ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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ON PATH DEPENDENCE OF THE QCD CORRELATION FUNCTIONS

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Gauge-invariant hadronic and vacuum correlation functions in QCD contain the systems of Wilson lines and loops having complicated geometrical structure. Path dependence propagates, therefore, into such important properties of the quantum correlators as the renormalization-group behaviour, light-cone peculiarities, evolution, etc. In the present paper, I briefly overview several instructive examples of the manifestations of the structure of paths in the hadronic and vacuum correlation functions with explicit transverse momentum/distance dependence. In particular, the transverse-momentum-dependent (TMD) parton densities and the skewed jet quenching parameter in Euclidean and Minkowski space–time are addressed.

Калибровочно-инвариантные адронные и вакуумные корреляционные функции в КХД включают системы вильсоновских линий и петель, имеющих сложную геометрическую структуру. Зависимость от путей влияет, следовательно, на такие важные свойства квантовых корреляторов, как их ренормгрупповое поведение, особенности на световом конусе, эволюция и т. д. В данной работе кратко обсуждается несколько полезных примеров проявления структуры путей в адронных и вакуумных корреляционных функциях с явной зависимостью от поперечных импульсов или координат. В частности, рассмотрены поперечно-зависимые партонные функции плотности и асимметричный параметр квенчинга струй в евклидовом пространстве и пространстве Минковского.

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1. INTRODUCTION: PATH AND SHAPE VARIATIONS IN THE WILSON LOOP SPACE

Loop space consists of colorless gauge-invariant field functionals — expectation values of the products of n Wilson loops [1, 2], defined, in general, in the manifold of arbitrary integration paths $\{\Gamma_i\}$:

$$\mathcal{W}_n\left[\Gamma_1,\ldots,\Gamma_n\right] = \left\langle 0 \left| \mathcal{T}\frac{1}{N_c}\,\mathrm{Tr}\,\Phi\left(\Gamma_1\right)\cdots\frac{1}{N_c}\,\mathrm{Tr}\,\Phi\left(\Gamma_n\right)\right| 0 \right\rangle, \qquad (1)$$

$$\Phi(\Gamma_i) = \mathcal{P}\exp\left[ig\oint\limits_{\Gamma_i} dz^{\mu}\mathcal{A}_{\mu}(z)\right].$$

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The gauge fields $\mathcal{A}_\mu$ belong to the fundamental representation of non-Abelian gauge group $SU(N_c)$. Although the Wilson loops (2) are gauge invariant, their definition gives rise to the functional dependence on path and to the additional singularities due to nontrivial behavior in vicinity of obstructions, cusps or self-intersections. Moreover, the renormalization and conformal properties of the Wilson loops possessing light-like segments (or lying completely on the light cone) are known to be more intricate than those of the Wilson loops defined on off-light-cone integration contours. Therefore, study of the geometrical and dynamical properties of the loop space which can include, in general case, cusped light-like Wilson exponentials, will provide us with fundamental information on the renormalization group behavior and evolution of the various gauge-invariant quantum correlation functions.

Finding an appropriate and complete set of equations of motion in the loop space is not straightforward. It is known that the Wilson loops obey the nonperturbative Makeenko–Migdal (MM) equations [3, 4]:

$$\partial_x^\nu \frac{\delta}{\delta\sigma_{\mu\nu}(x)} W_1[\Gamma] = N_c g^2 \oint_\Gamma dz^\mu\, \delta^{(4)}(x-z)\, W_2[\Gamma_{xz}\Gamma_{zx}], \tag{2}$$

supplied with the Mandelstam constraints

$$\sum a_i \mathcal{W}_{n_i}[\Gamma_1^i, \ldots, \Gamma_{n_i}^i] = 0. \tag{3}$$

The MM set of equations follows from the Schwinger–Dyson equations being applied to the Wilson functionals

$$\delta\langle\alpha'|\alpha''\rangle = \frac{i}{\hbar}\langle\alpha'|\delta S|\alpha''\rangle, \tag{4}$$

where the quantum action operator S defines variations of arbitrary matrix elements.

The differential operations in the loop space are the area and path derivatives [3] *:

$$\frac{\delta}{\delta\sigma_{\mu\nu}(x)}\,\Phi(\Gamma) \equiv \lim_{|\delta\sigma_{\mu\nu}(x)|\to 0} \frac{\Phi(\Gamma\delta\Gamma) - \Phi(\Gamma)}{|\delta\sigma_{\mu\nu}(x)|}, \tag{5}$$

$$\partial_\mu \Phi(\Gamma) = \lim_{|\delta x_\mu|} \frac{\Phi(\delta x_\mu^{-1}\Gamma\delta x_\mu) - \Phi(\Gamma)}{|\delta x_\mu|}. \tag{6}$$

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*Another definition of the area derivative is also possible $\frac{\delta}{\delta\sigma_{\mu\nu}[x(\tau)]} = \int_{\tau+0}^{\tau-0} d\tau'(\tau' - \tau)\frac{\delta}{\delta x_\mu(\tau')}\frac{\delta}{\delta x_\mu(\tau)}$, which had been used in, e.g., [5], to address the similar problems from an alternative point of view.

