. (26) [47].

Now one can conclude that the integral (33) is convergent but the last integral term in Eq. (34) needs a regularization. The regularizing procedure of the Pauli–Villars type is based on the subtraction of appropriate counterterms in integrals (30), (33)–(35). These counterterms can be obtained by the substitution $p^2 \to p^2 + M^2$ into the denominator of asymptotics (38),

$$f_R = f - f_M, \qquad u_R = u - u_M. \tag{39}$$

If the regularizing mass $M \gg m(t)$ can be chosen rather large, $M \gg \Lambda$ (Λ is «the computer cut-off parameter»), the influence of counterterms on the results of numerical calculations is negligible.

The numerical investigation of the KE (23) and observable densities (30), (33)–(35) will be presented in Subsec. 2.4.

2. FERMION FIELD

2.1. Quasiparticle Representation. The material of this subsection is based on papers [53–55].

Equations of motion for fermion fields with the variable mass are

$$\begin{aligned} [i\gamma^\mu \partial_\mu - m(t)]\psi(x) &= 0, \\ \bar{\psi}(x)[i\gamma^\mu \overleftarrow{\partial}_\mu + m(t)] &= 0, \end{aligned} \tag{40}$$

where $\bar{\psi} = \psi^\dagger\gamma^0$. The corresponding Hamiltonian is $(k = 1, 2, 3)$

$$H(t) = i\int d^3x\ \psi^\dagger\dot{\psi} = \int d^3x\ \bar{\psi}\{-i\gamma^k\partial_k + m(t)\}\psi. \tag{41}$$

By analogy to the scalar case, we use the following decompositions of field functions in the discrete momentum space:

$$\begin{aligned}\psi(x) &= \frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\sum_{\alpha=1,2}\big\{\mathrm{e}^{i\mathbf{px}}\,a_\alpha(\mathbf{p},t)\,\mathsf{u}^\alpha(\mathbf{p},t) + \mathrm{e}^{-i\mathbf{px}}\,b^\dagger_\alpha(\mathbf{p},t)\,\mathsf{v}^\alpha(\mathbf{p},t)\big\},\\ \bar{\psi}(x) &= \frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\sum_{\alpha=1,2}\big\{\mathrm{e}^{-i\mathbf{px}}\,a^\dagger_\alpha(\mathbf{p},t)\,\bar{\mathsf{u}}^\alpha(\mathbf{p},t) + \mathrm{e}^{i\mathbf{px}}b_\alpha(\mathbf{p},t)\,\bar{\mathsf{v}}^\alpha(\mathbf{p},t)\big\}.\end{aligned} \tag{42}$$

The OR is intended to derive equations of motion for creation and annihilation operators. It is based on the primary equations (40) and free u, v spinors with the substitution $m \to m(t)$. Thus, the following equations for the spinors are postulated in the OR:

$$\begin{aligned}[\gamma p - m(t)]\,\mathsf{u}(\mathbf{p},\mathsf{t}) &= 0,\\ [\gamma p + m(t)]\,\mathsf{v}(\mathbf{p},\mathsf{t}) &= 0\end{aligned} \tag{43}$$

with $p^0 = \omega(\mathbf{p}, t)$. These definitions create the set of standard orthogonality conditions [56] depending on time now parameterically

$$\begin{gathered}\bar{\mathsf{u}}^\alpha(\mathbf{p},t)\,\mathsf{u}^\beta(\mathbf{p},t) = \frac{m(t)}{\omega(\mathbf{p},t)}\delta_{\alpha\beta}, \qquad \bar{\mathsf{v}}^\alpha(\mathbf{p},t)\,\mathsf{v}^\beta(\mathbf{p},t) = -\frac{m(t)}{\omega(\mathbf{p},t)}\,\delta_{\alpha\beta},\\ \mathsf{u}^{\dagger\alpha}(\mathbf{p},t)\,\mathsf{u}^\beta(\mathbf{p},t) = \mathsf{v}^{\dagger\alpha}(-\mathbf{p},t)\,\mathsf{v}^\beta(-\mathbf{p},t) = \delta_{\alpha\beta},\\ \bar{\mathsf{u}}^\alpha(\mathbf{p},t)\,\mathsf{v}^\beta(\mathbf{p},t) = \mathsf{u}^{\dagger\alpha}(\mathbf{p},t)\,\mathsf{v}^\beta(-\mathbf{p},t) = 0.\end{gathered} \tag{44}$$

Decompositions (42) and relations (44) lead immediately to the diagonal form of the Hamiltonian (41)

$$H(t) = \sum_{\mathbf{p},\alpha}\omega(\mathbf{p},t)[a^\dagger_\alpha(\mathbf{p},t)a_\alpha(\mathbf{p},t) + b^\dagger_\alpha(\mathbf{p},t)b_\alpha(\mathbf{p},t)] \tag{45}$$

with interpretation of $a^\dagger,\ a$ (and $b^\dagger,\ b$) as the creation and annihilation operators of quasiparticles obeying the standard anticommutation relations

$$\{a_\alpha(\mathbf{p},t), a^\dagger_\beta(\mathbf{p}',t)\} = \{b_\alpha(\mathbf{p},t), b^\dagger_\beta(\mathbf{p}',t)\} = \delta_{\mathbf{pp}'}\delta_{\alpha\beta}. \tag{46}$$

We are not interested in the subsequent diagonalization of the spin operator and such QPR can be named the *incomplete* representation.

Now in order to get equations of motion for creation and annihilation operators in the OR, let us substitute the decomposition (42) in Eqs. (40) and use relations (44). Then, as an intermediate result, we obtain the following closed set of equations of motion which is valid in a general case:

$$\begin{aligned}
&\dot a_\alpha(\mathbf{p},t) + U_1^{\alpha\beta}(\mathbf{p},t)\, a_\beta(\mathbf{p},t) + U_2^{\alpha\beta}(\mathbf{p},t)\, b^\dagger_\beta(-\mathbf{p},t) = -i\omega(\mathbf{p},t)\, a_\alpha(\mathbf{p},t),\\
&\dot a^\dagger_\alpha(\mathbf{p},t) - a^\dagger_\beta(\mathbf{p},t)\, U_1^{\beta\alpha}(\mathbf{p},t) + b_\beta(-\mathbf{p},t)\, U_2^{\beta\alpha}(\mathbf{p},t) = i\omega(\mathbf{p},t)\, a^\dagger_\alpha(\mathbf{p},t),\\
&\dot b_\alpha(-\mathbf{p},t) + a^\dagger_\beta(\mathbf{p},t)\, V_1^{\beta\alpha}(\mathbf{p},t) - b_\beta(-\mathbf{p},t)\, V_2^{\beta\alpha}(\mathbf{p},t) = \\
&\qquad\qquad = -\, i\omega(\mathbf{p},t)\, b_\alpha(-\mathbf{p},t),\\
&\dot b^\dagger_\alpha(-\mathbf{p},t) + V_1^{\alpha\beta}(\mathbf{p},t)\, a_\beta(\mathbf{p},t) + V_2^{\alpha\beta}(\mathbf{p},t)\, b_\beta(-\mathbf{p},t) = i\omega(\mathbf{p},t)\, b^\dagger_\alpha(-\mathbf{p},t).
\end{aligned} \tag{47}$$

The spinor construction is introduced here as

$$\begin{aligned}
U_1^{\alpha\beta} &= \mathsf{u}^{\dagger\alpha}(\mathbf{p},\mathsf{t})\,\dot{\mathsf{u}}^\beta(\mathbf{p},\mathsf{t}), & V_1^{\alpha\beta} &= \mathsf{v}^{\dagger\alpha}(-\mathbf{p},\mathsf{t})\,\dot{\mathsf{u}}^\beta(\mathbf{p},\mathsf{t}),\\
U_2^{\alpha\beta} &= \mathsf{u}^{\dagger\alpha}(\mathbf{p},\mathsf{t})\,\dot{\mathsf{v}}^\beta(-\mathbf{p},\mathsf{t}), & V_2^{\alpha\beta} &= \mathsf{v}^{\dagger\alpha}(-\mathbf{p},\mathsf{t})\,\dot{\mathsf{v}}^\beta(-\mathbf{p},\mathsf{t}).
\end{aligned} \tag{48}$$

The matrices U_2 and V_1 describe the transitions between states with positive and negative energies and different spins, while the antiunitary matrices U_1 and V_2 show the spin rotations only

$$U_1^\dagger = -U_1, \qquad V_2^\dagger = -V_2, \qquad V_2^\dagger = -U_2. \tag{49}$$

Equations (47) are compatible with the canonical commutation relations (46).

Let us write now the $u,\ v$ spinors in an explicit form, according to [57]:

$$\begin{aligned}
\mathsf{u}^{\dagger\,1}(\mathbf{p},t) &= A(\mathbf{p})\,[\,\omega_+, 0, p_3, p_-],\\
\mathsf{u}^{\dagger\,2}(\mathbf{p},t) &= A(\mathbf{p})\,[\,0, \omega_+, p_+, -p_3\,],\\
\mathsf{v}^{\dagger\,1}(-\mathbf{p},t) &= A(\mathbf{p})\,[-p_3, -p_-, \omega_+, 0\,],\\
\mathsf{v}^{\dagger\,2}(-\mathbf{p},t) &= A(\mathbf{p})\,[-p_+, p_3, 0, \omega_+],
\end{aligned} \tag{50}$$

where $p_\pm = p_1 \pm ip_2,\ \omega_+ = \omega + m(t)$ and $A(\mathbf{p}) = [2\omega\omega_+]^{-1/2}$. Spin rotation matrices (48) in this representation are equal to zero

$$U_1 = V_2 = 0. \tag{51}$$

For the remaining matrices (48) we have $U_2 = -V_1 = U$, where U is the Hermitian matrix

$$U(\mathbf{p},t) = \frac{\dot m(t)}{2\omega^2(\mathbf{p},t)} \begin{bmatrix} p_3 & p_- \\ p_+ & -p_3 \end{bmatrix}. \tag{52}$$

Thus, the system of equations of motion (47) reduces to the following one:

$$\begin{aligned}&\dot{a}_\alpha(\mathbf{p},t)+U_{\alpha\beta}(\mathbf{p},t)b^\dagger_\beta(-\mathbf{p},t)=-i\omega(\mathbf{p},t)a_\alpha(\mathbf{p},t),\\&\dot{b}_\alpha(-\mathbf{p},t)-a^\dagger_\beta(\mathbf{p},t)U_{\beta\alpha}(\mathbf{p},t)=-i\omega(\mathbf{p},t)b_\alpha(-\mathbf{p},t).\end{aligned}\tag{53}$$

2.2. Kinetic Equation. Equations of motion (53) do not contain the spin rotation matrices (51) and they are similar to Eqs. (15); therefore, the KE derivation meets no problem now. To be specific, let us introduce the one-particle correlation functions

$$\begin{aligned}&g_{\alpha\beta}(\mathbf{p},t)=\langle 0|a^\dagger_\beta(\mathbf{p},t)a_\alpha(\mathbf{p},t)|0\rangle,\\&\tilde{g}_{\alpha\beta}(\mathbf{p},t)=\langle 0|b_\beta(-\mathbf{p},t)b^\dagger_\alpha(-\mathbf{p},t)|0\rangle.\end{aligned}\tag{54}$$

The differentiation of (54) with respect to time leads to the following matrix equations:

$$\begin{aligned}&\dot{g}(\mathbf{p},t)=-U(\mathbf{p},t)\,G(\mathbf{p},t)-G^\dagger(\mathbf{p},t)U(\mathbf{p},t),\\&\dot{\tilde{g}}(\mathbf{p},t)=G(\mathbf{p},t)U(\mathbf{p},t)+U(\mathbf{p},t)\,G^\dagger(\mathbf{p},t),\end{aligned}\tag{55}$$

where the auxiliary function was introduced

$$G_{\alpha\beta}(\mathbf{p},t)=\langle 0|a^\dagger_\beta(\mathbf{p},t)b^\dagger_\alpha(-\mathbf{p},t)|0\rangle.\tag{56}$$

Together with Eqs. (55), the corresponding equation of motion

$$\dot{G}(\mathbf{p},t)=U(\mathbf{p},t)g(\mathbf{p},t)-\tilde{g}(\mathbf{p},t)U(\mathbf{p},t)+2i\omega(\mathbf{p},t)G(\mathbf{p},t)\tag{57}$$

forms a closed set of equations for correlation functions. With the help of Eqs. (57), one can exclude the auxiliary correlator from the system (55)

$$\begin{aligned}&\dot{g}(\mathbf{p},t)=2U(\mathbf{p},t)\int\limits_{t_0}^{t}dt'[\,\tilde{g}(\mathbf{p},t')U(\mathbf{p},t')-U(\mathbf{p},t')g(\mathbf{p},t')]\,\cos 2\theta(\mathbf{p},t',t),\\&\dot{\tilde{g}}(\mathbf{p},t)=2\int\limits_{t_0}^{t}dt'[U(\mathbf{p},t')g(\mathbf{p},t')-\,\tilde{g}(\mathbf{p},t')U(\mathbf{p},t')]U(\mathbf{p},t)\,\cos 2\theta(\mathbf{p},t',t),\end{aligned}\tag{58}$$

using zero initial conditions. The subsequent transformation is based on the relation

$$\mathrm{Tr}\,\{U(t)AU(t')\}=\frac{1}{4}\lambda(\mathbf{p},t)\lambda(\mathbf{p},t')TrA\tag{59}$$

for an arbitrary second-rank matrix A, which follows from Eq. (52). The function ($p = |\mathbf{p}|$)

$$\lambda(\mathbf{p}, t) = \frac{\dot{m}(t)p}{\omega^2(\mathbf{p}, t)} \tag{60}$$

plays a role of some analog of Eq. (16).

