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NONLOCAL QUARK MODEL BEYOND MEAN FIELD *

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A nonlocal chiral quark model is extended beyond mean field using a strict $1/N_c$ expansion scheme. The nonlocal interaction has the advantage that all diagrams are finite and leads to a unique evaluation of $1/N_c$ corrections. The parameters of the nonlocal model are refitted making use of the physical values of the pion mass and the weak pion decay constant. The size of the $1/N_c$ correction to the quark condensate is carefully studied in nonlocal and local Nambu–Jona-Lasinio models. It is found that even the sign of the corrections can be different.

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INTRODUCTION

A quantum field theoretical description of strong interactions in the nonperturbative regime is one of the most interesting and challenging problems of present-day theoretical physics. Quantum chromodynamics (QCD) is well known only at the *perturbative* level, whereas the low-energy region is in the nonperturbative regime. The only nonperturbative *ab initio* calculations are performed in lattice QCD, but their range of applicability is still limited. To gain some analytical insights to nonperturbative QCD, continuum approaches, even using effective models, are legitimate tools. They may provide a theoretical interpretation of results from lattice QCD and allow their extrapolation to otherwise inaccessible domains.

One of the successful models for a description of chiral dynamics is the quark Nambu–Jona-Lasinio model. This model provides a mechanism for spontaneous chiral symmetry breaking by quark condensate formation.

Usually, the NJL model is formulated at the mean-field (MF) level. However, there are physical problems where the MF formulation is not sufficient. Large corrections to a MF behavior can be expected, e.g., in the description of broad resonances from their coupling to intermediate meson states.

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There are different schemes to go beyond the MF level [1–5]. One of the most promising beyond MF schemes is based on a strict expansion in the inverse number of quark colors, $1/N_c$, which is a natural expansion parameter for gauge theories [6]. The local NJL model is nonrenormalizable, and therefore it is necessary to introduce an additional cutoff parameter when going beyond the MF level. This problem is absent in a nonlocal version of the NJL model where the nonlocality leads to an effective regularization which renders the (multi-)loop quark diagrams convergent. In the present contribution we discuss an extension of the nonlocal model beyond mean field within a strict $1/N_c$ expansion scheme.

1. MF APPROXIMATION

The quark sector of the nonlocal chiral quark model is described by the Lagrangian

$$\mathcal{L}_q = \bar{q}(x)(i\not{D} - m_c)q(x) + \frac{G}{2}[J_\sigma^2(x) + \mathbf{J}_\pi^2(x)], \tag{1}$$

where m_c is the current quark mass, and $D_\mu = \partial_\mu - iA_\mu$ is the covariant derivative with a background gluon field $A_\mu \equiv A_\mu^a\,(\lambda^a/2) = \delta_{\mu 0}A_0$. The nonlocal quark currents are

$$J_M(x) = \int d^4(x_1x_2) f(x_1) f(x_2) \bar{q}(x - x_1)\mathbf{\Gamma}_M q(x + x_2), \tag{2}$$

where $\mathbf{\Gamma}_\sigma = 1$, $\mathbf{\Gamma}_\pi = i\gamma^5\tau^a$ with $a = 1, 2, 3$. Spontaneous breaking of chiral symmetry leads to the formation of a quark condensate and generates a dynamical contribution to the quark mass. As a result, the Euclidean quark propagator takes the form

$$S_p = (i\not{p} + m(p^2))^{-1}, \quad m(p^2) = m_c + m_d f^2(p^2),$$

where $f^2(p^2) = \exp(-p^2/\Lambda^2)$ is a (Fourier transformed) Gaussian form-factor and m_d is an order parameter for dynamical chiral symmetry breaking. The chiral condensate is obtained from the nonperturbative part of the quark propagator, $S_p^{np} = S_p - S_p^c$, $S_p^c = (i\not{p} + m_c)^{-1}$.