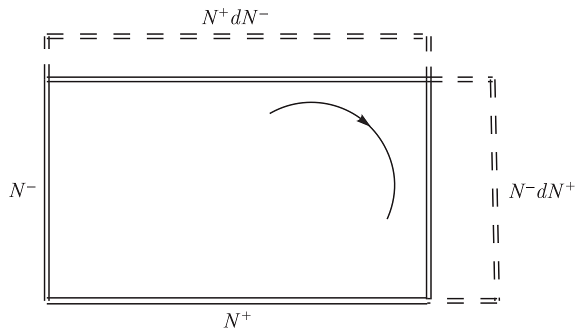

Fig. 1. Area variations allowed for a light-cone Wilson rectangle: we consider only those area variations which conserve the angles between the sides

However, the MM equations in the form (2) with the differential operations (5), (6) do not take into account possible discontinuities of the slope of the integration trajectories on which the Wilson loops are defined. In our recent works [6], we developed an approach which allows one to apply the Schwinger method to certain classes of the cusped loops (partially) lying on the light-like rays. The loops of this kind occur, in the first place, in investigation of the duality between the n-gluon scattering amplitudes and n-polygonal Wilson loops in $\mathcal{N} = 4$ super-Yang–Mills theory [7]. Moreover, analogous light-like configurations arise in the soft parts of transverse-momentum-dependent PDFs (see the next Section).

In order to describe the shape variations of such Wilson loops which correspond to the classically conformal invariant transformations, we proposed an evolution equation which is valid for the planar light-like Wilson rectangular loops, Fig. 1:

$$\mu \frac{d}{d\mu} \left(\frac{\delta}{\delta \ln \sigma} \ln \mathcal{W}[\Gamma] \right) = - \sum \Gamma_{\rm cusp}, \tag{7}$$

where the area differentials are defined in the transverse $\vec{z}_\perp = 0$:

$$d\sigma^{+-} = N^+ dN^-, \quad d\sigma^{-+} = -N^- dN^+, \tag{8}$$

so that the only allowed shape variations are presented in Fig. 1. The area logarithmic derivation operator then reads

$$\frac{\delta}{\delta \ln \sigma} \equiv \sigma_{\mu\nu} \frac{\delta}{\delta \sigma_{\mu\nu}} = \sigma_{+-} \frac{\delta}{\delta \sigma_{+-}} + \sigma_{-+} \frac{\delta}{\delta \sigma_{-+}}. \tag{9}$$

The r.h.s. of Eq. (7) is defined by the sum of the light-cone cusp anomalous dimensions. The latter appears to be a fundamental ingredient of an effective quantum action for the Wilson loops with discontinuities of the slope. Equation (7) suggests, in fact, the duality of the energy (or rapidity, e.g., in the TMD case) evolution and geometrical shape variations of the underlying paths. We made, therefore, a few steps in the understanding of the relationship between the geometrical properties of the loop space in terms of the area differential evolution equations, from one side, and the dynamics accumulated in the cusps — the angles between the light-like straight lines, from another side. Thus, in the loop space, the (external) dynamics can be taken into account by introducing the obstructions to the initially smooth loops, with those obstructions resembling the sources within the Schwinger field-theoretical picture. We have demonstrated that the universal Schwinger quantum dynamical principle is a useful tool to study some special classes of the elements of the loop space, in particular, the cusped Wilson exponentials (null polygons) on the light cone.

2. STRUCTURE OF PATHS IN TRANSVERSE-MOMENTUM-DEPENDENT PARTON DENSITIES

In this section, we address a bit more complicated systems of the Wilson lines, namely those which enter the operator definitions of the fully gauge-invariant TMDs. These objects arise naturally in the QCD factorization approach to semi-inclusive hadronic processes, such as SIDIS, Drell–Yan, etc. In the classical deep inelastic $lH \to l'X$ scattering experiments, by measuring the momentum of the outgoing lepton l', we learn about the longitudinal distribution of partons inside the nucleon. In a reference frame, where the nucleon moves (infinitely) fast, this information is accumulated in the parton distribution functions (PDFs) $f_a(x, Q^2)$ in terms of the partonic degrees of freedom: e.g., the Bjorken variable $x_{\rm Bj}$ relates to the fraction of the longitudinal momentum P of the parent hadron possessed by a parton of the flavor a. Such collinear (integrated) PDFs can be properly defined as completely gauge-invariant (nonperturbative) hadronic matrix elements

$$f(x; n^+, n^-, \mu^2;) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} \, {\rm e}^{-ik^+\xi^-} \times \\ \times \left\langle p \,|\bar{\psi}(\xi^-) \mathcal{W}_{n^-}[\xi^-, 0^-] \gamma^+ \psi(0^-)|\, p\right\rangle_{\xi^+, \xi_\perp = 0}, \quad x = \frac{k^+}{p^+}, \quad (10)$$

with the renormalization-group properties controlled by the DGLAP evolution equations (for review and references see [8]). The light-cone components of a four-vector a^μ are

$$a^\pm = \frac{1}{\sqrt{2}} (a^0 \pm a^z).$$

Note that the implicit dependence on the light-like vectors $n^\pm$ is understood in this definition. Generic semi-infinite Wilson lines evaluated along a given four-vector w are defined as