Using isotropy of the considered system, we will limit ourselves to the spin-averaged scalar distributions

$$f(\mathbf{p}, t) = \frac{1}{2}\,\mathrm{Tr}\, g(\mathbf{p}, t), \qquad \tilde{f}(-\mathbf{p}, t) = 1 - \frac{1}{2}\,\mathrm{Tr}\,\tilde{g}(\mathbf{p}, t). \tag{61}$$

Calculating the trace of (58) we get

$$\dot{f}(\mathbf{p}, t) = \dot{\tilde{f}}(-\mathbf{p}, t) = \frac{1}{2}\lambda(\mathbf{p}, t)\int\limits_{t_0}^{t} dt'\lambda(\mathbf{p}, t')[1 - \tilde{f}(-\mathbf{p}, t') - \\ - f(\mathbf{p}, t')]\,\cos 2\theta(\mathbf{p}; t, t'). \tag{62}$$

In the case of the vacuum initial state, $\tilde{f}(-\mathbf{p}, t) = f(\mathbf{p}, t)$, we have

$$\dot{f}(\mathbf{p}, t) = 2\lambda(\mathbf{p}, t)\int\limits_{t_0}^{t} dt'\lambda(\mathbf{p}, t')[1 - 2f(\mathbf{p}, t')]\,\cos[2\theta(t, t')], \tag{63}$$

The KEs (23) and (63) are similar but differ by statistical factors $1 \pm 2f$ (the Bose enhancement or the Fermi suppression) and by structure of the factors (16) and (60). The corresponding linear equations for the non-Hamiltonian dynamical system become

$$\dot{f} = \frac{1}{2}\lambda u, \qquad \dot{u} = \lambda[1 - 2f] - 2\omega v, \qquad \dot{v} = 2\omega u. \tag{64}$$

This system possesses one first integral of motion (see [38])

$$(1 - 2f)^2 + v^2 + u^2 = 1. \tag{65}$$

This relation represents an ellipsoid in the phase space of (f, u, v) variables. After exclusion of the function f from Eq. (64), we obtain the system of non-linear equations

$$\begin{aligned} \dot{u} &= \lambda\sqrt{1 - u^2 - v^2} - 2\omega v, \\ \dot{v} &= 2\omega u. \end{aligned} \tag{66}$$

It can easily be proved that the KE (63) is invariant with respect to time inversion. Equations analogous to Eqs. (26) and (64) were obtained in [53] (see also [26]) for the conformal flat space-time. The KE (63) to the case of spinor QED was generalized in [54, 55]. In the general case, the spin correlation functions (54) and (56) can be decomposed in respect of the Pauli matrices (e.g., [58]).

2.3. Observables and Regularization. The total particle number density and energy density in the considered case are distinguished from the corresponding expressions (30) and (33) for the scalar system by the spin degeneration factor $g = 4$ (for an equal number of particles and antiparticles)

$$n(t) = 4 \int [dp] f(\mathbf{p}, t), \tag{67}$$

$$\epsilon(t) = \langle 0|T_{00}|0\rangle = 4 \int [dp] \omega(\mathbf{p}, t) f(\mathbf{p}, t), \tag{68}$$

where T_{00} is the zero component of the energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}[\bar{\psi}\gamma_\mu(\partial_\nu\psi) - (\partial_\nu\bar{\psi})\gamma_\mu\psi]. \tag{69}$$

By definition, the entropy density of the fermion system is equal to

$$S(t) = -4 \int [dp] \big[f \ln f + (1-f) \ln (1-f)\big]. \tag{70}$$

Finally, the pressure is

$$P(t) = \frac{1}{3}\langle T_{kk}\rangle. \tag{71}$$

Using (69) and (40), this relation can be reduced to the following form:

$$P(t) = \frac{1}{3}\{\epsilon(t) - m(t)\langle\bar{\psi}(x)\psi(x)\rangle\}. \tag{72}$$

Here the correlation function is calculated by means of relations (44)

$$P(t) = \frac{1}{3}\epsilon(t) + \frac{1}{3}\int [dp] \; \frac{m^2(t)}{\omega(\mathbf{p}, t)} \; [1 - 2f(\mathbf{p}, t)] + P_{\rm pol}(t), \tag{73}$$

where the last term takes into account the contribution of the vacuum polarization,

$$P_{\rm pol}(t) = -\frac{2}{3}m(t)\int [dp] \; \frac{p}{\omega(\mathbf{p}, t)} \; u(\mathbf{p}, t). \tag{74}$$

This result was obtained under additional conditions for «observable» correlation functions

$$\langle 0|a^\dagger_\alpha(\mathbf{p}, t) a_\beta(\mathbf{p}', t)|0\rangle = \langle 0|b^\dagger_\alpha(\mathbf{p}, t) b_\beta(\mathbf{p}', t)|0\rangle = \delta_{\alpha\beta}\delta_{\mathbf{p}\mathbf{p}'}, \tag{75}$$

which is a consequence of space-homogeneity and isotropy (absence of the spin moment) of the system. The auxiliary correlation function (56) is not connected with the spin moment and, therefore, it remains off-diagonal with respect to spin indices.

The same regularization procedure can be realized here for the calculation of divergence integrals, as presented in Subsec. 1.3 for the scalar bosons. Equation (25) for the distribution function in the low-density approximation is valid also after the replacement $\Delta(\mathbf{p},t) \to \lambda(\mathbf{p},t)$. Asymptotics of the factor (60) is equal to $\lambda^{\infty}(\mathbf{p},t) = \dot{m}(t)/p$. Using Eq. (25) and the rules of Subsec. 1.3, one can derive the following expressions for counterterms for the case of fermion fields:

$$\begin{aligned} f_M^{(4)}(\mathbf{p},t) &= \left[\frac{\dot{m}(t)}{4(p^2+M^2)}\right]^2, \\ u_M^{(3)}(\mathbf{p},t) &= \frac{\ddot{m}(t)}{4(p^2+M^2)^{3/2}}. \end{aligned} \tag{76}$$

However, these counterterms can be ignored in computer calculations.

It is known [26] that the phase density of pairs created in an electric field for the whole period of its action is related to long-time asymptotics of solutions of some oscillator equations. For the inertial mechanism, analogous derivation results in the following relation [22]:

$$\lim_{t\to+\infty} f(\mathbf{p},t) = \frac{\cosh \tau_\pi[(m_i - m_f)] - \cosh \tau_\pi[(\omega_i - \omega_f)]}{2\sinh(\tau_\pi \omega_i)\sinh(\tau_\pi \omega_f)}, \tag{77}$$

where $\tau_\pi = \pi\tau/2$, the indices i, f correspond to the initial and final states. This relation is convenient for calculation of observables for long pulses $\tau \gg m$, see, e.g., Fig. 4 where the direct solution of kinetic equation is a very robust numerical problem.

2.4. Numerical Results. Here numerical investigations of the KEs are presented for bosons (23), fermions (63), and appropriate densities of observable variables are estimated (Subsecs. 1.3 and 2.3). As an example, two variants of time-dependent masses are considered. The first case qualitatively corresponds to a typical meson mass change under the phase transition within the NJL model [59]

$$m(t) = (m_0 - m_f)\exp[-(t/\tau)^2] + m_f, \qquad t \geqslant 0, \tag{78}$$

with the parameters m_0 (initial mass), m_f (final mass), and τ (transition time). Another variant suggested in [22, 23] allows an analytical solution of the Dirac equation

$$m(t) = \frac{m_f + m_0}{2} + \left(\frac{m_f - m_0}{2}\right)\tanh(2t/\tau). \tag{79}$$

Numerical results for solution of the KEs are presented in Figs. 1–8 for $m(t)$ defined by Eq. (78) with $\tau = 10$ fm/c and in Figs. 9, 10 for the mass (79) with $\tau = 20$ fm/c. Masses are specified in natural units, $m_h = 197$ MeV/c^2.

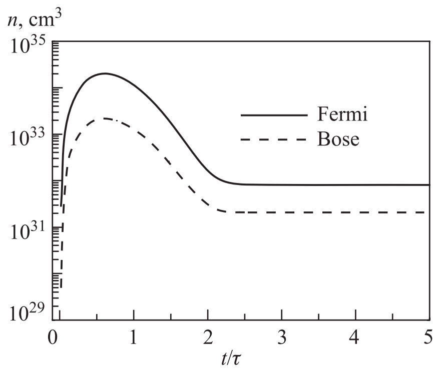

Fig. 1. Time dependence of the pair density for bosons and fermions at $m_0 = 1$

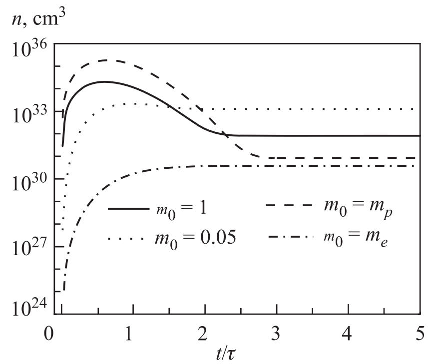

Fig. 2. The pair density evolution of fermions for various initial masses (p — proton, e — electron)

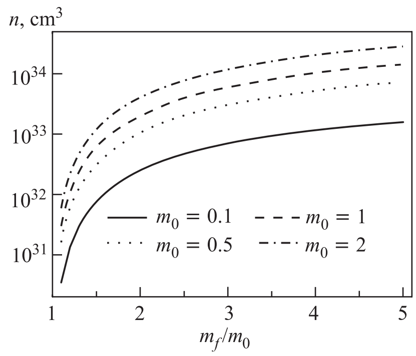

Fig. 3. The residual pair density versus the mass ratio

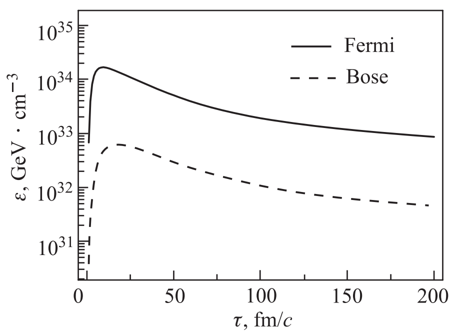

Fig. 4. The residual energy density versus the transition time τ, $m_0 = 1$

At a glimpse, the time dependence of quasiparticle density for bosons and fermions repeats qualitatively the curve $\dot{m}(t)$; however, when $\dot{m}(t) \to 0$ the densities go asymptotically to certain finite values n_r (residual density, Fig. 1, $t \gg \tau$), which characterize the real (free) particles (at the active stage of the process, $\dot{m}(t) \neq 0$, one may talk about quasiparticles only). The $n(t)$ dependence for fermions on the initial mass value is shown in Fig. 2 in the range from the electron mass to proton one. This dependence is nonmonotonic: With increasing m_0 the residual density reaches the maximum at $m_0 \sim 10$ MeV and then begins to decrease. This effect appreciably depends on the variant used for the mass change: For the case (79) it manifests itself much more clearly than for the model (78). The residual particle density dependence on the mass change is presented in Fig. 3. Qualitative behavior of the energy density is similar to that of particle density, showing a smooth asymptotic decrease (see Fig. 4). In Fig. 5, the

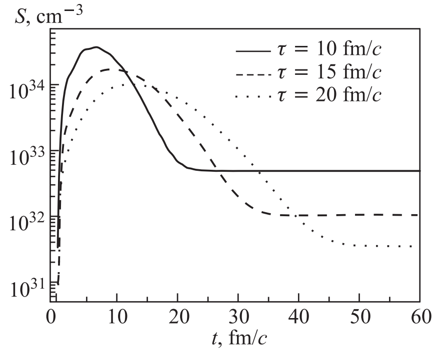

Fig. 5. Entropy density evolution of bosons for different values of the relaxation time τ, $m_0 = 1$

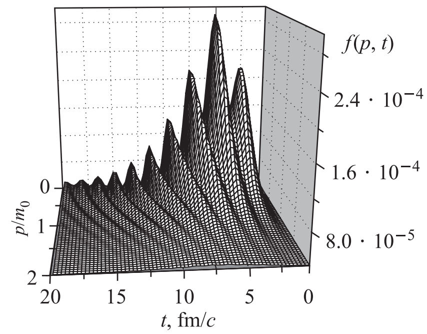

Fig. 6. The distribution function of bosons in time-momentum variables, $m_0 = 1$, $\tau = 10$ fm/c

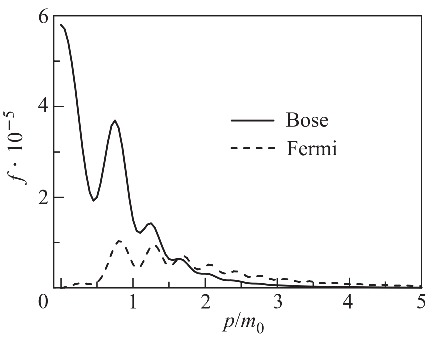

Fig. 7. Distribution functions at the time $t = \tau = 10$ fm/c, $m_0 = 1$

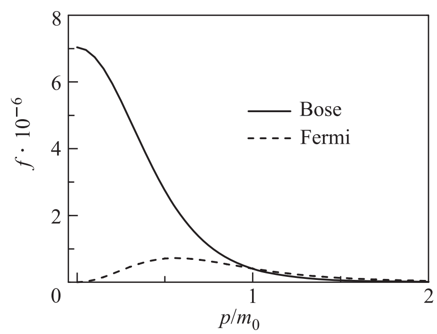

Fig. 8. Asymptotic form of distribution functions at $t \gg \tau$

time dependence of the boson entropy density is presented for different values of the relaxation time. Nonmonotonic behavior of entropy is caused by the fact that the system is opened and treated in the nondissipative approximation.

Momentum spectra of particles at different stages of the interaction process are shown in Figs. 6–8. The maximal number of bosons is created with zero momentum, whereas there are no fermions with $p = 0$. This feature differs qualitatively from the case of the Schwinger mechanism of particle creation [38].

It is important that the formation of appreciably nonmonotonic distributions with the «fast» mass changing (78), Fig. 7, assists in the development of plasma oscillations. For smoother mass changing (79) this effect becomes much less pronounced.

The most interesting features are observed in the pressure behavior, Fig. 9, 10. For both variants of the mass evolution and independently of particle statistics,

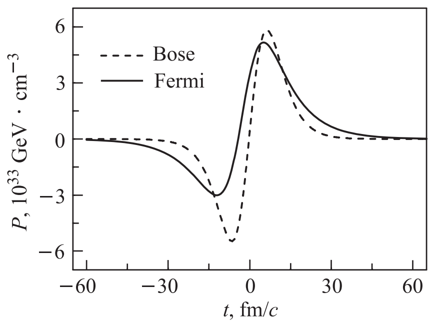

Fig. 9. Pressure evolution corresponding to mass changing (79), $m_0 = 1$, $\tau = 20$ fm/c

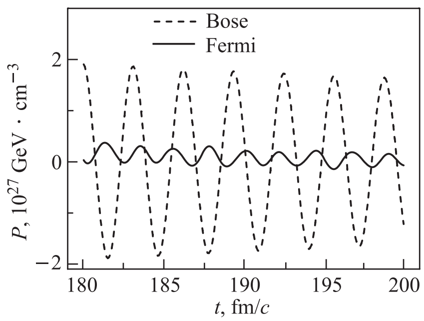

Fig. 10. Asymptotic oscillations of pressure at $t \gg \tau$ without «switching off» of vacuum polarization effects

the pressure is negative at the beginning of the process, then it changes its sign in the reflection point of $m(t)$ and gradually decreases. However, contrary to other observables, the pressure has no constant asymptotics and at $t \gg \tau$ looks like almost undamped oscillations, Fig. 9. It is distinctive for pressure of the bosonic quasiparticle system which strongly oscillates around zero, Fig. 10. The reason is that unlike the other considered quantities, the pressure is not completely determined by the quasiparticles distribution function $f(\mathbf{p}, t)$, but it depends also on the function $u(\mathbf{p}, t)$, which describes vacuum polarization effects. At the operator language, this means incomplete diagonalization of the energy-momentum tensor in the Fock space: Averaged over the initial vacuum, its spatial components include the contribution of anomalous correlators like $\langle 0 \,| a_p^\dagger \, a_p^\dagger \,| 0 \rangle$.

