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THE MODEL-INDEPENDENT ANALYSIS INDICATIONS FOR SPECTROSCOPY OF SCALAR MESONS. PROOF FOR THE $K_0^*(900)$

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In a model-independent approach the data on $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, $\eta\eta'$ in the $I^G J^{PC} = 0^+0^{++}$ channel and on the $K\pi$ scattering in the $I(J^P) = (1/2)(0^+)$ channel are analyzed jointly for studying the status and QCD nature of the f_0 and the K_0^* mesons. It is shown that in the 1500-MeV region, there are two states, wide (interpreted as a glueball) and narrow ($q\bar{q}$). In the $K\pi$-scattering data analysis, the proof for the $K_0^*(900)$ is given.

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INTRODUCTION

The scalar mesons have an especial status because they possess the vacuum quantum numbers and, therefore, they can mediate the vacuum influence on hadronic spectrum. Therefore, the scalars are highly exciting and discussed topic in the light-hadron physics. However, both present knowledge about nature of discovered scalar states is still rather incomplete and their parameters are known with rather large spread [1]. For example, when observing the $f_0(1500)$, in many works analyzing mainly mesons production and decay processes and cited in the PDG issue [1], rather narrow $f_0(1500)$ was observed, whereas in the model-independent analysis of $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta$, $\eta\eta'$ [2], the wide state $f_0(1500)$ was obtained. To explain the different manifestations of the $f_0(1500)$ in processes of the various types indicated above, it is reasonable to suppose that the wide $f_0(1500)$, observed in the multichannel $\pi\pi$ scattering, indeed, is a superposition of two states, wide and narrow. The latter is observed just in processes of decay and production of mesons.

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In the scalar-isodoublet sector, except the known $K_0^*(1430)$, now there is discussed a possible broad meson $K_0^*(900)$ [1]. However, in some analyses, one saw a pole, corresponding to this state, in others did not see [1].

Here, we used our model-independent method [2], based only on analyticity and unitarity directly applied to analysis of experimental data.

1. ANALYSIS OF THE SCALAR–ISOSCALAR SECTOR

Our method can be used only for the 2- and 3-channel cases. Therefore, when analyzing data on the isoscalar S waves of $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta, \eta\eta'$ (for references to the used data see [2]), we performed independently 2 variants of the 3-channel analysis: variant I — the combined analysis of $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and variant II — the analysis of $\pi\pi \to \pi\pi$, $K\overline{K}$, $\eta\eta'$.

The 3-channel S matrix is determined on the 8-sheeted Riemann surface. The elements $S_{\alpha\beta}$ ($\alpha, \beta = 1 - \pi\pi, 2 - K\overline{K}, 3 - \eta\eta$ or $\eta\eta'$) have the right cuts along the real axis of the complex-s plane (s is the invariant total energy squared), starting with the channel thresholds s_α, and the left cuts. The sheets are numbered as follows: signs $(\mathrm{Im}\sqrt{s-s_1}, \mathrm{Im}\sqrt{s-s_2}, \mathrm{Im}\sqrt{s-s_3}) = +++, -++, --+, +-+, +--, ---, -+-, ++-$ correspond to sheets I, …, VIII, respectively. In the 3-channel case, we have 7 types of resonances (depending on their nature) corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} — (**a**); S_{22} — (**b**); S_{33} — (**c**); S_{11} and S_{22} — (**d**); S_{22} and S_{33} — (**e**); S_{11} and S_{33} — (**f**); S_{11}, S_{22}, and S_{33} — (**g**). Then, starting from the resonance zeros on sheet I, one can obtain an arrangement (pole clusters) of poles and zeros of resonances on the whole Riemann surface using formulas from [2] of the analytic continuations of the S-matrix elements to unphysical sheets. To use the representation of resonances by the pole clusters, a procedure of uniformization of the S matrix is carried out. We used a new uniformizing variable, neglecting the $\pi\pi$-threshold branch-point and taking into account the threshold branch-points related to 2 remaining channels and the left branch-point at $s = 0$: $w = (\sqrt{(s-s_2)s_3} + \sqrt{(s-s_3)s_2})/\sqrt{s(s_3-s_2)}$, where $s_2 = 4m_K^2$, and $s_3 = 4m_\eta^2$ or $(m_\eta + m_{\eta'})^2$ in variants I or II, respectively. The Le Couteur–Newton relations [3] are used, whose forms on the w plane can be found in [2]. These relations express the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(\sqrt{s-s_1}, \ldots, \sqrt{s-s_N})$ that is a real analytic function with the only branch-points at $\sqrt{s-s_\alpha} = 0$.