$$\mathcal{W}_w[0,\infty] \equiv \mathcal{P}\exp\left[-ig\int_0^\infty d\tau\, w_\mu\, \mathcal{A}^\mu(\xi + w\tau)\right], \tag{11}$$

where, in the case under consideration, the vector w can be either light-like $w_L = n^\pm$, $(n^\pm)^2 = 0$, or transverse $w_T = \mathbf{l}$. It is worth noting that one can relate the moments of the collinear PDFs

$$M^N = \int dx\, x^{N-1} f(x)$$

to the matrix elements of the local twist-two operators

$$\mathcal{O}^N = \frac{(p^+)^{-N}}{2}\langle\, p\,|\bar\psi(0)\{\gamma^+ iD^+\cdots iD^+\}_{\rm sym}\,\psi(0)|\,p\,\rangle$$

arising in the operator product expansion on the light cone, thus making them well-defined objects from the field-theoretical point of view [9].

Another attractive feature of the collinear PDFs (10) is that they imply a clear interpretation as the probability for a parton inside the nucleon to have the longitudinal momentum $k_{\rm long} = xP_{\rm long}$. This interpretation is most naturally established when QCD is canonically quantized (and subsequently renormalized) on equal-«light-cone-time» surfaces $\xi^+ = 0$ in a class of singular noncovariant gauges [10]. This parton number interpretation holds, in the light-cone gauge $A^+ = 0$, in higher order calculations as well [11–13]. However, an important problem must be solved in order to make the calculations in the light-cone gauge feasible. Even after imposing the gauge condition $A^+ = 0$, the gauge in not completely fixed: one may still perform a ξ^--independent gauge transformation $U(\xi^+, \boldsymbol{\xi}_\perp)$ by virtue of

$$\partial^+ U(\xi^+, \boldsymbol{\xi}_\perp) = \frac{\partial}{\partial x^-}\, U(\xi^+, \boldsymbol{\xi}_\perp) = 0.$$

Therefore, fixing the gauge is equivalent to imposing certain boundary conditions on the gauge field (see, e.g., [12,14–16]). Going over to the conjugate momentum space, one observes that the ambiguity in the behavior of the gauge field at light-cone infinity $\xi^- \to \infty$ maps over ambiguity of the gluon Green function at small $q^+ \to 0$. A key issue is, therefore, how to get rid of extra complications due to the emergent «spurious» singularity $\sim [q^+]^{-1}$ in the free gluon propagator

$$D^{\mu\nu}(q) = \frac{i}{q^2 + i0}\left(-g^{\mu\nu} + \frac{(n^-)^\mu q^\nu + (n^-)^\nu q^\mu}{[q^+]}\right). \tag{12}$$

The uncertainty of the pole prescription in Eq. (12) corresponds to the residual gauge freedom and can be treated without changing the gauge-fixing constraint $A^+ = 0$.

There are several possible pole-prescription-fixing procedures that are compatible with the light-cone gauge and have been shown to give correct results (at least, up to the $O(\alpha_s^2)$-order). In particular, the principal value prescription

$$\frac{1}{[q^+]_\eta^{\rm PV}} = \lim_{\eta\to 0} \frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)$$

was used in [17] to evaluate the DGLAP kernel in the next-to-leading order. Nonsymmetrical advanced and retarded pole prescriptions are also possible [14, 15]. Although these methods work in some situations, the only pole prescription which is consistent with the equal-time canonical quantization in the light-cone gauge is the Mandelstam–Leibbrandt one [18]:

$$\frac{1}{[q^+]} \to \lim_{\eta\to 0} \frac{1}{[q^+]_{\rm ML}} = \\ = \lim_{\eta\to 0} \frac{(q\cdot n^+)}{(q\cdot n^+)(q\cdot n^-) + i\eta} \doteq \lim_{\eta\to 0} \frac{1}{(q\cdot n^-) + i\eta(q\cdot n^+)}, \quad (13)$$

where and in what follows $n^\pm$ are the light-like vectors $(n^\pm)^2 = 0$, $n^+n^- = 1$, and $\doteq$ means equality in the sense of the theory of distributions. It was shown that the free gluon propagator supplied with the ML pole prescription can be directly derived following the equal-time quantization procedure and is compatible with well-established results at least up to the $O(\alpha_s)$-order [11, 15, 19]. The main difference between the q^--independent prescriptions and the ML one (13) originates in the different situation of poles in the q^0 plane, as it is shown and explained in Fig. 1. Thanks to this feature, one can perform the Wick rotation of the integration contour to the Euclidean momentum space, and the ultraviolet divergences can be analyzed by means of the usual power counting procedure in the Euclidean space. This observation anticipates the absence of overlapping divergences in the loop calculations with the ML prescription.