Thus, if the process of particle creation stops when the time mass evolution is completed ($\dot{m}(t) \to 0$), the vacuum polarization effects are not «switched off» simultaneously but continue to influence some observables, e.g., pressure. As a consequence, in such nondissipative nonequilibrium model it is impossible to determine unambiguously the equation of state [60].

3. MASSIVE VECTOR BOSONS

3.1. The Complete QPR. The simplest version of quantum field theory of neutral massive vector bosons is given by the Lagrangian density [56]

$$\mathcal{L}(x) = -\frac{1}{2}\,\partial_\mu u_\nu\,\partial^\mu u^\nu + \frac{1}{2}\,m^2(t) u_\nu u^\nu, \tag{80}$$

which corresponds to the equation of motion

$$[\partial_\mu \partial^\mu + m^2(t)] u_\nu = 0 \tag{81}$$

with the additional «external» constraint

$$\partial_\mu u^\mu = 0. \tag{82}$$

An alternative way is to proceed from the Wentzel Lagrangian [61]

$$\mathcal{L}(x) = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m^2(t)u_\nu u^\nu, \tag{83}$$

where the strength tensor $F_{\mu\nu} = \partial_\nu u_\mu - \partial_\mu u_\nu$. Here the constraint (82) is a consequence of dynamical equations. Lagrangians (80) and (83) result in different energy-momentum tensors (e.g., see [61, 62]).

The transition to the QPR is carried out by the standard decomposition of free fields and momenta in the discrete momentum space with the replacement $m \to m(t)$ in the dispersion law (see Subsec. 1.1),

$$\begin{aligned} u_\mu(x) &= \frac{1}{\sqrt{2V}}\sum_{\mathbf{p}}\frac{1}{\sqrt{\omega(\mathbf{p},t)}}\left\{a_\mu(\mathbf{p},t)\,\mathrm{e}^{i\mathbf{px}} + a^*_\mu(\mathbf{p},t)\,\mathrm{e}^{-i\mathbf{px}}\right\},\\ \pi_\mu(x) &= -\frac{i}{\sqrt{2V}}\sum_{\mathbf{p}}\sqrt{\omega(\mathbf{p},t)}\left\{a_\mu(\mathbf{p},t)\,\mathrm{e}^{i\mathbf{px}} - a^*_\mu(\mathbf{p},t)\,\mathrm{e}^{-i\mathbf{px}}\right\}, \end{aligned} \tag{84}$$

where $a_\mu(\mathbf{p},t)$ are the classical amplitudes. Unlike the scalar case, the consistent quantization is possible only after including the constraint (82).

The substitution of the field operators (84) into the Hamiltonian

$$H = -\frac{1}{2}\int d\mathbf{x}\left(\pi_\mu\pi^\mu + \nabla\mathbf{u}_\mu\nabla\mathbf{u}^\mu + \mathbf{m^2(t)u_\mu u^\mu}\right) \tag{85}$$

gives directly the diagonal form in the Fock space

$$H = -\sum_{\mathbf{p}}\omega(\mathbf{p},t)\,a^*_\mu(\mathbf{p},t)\,a_\mu(\mathbf{p},t). \tag{86}$$

However, this quadratic form is not positively defined. To correct it, one needs to exclude the $\mu = 0$ component by the additional condition (82). The equations of motion for amplitudes $a_\mu(\mathbf{p},t)$ are similar to the scalar case (15)

$$\dot a_\mu(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\,a^*_\mu(-\mathbf{p},t) - i\omega(\mathbf{p},t)\,a_\mu(\mathbf{p},t), \tag{87}$$

where $\Delta(\mathbf{p},t)$ is defined by Eq. (16). Using Eqs. (87), the condition (82) may be transformed now to the following relation ($i = 1, 2, 3$):

$$\omega(\mathbf{p},t)\,a_0(\mathbf{p},t) = p_i\,a_i(\mathbf{p},t). \tag{88}$$

This equation allows one to exclude the $\mu = 0$ component from the Hamiltonian (86) and quantize other components, which gives

$$H = \sum_{\mathbf{p},i,k} \frac{1}{\omega(\mathbf{p},t)} \Big[\omega^2(\mathbf{p},t)\,\delta_{ik} - p_i p_k\Big]\, a_i^\dagger(\mathbf{p},t)\, a_k(\mathbf{p},t), \tag{89}$$

with the corresponding commutation relation and the vacuum state

$$\Big[a_i(\mathbf{p},t), a_k^\dagger(\mathbf{p}',t)\Big] = \delta_{ik}\delta_{\mathbf{pp}'}, \quad a_i(\mathbf{p},t\to-\infty)|0\rangle = 0. \tag{90}$$

The next step is the diagonalization of the quadratic form (89) by means of the linear transformation [56]

$$\mathbf{a}_i(\mathbf{p},t) = E_{ik}\alpha_k(\mathbf{p},t) \equiv (\mathbf{e}_1)_i\alpha_1(\mathbf{p},t)+ \\ +(\mathbf{e}_2)_i\alpha_2(\mathbf{p},t) + (\mathbf{e}_3)_i\frac{\omega}{m(t)}\alpha_3(\mathbf{p},t), \tag{91}$$

where $\{\mathbf{e}_1(\mathbf{p}), \mathbf{e}_2(\mathbf{p}), \mathbf{e}_3(\mathbf{p})\}$ is the local orthogonal basis constructed on the vector $\mathbf{e}_3 = \mathbf{p}/|p|$. These real unit vectors form the triad,

$$e_{ik}e_{jk} = e_{ki}e_{kj} = \delta_{ij}, \qquad e_{ik} = (\mathbf{e}_i)_k. \tag{92}$$

The transformation (91) establishes the positively-defined Hamiltonian

$$H = \sum_{\mathbf{p}} \omega(\mathbf{p},t)\Big[\alpha_i^\dagger(\mathbf{p},t)\alpha_i(\mathbf{p},t) + \alpha_i(\mathbf{p},t)\alpha_i^\dagger(\mathbf{p},t)\Big]. \tag{93}$$

The presence of the ω/m factor in the nonunitary matrix E in Eq. (91) leads to violation of the unitary equivalence between the a representation (89) and α representations (93). Equations of motion for these new amplitudes follow from a combination of Eqs. (87) and (91)

$$\dot{\alpha}_i(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\alpha_i^\dagger(-\mathbf{p},t) - i\omega(\mathbf{p},t)\alpha_i(\mathbf{p},t) + \eta_{ij}(\mathbf{p},t)\alpha_j(\mathbf{p},t). \tag{94}$$

The spin rotation matrix η_{ij} is defined as

$$\eta_{ik}(\mathbf{p},t) = -\Delta_m\delta_{i3}\delta_{k3} \tag{95}$$

with $\Delta_m = -\dot{m}/m + \Delta$. This relation shows a particular role of the third component.

Together with the Hamiltonian (86), the total momentum operator takes also the diagonal form. However, the spin operator

$$S_i = \varepsilon_{ijk}\int d\mathbf{x}\,\Big[u_k\pi_j + \pi_j u_k - u_j\pi_k - \pi_k u_j\Big] \tag{96}$$

has nondiagonal terms in the spin space in terms of the operator α_i

$$S_k = i\varepsilon_{ijk} \sum_{\mathbf{p}} \left[\alpha_i^\dagger(\mathbf{p}, t)\alpha_j(\mathbf{p}, t) - \alpha(\mathbf{p}, t)\alpha_j^\dagger(\mathbf{p}, t) \right], \tag{97}$$

where ε_{ijk} is the unit antisymmetric tensor. In particular, the spin projection on the p_3 axis is

$$S_3 = i \sum_{\mathbf{p}} \Big[\alpha_1^\dagger(\mathbf{p}, t)\alpha_2(\mathbf{p}, t) - \alpha_2^\dagger(\mathbf{p}, t)\alpha_1(\mathbf{p}, t) + \\ + \alpha_2(\mathbf{p}, t)\alpha_1^\dagger(\mathbf{p}, t) - \alpha_1(\mathbf{p}, t)\alpha_2^\dagger(\mathbf{p}, t) \Big]. \tag{98}$$

Thus, this representation can be named the *incomplete* quasiparticle representation since the spin projection is not fixed. The operator (98) can be diagonalized by a linear transformation to the new basis [56]

$$c_i(\mathbf{p}, t) = R_{ik}\alpha_k(\mathbf{p}, t), \tag{99}$$

with the unitary matrix

$$R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & i & 0 \\ -i & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}. \tag{100}$$

As a result, the new amplitudes $c_i(\mathbf{p}, t)$ correspond to creation and annihilation operators of vector quasiparticles with the total energy, 3-momentum and spin projection into the chosen direction,

$$H(t) = \sum_{\mathbf{p}} \omega(\mathbf{p}, t) \left[c_i^\dagger(\mathbf{p}, t)c_i(\mathbf{p}, t) + c_i(\mathbf{p}, t)c_i^\dagger(\mathbf{p}, t) \right], \tag{101}$$

$$\mathbf{\Pi}(t) = \sum_{\mathbf{p}} \mathbf{p} \left[c_i^\dagger(\mathbf{p}, t)c_i(\mathbf{p}, t) + c_i(\mathbf{p}, t)c_i^\dagger(\mathbf{p}, t) \right], \tag{102}$$

$$S_3(t) = \sum_{\mathbf{p}} \Big[c_1^\dagger(\mathbf{p}, t)c_1(\mathbf{p}, t) - c_1(\mathbf{p}, t)c_1^\dagger(\mathbf{p}, t) + \\ + c_2(\mathbf{p}, t)c_2^\dagger(\mathbf{p}, t) - c_2^\dagger(\mathbf{p}, t)c_2(\mathbf{p}, t) \Big]. \tag{103}$$

This c representation will be referred to as the *complete* quasiparticle representation. The equations of motion for these amplitudes follow from Eqs. (94), (99)

$$\dot{c}_i(\mathbf{p}, t) = \frac{1}{2}\Delta(\mathbf{p}, t)c_i^\dagger(-\mathbf{p}, t) - i\omega(\mathbf{p}, t)c_i(\mathbf{p}, t) + \eta_{ij}(\mathbf{p}, t)c_j(\mathbf{p}, t), \tag{104}$$

where the matrix η_{ij} is fixed by Eq. (95).

The transition to this representation from the initial a representation is defined by the combination of transformations (91) and (99)

$$c(\mathbf{p},t) = U(\mathbf{p},t)a(\mathbf{p},t) \tag{105}$$

with the nonunitary operator ($\mathbf{e}^{(\pm)} = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$)

$$U(\mathbf{p},t) = R \cdot E^{-1}(\mathbf{p},t) = \begin{bmatrix} e_1^{(+)} & e_2^{(+)} & e_3^{(+)} \\ e_1^{(-)} & e_2^{(-)} & e_3^{(-)} \\ \dfrac{m}{\omega}e_{31} & \dfrac{m}{\omega}e_{32} & \dfrac{m}{\omega}e_{33} \end{bmatrix}. \tag{106}$$

To solve the quantization problem, the equation of motion should be taken into account. The commutation relation has the noncanonical form

$$\left[c_i(\mathbf{p},t), c_j^+(\mathbf{p}',t)\right] = Q_{ik}(\mathbf{p},t)Q_{jk}(\mathbf{p},t)\delta_{\mathbf{pp}'}, \tag{107}$$

where the matrices $Q_{il}(\mathbf{p},t)$ are defined by the equations

$$\dot{Q}_{ij}(\mathbf{p},t) = \eta_{ik}(\mathbf{p},t)Q_{kj}(\mathbf{p},t) \tag{108}$$

with the initial conditions

$$\lim_{t\to-\infty} Q_{ij}(\mathbf{p},t) = \delta_{ij}. \tag{109}$$

So the commutation relation is transformed to the canonical form only in the asymptotic limit $t \to -\infty$. Relation (107) provides the definition of positive-energy quasiparticle excitations of vacuum to be treated as some time-dependent energy reservoir.

3.2. Kinetic Equations. The standard procedure to derive the KE [29] is based on the Heisenberg-type equations of motion (87) or (104). Let us introduce one-particle correlation functions of vector bosons in the initial a representation

$$F_{ik}(\mathbf{p},t) = \langle 0|a_i^\dagger(\mathbf{p},t)a_k(\mathbf{p},t)|0\rangle, \tag{110}$$

where the averaging procedure is performed over the in-vacuum state [26]. By differentiating the first equation with respect to time, we obtain

$$\dot{F}_{ik}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\left\{\Phi_{ik}(\mathbf{p},t) + \Phi_{ik}^\dagger(\mathbf{p},t)\right\}, \tag{111}$$

where the auxiliary correlation function is introduced as

$$\Phi_{ik}(\mathbf{p},t) = \langle 0|a_i(-\mathbf{p},t)a_k(\mathbf{p},t)|0\rangle. \tag{112}$$

Equation of motion for this function can be obtained by analogy with Eq. (111). We write out the answer in the integral form

$$\Phi_{ik}(\mathbf{p},t) = \frac{1}{2}\int\limits_{-\infty}^{t} dt'\Delta(\mathbf{p},t')\,[\delta_{ik} + 2F_{ik}(\mathbf{p},t')]\ \mathrm{e}^{2i\theta(\mathbf{p};t,t')}. \qquad (113)$$

The asymptotic condition $F_{ik}(\mathbf{p},-\infty) = 0$ (the absence of quasiparticles in the initial time) has been used here. The substitution of Eq. (113) into Eq. (111) leads to the resulting KE

$$\dot{F}_{ik}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\int\limits_{-\infty}^{t} dt'\Delta(\mathbf{p},t')[\delta_{ik} + 2F_{ik}(\mathbf{p},t')]\ \cos\,[2\,\theta(\mathbf{p};t,t')]. \quad (114)$$

This KE is a natural generalization of the corresponding KE for scalar particles (Subsec. 1.2).

However, there is a number of problems that are specific for the theory of massive bosons: The energy is not positively-defined, the spin operator has nondiagonal terms in the space of spin states, etc. This circumstance hampers the physical interpretation of the distribution function (110). In order to overcome this difficulty, it is necessary to proceed to the complete QPR where the system has well-defined values of energy, spin, etc. The simplest way to derive the KE in this QPR is based on the application of the transformation (105) directly to the KE (114).