In the S matrix, $S = S_B S_{\mathrm{res}}$ (S_B describes the background; S_{res}, the resonance contributions), the d-function for S_{res} is $d_{\mathrm{res}}(w) = w^{-M/2} \prod_{r=1}^{M} (w + w_r^*)$, where M is the number of resonance zeros. For S_B, the d-function has the form: $d_B = \exp\left[-i(a + \sum_{n=1}^{3} (\sqrt{s-s_n}/2m_n)(\alpha_n + i\beta_n))\right]$, where $\alpha_n = a_{n1} + a_{n\sigma}(s-s_\sigma)/s_\sigma\ \theta(s-s_\sigma) + a_{nv}(s-s_v)/s_v\theta(s-s_v)$ and $\beta_n = b_{n1} + b_{n\sigma}(s-s_\sigma)/s_\sigma\theta(s-s_\sigma) + b_{nv}(s-s_v)/s_v\theta(s-s_v)$ with s_σ — the $\sigma\sigma$ threshold, s_v — the combined threshold of the channels $\eta\eta'$, $\rho\rho$, $\omega\omega$. In variant II, the terms $a'_{n\eta}(s-4m_\eta^2)/4m_\eta^2\theta(s-4m_\eta^2)$ and $b'_{n\eta}(s-4m_\eta^2)/4m_\eta^2\ \theta(s-4m_\eta^2)$ should be added to α'_n and β'_n, respectively (the quantities related to variant II are primed).

We have considered the scalar resonances discussed in the PDG issue [1], assuming 2 states in the 1500-MeV region. In variant I (the total $\chi^2/\mathrm{dof} = 299.485/(301-45) \approx 1.17$), the $f_0(600)$ is described by the cluster of type (**a**) (the poles, in MeV, at $\sqrt{s_r} = 569.5-i518$ on sheet II, at $639.6-i518$ on sheet III, at $644.5-i518$ on sheet VI, and at $574.4-i518$ on sheet VII); $f_0(1370)$, (**c**) (the poles at $1394.2-i286.9$, $1394.2-i271.1$, $1394.2-i157.7$, $1394.2-i173.5$ on sheets $\mathrm{V},\ldots,\mathrm{VIII}$, respectively); $f_0(1500)$, (**c**) (the poles at $1499.2-i70.4$, $1499.2-i60.4$, $1499.2-i46.8$, $1499.2-i56.8$ on sheets $\mathrm{V},\ldots,\mathrm{VIII}$); $f'_0(1500)$, (**g**) (the poles at $1502.4-i345.1$, $1496.2-i133.5$, $1502.4-i227.7$, $1496.1-i139.7$, $1510.9-i186.6$, $1502.1-i97.8$, $1502.4-i343.3$ on sheets $\mathrm{II},\ldots,\mathrm{VIII}$); $f_0(1710)$, (**c**) (the poles at $1726.7-i139.9$, $1726.7-i136.1$, $1726.7-i99.9$, $1726.7-i103.7$ on sheets $\mathrm{V},\ldots,\mathrm{VIII}$). The $f_0(980)$ is represented by the poles at $1007.5-i36.3$ on sheet II and $971.4-i55.8$ on sheet III. The background parameters are: $a = 0.1185$, $a_{11} = 0.2806$, $a_{1\sigma} = -0.0131$, $a_{1v} = 0$, $b_{11} = b_{1\sigma} = 0$, $b_{1v} = 0.0504$, $a_{21} = -0.9792$, $a_{2\sigma} = -0.416$, $a_{2v} = -6.644$, $b_{21} = 0.0289$, $b_{2\sigma} = 0$, $b_{2v} = 6.955$, $b_{31} = 0.6417$, $b_{3\sigma} = 0.6104$, $b_{3v} = 0$; $s_\sigma = 1.638$ GeV^2, $s_v = 2.085$ GeV^2.