It is worth noting that albeit the pole-prescription issues mentioned above may reveal themselves in the course of the calculation, they are nonvisible in the case of the collinear PDFs, by virtue of the cancelation of the soft divergences in the virtual and the real gluon exchange graphs. However, those issues are crucial and unavoidable in unintegrated PDFs [9], which are introduced in the factorization approach to the semi-inclusive processes. The picture of the nucleon revealed in the DIS experiments, being essentially one-dimensional, is still incomplete: in this «longitudinal» picture the transverse degrees of freedom of the partons are eliminated by definition and the 3D structure remains inaccessible. The study of semi-inclusive processes, such as semi-inclusive deep inelastic scattering (SIDIS), the Drell–Yan (DY) process, hadron–hadron collisions, or lepton–lepton annihilation to hadrons, where (at least) one more final or initial hadron is detected and its transverse momentum (and, possibly, its spin) is observed, calls for the introduction of more involved quantities — unintegrated transverse-momentum-dependent (TMD) distribution and fragmentation functions (see [20] and references therein).

Let us present a brief account of how the problem of the emergent singularities beyond the tree approximation is approached in different operator definitions of the (quark) TMDs. At the one-loop level, the following three classes of singularities are expected: (i) simple ultraviolet poles which must be removed by the standard renormalization procedure; (ii) pure rapidity divergences, which depend on an additional rapidity parameter, but do not jeopardize the renormalizability of the TMDs, and can be safely resumed by means of the Collins–Soper equation; (iii) highly undesirable overlapping divergences: they contain the UV and rapidity poles simultaneously and thus break down the standard renormalizability of TMDs, calling for a generalized renormalization procedure in order to enable the construction of a consistent operator definition of the TMDs. Before getting started with the analysis of the divergences, let us try to learn something from the tree approximation. The simplest «unsubtracted» definition of «a quark in a quark» TMD with only the light-like Wilson lines along the vectors n^+, n^-, which allows a *parton number interpretation* in the light-cone gauge, reads

$$\mathcal{F}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \, \mathrm{e}^{-ik\cdot\xi} \times$$
$$\times \left\langle p \left| \bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp) \mathcal{W}^\dagger_{n^-}[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp] \mathcal{W}^\dagger_{\mathbf{l}}[\infty^-, \xi_\perp; \infty^-, \infty_\perp] \times \right.\right.$$
$$\left.\left. \times \gamma^+ \mathcal{W}_{\mathbf{l}}[\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] \mathcal{W}_{n^-}[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi(0^-, \mathbf{0}_\perp) \right| p \right\rangle \quad (14)$$

with $\xi^+ = 0$. Although the formal integration of definition (14) over $\mathbf{k}_\perp$ yields the collinear PDF, Eq. (10),

$$\int d^2\, \mathbf{k}_\perp \mathcal{F}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right) = f(x; n^+, n^-, \mu^2), \quad (15)$$

this is only well-justified in the tree approximation, because the rapidity divergences in the loop corrections prevent a straightforward reduction. It is worth noting that the normalization of the above TMD

$$\mathcal{F}^{(0)}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right) = \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \times$$
$$\times \, \mathrm{e}^{-ik^+\xi^- + i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp} \langle p \,|\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)\gamma^+\psi(0^-, \mathbf{0}_\perp)|\, p\rangle = \delta(1-x)\delta^{(2)}(\mathbf{k}_\perp) \quad (16)$$

can be obtained following the canonical quantization procedure in the light-cone gauge, where the longitudinal Wilson lines disappear and the equal-time commutation relations for the quark creation and annihilation operators $\{a^\dagger(k,\lambda), a(k,\lambda)\}$ lead immediately to the parton number interpretation of the TMD:

$$\mathcal{F}^{(0)}\left(x,\mathbf{k}_\perp;n^+,n^-,\mu^2\right) \sim \langle\, p\,|\, a^\dagger(k^+,\mathbf{k}_\perp;\lambda)a(k^+,\mathbf{k}_\perp;\lambda)\,|\,p\,\rangle. \qquad (17)$$

The use of the «tilted» gauge links does not meet this requirement.

Going beyond the tree approximation, one encounters a bunch of singularities mentioned above. To overcome the problems related with them, different frameworks have been proposed.

Adopting the covariant Feynman gauge, Ji, Ma, and Yuan developed a framework which makes use of the tilted (off-the-light-cone) longitudinal gauge links lined up along the vector $n_B^2 \neq 0$ [21]. In a covariant gauge, the transverse gauge links at light-cone infinity cancel, and the rapidity cutoff $\zeta = (2p\cdot n_B)^2/|n_B^2|$ is introduced to control the deviation of the longitudinal gauge links from the light-like direction. A subtracted soft factor contains the nonlight-like gauge links as well. Within this approach, the off-the-light-cone unsubtracted TMDs, where the light-like vector n^- in the longitudinal Wilson lines is replaced by the tilted vector

$$n_B = (-\mathrm{e}^{2y_B}, 1, \mathbf{0}_\perp),$$

do not satisfy the relation (15), even in the tree approximation — cf. Fig. 2 and the caption. However, one can design a «secondary factorization» method which allows the expression of this TMD (transformed to the impact parameter space $\mathcal{F}(x,\mathbf{b}_\perp)$) in terms of a convolution of collinear PDFs and perturbative coefficient