Similarly to the definitions (110), we introduce correlation functions of vector particles in the complete QPR

$$f_{ik}(\mathbf{p},t) = \langle 0|c_i^{\dagger}(\mathbf{p},t)c_k(\mathbf{p},t)|0\rangle. \qquad (115)$$

They are connected with the primordial correlation functions (110) by relations of the type

$$f_{ik}(\mathbf{p},t) = U_{in}^{\dagger}(\mathbf{p},t)U_{km}(\mathbf{p},t)F_{nm}(\mathbf{p},t). \qquad (116)$$

As a result, the KE (114) becomes

$$\dot{f}_{ik}(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\int\limits_{-\infty}^{t} dt'\Delta(\mathbf{p},t')M_{ikjl}(\mathbf{p},t,t')\big[\,\delta_{jl} + 2f_{jl}(\mathbf{p},t')\big]\times$$
$$\times\cos 2\theta(\mathbf{p};t,t') - \Delta_m(\mathbf{p},t)[\delta_{i3}f_{3k}(\mathbf{p},t) + \delta_{k3}f_{i3}(\mathbf{p},t)], \quad (117)$$

where

$$M_{ikjl}(t,t') = \delta^{\perp}_{ij}\delta^{\perp}_{kl} + \\ + \frac{\omega(t')}{\omega(t)}\frac{m(t)}{m(t')}\left[\delta_{i3}\delta_{j3}\delta^{\perp}_{kl} + \delta_{k3}\delta_{l3}\delta^{\perp}_{ij} + \frac{\omega(t')}{\omega(t)}\frac{m(t)}{m(t')}\delta_{i3}\delta_{k3}\delta_{j3}\delta_{l3}\right] \quad (118)$$

and $\delta^{\perp}_{ik} = \delta_{ik} - \delta_{i3}\delta_{k3}$.

As was expected, distribution functions $f_{\alpha\beta}(\mathbf{p},t)$ satisfy the same KE (114) for $\alpha = 1,2$. The feature of the complete QPR becomes apparent only in tensor components of the distribution function $f_{ik}(\mathbf{p},t)$ which contains the preferred values of the spin index i and (or) $k = 3$. Let us select the KE for diagonal components of the correlation function (115) which has the direct physical meaning of transversal ($i = 1,2$)

$$\dot{f}_i(\mathbf{p},t) = \frac{1}{2}\Delta(\mathbf{p},t)\int\limits_{-\infty}^{t} dt'\Delta(\mathbf{p},t')\big[1 + 2f_i(\mathbf{p},t')\big]\cos 2\theta(\mathbf{p};t,t') \quad (119)$$

and longitudinal components of the distribution function

$$\dot{f}_3(\mathbf{p},t) = -2\Delta_m(\mathbf{p},t)f_3(\mathbf{p},t) + \frac{1}{2}\Delta(\mathbf{p},t)\frac{m^2(t)}{\omega^2(t)}\times \\ \times\int\limits_{-\infty}^{t} dt'\Delta(\mathbf{p},t')\frac{\omega^2(t')}{m^2(t')}\big[2f_3(\mathbf{p},t') + Q(\mathbf{p},t')\big]\cos 2\theta(\mathbf{p};t,t'). \quad (120)$$

Here the shorthand notation $f_{ii} = f_i$ has been introduced for diagonal components of the matrix correlation functions (115) and

$$\Delta = \frac{m\dot{m}}{\omega^2}, \qquad \Delta_m = -\Delta\frac{\mathbf{p}^2}{m^2}. \quad (121)$$

Longitudinal and transversal distribution functions are connected by the relation

$$f_3(\mathbf{p},t) = Q(\mathbf{p},t)f_1(\mathbf{p},t), \quad (122)$$

where $Q(\mathbf{p},t)$ is the function entering into the commutation relation for the longitudinal bosons,

$$[c_3(\mathbf{p},t), c_3^{\dagger}(\mathbf{p}',t)] = Q(\mathbf{p},t)\,\delta_{pp'}, \\ Q(\mathbf{p},t) = \exp\left[-2\int\limits_{t_0}^{t}\Delta_m(t')dt'\right] = \left[\frac{m(t)}{m(t_0)}\frac{\omega(t_0)}{\omega(t)}\right]^2 \quad (123)$$

with the corresponding initial values $m(t_0)$ and $\omega(t_0)$.

Due to relation (122), it is sufficient to solve only Eq. (119). The KE for transversal bosons is transformed from the integro-differential form to a set of ordinary differential equations similarly to the cases considered above (Subsecs. 1.2 and 2.2):

$$\dot{f}_k = \frac{1}{2}\Delta u_k, \qquad \dot{u}_k = \Delta(1 + 2f_k) - 2\omega v_k, \qquad \dot{v}_k = 2\omega u_k. \tag{124}$$

The general initial condition for all diagonal components of the distribution function

$$\lim_{t\to-\infty} f_k(t) = \lim_{t\to-\infty} u_k(t) = \lim_{t\to-\infty} v_k(t) = 0 \tag{125}$$

satisfies the following requirement:

$$\lim_{t\to-\infty} m(t) = m_0, \quad \text{or} \quad \lim_{t\to-\infty} \dot{m}(t) = 0. \tag{126}$$

The main operating characteristic of the vacuum creation process is the total number density of vector bosons,

$$\begin{aligned} n_{\text{tot}}(t) &= \frac{1}{\pi^2}\int\limits_0^\infty p^2 dp\,\big[2f_1(p,t) + f_3(p,t)\big], \\ &= \frac{1}{\pi^2}\int\limits_0^\infty p^2 dp\, f_1(p,t)\big[2 + Q(p,t)\big], \end{aligned} \tag{127}$$

where isotropy of the system was assumed, $p = |\mathbf{p}|$. As will be shown in Subsec. 3.3, the integral (127) is convergent. It essentially differs from results based on the Wentzel Lagrangian (83). In particular, as was shown in [16,17,32], the Wentzel approach leads to infinite particle density. Authors of these papers give some arguments in favor of that allowing for quasiparticle collisions will result in finite physical quantities and bring the estimated energy budget of the Universe to agreement with observation data [17,63].

We will consider below the case when the time dependence of the vector boson mass is defined by conformal evolution of the Universe. The set of equations similar to (124) was obtained first in [80] within an alternative model Lagrangian (83) in the Friedman–Robertson–Walker (FRW) space-time. The quantization procedure is quite ordinary in this approach. It would be useful to compare the predictions of these two models in detail.

3.3. EoS for the Isotropic Case. The relations for the energy density and pressure can be derived from the energy-momentum tensor corresponding to the Lagrangian (80)

$$T_{\mu\nu} = -\partial_\mu u_\alpha \partial_\nu u^\alpha - \partial_\nu u_\alpha \partial_\mu u^\alpha - g_{\mu\nu}\mathcal{L}. \tag{128}$$

Taking into account the isotropy of the system, we obtain the EoS for the massive vector boson gas [16]

$$\begin{aligned} \varepsilon(t) &= 2\int [dp]\,\omega(2+Q)f_1, \\ P(t) &= \frac{2}{3}\varepsilon(t) - \frac{4}{3}m^2\int \frac{[dp]}{\omega}(2+Q)f_1 + \delta P_{\text{vac}}(t), \end{aligned} \tag{129}$$

where $\delta P_{\text{vac}}(t)$ is the contribution to pressure induced by the vacuum polarization,

$$\delta P_{\text{vac}}(t) = -\frac{2}{3}\int \frac{[dp]}{\omega}\left(2\omega^2 + m^2\right)\left[1 + Q\left(\frac{1}{2} + \frac{\mathbf{p}^2}{m^2}\right)\right]u_1. \tag{130}$$

In order to prove the convergence of the integrals (129), (130), we investigate the asymptotic behavior of the solution of the system of equations (124). This system can be solved exactly in the limit $p \gg m$ for the case $\alpha = 1/2$ (the parameter α is defined in Sec. 4)

$$\dot{f} = \frac{m_H^2}{4t_H}\frac{1}{p^2}u, \qquad \dot{u} = \frac{m_H^2}{2t_H}\frac{1}{p^2}(1+2f) - 2pv, \qquad \dot{v} = 2pu. \tag{131}$$

The asymptotic solution of Eqs. (131) with the initial conditions (125) is

$$f(p,t) = \frac{v(p,t)}{(2p/m_0)^3} \sim \frac{\sin^2 p(t-t_0)}{16(p/m_0)^6}, \qquad u(p,t) \approx \frac{\sin 2p(t-t_0)}{4(p/m_0)^3}, \tag{132}$$

where $m_0 = m(t_0) = m_H^{2/3} t_H^{-1/3}$ and $t_0 = 1/m_0$. The numerical study of Eqs. (124) shows that the basic features of the solutions (132) for $\alpha = 1/2$ are conserved also for other $\alpha > 0$. It corresponds to the results of Subsec. 1.3.

According to Eq. (132), the particle number density and energy density (see Eqs. (127) and (129), respectively) are convergent, but the vacuum polarization contribution to pressure (130) is divergent. Moreover, irrelevant fast vacuum oscillations of the pressure are observed here. Such a behavior of the pressure for a plasma created from vacuum is not specific for the present theory but it is inherent in the models where an electron–positron plasma is created in strong time-dependent electric fields as investigated in [64]. The standard regularization procedure of similar integrals with some unknown functions satisfying ordinary differential equations is based on the study of asymptotic decompositions of these functions in power series of the inverse momentum, $1/p^N$ (see Subsec. 1.3). In the considered case, such a procedure is not effective because the solution (131) has quickly oscillating factors («Zitterbewegung»), whose asymptotic decomposition leads to secular terms. Therefore, for numerical calculations we regularize the pressure by a momentum cut-off at $p = 10m_0$ and separate its stable part by means of the time averaging procedure

$$\langle P\rangle = \frac{1}{(t-t_0)}\int\limits_{t_0}^{t} p(t)dt. \tag{133}$$

Such «coarse graining» procedure was proposed in [65] in order to exclude the «Zitterbewegung» from the description of vacuum particle creation. In reality, these fast oscillations are smoothed out due to dissipative processes which are not taken into consideration here (the first attempt to derive the collision integral for the scalar quasiparticle gas in a strong electric field was made in [42]).

4. APPLICATION TO CONFORMAL COSMOLOGY MODELS

The description of the vacuum creation of particles in time-dependent gravitational fields of cosmological models goes back to [45, 66–68] and has been reviewed, e.g., in monographs [34, 69, 70]. The particularity of our work consists in the consideration of vacuum generation of particles at conditions of the early Universe in the framework of a conformal-invariant cosmological model [16, 17]. Thus, the space-time is assumed to be conformably flat and the expansion of the Universe in the Einstein frame (with metric $\tilde{g}_{\mu\nu}$) with constant masses $\tilde{m}$ can be replaced by the change of masses in the Jordan frame (with metric $g_{\mu\nu}$) due to the evolution of the cosmological (scalar) dilaton background field [14, 71]. This mass change is defined by the conformal factor $\Omega(x)$ of the conformal transformation

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x)g_{\mu\nu}. \tag{134}$$

Since mass terms generally violate the conformal invariance, a space-time dependent mass term

$$m(x) = \frac{1}{\Omega(x)}\tilde{m} \tag{135}$$

has been introduced which formally keeps the conformal invariance of the theory [34]. In the important particular case of the isotropic FRW space-time, the conformal factor is equal to the scale factor, $\Omega(x) = a(\tilde{t})$, and hence $m(\tilde{t}) = a(\tilde{t})m_{\rm obs}$, where $\tilde{t}$ is «the Einstein time» and $m_{\rm obs}$ is the observable present-day mass. Such a dependence was used, e.g., in [26] for the FRW metric. On the other hand, the scaling factor $a(\tilde{t})$ is defined by the cosmic equation of state. For a barotropic fluid, this EoS has the form

$$P_{\rm ph} = (\gamma - 1)\varepsilon_{\rm ph} = c_s^2\varepsilon_{\rm ph}, \tag{136}$$

where $P_{\rm ph}$ and $\varepsilon_{\rm ph}$ are phenomenological pressure and energy density (in contrast to «dynamical» P and ε, see Subsec. 3.3); γ is the barotropic parameter; c_s is the sound velocity. The solution of the Friedman equation for such EoS leads to the following scaling factor:

$$a(\tilde{t}) \sim \tilde{t}^{\;2/3\gamma}. \tag{137}$$

Kinetics of the vacuum creation of massive vector bosons (Subsec. 3.2) was constructed in the flat Jordan frame with the proper conformal time t which is necessary to introduce now in Eq. (137). The transition to the conformal time is defined by the relation $dt = d\tilde{t}/a(\tilde{t})$. From this relation and Eq. (137) it follows

$$\tilde{t} \sim \left[\left(1 - \frac{2}{3\gamma}\right) t\right]^{3\gamma/(3\gamma-2)}. \tag{138}$$

The substitution of this relation into Eq. (137) establishes the mass evolution law in the terms of the conformal time

$$m(t) = (t/t_H)^{\alpha}\, m_W, \qquad \alpha = \frac{2}{3\gamma - 2}, \tag{139}$$

where $t_H = [(1+\alpha)H]^{-1}$ is the scaling factor (the age of the Universe); $H = 70$ km/s/Mpc is the the present-day Hubble constant and the W-boson mass is taken as $m_W = 80$ GeV. Values of the α parameter for some popular EoS are: $\gamma = 2$, $\alpha = 1/2$ (stiff fluid); $\gamma = 4/3$, $\alpha = 1$ (radiation); $\gamma = 1$, $\alpha = 2$ (dust); $\gamma < 2/3$ (quintessence); $\gamma = 0$, $\alpha = -1$ (cold matter including baryon mass and dark matter) [72, 73].

Due to back reactions and the dynamical mass generation during the cosmic evolution the detailed mass history remains to be worked out. The central question, however, is whether the number density of produced W bosons could be of the same order as that of the cosmic microwave background (CMB) photons, $n_{\rm CMB} \sim 465$ cm^{-3}. If this question may be answered positively, the vacuum pair creation of W bosons from a time-dependent scalar field (mass term) could be suggested as a mechanism for the generation of matter and radiation in the early Universe.

The numerical analysis of Eqs. (124) for massive vector bosons is performed by the standard Runge–Kutta method on a one-dimensional momentum grid. As one can see in Fig. 11, the creation process ends very quickly and the particle density saturates at some finite value. The momentum distribution of particles is formed also very early when $m(t) \approx m_0$ and frozen in such a form so later on, for times $t \gg t_0$, most of particles have very small momenta $p \ll m(t)$. The spectrum of created bosons is essentially nonequilibrium; hence we should continue the analysis of relevant dissipative mechanisms and other observable manifestations of a nonequilibrium state (e.g., CMB photons; in this connection, see, for example, [74, 75]).