In variant II ($\chi^2/\mathrm{dof} = 274.604/(293-42) \approx 1.09$), the $f_0(600)$ is described by the cluster of type (**a′**) (the poles at $559.0-i529.5$ on sheet II, at $564.9-i529.5$ on sheet III, at $542.0-i529.5$ on sheet VI, and at $536.1-i529.5$ on sheet VII); $f_0(1370)$, (**b′**) (the poles at $1420.3-i164.3$, $1420.3-i179.7$, $1436.9-i179.7$, $1436.9-i164.3$ on sheets $\mathrm{III},\ldots,\mathrm{VI}$, respectively); $f_0(1500)$, (**c′**) (the poles at $1501.0-i68.5$, $1501.0-i61.3$, $1501.0-i48.9$, $1501.0-i56.18$ on sheets $\mathrm{V},\ldots,\mathrm{VIII}$); $f'_0(1500)$, (**d′**) (the poles at $1495.7-i198.9$, $1502.8-i235.5$, $1495.7-i193.1$, $1495.7-i198.9$, $1495.6-i193.9$, $1495.7-i193.1$ on sheets $\mathrm{II},\ldots,\mathrm{VII}$); $f_0(1710)$, (**c′**) (the poles at $1754.5-i150.2$, $1754.5-i116.2$, $1754.5-i79.0$, $1754.5-i113.0$ on sheets $\mathrm{V},\ldots,\mathrm{VIII}$). The $f_0(980)$ is described by the poles at $1008.2-i31.5$ on sheet II and $986.4-i55.9$ on sheet III. The poles, corresponding to the $f_0(1500)$, on sheets IV, VI, VIII, and V in variant I are of the 2nd and 3rd order, respectively, and on sheets IV and V in variant II are of the 2nd order (this is an approximation). The fact that the $f_0(980)$ is described in both variants only by the poles on sheets II and III without the corresponding poles on sheets VI and VII, as it was expected for standard clusters, indicates that the $f_0(980)$ is a non-$q\bar{q}$ state and might be, e.g., the bound $\eta\eta$ state [2]. The background parameters are: $a' = 0.2278$, $a'_{11} = 0$, $a'_{1\eta} = -0.0765$,

$a'_{1\sigma} = 0.0568$, $a'_{1v} = 0.0593$, $b'_{11} = b'_{1\eta} = b'_{1\sigma} = 0$, $b'_{1v} = 0.0442$, $a'_{21} = -3.027$, $a'_{2\eta} = 0$, $a'_{2\sigma} = 0.046$, $a'_{2v} = -4.665$, $b'_{21} = 0$, $b'_{2\eta} = -0.7386$, $b'_{2\sigma} = 2.4113$, $b'_{2v} = 1.904$, $b'_{31} = 0.4085$, $s_\sigma = 1.638$ GeV^2, $s_v = 2.126$ GeV^2. Here a pseudobackground ($b'_{2\eta} = -0.7386$) arises at describing the $K\overline{K}$ scattering that implies a necessity of the explicit allowance for the $\eta\eta$-threshold branch-point.