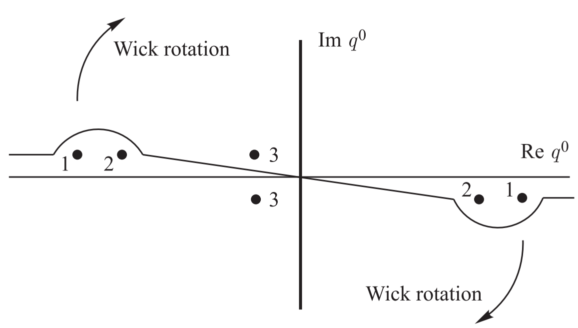

Fig. 2. Location of the poles in the complex q^0 plane: the poles of the light-cone gluon propagator with the ML prescription (1) and the poles of the propagator in a covariant gauge (2) belong to the same quadrants, so that the clock-wise Wick rotation is allowed. The poles of the light-cone propagator with the principal-value prescription (3), in contrast, impede that rotation

functions at small $\mathbf{b}_\perp$ [21]. Another subtraction method, also in covariant gauges, but without the explicit off-the-light-cone regularization in the unsubtracted TMD, was developed in [22,23]. The corresponding geometry of the light-like and tilted Wilson lines in the soft factors is shown in Fig. 3, lower panel.

In our works [24], we proposed to study the renormalization-group properties of the unsubtracted quark TMD (14) and to make use of its one-loop anomalous dimensions in order to reveal the simplest *minimal* geometry of the gauge links in the soft factor which allows one to get rid of the mixed rapidity-dependent terms. We showed (in the leading $O(\alpha_s)$-order) that the extra contribution to the anomalous dimension is nothing but the cusp anomalous dimension [19,26].

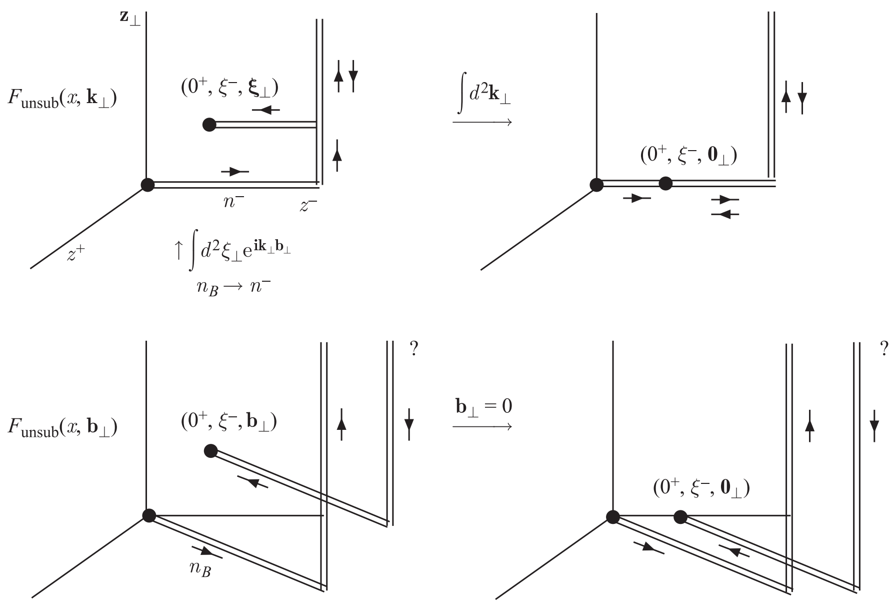

Fig. 3. Geometrical structure of integration contours in the unsubtracted TMDs with the light-like (upper panel) and off-the-light-cone (lower panel) longitudinal gauge links and their symbolic reduction to the collinear PDFs. In the former case, the transverse gauge links cancel completely after the $\mathbf{k}_\perp$-integration, while the longitudinal gauge links reduce to the one-dimensional light-like connector $\mathcal{W}_n[\xi^-, 0^-]$. In the off-the-light-cone situation, the cancelation of the transverse gauge links at infinity is not, at least, straightforward. Moreover, the integrated configuration contains two nonvanishing off-the-light-cone gauge links which are not equivalent to the simple connector $\mathcal{W}_n[\xi^-, 0^-]$. Beyond the tree level, the renormalization group properties of those two objects are also different. I put the interrogation marks next to the transverse gauge links at infinity since I am not aware of any consistent treatment of them in the TMD formulations with off-the-light-cone (tilted) Wilson lines. In contrast, the transverse gauge links appear naturally in the «light-cone» frameworks

Note that in these works we adopted the light-cone gauge supplied with the q^--independent pole prescriptions. In subsequent works we showed that our approach works in the case of the ML pole prescription as well [24], and that it can be consistently used to formulate a generalized definition of the quark TMD with a *nonminimal spin-dependent term* in the Wilson lines [25].

Compared to the Ji–Ma–Yuan approach, we followed a different strategy. Making the assumption that the parton number interpretation (17) must hold for TMDs in the light-cone gauge (like it holds in the collinear PDFs), we are in a position to *derive* a gauge-invariant operator definition of the TMD. In other words, starting from the requirement of the probability interpretation in the light-cone gauge and adding, step by step, the *minimally necessary* gauge links, we would end up with a gauge-invariant operator definition of the TMD without undesirable singularities. The generalized definition of the quark TMD we proposed, reads

$$\tilde{\mathcal{F}}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right) = \frac{\mathcal{F}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right)}{S_F(n^+, n^-) L_F^{-1}(n^+)}, \tag{18}$$

where the soft factor S_F and the self-energy factor L_F are defined in [24, 28], see also Figs. 3 and 4.