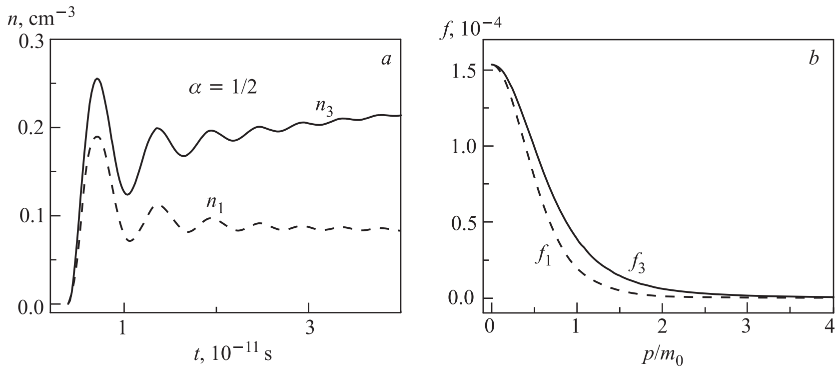

Fig. 11. Time evolution of the number density for transversal n_1 and longitudinal n_2 vector bosons with the initial condition $m_0 t_0 = 1$ for $\alpha = 1/2$ (*a*) and the corresponding momentum distribution at the time $t \gg t_0$ (*b*)

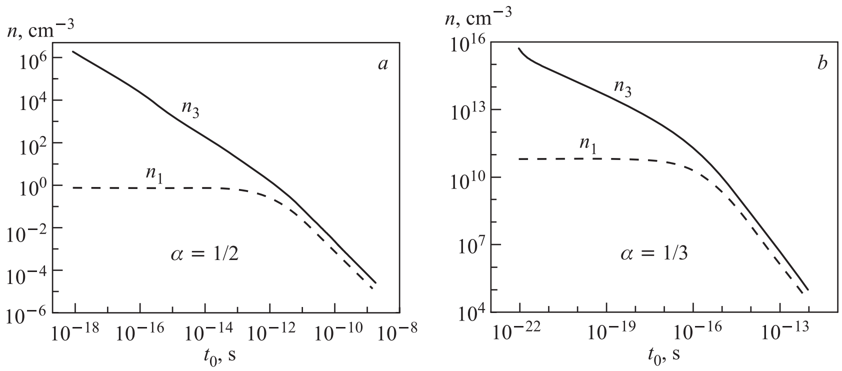

Fig. 12. The dependence of the number density of residual vector bosons on the initial time t_0: $\alpha = 1/2$ (*a*), $\alpha = 1/3$ (*b*)

The dependence of the corresponding final value of density on the initial time is shown in Fig. 12. The final density n_1 of the transversal vector bosons with spin projection ± 1 reaches a maximum when for very early initial times we are close to the birth of the Universe. However, in the same limit, the density f_3 of the longitudinal particles with zero spin projection grows beyond all bounds. The choice of the EoS changes drastically the number of created particles, thus resulting in values which are too small ($\alpha = 1/2$) or too large ($\alpha = 1/3$) in comparison with the observed CMB photon densities. In order to improve this model, we should use an improved EoS, assuming that the barotropic parameter γ characterizing the evolution of particle masses can be changed during the time evolution. Such a time-dependence could be induced by action of the back-

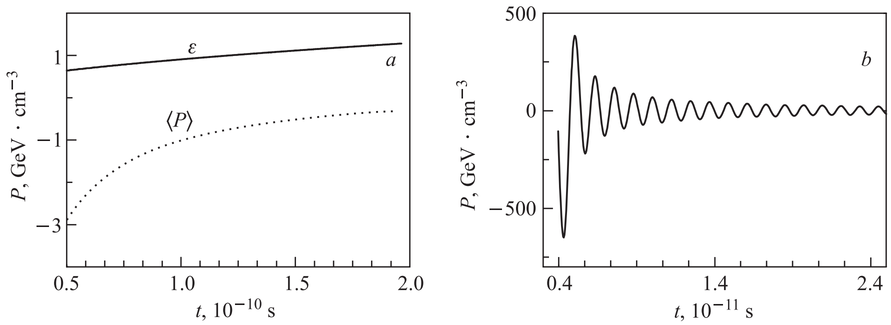

Fig. 13. The time dependence of the energy density and the mean value of pressure (133) at $t \gg t_0$ (a) as well as pressure (129) (b) with the initial condition $m_0 t_0 = 1$ for $\alpha = 1/2$

reaction of created particles on the scalar field. Furthermore, we could use another space-time model, e.g., the Kasner space-time [76] instead of the conformally flat de Sitter one. As compared to the earlier work [16], the main achievement of this approach is that there is no divergence in the distribution function; thus we do not need to introduce any ambiguous regularization procedure.

As shown in Fig. 13, the mean pressure remains negative and its magnitude becomes negligible in comparison with the energy density (as to the role of the negative pressure in cosmology, see, e.g., [77]). These features can lead to violation of the energy dominance condition $\epsilon + P \geqslant 0$ that corresponds to accelerated expansion of the Universe. Such a kind of models is widely discussed (e.g., [78]). For large time moments, the energy density grows but the pressure stays very small, $P \simeq 0$. The energy growth under condition $P \simeq 0$ results in the conclusion that the massive vector boson–antiboson gas created from the vacuum is cold. It can be seen directly from Eq. (129) that at large times $\varepsilon(t) \simeq m(t) n_{\rm tot}(t)$, because of $\omega(t) \simeq m(t)$. This EoS of the massive vector boson gas ($\varepsilon \neq 0$ and $p \simeq 0$) corresponds to dust-like matter [79], which would characterize the evolution of the Universe when the vector boson gas is the dominant component of its matter (energy) content. At a qualitative level, this conclusion is valid independently of the specific choice of the EoS and, in particular, in the case of the dust-like EoS. It would be interesting to obtain a formula like (139) as a result of the solution of the Friedman equation with the EoS (129), (130) (such a procedure represents the back reaction problem) and investigate self-consistently the production of vector bosons in the Universe. Let us remark that vacuum creation of massive vector bosons in the FRW metric was first considered in [80].

In order to investigate features of the fermion vacuum creation in the conformal model of the Universe, let us use the corresponding basic KE (63) and, as an example, choose the system of heavy top quarks with the mass $m_q = 170$ GeV

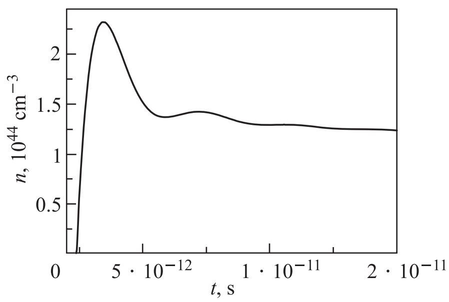

Fig. 14. The time dependence of fermion pair density for a mass evolution with $\alpha = 1/2$ and $m_q = 170$ GeV

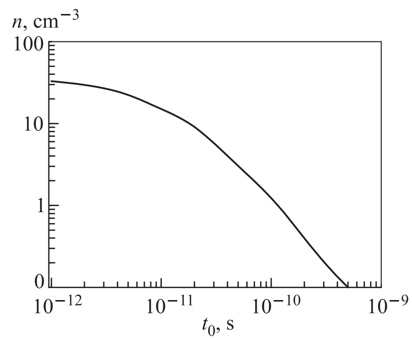

Fig. 15. The final density of fermions for the conformal time dependence of mass (139) as a function of the initial time t_0 for $\alpha = 1/2$

(with the change $m_w \to m_q$ in Eq. (139)). The values of the parameter α are the same as for the vector boson. As one can see in Fig. 14, the creation process is completed very quickly and the particle density saturates at some finite value.

Finally, Fig. 15 shows the dependence of the residual density on the initial time t_0 for $m(t)$ given by Eq. (139). At the qualitative level, the same picture will take place for the vacuum creation of neutralino, which can be the main component of dark matter (e.g., [81]). An analogous problem in early cosmology was considered in [15].

5. SYSTEMS WITH METASTABLE VACUUM

5.1. Formulation of the Problem. Here a rather general mechanism of the mass formation as a result of self-consistent dynamics of mean-field and quantum fluctuations will be considered. The separation of the quasi-classical background field is a common procedure of different nonperturbative approaches in QFT [26, 34, 82, 83]. In the framework of this procedure quantum fluctuations can be described by the perturbation theory.

There is a class of physics problems in which the strong background field creates particles which in turn influence the background field (the back reaction problem). In this respect it is worthy to mention such problems as the decay of disoriented chiral condensate [84], the resonant decay of CP-odd metastable states [4,85], the preequilibrium QGP evolution [29,36,37,86], the phase transition in systems with the broken symmetry [87], etc.

The construction of general kinetic theory of such a kind for various potentials is presented in Subsec. 5.2. We will derive the closed system of equations for the self-consistent description of the back reaction problem, including the KE with nonperturbative source term describing the particle creation in the quasi-classical background field and the equation of motion for this background field. We use the OR to derive the KE. As an illustrative examples, in Subsec. 5.3 the one-component scalar theory with Φ^4 and double-well potential are considered. In these examples, we study some features of the proposed approach. In particular, the problem of the stable vacuum state definition and possibility to emerge tachyonic regimes is discussed. As a less trivial example, the pseudoscalar sector of the Witten–De Vecchia–Veneziano model will be considered. Similar analysis was carried out in some other models of such a kind (e.g., [4, 85, 88–90]). In this section, we follow paper [1].

5.2. The Set of Basic Equations. Let us consider now the scalar field Lagrangian with a self-interaction potential $V[\Phi]$

$$\mathcal{L}[\Phi] = \frac{1}{2}\partial_\mu\Phi\ \partial^\mu\Phi - \frac{1}{2}m_0^2\Phi^2 - V[\Phi], \qquad (140)$$

where m_0 is the bare mass and the potential $V[\Phi]$ is an arbitrary continuous function with at least one minimum that is necessary for a correct definition of the vacuum state. It is assumed that the field Φ may be decomposed into the quasi-classical space-homogeneous time-dependent background field $\phi_0(t)$ and fluctuation part $\phi(x)$

$$\Phi(x) = \phi_0(t) + \phi(x). \qquad (141)$$

In accordance with the definition of fluctuations, we have $\langle\phi\rangle = 0$ and $\langle\Phi\rangle = \phi_0$, where the symbol $\langle\ldots\rangle$ denotes some averaging procedure. The background field $\phi_0(t)$ can be treated as quasi-classical one at the condition [91]

$$|\dot{\phi}_0| \gg 1/(\delta t)^2, \qquad (142)$$

where δt is the characteristic time of the field averaging.

We consider the case of quite small fluctuations in the vicinity of the background field. Therefore, the potential energy expansion in powers of $\phi(x)$ can be performed

$$V[\Phi] = V[\phi_0] + R_1\phi + \frac{1}{2}R_2\phi^2 + V_r[\phi_0, \phi], \qquad (143)$$

where

$$R_1 = R_1[\phi_0] = \frac{dV[\phi_0]}{d\phi_0}, \qquad R_2 = R_2[\phi_0] = \frac{d^2V[\phi_0]}{d\phi_0^2} \qquad (144)$$

and $V_r[\phi_0, \phi]$ is a residual term containing the higher order contributions to be neglected in the current consideration (nondissipative approximation). The decomposition (143) can be finite (for polynomial theories) or infinite. After field decomposition (141) the equation of motion

$$\partial_\mu \partial^\mu \Phi + m_0^2 \Phi + \frac{dV[\Phi]}{d\Phi} = 0 \tag{145}$$

can be rewritten in the following form:

$$(-\partial_\mu \partial^\mu - m^2)\phi = Q[\phi_0, \phi], \tag{146}$$

where the relation

$$m^2(t) = m^2[\phi_0] = m_0^2 + R_2[\phi_0] \tag{147}$$

defines the time-dependent in-medium mass. The term in the r.h.s. of (146) is

$$\begin{aligned} Q[\phi_0, \phi] &= \ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + Q_2[\phi_0, \phi], \\ Q_2[\phi_0, \phi] &= \frac{1}{2} \frac{dR_2[\phi_0]}{d\phi_0} \, \phi^2. \end{aligned} \tag{148}$$

As a result of averaging of Eq. (146), the equation of motion for the background field is obtained

$$\ddot{\phi}_0 + m_0^2 \phi_0 + R_1[\phi_0] + \langle Q_2[\phi_0, \phi] \rangle = 0, \tag{149}$$

where the time independence of the averaging procedure is taken into account.

The space-homogeneity assumption implies that the function $\langle Q_2[\phi_0, \phi] \rangle$ in Eq. (149) can depend only on time. As follows from Eqs. (148) and (149), the source term in the r.h.s. of Eq. (146) is exclusively defined by fluctuations,

$$Q[\phi_0, \phi] = Q_2[\phi_0, \phi] - \langle Q_2[\phi_0, \phi] \rangle. \tag{150}$$

On the other hand, in a nonstationary situation the field function $\phi(x)$ allows the decomposition:

$$\phi(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \left\{ \phi^{(+)}(\mathbf{p}, t) \, \mathrm{e}^{-i\mathbf{px}} + \phi^{(-)}(\mathbf{p}, t) \, \mathrm{e}^{i\mathbf{px}} \right\}, \tag{151}$$

where $\phi^{(\pm)}(\mathbf{p}, t)$ are the positive and negative frequency solutions of the equation of motion

$$\ddot{\phi}^{(\pm)}(\mathbf{p}, t) + \omega^2(\mathbf{p}, t) \phi^{(\pm)}(\mathbf{p}, t) = -Q[\phi_0, \phi; \pm\mathbf{p}] \tag{152}$$

with

$$\omega^2(\mathbf{p}, t) = m^2(t) + \mathbf{p}^2 \tag{153}$$

and with the Fourier image $Q[\phi_0, \phi; \mathbf{p}]$ of the function $Q[\phi_0, \phi]$,

$$Q[\phi_0, \phi] = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} Q[\phi_0, \phi; \mathbf{p}] \, \mathrm{e}^{-i\mathbf{px}}. \tag{154}$$

The function $Q[\phi_0, \phi; \mathbf{p}]$ gives a nonlinear contribution to Eq. (152). We suppose that there exists a finite limit $\lim\limits_{t\to-\infty} \phi^{(\pm)}(\mathbf{p}, t) = \phi_-^{(\pm)}(\mathbf{p})$ in the infinite past and assume that solutions $\phi^{(\pm)}(\mathbf{p}, t)$ become asymptotically free $\phi^{(\pm)}(\mathbf{p}, t) \to \mathrm{e}^{\pm i\omega_- t}$, where $\omega_-(\mathbf{p}) = \lim\limits_{t\to-\infty} \omega(\mathbf{p}, t)$. The existence of the last limit is based on the adiabatic hypothesis about switching off the self-interaction in Eq. (147).