The masses and total widths of the f_0 resonances

State	Variant I		Variant II	
	$m_{\rm res}$, MeV	$\Gamma_{\rm tot}$, MeV	$m'_{\rm res}$, MeV	$\Gamma'_{\rm tot}$, MeV
$f_0(600)$	769.8±15	1036.0±26	770.0±9.8	1059.0±22
$f_0(980)$	1008.2±4	72.6±12	1008.7±3	63.0±8
$f_0(1370)$	1404.9±16	347.0±30	1431.6±9	359.4±22
$f_0(1500)$	1500.3±14	113.6±22	1502.0±14	112.2±22
$f'_0(1500)$	1541.1±13.9	686.6±26	1508.9±11.9	397.8±22
$f_0(1710)$	1729.8±15	207.4±34	1758.1±12	226.0±18

Masses and total widths of states, calculated from the pole positions with using the denominator of the resonance part of amplitude in the form $m_{\rm res}^2 - s - i\sqrt{s}\Gamma_{\rm tot}$, are presented in the Table. For states of types (**a**), (**b**), and (**c**), one ought to take the poles on sheets II, IV, and VIII, respectively; for resonances of types (**d**) and (**g**), the poles can be used on sheets II, IV and for the (**g**) resonance, also on sheet VIII, because the analytic continuations of the S-matrix elements only to these sheets have the forms [2] $\propto 1/S_{11}^{\rm I}$, $\propto 1/S_{22}^{\rm I}$ and $\propto 1/S_{33}^{\rm I}$, respectively ($S_{\alpha\beta}^{\rm I}$ is the S-matrix element on the physical sheet). The pole positions of states only on these sheets are at the same points of the complex-energy plane, as the states zeros on the physical sheet, and are not shifted due to the coupling of channels.

2. ANALYSIS OF THE $K\pi$-SCATTERING DATA

When analyzing data [4] for the phase shift and module of the $K^-\pi^+$-scattering amplitude in the $I(J^P) = 1/2(0^+)$ channel, we applied the 2-channel model-independent method taking into account in the uniformizing variable [2] $z = (\sqrt{s - s_1} + \sqrt{s - s_2})/\sqrt{s_2 - s_1}$ the branch-points at s_1 and s_2 related to the thresholds of the $K^-\pi^+$ and $K^0\eta$ channels, respectively, assuming that the influence of remaining channels and the left cuts can be accounted via the background. The S matrix is determined on the 4-sheeted Riemann surface. The sheets are numbered as follows: signs $(\mathrm{Im}\sqrt{s-s_1}, \mathrm{Im}\sqrt{s-s_2}) = ++, -+, --, +-$ correspond to sheets $\mathrm{I}, \ldots, \mathrm{IV}$, respectively. In the 2-channel case, there are 3 types of resonances (and corresponding pole-clusters) represented by a pair of conjugate zeros on sheet I only in S_{11} — the type (**a**), only in S_{22} — (**b**), and in each of S_{11} and S_{22} — (**c**).

In the S-matrix element, taken as $S = S_{\rm res}\exp[2i\delta_{\rm bg}]$, the resonance part $S_{\rm res} = d(-z^{-1})/d(z)$ has no cuts on the z-plane, and $d(z) = z^{-M}\prod_{n=1}^{M}(1 - z_n^* z)(1 + z_n z)$ with M — the number of conjugate-zeros pairs corresponding to resonances. The phase $\delta_{\rm bg} = \sqrt{(s-s_0)/s}(a+ib)$ describes the background: a relates to its elastic part; b, to the inelastic one.