The following conjecture concerning the generic structure of divergences of the TMD beyond the tree approximation is in order:

The contribution of the overlapping singularities to the renormalized TMD can be expressed either in terms of a finite number of the cusp anomalous dimensions which are known in the theory of Wilson lines/loops up to the $O(\alpha_s^2)$-order — in this case, their treatment consists of the subtraction of (a finite number of) corresponding *cusped soft factors*; or those singularities depend on the *degenerate rapidities* — in that case, one has to subtract the self-energy soft factors which consist, in contrast, of the «smooth» infinite gauge links without any obstructions (cusp or intersections). The conjecture is, therefore, that there is no other sort of unphysical singularities in the loop corrections to the TMDs in any order of α_s.

In the leading order, we demonstrated the validity of the above statement in our works [24, 27, 28]. An important question remains, however, how our TMD can be built into an appropriate factorization formula for semi-inclusive hadronic tensor. In our approach, the following factorization formula is supposed to be valid

$$W^{\mu\nu} = |H(Q,\mu)^2|^{\mu\nu} \frac{\mathcal{F}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right)}{S_F(n^+, n^-) L_F^{-1}(n^+)} \otimes \frac{\mathcal{D}_{\rm n}(z, z\mathbf{k}'_\perp; n^+, n^-, \mu)}{S_D(n^+, n^-) L_D^{-1}(n^-)} + \ldots, \tag{19}$$

where the geometrical structure of the soft factor is consistent with that of the collinear PDF [19, 29] and does not break the number interpretation. The explicit proof of the conjectured factorization and of absence of double counting is in progress.

Recently, Collins proposed a new definition of the (quark) TMD [30] which is assumed to be a part of the factorization formula for the semi-inclusive hadronic tensor (up to the power corrections)

$$W^{\mu\nu} = |H(Q,\mu)^2|^{\mu\nu} \mathcal{F}^{[\mathrm{Col}]}(x, \mathbf{k}_\perp; \mu, \zeta_F) \otimes \mathcal{D}^{[\mathrm{Col}]}(z, z\mathbf{k}'_\perp; \mu, \zeta_D) + \ldots, \quad (20)$$

where all soft factors are absorbed into the TMD distribution $\mathcal{F}^{\mathrm{Col}}$ and the fragmentation $\mathcal{D}^{\mathrm{Col}}$ functions, so that there are no separate soft factors in factorized structure functions, e.g.,

$$\mathcal{F}^{[\mathrm{Col}]}(x, \mathbf{b}_\perp; \mu, \zeta_F) = \mathcal{F}\left(x, \mathbf{k}_\perp; n^+, n^-, \mu^2\right) \sqrt{\frac{S(n^+, n_B)}{S(n^+, n^-) S(n_A, n^-)}}. \quad (21)$$

Here the soft factors depend on the light-like $n^\pm$ or the tilted $n_{A,B}$ vectors (for details, see [30]). Note that the TMD (21) is defined in the impact parameter space

$$\mathcal{F}(x, \mathbf{b}_\perp) = \int d^2\mathbf{k}_\perp \, \mathrm{e}^{-i\, \mathbf{k}_\perp \mathbf{b}_\perp} \, \mathcal{F}(x, \mathbf{k}_\perp),$$

so that it is, in fact, a «semi-integrated PDF» and the reduction to the collinear case corresponds to the limit $\mathbf{b}_\perp \to 0$. The geometry of the gauge links in the soft factors is presented and explained in Fig. 4.

Several open questions are still to be answered:

• How should one prove the complete gauge invariance of the TMD (21)? It is formulated in the covariant Feynman gauge, where the transverse gauge links at light-cone infinity vanish. What will change, if we adopt some physical (axial) gauge?

• In particular, how should one treat the T-odd effects in the axial gauges given that the structure of the transverse gauge links at light-cone infinity is not yet clarified in the TMD (21)?

• After reduction to the collinear PDF (in the case of TMD (21), this corresponds to the limit $\mathbf{b}_\perp \to 0$), there is neither a mutual compensation of the longitudinal, nor that of the transverse gauge links (if introduced in the usual manner), see Fig. 4. Hence, the geometrical structure of the gauge links in the collinear PDF obtained from the TMD (21) seems too cumbersome to be simply included in the standard DIS factorization scheme (see, e.g., [19, 29]).