After the decompositions (141) and (143), the Hamiltonian density is

$$H[\Phi] = H[\phi_0] + H_1[\phi_0, \phi] + H_2[\phi_0, \phi] + V_r[\phi_0, \phi], \tag{155}$$

where $H_0[\phi_0]$ is the background field Hamiltonian, and the terms H_1 and H_2 are the Hamiltonian functions of the first and second order with respect to the fluctuation field

$$H[\phi_0] = H_0[\phi_0] + V[\phi_0] = \frac{1}{2}\dot\phi_0^2 + \frac{1}{2}m_0^2\phi_0^2 + V[\phi_0], \tag{156}$$

$$H_1[\phi_0, \phi] = \dot\phi_0\dot\phi + (m_0^2\phi_0 + R_1[\phi_0])\phi, \tag{157}$$

$$H_2[\phi_0, \phi] = \frac{1}{2}\dot\phi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2. \tag{158}$$

The quasiparticle representation for fluctuations is constructed by means of the decompositions (8). In order to obtain equation of motion for operator $a(\mathbf{p}, t)$, let us write the corresponding action with the Hamiltonian (156)–(158) (this way is alternative to the Hamiltonian approach of Subsec. 1.1)

$$S[\phi] = \int d^4x\ \{\pi\dot\phi - H_1 - H_2 - V_r\}. \tag{159}$$

If decompositions (8) are substituted here, we get

$$\begin{aligned} S[\phi] = \frac{1}{2}\sum_{\mathbf{p}} \int dt\ \Big\{ i\left[a^\dagger(\mathbf{p}, t)\dot a(\mathbf{p}, t) - a(\mathbf{p}, t)\dot a^\dagger(\mathbf{p}, t)\right] - \\ - \frac{\dot\omega(\mathbf{p}, t)}{\omega(\mathbf{p}, t)}\left[a^\dagger(\mathbf{p}, t)a^\dagger(-\mathbf{p}, t) - a(-\mathbf{p}, t)a^\dagger(\mathbf{p}, t)\right] - \\ -\omega(\mathbf{p}, t)[a^\dagger(\mathbf{p}, t)a(\mathbf{p}, t) + a(\mathbf{p}, t)a^\dagger(\mathbf{p}, t)] - 2V_{r\mathbf{p}}[\phi_0, \phi]\Big\} + S_1[\phi], \end{aligned} \tag{160}$$

where $S_1[\phi]$ is the part of the action corresponding to the Hamiltonian (157) and $V_{\mathbf{p}}[\phi_0, \phi]$ is the Fourier image of the residual potential term. Variation with respect to $a(\mathbf{p}, t)$ and subsequent transition to the occupation number representation lead to the Heisenberg-type equations of motion ($\mathbf{p} \neq 0$)

$$\dot a(\mathbf{p}, t) = \frac{1}{2}W(\mathbf{p}, t)a^\dagger(-\mathbf{p}, t) - i[H_2 + V_r, a(\mathbf{p}, t)], \tag{161}$$

where

$$W(\mathbf{p},t) = \frac{\dot\omega(\mathbf{p},t)}{\omega(\mathbf{p},t)} = \frac{\dot R_2[\phi_0]}{2\omega^2}. \tag{162}$$

In Eq. (161) the condensate contribution generated by the action part $S_1[\phi]$ is omitted because it corresponds to $\mathbf{p} = 0$ (an appropriate mechanism of the condensate state with $\mathbf{p} = 0$ and excitations is absent in the present model).

Let us introduce the distribution function of quasiparticles, according to Eq. (18). Using methods of Sec. 1 and Eq. (161), we get the kinetic equation in the nondissipative approximation, $V_r[\phi_0,\phi] \to 0$,

$$\dot f = \frac{1}{2}Wu, \quad \dot u = W(1+2f) - 2\omega v, \quad \dot v = 2\omega u, \tag{163}$$

which is an analog of (26) with the replace of Δ by W. To rewrite Eq. (149) for the background field in the nondissipative approximation, one has to calculate the averaged value $\langle in|\phi^2(x)|in\rangle$. In the space-homogeneous case one can obtain

$$\langle in|\phi^2(x)|in\rangle = \frac{1}{2}\int \frac{d\mathbf{p}}{\omega(\mathbf{p},t)}[1 + 2f(\mathbf{p},t) + u(\mathbf{p},t)]. \tag{164}$$

Then Eq. (149) is reduced to

$$\ddot\phi_0 + m_0^2\phi_0 + R_1[\phi_0] + \frac{1}{2}\frac{dR_2}{d\phi_0}\int \frac{d\mathbf{p}}{\omega(\mathbf{p},t)}\left[f(\mathbf{p},t) + \frac{1}{2}u(\mathbf{p},t)\right] = 0 \tag{165}$$

(the vacuum term is omitted in integrand).

The KEs (163) and (165) form the closed set of nonlinear equations describing the back-reaction problem. In the case of $v[\Phi_0,\Phi] = 0$, this set of equations directly follows from nonperturbative dynamics and assumption (141). For the description of the particle creation we will use the particle density (30) as well as the background field energy $\epsilon_{\rm cl}$ and energy of created quasiparticles ϵ_q

$$\begin{aligned}\epsilon_{\rm cl} &= \frac{1}{2}\dot\phi_0^2 + \frac{1}{2}m_0^2\phi_0^2 + V(\phi_0),\\ \epsilon_q &= \int \frac{d^3p}{(2\pi)^3}\omega(\mathbf{p},t)f(\mathbf{p},t).\end{aligned} \tag{166}$$

The conservation of the full system energy can be shown analytically.

The constructed formalism allows the consideration of certain initial states at the time $t = t_0$: Initial excitation of the background field $\phi_0(t_0)$ and $\dot\phi_0(t_0)$ is given under the additional condition that either $f(\mathbf{p},t_0) = 0$ or $f(\mathbf{p},t_0) \neq 0$, where $f(\mathbf{p},t_0)$ is some initial plasma distribution.

5.3. Examples. *5.3.1.* Φ^4 *Potential.* The separation of the background field (141) in the potential

$$V[\Phi] = \frac{1}{4}\lambda\Phi^4, \qquad \lambda > 0 \tag{167}$$

leads to the following decomposition coefficients (144):

$$\begin{aligned} R_1[\phi_0] &= \lambda\phi_0^3, \qquad R_2[\phi_0] = 3\lambda\phi_0^2, \\ V[\phi_0] &= \frac{1}{4}\lambda\phi_0^4, \quad V_r[\phi_0, \phi] = \lambda(\phi_0 + \phi/4)\phi^3. \end{aligned} \tag{168}$$

Thus, the time-dependent quasiparticle mass of the fluctuating field (147) is equal to

$$m^2(t) = m_0^2 + 3\lambda\phi_0^2. \tag{169}$$

If $\lambda < 0$ and the excitation is strong enough, it is possible that a tachyonic mode will arise that corresponds to an unstable state [92]. The mass (169) determines the factor (162)

$$W(\mathbf{p}, t) = \lambda \frac{2\phi_0\dot{\phi}_0}{\omega^2(\mathbf{p}, t)}. \tag{170}$$

The KE (163) with this factor is correct in the nondissipative approximation where the residual potential V_r is neglected.

Let us write down also the equation of motion for the background field (165) in this approximation:

$$\ddot{\phi}_0 + M^2(t)\phi_0 + \lambda\phi_0^3 = 0 \tag{171}$$

with the corresponding mass to be equal to

$$M^2(t) = m_0^2 + 3\lambda \int \frac{d^3k}{\omega(\mathbf{p}, t)} \left[f(\mathbf{p}, t) + \frac{1}{2}u(\mathbf{p}) \right], \tag{172}$$

i.e., the mass of the condensate excitations is defined by both the distribution of quasiparticles and the vacuum polarization.

In numerical calculations we apply zero initial conditions for the distribution function and nonzero ones for the background field $\phi_0(t_0) = 1.2$ (here and below we use the units $\hbar = c = 1$). Parameter values are chosen by analogy with [87], where the authors offered an alternative method for describing quantum systems under action of the strong background field (the so-called Cornwall–Jackiw–Tomboulis method [93]). As is seen in Fig. 16, at the early evolution stage all energy is mainly concentrated in field oscillations. For $t < 50$, the particle number density grows slowly. However, density drastically increases at $t \sim 50$ and after this time the quantum energy dominates over classical one.

The case $\lambda < 0$ (absolutely unstable potential) is associated with a tachyonic regime which is realized for high enough excitations when the initial amplitude $\phi_0(t_0)$ satisfies the condition $m_0^2 + 3\lambda\phi_0^2(t_0) \leqslant 0$.

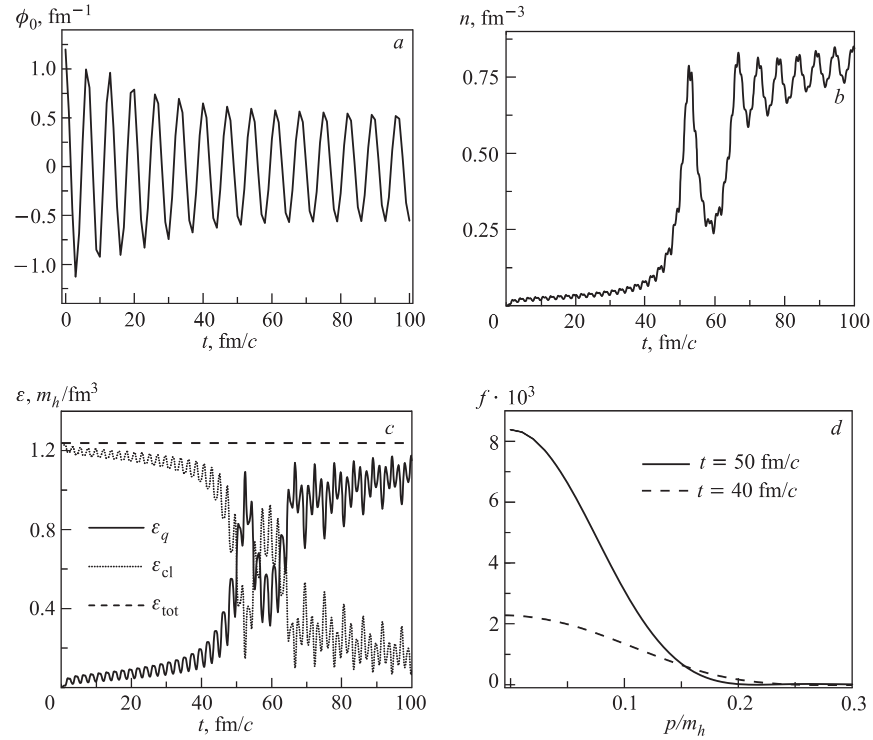

Fig. 16. Time evolution for the symmetric Φ^4 potential: *a*) the mean field; *b*) the particle density; *c*) the energy density; *d*) the momentum spectra of particles at time moments $t = 40$ fm/c and $t = 50$ fm/c. Parameter values used are: $\lambda = 1$, $m_0 = m_h = 197$ MeV, $\phi_0(0) = 1.2$ fm^{-1}, $\dot{\phi}_0(0) = 0$

5.3.2. Double-Well Potential. The potential

$$V[\Phi] = \frac{1}{4}\lambda\Phi^4 - \frac{1}{2}\mu^2\Phi^2, \qquad \lambda > 0 \tag{173}$$

leads to the same equation (171) for the background field but with a new mass (we put here $m_0 = 0$ and $\mu^2 > 0$)

$$M^2(t) = -\mu^2 + 3\lambda \int \frac{d\mathbf{p}}{\omega(t)} \left[f(\mathbf{p},t) + \frac{1}{2} u(\mathbf{p},t) \right]. \tag{174}$$

The factor (162) equals

$$W(\mathbf{p},t) = \lambda \frac{3\phi_0\dot{\phi}_0}{2\omega^2(\mathbf{p},t)}, \tag{175}$$

where now the quasiparticle frequency (153) includes the time-dependent mass

$$m^2(t) = -\mu^2 + 3\lambda\phi_0^2. \tag{176}$$

The vicinity of the central point $\phi_0(t) = 0$ is an unstable region. In this region the group velocity $v_g = d\omega(k)/dk = k/\omega(k)$ is either superluminous for $k > k_c$, where k_c is the root of the equation $\omega(k,t) = 0$ or undefined for $k < k_c$. Thus, it is the tachyonic region.

Let us denote the minimum position of the potential (173) as $\Psi_\pm = \pm\Psi_0 = \pm\mu/\sqrt{\lambda}$ and put the new origin of the reference frame in one of these points, $\Phi = \Psi_\pm + \Psi$. We discriminate now the background component ϕ_0 and the field Ψ, i.e., $\Psi = \phi_0 + \phi$. We will omit the sign indices $(\pm)$, which identify the branch of $\Psi_\pm$ which the fields Ψ, ϕ_0, and ϕ belong to. Only rather a small range of excitations in the vicinity of the stable points $\Psi_\pm$ will be considered below, $|\phi(x)|$, $|\phi_0(t)| \ll \sqrt{2}\Psi_0$, where $\pm\sqrt{2}\Psi_0$ are the roots of the equation $V[\Psi] = 0$. Using Eq. (145) and methods described in Subsec. 5.2, one can obtain the following system of equations of motion:

$$\begin{gathered}\ddot{\phi}_0 + 2\mu^2\phi_0 + 3\lambda\Psi_\pm\phi_0^2 + \lambda\phi_0^3 + 3\lambda(\Psi_\pm + \phi_0)\langle\phi^2\rangle + \lambda\langle\phi^3\rangle = 0,\\ -[\partial_\mu\partial^\mu + m_\pm^2(t)]\phi + 3\lambda(\Psi_\pm + \phi_0)[\langle\phi^2\rangle - \phi^2] + \lambda[\langle\phi^3\rangle - \phi^3] = 0,\end{gathered} \tag{177}$$

where

$$m_\pm^2(t) = 2\mu^2 + 3\lambda\phi_0(\phi_0 + 2\Psi_\pm). \tag{178}$$

In the nondissipative approximation only linear terms should be kept in Eq. (177). This leads to the KE (163) with the factor defined by the time-dependent mass (178)

$$W_\pm(\mathbf{p},t) = \frac{\dot{\omega}(\mathbf{p},t)}{2\omega(\mathbf{p},t)} = \frac{3\lambda\dot{\phi}_0(\phi_0 + \Psi_\pm)}{2\omega^2(\mathbf{p},t)}. \tag{179}$$

The mean values $\langle\phi^2\rangle$ and $\langle\phi^3\rangle$ are calculated either in the minimal order of the perturbation theory (for $\lambda \ll 1$) or in the random phase approximation. We use the result (164) for $\langle\phi^2\rangle$ and $\langle\phi^3\rangle = 0$. Thus, we have

$$\begin{aligned}\ddot{\phi}_0 + \lambda\phi_0\left[(2 + 3\phi_0)\Psi_\pm + \phi_0^2\right] + \\ + 3\lambda(\Psi_\pm + \phi_0)\int\frac{d\mathbf{p}}{2\,\omega(\mathbf{p},t)}\left[2f(\mathbf{p},t) + v(\mathbf{p},t)\right] = 0.\end{aligned} \tag{180}$$

Numerical results for the set of parameters corresponding to [87] are presented in Fig. 17. The stationary regime is achieved faster than in the case of the symmetric potential.