2. BEYOND MF

Corrections to the dynamical quark mass beyond MF can be accounted for in a systematic $1/N_c$ expansion scheme [2–4] for the quark selfenergy, $\Sigma_p^{N_c} = i\not{p}A_p + B_p$ (see Fig. 1), and the quark propagator

$$\left(S_p^{\mathrm{MF}+N_c}\right)^{-1} = S_p^{-1} + \Sigma_p^{N_c}, \quad S_p^{\mathrm{MF}+N_c} \approx S_p - S_p\Sigma_p^{N_c}S_p + \ldots \tag{3}$$

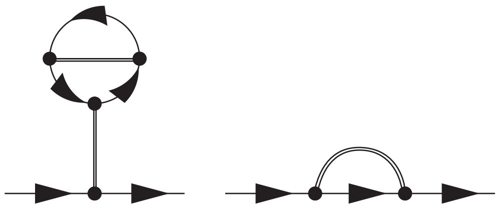

Fig. 1. $1/N_c$ corrections to the quark propagator

In order to arrive at a consistent approximation, one needs to take into account $1/N_c$ corrections to the meson propagator, see [4] for the NJL model and [5] for its nonlocal generalization. For the present model, we employ a diagrammatic technique developed in [7]. The $1/N_c$ corrections to meson properties will affect the results for the quark condensate via the readjustment of the model parameters (Λ, m_c, $G\Lambda^2$) which are to be chosen such that the physical values for pion mass $M_{\pi^\pm} = 139.57$ MeV and weak pion decay constant $f_\pi = 92.42$ MeV are obtained, while the dimensionless coupling $G\Lambda^2$ is left as a free parameter. Different parametrizations of the nonlocal model beyond mean field are given in the Table. The corresponding quark condensate is presented in Fig. 2. The parametrization No. 4 provides the highest (pseudo)threshold value of the external momentum in the quark loop. In the Table we also present the MF contributions to the pion mass and weak decay constant for different parameter sets. For lower values of the current (and dynamical) quark masses the $1/N_c$ corrections to the pion mass and weak pion decay constant amount to 15 and 20 MeV, respectively. For set No. 4 the corrections are only about 2 and 5 MeV.

Different parametrizations fitted to $M_{\pi^\pm}$ and f_π. The last two columns are the MF contributions to M_π, f_π

No.	Λ, MeV	m_c, MeV	m_d, MeV	$G\Lambda^2$	$M_\pi^{\rm MF}$, MeV	$f_\pi^{\rm MF}$, MeV
1	1479.2	2.82	139.2	13.35	155.5	72.6
2	934.8	5.58	211.2	14.89	144.6	83.4
3	705.9	8.64	269.1	17.06	142.5	87.1
4	**670.3**	**9.31**	**281.9**	**17.64**	**142.2**	**87.6**
5	580.5	11.78	322.5	19.72	141.7	88.7
6	500.8	14.95	373.8	22.83	141.4	89.6
7	445.3	18.15	424.0	26.33	141.2	90.0
8	404.4	21.37	473.4	30.20	141.1	90.3

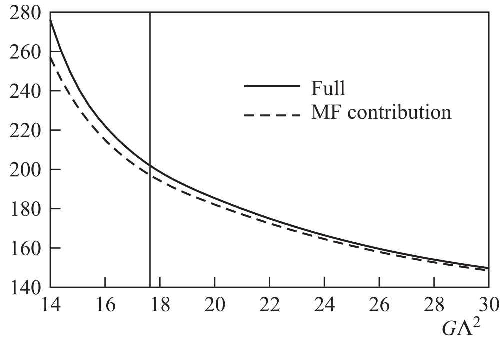

Fig. 2. Absolute value of the quark condensate to the power 1/3 in MeV. The solid line is the total result. The dashed line is the mean-field contribution. The vertical line corresponds to the dimensionless coupling in parameter set No. 4

Figure 2 shows that the $1/N_c$ correction to the absolute value of quark condensate is positive for all sets of model parameters. In the local NJL model of [4] it was found that this correction is negative. However, in the local NJL model due to its nonrenormalizability it is necessary to introduce different regularizations for the pure quark and the meson–quark loops, respectively. In [4] a Pauli–Villars regularization has been used for quark loops and a three-dimensional momentum cutoff Λ_M for meson–quark loops. In order to study the transition from the nonlocal model to a local one, let us construct a nonlocal model with three parameters:

1) parameter of nonlocality Λ,
2) parameter of quark loop regularization Λ_q,
3) parameter of meson loop regularization Λ_M.