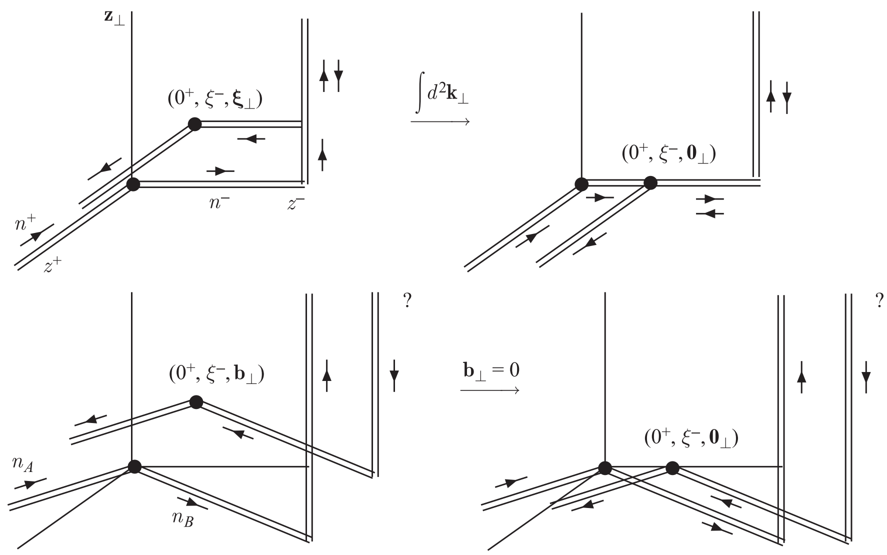

Fig. 4. Comparative geometry of the Wilson lines in unsubtracted soft factors and visualization of the reduction to the collinear case. Upper panel shows the soft factor in the momentum space, as proposed in [24]. Lower panel presents the tilted off-the-light-cone integration paths in the impact parameter space, as well as the result of the reduction to the collinear $\mathbf{b}_\perp \to 0$ configuration

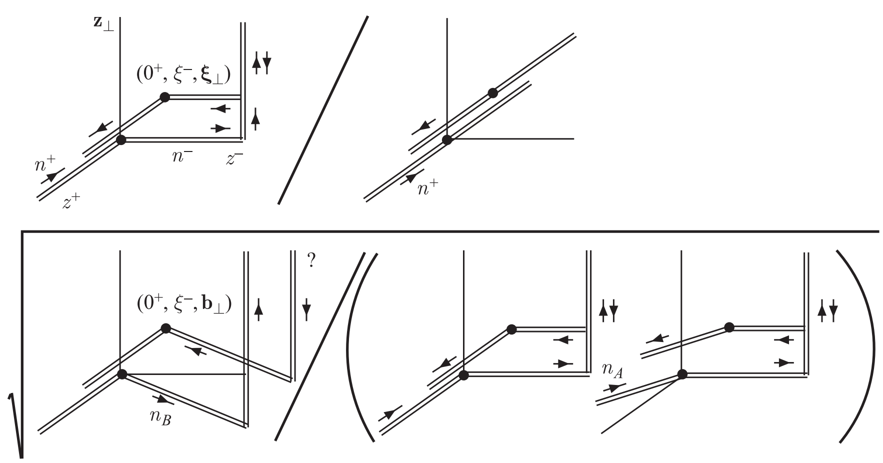

Fig. 5. Comparative geometry of the Wilson lines in the subtracted soft factors. Upper panel corresponds to the soft factor of the TMD distribution function which enters the factorization formula (19). Lower panel shows the longitudinal gauge links shifted off the light cone, which are used in the factorization approach (20

3. SKEWED JET QUENCHING PARAMETER IN EUCLIDEAN SPACE–TIME

The last example of the crucial role played by the geometrical layout in the theory of the Wilson loops is provided by the so-called jet quenching parameter which characterizes the transverse momentum gain of a jet propagating through the nuclear medium [31]. Jet quenching is observed in the heavy-ion collisions at RHIC [32] and at the LHC [33]. It is assumed that a dense, deconfined state of quarks and gluons is formed in such collisions, known as the quark–gluon plasma (QGP) [34]. For a recent review of jet quenching, see, e.g., [35], while [36] provides an up-to-date report on the quark–gluon plasma. We will not discuss the phenomenological aspects of this process, concentrating instead on the specific problems arising in calculation of the correlator of two infinite oppositely directed light-like Wilson lines. Perturbative calculations of the Wilson lines and loops call for a careful treatment of the angular dependence (e.g., cusps, self-intersections, etc.) and the possible divergences of various kinds both in Minkowski and Euclidean space–time (see, e.g., [2, 26, 37]). Here we address the issue of the angular dependence of the generic *skewed* correlator of two Wilson lines, defined first in Euclidean space, and then show how to transform to the Minkowskian geometry on the light cone. We do not specify the way how the expectation values in the medium

$$\langle \star | \mathcal{W}^\dagger \mathcal{W} | \star \rangle$$

must be evaluated, instead, we try to retrieve as much as possible information, making as less as possible conjectures about the properties of the two-gluon contraction in a medium.

