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FIELDS WITH CONTINUOUSLY DISTRIBUTED MASS

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We describe local field theories with continuously distributed mass. Such models can be realized as models in $d > 4$ space-time with Poincare invariance only in four-dimensional space-time. We also discuss some possible phenomenological consequences. Namely, we show that the Higgs boson phenomenology in the SM extension with continuously distributed Higgs boson mass can differ in a drastic way from the standard Higgs boson phenomenology.

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INTRODUCTION

In this report based on [1–3]*, we discuss local field theories with continuously distributed mass. We show that such models could be renormalizable. Moreover, such models can be realized as models in $d > 4$ space-time with Poincare invariance only in four-dimensional space-time. We also discuss some possible phenomenological consequences. Namely, we show that the Higgs boson phenomenology in the SM extension with continuously distributed Higgs boson mass can differ in a drastic way from the standard Higgs boson phenomenology, We also point out that the notion of an unparticle, introduced by Georgi [7], can be interpreted as a particular case of a field with continuously distributed mass.

1. SOME EXAMPLES

Let us start with N scalar fields $\phi_k(x)$ with masses m_k $(k = 1, 2, \ldots, N)$. For the field $\phi(x, m_k, c_k, N) = \sum\limits_{k=1}^{N} c_k \phi_k(x)$ free propagator has the form

$$D(p^2, m_k, c_k, N) = \sum_{k=1}^{N} \frac{|c_k|^2}{(p^2 - m^2 + i\epsilon)} = \int\limits_0^\infty \frac{\rho(t, c_k, m_k, N)}{p^2 - t + i\epsilon} dt, \qquad (1)$$

*See also [4–6].

where the spectral density is $\rho(t, c_k, m_k, N) = \sum_{k=1}^{N} |c_k|^2 \delta(t - m_k^2)$. In the limit $N \to \infty \rho(t, c_k, m_k, N) \to \rho(t)$ and the propagator $D(p^2, m_k, c_k, N) \to D(p^2) = \int_0^\infty \frac{\rho(t)}{p}^2 - t + i\epsilon dt$. For instance, for $m_k^2 = m_0^2 + k/N\Delta^2$ and $|c_k|^2 = 1/N$ we find that the limiting spectral density is $\rho(t) = 1/\Delta^2 \theta(t - m^2)\theta(m^2 + \Delta^2 - t)$. For the limiting spectral density $\rho(t) \sim t^{\delta-1}$ we find that $D(p^2) \sim (p^2)^{\delta-1}$, that corresponds to the case of unparticle propagator. In other words, for the limiting spectral density $\rho(t) \sim t^{\delta-1}$ the field $\phi(x, \rho(t))$ can be interpreted as unparticle *. One can introduce the self-interaction Lagrangian in standard way as

$$L_{\text{int}}(\phi(x, \rho(t)) = -\lambda(\phi(x, \rho(t)))^4. \tag{2}$$

For finite $\int_0^\infty \rho(t)\, dt$ the asymptotics of propagator $D(p^2) \sim 1/p^2$ and the model (2) is renormalizable. It should be noted that for Georgi noninteracting scalar unparticle the effective Lagrangian has the form

$$L_{\text{unp}} = \frac{1}{2}\partial_\mu \phi \left(-\frac{\partial^\mu \partial_\mu}{M^2} \right)^{-\delta} \partial^\mu \phi. \tag{3}$$

The fields with continuously distributed mass arise naturally in d-dimensional field theories. Consider five-dimensional scalar field with the Lagrangian

$$L_5 = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - \phi f(-\partial_4^2)\phi), \tag{4}$$

where $\mu = 0, 1, 2, 3$. The Lagrangian (4) is invariant only under the four-dimensional Poincare group and for arbitrary $f(-\partial_4^2)$ it is not invariant under five-dimensional Poincare group**. For the Lagrangian L_5 free propagator has the form

$$D_0 = \frac{1}{p_\mu p^\mu - f(p_4^2)}. \tag{5}$$

For the field $\phi(x, x_4 = 0)$ propagator is proportional to $\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{dp_4}{p_\mu p^\mu - f(p_4^2) + i\epsilon}$, that corresponds to the case of the field with continuously distributed mass. Usually in the literature only models in $d > 4$ space-time with d-dimensional Poincare group invariant Lagrangians are considered. However it should be stressed that from the experimental point of view we have to postulate the invariance

*The interpretation of the unparticle as a tower of massive particles was also proposed in [8].
**The Lagrangian (4) is invariant under five-dimensional Poincare group for $f(-\partial_4^2) = -\partial_4^2$.

of the Lagrangian only under the four-dimensional Poincare group. For the interaction

$$L_I = -\lambda\phi^4 \tag{6}$$

and for some functions $f(p_4^2)$ the model is renormalizable*. Remember that standard ϕ^4model is nonrenormalizable in $d > 4$ space-time.

It is possible to construct models where some fields exist in four-dimensional space-time (four-dimensional brane) and other fields live in $(d > 4)$ space-time. One of the simplest examples is the model [1] where scalar field $\phi(x, x_4)$ propagates in five-dimensional space-time and interacts with the four-dimensional fermion field $\psi(x)$. The action of the model has the form

$$S_t = S_1 + S_2 + S_i, \tag{7}$$

where

$$S_1 = \int \frac{1}{2}[\partial_\mu\phi(x, x_4)\partial^\mu\phi(x, x_4) - \phi(x, x_4)f(-\partial_4^2)\phi(x, x_4)]d^5x, \tag{8}$$

$$S_2 = \int \bar{\psi}(x)[i\gamma^\mu\partial_\mu]\psi(x)d^4x, \tag{9}$$

$$S_i = \int [h\phi(x, x_4 = 0)\psi\bar{(x)}\psi(x) - \lambda\phi^4(x, x_4 = 0)]d^4x, \tag{10}$$