Another formalism for describing the strong field problem in the quantum field system with the potential (173) is developed by J. Baacke et al. (see [87],

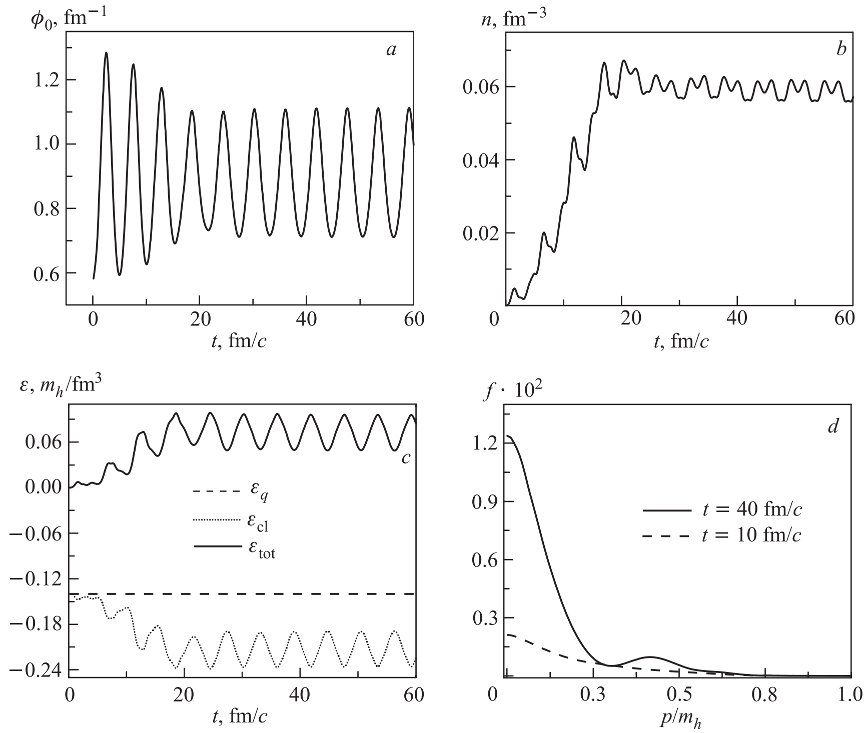

Fig. 17. Time evolution for the bistable Φ^4 potential (173): *a*) the mean field; *b*) the particle density; *c*) the energy density; *d*) the momentum spectra of particles at time $t = 10$ fm/c and $t = 40$ fm/c. Parameter values are $m_0 = 0$, $\mu = m_h$, $\lambda = 6$, $\phi_0(0) = 0.58$ fm^{-1}, $\dot{\phi}_0(0) = 0$

and papers cited therein). Here the general kinetic approach is developed for arbitrary highly-excited nonequilibrium states in the scalar QFT with self-interaction admitting the existence of unstable vacuum states. We restrict ourselves to the collisionless (nondissipative) approximation. However, attempts to go beyond this approximation have been made [42]. As a particular example, ϕ^4 and double-well potentials were investigated.

It would be of interest to study some other properties of the considered model, such features as excitation transitions between states with different vacua (in the same space-time point). Apparently, it is possible at high initial excitation $|\phi_0(0)| \geqslant \sqrt{2}\Psi(0)$ and as a consequence of the tunnelling process through the central barrier. The last problem is especially interesting in the generalized double-well potential model with nondegenerated vacuum states.

5.3.3. η-Meson System. We will use now the developed technique to consider a more complicated quantum field system, namely, the η-meson sector of the Witten–Di Vecchia–Veneziano model [2] which describes the low-energy dynamics of the nonet of pseudoscalar mesons in the large N_c limit of QCD. We drow our attention to the singlet state of this model with the following Lagrangian [85]:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + b^2 \mu^2 \cos\left(\frac{\eta}{b}\right) - \frac{a}{2}\eta^2, \tag{181}$$

where $b = \sqrt{3/2} b_\pi$, $b_\pi = 92$ MeV is the semileptonic pion decay constant, $\mu^2 = \dfrac{1}{3}(m_\pi^2 + 2m_K^2) \simeq 0.171$ GeV^2, m_π and m_K are π- and K-meson masses, $a = m_\eta^2 + m_{\eta'}^2 - 2m_K^2 \simeq 0.726$ GeV^2 for zero temperature. The corresponding total Hamiltonian density is given by

$$H = \frac{1}{2}\dot{\eta}^2 + \frac{1}{2}(\nabla\eta)^2 + \frac{1}{2}a\eta^2 + 2b^2\mu^2 \sin^2 \frac{\eta}{2b}, \tag{182}$$

where the constant addend $b^2\mu^2$ was discarded.

The Hamiltonian density (182) can be reduced to the form [2]

$$\begin{gathered} H = \frac{1}{2}\dot{\eta}^2 + \frac{1}{2}(\nabla\eta)^2 + \frac{1}{2}m_0^2\eta^2 + H_{in}, \\ H_{in} = 2b^2\mu^2\left[\sin^2\frac{\eta}{2b} - \left(\frac{\eta}{2b}\right)^2\right], \end{gathered} \tag{183}$$

where $m_0^2 = a + \mu^2$. The effective potential H_{in} is constructed in such a way that its formal decomposition with respect to the field function $\eta(x)$ does not contain the corresponding squared contribution to be associated with the mass term. However, the redefinition (183) leads to the absolutely unstable potential (Subsubsec. 5.3.1) with the corresponding tachyonic modes [2]. In our opinion, these modes have artificial character and can be eliminated by return to the original Hamiltonian density (182).

By analogy with Eq. (141), let us select the quasi-classical field $\eta_0(t)$ and quantum fluctuation part $\phi(x)$,

$$\eta(x) = \eta_0(t) + \phi(x). \tag{184}$$

Now we can implement the general formalism of Subsubsec. 5.3.2 to the η-meson system with the Hamiltonian density (182). The master equation (165) is now given by

$$\ddot{\eta}_0 + a\eta_0 + b\mu^2 \sin\left(\frac{\eta_0}{b}\right)\left[1 - \frac{1}{2b^2}\int \frac{d\mathbf{p}}{\omega}\left(f + \frac{1}{2}v\right)\right] = 0, \tag{185}$$

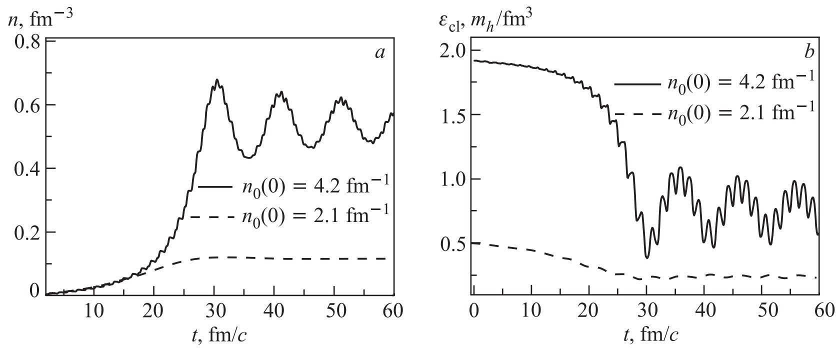

Fig. 18. The time evolution for the potential (183): the particle density (a) and the mean field energy (b). Parameter values are $m_0 = 0$, $\dot{\eta}_0(0) = 0$

where $\omega(\mathbf{p}, t)$ is the quasiparticle energy (153) with the mass (147)

$$m^2(t) = a + \mu^2 \cos \frac{\eta_0}{b}. \tag{186}$$

The KE (163) is defined via the factor (162) which can be presented as

$$W(\mathbf{p}, t) = -\frac{\mu^2}{4b\,\omega^2(\mathbf{p}, t)}\,\dot{\eta}_0\,\sin \frac{\eta_0}{b}. \tag{187}$$

The mass formula (186) allows one to fit parameters a, μ^2, and $m^2(t) > 0$ for any amplitude of the quasi-classical field $\eta_0(t)$, as is seen in Fig. 18. The qualitative behavior of the presented observable is similar to that for the bistable potential case. Oscillations of the condensate energy are more pronounced for the symmetric potential that is related with higher initial values of η_0.

It was shown that if one vacuum state among a set of vacuum states is fixed, it leads to some specific evolution of the system under action of the mean field. But there is an open question concerning occupation of other system states due to tunnelling process [70]. This problem follows also from our treatment.

SUMMARY

This review is mainly based on the results obtained by the authors in the last years and aims to summarize the information about the kinetic description of vacuum particle creation. The latter results for the inertial mechanism with the time-dependent particle masses are stipulated at the phenomenological level. Three basic quantum field models were considered here: massive scalar, vector and spinor fields. The constructed kinetic theory was applied (Sec. 4) to a conformal cosmology model for investigation of matter created from vacuum in an early period of the Universe evolution. In particular, it was shown that the density of the produced vector bosons is sufficient for explanation of the present-day density of CMB photons.

The obtained results can be used for a subsequent study of different aspects of matter dynamics created from the vacuum in the early Universe (the equation of state, the long wave-length acoustic excitations, the back-reaction problem, etc.). Only the single mechanism of a mass change was considered where this change is induced by the conformal expansion of the Universe whose action is switched on for the mass $m_0(t_0)$ at some arbitrary initial time t_0. For construction of a more elaborated theory, one should eventually take into account the inflation mechanism of mass generation acting during an earlier period of the Universe evolution [72, 94–96]. It would be of interest to consider the generation of particles of different masses and quantum statistics using Eq. (139) which is valid for all particles independently of their inner symmetry.

Interesting perspectives are opened up also by results of Sec. 5, where some simplest quantum field models are investigated for scalar fields with various self-interactions and a corresponding quasi-classical nonstationary field (the problem of a phase transition at the restoration of broken symmetry, particle tunnelling between states with a different vacua, etc.).

Certainly, the considered examples do not exhaust all variety of systems, where the vacuum generation of particles is induced by the inertial mechanism. In particular, a large class of experimentally controlled models of meson and quark subsystems evolved close to a phase transition has remained beyond our scope. In addition, the investigation of two-particle correlations is of interest for description of some delicate experimental effects in nuclear reactions [23]. The extension of the mean-field approximation to take into account the back-reaction and collisions is studied intensively with different methods in some scalar models [85, 87]. The important problem of relevant initial conditions also attracts attention in the context of heavy-ion collisions [97]. We did not worry much concerning relativistic invariant form of the developed formalism: it is an open question. The basis for its solution is the covariant Hamiltonian formalism in relativistic kinetic theory [13, 31, 32].

Finally, some effects discussed here take place also in condensed matter physics and can be interpreted in the conceptual framework of the inertial mechanism (e.g., [98, 99] and references therein). Such a kind of analogy is very promising for an experimental test of the given theory with more accessible realizations under condensed matter conditions.

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REFERENCES

1. *Smolyansky S. A., Skokov V. V., Prozorkevich A. V.* Kinetic Theory of the Quantum Field Systems with Unstable Vacuum // PEPAN Lett. 2005. V. 2. P. 50–60.

2. *Veneziano G.* $U(1)$ without Instantons // Nucl. Phys. B. 1979. V. 159. P. 213–224;
Di Vecchia P., Veneziano G. Chiral Dynamics in the Large N Limit // Nucl. Phys. B. 1980. V. 171. P. 253–272;
Di Vecchia P. et al. Large N Chiral Approach to Pseudoscalar Masses, Mixings and Decays // Nucl. Phys. B. 1981. V. 181. P. 318–334.

3. *Witten E.* Large N Chiral Dynamics // Ann. Phys. 1980. V. 128. P. 363–375;
Witten E. Theta Dependence in the Large N Limit of Four-Dimensional Gauge Theories // Phys. Rev. Lett. 1998. V. 81. P. 2862–2865.

4. *Blaschke D. B. et al.* A Kinetic Approach to η' Production from a CP-Odd Phase // Phys. Rev. D. 2002. V. 65. P. 054039.

5. *Volkov M. K.* Meson Lagrangians in a Superconductor Quark Model // Ann. Phys. 1984. V. 157. P. 282–303;
Volkov M. K. Low-Energy Meson Physics in the Quark Model of Superconductivity Type // Sov. J. Part. Nucl. 1986. V. 17. P. 186–203.

6. *Koch V.* Aspects of Chiral Symmetry // Intern. J. Mod. Phys. E. 1997. V. 6. P. 203–250.

7. *Baier R., Dirks M., Redlich K.* Thermal Dileptons from π–ρ Interactions in a Hot Pion Gas // Phys. Rev. D. 1997. V. 55. P. 4344–4354.

8. *Florkowski W.* Description of Hot Compressed Hadronic Matter Based on an Effective Chiral Lagrangian // Acta Phys. Polon. B. 1997. V. 28. P. 2079–2205;
Florkowski W. et al. Chirally Invariant Transport Equations for Quark Matter // Ann. Phys. 1996. V. 245. P. 445–463.

9. *Pisarski R.* Phenomenology of the Chiral Phase Transition // Phys. Lett. B. 1982. V. 110. P. 155–158.

10. *Furnstahl R. J., Hatsuda T., Lee S. H.* Applications of QCD Sum Rules at Finite Temperature // Phys. Rev. D. 1990. V. 42. P. 1744–1756.

11. *Cooper F., Mottola E.* Quantum Back Reaction in Scalar QED as an Initial Problem // Phys. Rev. D. 1989. V. 40. P. 456–464.

12. *Müller M.M.* Comparing Boltzmann vs. Kadanoff–Baym // J. Phys.: Conf. Ser. 2006. V. 35. P. 390–397.

13. *Smolyansky S. A. et al.* A Covariant Generalization of Real-Time Green's Functions Method // Ann. Phys. 1999. V. 277. P. 193–218.

14. *Fujii Y., Maeda K.-I.* The Scalar-Tensor Theory of Gravitation. Cambrige Univ. Press, 2004.

15. *Garbrecht B., Prokopec T., Schmidt M. G.* Particle Number in Kinetic Theory // Eur. Phys. J. C. 2004. V. 38. P. 135–143.

16. *Blaschke D. B. et al.* Cosmological Production of Vector Bosons and Cosmic Microwave Background Radiation // Phys. At. Nucl. 2004. V. 67. P. 1074–1086.

17. *Pervushin V. N., Proskurin D. V., Gusev A. A.* Cosmological Particle Origin in the Standard Model // Grav. Cosmology. 2002. V. 8. P. 181–189.

18. *Slavnov A. A.* Renormalizable Electroweak Model without Fundamental Scalar Mesons. hep-th/0601125.

19. *Sauter F.* Über das Verhalten eines Electrons im Homogenen Elektrischen Feld nach der Relativistische Theorie Diracs // Z. Phys. 1931. Bd. 69. S. 742–764.

20. *Heisenberg W., Euler H.* Consequences of Dirac's Theory of Positrons // Z. Phys. 1936. Bd. 98. S. 714–732.

21. *Schwinger J.* On Gauge Invariance and Vacuum Polarization // Phys. Rev. 1951. V. 82. P. 664–679.

22. *Andreev I. V.* Reaction of the Fermion Field on Spontaneous Change of Its Mass. hep-ph/0208255;

    Andreev I. V. Fermion Production and Correlations due to Time Variation of Effective Mass // Phys. At. Nucl. 2003. V. 66. P. 1335–1341.

23. *Andreev I. V., Weiner R. M.* Production of a Chaotic Squeezed State from a «Pion Liquid» and Overburning of Identical Pion Correlations // Phys. Lett. B. 1996. V. 373. P. 159;

    Asakawa M., Csorgo T. Strangeness Correlation: A Clue to Hadron Modification in Dense Matter // Heavy Ion Phys. 1996. V. 4. P. 233–240.