The local model corresponds to the limit

$$\Lambda \to \infty, \quad \Lambda_q = \Lambda_q^{\rm phys}, \quad \Lambda_M = \Lambda_M^{\rm phys}, \tag{4}$$

while the nonlocal model without regularization can be obtained by setting

$$\Lambda = \Lambda^{\rm phys}, \quad \Lambda_q \to \infty, \quad \Lambda_M \to \infty. \tag{5}$$

For definiteness, let us compare the local model [4] with the nonlocal one from [8] with parametrizations fixed in the MF approximation.

The next step is to consider the $1/N_c$ corrections and to investigate the role of the mesonic 3D cutoff Λ_M. For this purpose it is very instructive to study the ratio of the full quark condensate to the MF contribution $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle^{\rm MF}$. In Fig. 3 we compare the Λ_M dependence of this ratio for the local NJL model as given in [4] to that of the nonlocal model and its local limit. It is very interesting that

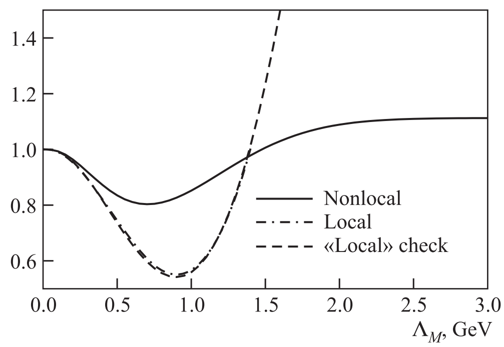

Fig. 3. The ratio $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle^{\rm MF}$ as a function of the meson cutoff Λ_M. The local result (dash-dotted line) is taken from Fig. 3, *a* [4]. «Local» check (dashed line) denotes the local limit of nonlocal calculations and the solid line is the nonlocal result

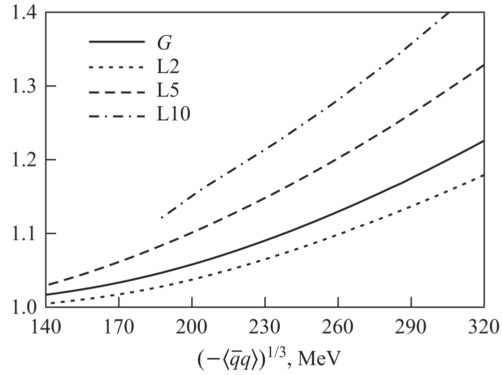

Fig. 4. The ratio $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle^{\rm MF}$ as a function of the quark condensate for different form-factors: Gaussian and n-Lorentzian with $n = 2, 5$ or 10

in the region below ~ 1.5 GeV these models predict a negative sign for the $1/N_c$ correction, whereas for large mesonic cutoff the sign is positive. However, in the nonlocal model the absolute value of the correction saturates for Λ_M larger than ~ 2.5 GeV, which is well above actual parametrizations for Λ_q and Λ_M in [4].

In order to study the dependence of the sign of the $1/N_c$ correction to the quark condensate on the form-factor, we consider n-Lorentzian form-factors, $f(p^2) = 1/(1 + (p^2/\Lambda^2)^n)$, with $n = 2, 5$ or 10 (see [9]). We found that the sign of $1/N_c$ correction is positive for all possible parametrizations, see Fig. 4.

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