where $x = (x_0, x_1, x_2, x_3)$, $\partial_4 = \partial/\partial x_4$, $d^5x = d^4x\, dx_4$, $d^4x = dx_0\, dx_1\, dx_2\, dx_3$. It should be stressed that the model (7)–(10) is a local one in four-dimensional space-time. The Feynman rules for the model (7)–(10) coincide with the Feynman rules for the four-dimensional Yukawa model, the single difference is that instead of the free propagator $(p^2 - m^2 + i\epsilon)^{-1}$ for the standard four-dimensional scalar field we have to use the effective propagator

$$D^{\text{eff}}(p^2) = (2\pi)^{-1} \int [p^2 - f(p_4^2) + i\epsilon]^{-1} dp_4 \tag{11}$$

for the four-dimensional field $\phi(x, x_4 = 0)$.

There are many generalizations to the case of vector fields. For instance, consider the Stueckelberg Lagrangian

$$L_0 = \sum_{k=1}^{N} \left[-\frac{1}{4e_k^2} F^{\mu\nu,k} F_{\mu\nu,k} + \frac{m_k^2}{2e_k^2}(A_{\mu,k} - \partial_\nu\phi_k)^2 \right], \tag{12}$$

where $F_{\mu\nu,k} = \partial_\mu A_{\nu,k} - \partial_\nu A_{\mu,k}$. The Lagrangian (12) is invariant under gauge transformations

$$A_{\mu,k} \to A_{\mu,k} + \partial_\mu\alpha_k, \tag{13}$$

*For instance, for $f(p_4^2) = m^2$ for $|p_4| \leqslant p_0$ and $f(p_4^2) = \infty$ for $|p_4| > p_0$.

$$\phi_k \to \phi_k + \alpha_k. \tag{14}$$

For the field $B_\mu = \sum_{k=1}^{N} A_{\mu,k}$ free propagator in transverse gauge is

$$D_{\mu\nu}(p) = \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\left(\sum_{k=1}^{N}\left(\frac{e_k^2}{p^2 - m_k^2}\right)\right). \tag{15}$$

In the limit $N \to \infty$

$$D_{\mu\nu}(p) \to \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2), \tag{16}$$

where

$$D(p^2) = \int\limits_0^\infty \frac{\rho(t)}{p^2 - t + i\epsilon} dt \tag{17}$$

and $\rho(t) \geqslant 0$. One can introduce the interaction of the field B_μ with fermion field ψ in standard way, namely

$$L_{\rm int} = \bar{\psi}\gamma_\mu \psi B^\mu. \tag{18}$$

The simplest generalization of the SM model consists in the the replacement of the $U(1)$ gauge field propagator

$$\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2} \to \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2). \tag{19}$$

This generalization preserves the renormalizability for finite $\int\limits_0^\infty \rho(t) dt$ because the ultraviolet asymptotics of $D(p^2)$ coincides with free propagator. For $\rho(t) \sim t^{\delta - 1}$ we reproduce the case of vector unparticle.

2. POSSIBLE PHENOMENOLOGICAL CONSEQUENCES

Consider the SM in unitary gauge, and instead of free propagator $(p^2 - m_H^2)^{-1}$ let us use the propagator $D(p^2) = \int\limits_0^\infty \rho(t)(p^2 - t)^{-1}\, dt$. For finite $\int\limits_0^\infty \rho(t)\, dt$ the model is renormalizable. For Breit–Wigner spectral density*

$$\rho_{\rm BW}(t) = \left(\frac{1}{\pi}\right)\Gamma m_H[(t - m_H^2)^2 + \Gamma^2 m_H^2]^{-1} \tag{20}$$

*For the spectral density (20) $\int \rho_{\rm BW}(t)\, dt = 1$, $\lim\limits_{\Gamma \to 0} \rho_{\rm BW}(t) = \delta(t - m_H^2)$.

one can interpret Γ as an internal decay width of the Higgs boson [1]. For $\Gamma \gg \Gamma_t$, where Γ_t is the standard Higgs boson decay width, the Higgs boson will decay mainly into invisible modes that makes the Higgs boson discovery at the LHC extremely difficult.

Similar gauge invariant generalization of the SM is the following. Let us add to the SM fields $SU_c(3) \otimes SU_L(2) \otimes U(1)$ singlet scalar field $\phi(x, \rho(t))$ with continuously distributed mass. The interaction of the field $\phi(x, \rho(t))$ with the Higgs doublet field $H(x)$ has the form

$$L_{\text{int}}(\phi_{\text{int}}(x, \rho(t)), H(x)) = -\lambda_2(\phi_{\text{int}}(x, \rho(t))H^+(x)H(x). \tag{21}$$

After electroweak symmetry breaking, the singlet field $\phi_{\text{int}}(x, \rho(t))$ will mix with the standard Higgs boson. As a result of the mixing, the Higgs boson will have invisible decay modes as in previous example.

Another example is the Z' vector boson model with continuously distributed mass. One of the possible effects due to nonzero internal decay width of the Z' boson is the existence of rather broad resonance structure in Drell–Yan reaction $pp \to Z' + \ldots \to l^+l^- + \ldots$

CONCLUSION

In this report we described quantum field theories with continuously distributed masses. It is possible to interpret such models as quantum field theory models in $d > 4$ space-time. The most interesting example is the Higgs boson with continuously distributed mass. The Higgs boson phenomenology for such model for $\Gamma \gg \Gamma_t$ is different from the standard Higgs boson phenomenology, namely, Higgs boson decays mainly into invisible modes that makes the LHC Higgs boson discovery very untrivial.

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