24. *Pervushin V. N. et al.* The Kinetic Description of Vacuum Particle Creation in the Oscillator Representation // Intern. J. Mod. Phys. A. 2005. V. 20. P. 5689–5704.

25. *Pervushin V. N., Smirichinski V. I.* Bogoliubov Quasiparticles in Constrained Systems // J. Phys. A. 1999. V. 21. P. 6191–6201.

26. *Grib A. A., Mamaev S. G., Mostepanenko V. M.* Vacuum Quantum Effects in Strong External Fields. Friedman Lab. Publ. SPb., 1994.

27. *Bialynicki-Birula I., Górnicki P., Rafelski J.* Phase Space Structure of the Dirac Vacuum // Phys. Rev. D. 1991. V. 44. P. 1825–1835.

28. *Krekora P., Su Q., Grobe R.* Relativistic Electron Localization and the Lack of Zitterbewegung // Phys. Rev. Lett. 2004. V. 93. P. 043004.

29. *Schmidt S. M. et al.* A Quantum Kinetic Equation for Particle Production in the Schwinger Mechanism // Intern. J. Mod. Phys. E. 1998. V. 7. P. 709–722.

30. *Blaschke D. B. et al.* Kinetic Description of Vacuum Creation of Massive Vector Bosons // Phys. At. Nucl. 2005. V. 68. P. 1087–1094.

31. *Zakharov A. F., Zinchuk V. A., Pervushin V. N.* Tetrad Formalism and Frames of References in General Relativity // Part. Nucl. 2006. V. 37. P. 104–134 (Translated from: Fiz. Elem. Chast. At. Yadra. 2006. V. 37. P. 183–244).

32. *Barbashov B. M. et al.* Hamiltonian General Relativity in Finite Space and Cosmological Potential Perturbations // Intern. J. Mod. Phys. A. 2006. V. 21. P. 5957–5990.

33. *Behnke D. et al.* Description of Supernova Data in Conformal Cosmology without Cosmological Constant // Phys. Lett. B. 2002. V. 530. P. 22–26.

34. *Birrell N. D., Davies P. C. W.* Quantum Fields in Curved Spaces. Cambridge: Cambridge Univ. Press, 1982.

35. *Fradkin E. S., Gitman D. M.* Vacuum Instability in External Fields // Phys. Rev. D. 1996. V. 53. P. 7162–7175.

36. *Schmidt S. M. et al.* Non-Markovian Effects in Strong-Field Pair Creation // Phys. Rev. D. 1999. V. 59. P. 094005.

37. *Vinnik D. V. et al.* Plasma Production and Thermalization in a Strong Field // Eur. Phys. J. C. 2001. V. 22. P. 341–349.

38. *Vinnik D. V. et al.* Kinetic Description of Vacuum Particle Production in Collisions of Ultrarelativistic Nuclei // Phys. At. Nucl. 2001. V. 64. P. 775–786.

39. *Prozorkevich A. V., Smolyansky S. A., Ilyin S. V.* A Vlasov Type Kinetic Equation for Quarks in Strong Gluon Fields // Progress in Nonequilibrium Green's Functions II / Eds. M. Bonitz and D. Semkat. Singapore, 2003. P. 401–408.

40. *Smolyansky S. A. et al.* Nonperturbative Dynamics of Expanding Flux Tube in the Theory of Heavy Ion Collisions // Proc. of the 8th Intern. Workshop «Relativistic Nuclear Physics: from Hundreds MeV to TeV», May 23–28, 2005. Dubna, 2006. P. 183–188.

41. *Kadanoff L. P., Baym G.* Quantum Statistical Mechanics. N. Y., W. A. Benjamin Inc., 1962.

42. *Smolyansky S. A. et al.* Collision Integrals in the Kinetics of Vacuum Particle Creation in Strong Fields // Proc. of Conf. «Progress in Nonequilibrium Green's Functions II». Singapore, 2003. P. 384–392.

43. *Tagirov E. A., Chernikov N. A.* Quantum Theory of a Scalar Field in the de-Sitter Space // Ann. Inst. Henri Poincare. 1968. V. 9. P. 109–141.

44. *Bronnikov K. A., Tagirov E. A., Chernikov N. A.* Quantum Theory of a Scalar Field in Isotropic World. JINR Preprint 2-4151. Dubna, 1968.

45. *Parker L.* Quantized Fields and Particle Creation in Expanding Universes // Phys. Rev. 1969. V. 183. P. 1057–1068;

    Parker L. Conformal Energy-Momentum Tensor in Riemannian Space–Time // Phys. Rev. D. 1973. V. 7. P. 976–983.

46. *Popov V. S., Marinov M. S.* Pair Production in Electromagnetic Field (Case of Arbitrary Spin) // Yad. Fiz. 1972. V. 16. P. 809–822.

47. *Zeldovich Ya. B., Starobinsky A. A.* Particle Production and Vacuum Polarization in an Anisotropic Gravitational Field // Sov. Phys. JETP. 1972. V. 34. P. 1159.

48. *Casher A., Kogut J., Susskind L.* Vacuum Polarization and the Absence of Free Quarks // Phys. Rev. D. 1974. V. 10. P. 732–745.

49. *Prozorkevich A. V. et al.* The Pair Creation in the Optical Laser Fields // Proc. of SPIE. 2004. V. 5476. P. 68–72.

50. *Blaschke D. B. et al.* Observable Manifestation of an Electron–Positron Plasma Created by the Field of an Optical Laser // J. Phys.: Conf. Ser. 2006. V. 35. P. 121–126.

51. *Blaschke D. B. et al.* Pair Production and Optical Laser // Phys. Rev. Lett. 1996. V. 96. P. 140–402.

52. *Fedoruk M. V.* Asymptotic: Integrals and Series. M.: Nauka, 1987.

53. *Mamaev S. G., Mostepanenko V. M., Frolov V. M.* Fermion Pairs Production with Nonstationary Gravitational Field // Yad. Fiz. 1976. V. 23. P. 1118–1127.

54. *Filatov A. V., Prozorkevich A. V., Smolyansky S. A.* Pair Creation by Electromagnetic Wave in a Collisionless Plasma // Proc. of SPIE. V. 6165. P. 616509.

55. *Pervushin V. N., Skokov V.V.* Kinetic Description of Fermion Production in the Oscillator Representation // Acta Phys. Polon. B. 2006. V. 37. P. 2587–2600.

56. *Bogolyubov N. N., Shirkov D. V.* Introduction to the Theory of Quantum Fields. 3-rd ed. Wiley, 1980.

57. *Rayder L. H.* Quantum Field Theory. Cambridge: Cambridge Univ. Press, 1985.

58. *Silin V. P.* Introduction in the Kinetic Theory of Gases. M.: Nauka, 1971.

59. *Rehberg P., Kalinovsky Yu., Blaschke D. B.* Critical Scattering and Two Photon Spectra for a Quark/Meson Plasma // Nucl. Phys. A. 1997. V. 622. P. 478–496.

60. *Blaschke D. B. et al.* Low-Momentum π-Meson Production from Evolving Quark Condensate // Proc. of the Intern. Conf. on Dense Matter, Dubna, Aug. 21 — Sept. 1, 2006; Part. Nucl., Lett. (in press).

61. *Wentzel G.* Quantum Theory of Fields. N. Y., 1949.

62. *Pavel H.-P., Pervushin V. N.* Reduced Phase Space Quantization of Massive Vector Theory // Intern. J. Mod. Phys. A. 1999. V. 14. P. 2285.

63. *Gusev A. A. et al.* Cosmological Creation of W and Z Bosons and the Large-Scale Structure of the Universe // Problems of Gauge Theories / Ed. by Barbashov B. M., Nestetrenko V. V. Dubna, 2004. P. 127–131.

64. *Mamaev S. G., Trunov N. N.* Vacuum Polarization and Particle Creation in Nonstationary Homogeneous Electromagnetic Field // Yad. Phys. 1979. V. 30. P. 1301–1311.

65. *Rau J., Müller B.* From Reversible Quantum Microdynamics to Irreversible Quantum Transfer // Phys. Rep. 1996. V. 272. P. 1–59.

66. *Grib A. A., Mamaev S. G.* Field Theory in the Friedman Space // Yad. Fiz. 1969. V. 10. P. 1276–1281.

67. *Sexl R. U., Urbantke H. K.* Production of Particles by Gravitational Fields // Phys. Rev. 1969. V. 179. P. 1247–1250.

68. *Zeldovich Ya. B.* Particle Creation in Cosmology // Pis'ma Zh. Exp. Teor. Fiz. 1970. V. 12. P. 443–447;

    Zeldovich Ya. B., Starobinsky A. A. Particle Creation and Vacuum Polarization in Anisotropic Gravitational Field // Zh. Exp. Teor. Fiz. 1971. V. 61. P. 2161–2175.

69. *Zeldovich Ya. B., Novikov I. D.* Structure and Evolution of the Universe. Chicago: Chicago Univ. Press, 1983.

70. *Linde A.* Particle Physics and Inflationary Cosmology. Harwood Acad. Publ., 1990.

71. *Blaschke D., Dabrowski M. P.* Conformal Relativity versus Brans–Dicke and Superstring Theories. hep-th/0407078.

72. *Dolgov A.* Particle Production in Cosmology and Imaginary Time Method // Multiple Facets of Quantization and Supersymmetry. Singapore., 2002. P. 104.

73. *Gershtein S. S., Logunov A. A., Mestvirishvili M. A.* Gravitational Field Self-Limitation and Its Role in the Universe // Phys. Usp. 2006. V. 49. P. 1207–1255.

74. *Komatsu E., Spergel D. N.* Acoustic Signatures in the Primary Microwave Background Bispectrum // Phys. Rev. D. 2001. V. 63. P. 063002.

75. *Vernizzi F., Melchiorri A., Durrer R.* Cosmic Microwave Background Anisotropies from Pre-Big Bang Cosmology // Phys. Rev. D. 2001. V. 63. P. 063501.

76. *Smolyansky S. A. et al.* Kinetic Description of Particles Creation in Kasner Space-Time // Proc. of Intern. Workshop «Hot Point in Astrophysics». Dubna, 2000. P. 364–370.

77. *Rama S. K.* Holographic Principle in the Closed Universe: a Resolution with Negative Pressure Matter // Phys. Lett. B. 1999. V. 457. P. 268–247.

78. *Rubakov V. A.* Phantom without Pathologies in Ultra-Violet Region // Theor. Math. Phys. 2006. V. 149. P. 409–426.

79. *Dabrowski M. P., Stelmach J.* Observable Quantities in Cosmological Models with Strings // Astrophys. J. 1989. V. 97. P. 978–985.

80. *Veryaskin A. V., Lapchinsky V. G., Rubakov V. A.* Quantization of Massive Vector Field in Homogeneous and Isotropic Space. Preprint P-0198. Inst. of Nucl. Res. M., 1981;

Veryaskin A. V., Lapchinsky V. G., Rubakov V. A. About Spontaneously Broken Symmetry in Closed Cosmological Fridman Model // Theor. Math. Phys. 1980. V. 45. P. 407.

81. *Vysotskii M. I., Nevzorov R. B.* Selected Problems of Supersymmetry Phenomenology // Phys. Usp. 2001. V. 44., No. 9. P. 919–930.

82. *Cooper F. et al.* Particle Production in the Central Rapidity Region // Phys. Rev. D. 1993. V. 48. P. 190–208.

83. *Shuryak E. V.* The QCD Vacuum, Hadrons and the Superdense Matter. World Sci. Publ. Co., 1988.

84. *Kaiser D. I.* Larger Domains from Resonant Decay of Disoriented Chiral Condensates // Phys. Rev. D. 1999. V. 59. P. 117901.

85. *Ahrensmeier D., Baier R., Dirk D.* Resonant Decay of Parity Odd Bubbles in Hot Hadronic Matter // Phys. Lett. B. 2000. V. 484. P. 58–63.

86. *Bloch J. C. et al.* Pair Creation: Back-Reaction and Damping // Phys. Rev. D. V. 61. P. 117502.

87. *Baacke J., A. Heinen A.* Nonequilibrium Evolution of Φ^4 Theory in $1+1$ Dimensions in the Two-Particle Point-Irreducible Formalism // Phys. Rev. D. 2003. V. 67. P. 105020.

88. *Friedberg R., Lee T. D.* Fermion-Field Nontopological Solitons: II. Models for Hadrons // Phys. Rev. D. 1977. V. 16. P. 1096–1118.

89. *Loh S. et al.* Dynamical Fragmentation of Flux Tubes in the Friedberg–Lee Model // Nucl. Phys. A. 1997. V. 619. P. 321–345.

90. *Hamazaki T. et al.* Self-Excitation of the Tunneling Scalar Field in False Vacuum Decay // Phys. Rev. D. 1996. V. 53. P. 2045–2061.

91. *Berestetsky V. B., Livshitz E. M., Pitaevsky L. P.* Quantum Electrodynamics. M.: Nauka, 1980.

92. *Andreev A. Yu., Kirzhnitz D. V.* Tachyons and the Instability of Physical Systems // Phys. Usp. 1996. V. 39. P. 1071–1076.

93. *Cornwall J. M., Jackiw R., Tomboulis E.* Effective Action for Composite Operators // Phys. Rev. D. 1974. V. 10. P. 2428–2445.

94. *Green P. B., Kofman L.* Preheating of Fermions // Phys. Lett. B. 1999. V. 448. P. 6–12.

95. Proc. of the 18th IAP Astrophys. Colloquium «On the Nature of Dark Energy». July 1–5, 2002 / Eds.: Ph. Brax, J. Martin, J.-Ph. Uzan. Paris, 2002.

96. *Casas J. A., Garcia-Bellido J., Quiros M.* Scalar-Tensor Theories of Gravity with ϕ-Dependent Masses // Class. Quant. Grav. 1992. V. 9. P. 1371–1384;

    Anderson G. W., Carroll S. M. Dark Matter with Time-Dependent Mass. astro-ph/9711288.

97. *Berges J.* Controlled Nonperturbative Dynamics of Quantum Fields Out of Equilibrium // Nucl. Phys. A. 2002. V. 699. P. 847–886;

    Berges J., Serreau J. Parametric Resonance in Quantum Field Theory // Phys. Rev. Lett. 2003. V. 91. P. 111601;

    Berges J., Borsanyi Sz., Wetterich C. Isotropization Far from Equilibrium // Nucl. Phys. B. 2005. V. 727. P. 244.

98. *Green A. G., Sondhi S. L.* Nonlinear Quantum Critical Transport and the Schwinger Mechanism for a Superfluid-Mott-Insulator Transition of Bosons // Phys. Rev. Lett. 2005. V. 95. P. 267001.

99. *Oka T., Aoki H.* Ground-State Decay Rate for the Zener Breakdown in Band and Mott Insulators // Ibid. P. 137601.