First we carried out the analysis considering only one resonance $K_0^*(1450)$ of type (**a**). It is possible to obtain a satisfactory description with the total $\chi^2/\text{dof} = 92.09/(68-6) \approx 1.48$. The mass and total width, calculated from the pole on sheet II, were 1428 and 282 MeV, respectively. The background parameters were $a = 0.6951$ and $b = -0.0614$. A negative sign of the quantity b means the increasing inelastic part of the background. *The increasing inelastic background part implies a necessity to consider explicitly some physical phenomenon.* Here this implies the explicit consideration of one more resonance of the expected type (**b**). Indeed, for the reasonable description, two resonances, $K_0^*(900)$ of type (**b**) and $K_0^*(1450)$ of type (**a**), should be considered. Then the total $\chi^2/\text{dof} = 75.707/(68-9) \approx 1.28$, i.e., the description is better than without $K_0^*(900)$, and, furthermore and this is principal, *the background parameter b equals zero in this case*. Furthermore, this elastic background means that the influence of other channels such as $K\pi\pi\pi$ and $K\pi\sigma$ is negligible, except for the $K^0\eta'$, at opening of which there is a small deviation of our curve for the module of amplitude from the data.

The obtained poles of pole-clusters on the lower $\sqrt{s}$-half-plane are (in MeV): $859.9-i221.6$ on sheet III and $885.6-i280.8$ on sheet IV for $K_0^*(900)$; $1441.7-i172.3$ on sheet II and $1430-i144$ on sheet III for $K_0^*(1450)$. These pole clusters mean that the $K_0^*(1450)$ is coupled mainly with the $K\pi$ channel, whereas the $K_0^*(900)$ is coupled weaker with this channel than with other ones such as the $K\eta$ and $K\eta'$ channels. The masses and total widths, calculated from the pole positions, are respectively (in MeV): 929 ± 40 and 561.6 ± 55 for $K_0^*(900)$, 1452 ± 12 and 344.6 ± 15 for $K_0^*(1450)$.

CONCLUSIONS

1. It is shown that in the $I^GJ^{PC} = 0^+0^{++}$ sector there might be 2 states in the 1500-MeV region: $f_0(1500)$ ($m_{\rm res} = 1502$ MeV, $\Gamma_{\rm tot} = 112$ MeV) and $f_0'(1500)$ ($m_{\rm res} = 1509$ MeV, $\Gamma_{\rm tot} = 398$ MeV). The $f_0(1500)$ has the large $s\bar{s}$ component. All conclusions on the nature and parameters of other considered states are as in the previous model-independent 3-channel analyses [2], i.e., an additional confirmation of the wide σ-meson is obtained with the mass $m_\sigma \approx m_\rho$ that accords with a prediction by Weinberg [5]; the $f_0(980)$ is a non-$q\bar{q}$ state and

might be, e.g., the bound $\eta\eta$ state, the $f_0(1370)$ and $f_0(1710)$ have the dominant $s\bar{s}$ component, and the $f_0'(1500)$ is the glueball.

2. In the analysis of $K\pi$-scattering data [4], a proof for the $K_0^*(900)$ ($m_{\rm res} = 929$ MeV, $\Gamma_{\rm tot} = 564$ MeV) is obtained. These values of the mass and width correspond most near to the ones from work [6], obtained in the analysis of the $K\pi$ scattering using the so-called interfering Breit–Wigner amplitudes. However, unlike the indicated work, *we did not need the repulsive background*, not very clear in the $K\pi$ scattering. This state should be coupled weaker with the $K\pi$ channel than with the $K\eta$ and/or $K\eta'$ ones. The parameters of second K_0^* resonance accord with the values cited in the PDG tables.

3. Since the approximately equal masses of the $f_0(980)$ and $a_0(980)$, the $s\bar{s}$ dominance in the wave function of the $f_0(980)$ and also the too big mass of the $K_0^*(1450)$ were main difficulties when trying to put these states into one nonet, then exclusion of the $f_0(980)$ from the assignment to the lowest nonet as the non-$q\bar{q}$ state and discovery of the $K_0^*(900)$ moves off a number of problems in this task. We can take more surely as the lowest nonets the following: $a_0(980)$, $K_0^*(900)$, $f_0(600)$, $f_0(1370)$ and $a_0(1450)$, $K_0^*(1450)$, $f_0(1500)$, $f_0(1710)$ [2].

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