Namely, we consider the following object in Euclidean space:

$$\tilde{P}(\mathbf{z}_\perp; v, \bar{v}) = \langle \star | \frac{1}{N_c} \mathrm{Tr}\, \{ \mathcal{W}^\dagger_{\bar{v}}[\mathbf{z}_\perp] \mathcal{W}_v[\mathbf{0}_\perp] \} | \star \rangle, \qquad (22)$$

where

$$\mathcal{W}_v[\mathbf{0}_\perp] = \mathcal{P} \exp \left[i g v_\mu \int\limits_{-\infty}^{\infty} d\sigma \mathcal{A}_\mu(y) \right], \quad y = v\sigma, \qquad (23)$$

and

$$\mathcal{W}^\dagger_{\bar{v}}[\mathbf{z}_\perp] = \mathcal{P} \exp \left[-i g \bar{v}_\mu \int\limits_{-\infty}^{\infty} d\sigma' \mathcal{A}_\mu(y') \right], \quad y' = \bar{v}\sigma' + \mathbf{z}_\perp, \qquad (24)$$

where the directions of the Euclidean vectors v and $\bar{v}$ are determined by the angles ϕ, $\bar{\phi}$

$$v_\mu = (v^0, v^z, \mathbf{0}_\perp) = L(\cos\phi/2, \sin\phi/2, \mathbf{0}_\perp), \tag{25}$$

$$\bar{v}_\mu = (\bar{v}^0, \bar{v}^z, \mathbf{0}_\perp) = -L(\cos\bar{\phi}/2, \sin\bar{\phi}/2, \mathbf{0}_\perp), \tag{26}$$

$$v^2 = \bar{v}^2 = L^2. \tag{27}$$

This object is more general than is needed for the straightforward calculation of the jet quenching parameter, but contains all peculiarities we are interested in. Let us define an asymmetric function of the Euclidean vectors $(v, \bar{v})$

$$\rho(v, \bar{v}) = \frac{1}{L} \int d^2k_\perp \mathbf{k}_\perp^2 \int d^2z_\perp \mathrm{e}^{ik_\perp \cdot z_\perp} \tilde{P}(\mathbf{z}_\perp; v, \bar{v}), \tag{28}$$

which formally resembles the skewed analogue of the physical $\hat{q}$, and we assume that an appropriate transition procedure

$$\rho(v, \bar{v}) \to \hat{q}_{\mathrm{LC}} \tag{29}$$

exists.

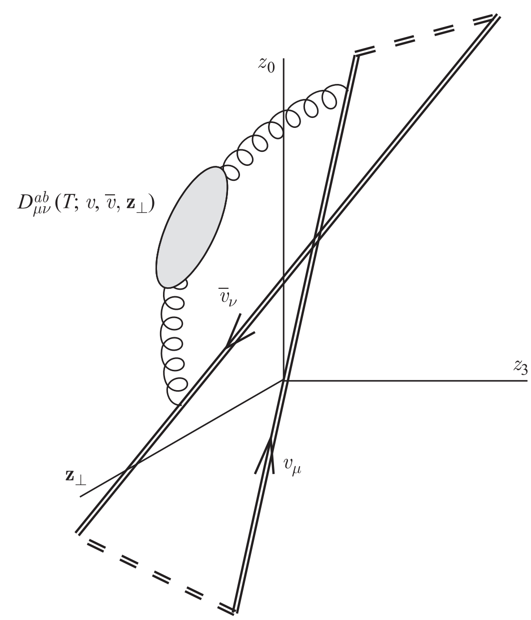

Fig. 6. Skewed configuration of the Wilson lines

Naively, the realistic situation is supposed to be achieved by making the transformation of the angles to the Minkowski geometry $(\phi, \bar{\phi}) = i(\psi, \bar{\psi})$ and setting them equal. The light-cone case can be obtained, formally, by taking the limit of large Minkowskian angles ϕ and $\bar{\phi}$. We will see, however, that this straightforward strategy does not work in our case. Instead, we will keep the two angles different after transformation to Minkowski space–time and, given that the angular dependence gets factorized into a covariant multiplier, demonstrate that the light-cone limit can be consistently performed in the skewed layout. Another important change as compared to the standard definition of $P(\mathbf{k}_\perp)$ is that we evaluate the line integrals in the Wilson functionals along the *infinite* paths, keeping IR singularities under control, if needed, by an additional energy scale λ. In the dimensional regularization, λ is introduced formally as an energy parameter in the integration measure. On the other hand, the length L provides the natural longitudinal scale. Recall that the length of an integration contour in the coordinate space corresponds to the inverse mass of the eikonalized particle in the momentum space $L \sim m^{-1}$ [26]. Let us note that the contribution under consideration is UV finite due to the space-like separation of the Wilson lines. This is not the case anymore in the NLOs.

The leading nontrivial term of the weak-field expansion of the skewed probability distribution (22) reads

$$\tilde{P}^{(1)}(\mathbf{z}_\perp; v, \bar{v}) = -(ig)^2 (v_\mu \bar{v}_\nu) \int\limits_{-\infty}^{\infty} d\sigma \int\limits_{-\infty}^{\infty} d\sigma' \, \mathrm{Tr} \frac{1}{N_c} \times \\ \times \langle \star | \mathcal{P} \left[\mathcal{A}_\mu (v\sigma + \mathbf{z}_\perp) \, \mathcal{A}_\nu (\bar{v}\sigma') \right] | \star \rangle. \quad (30)$$

The most general Lorentz and colour structure of the two-gluon correlator, which takes into account both perturbative and possible nonperturbative contributions [38]

$$\langle \star | [A^a_\mu (v\sigma + \mathbf{z}_\perp) A^b_\nu (\bar{v}\sigma')] | \star \rangle = \delta^{ab} \; D_{\mu\nu}(v\sigma - \bar{v}\sigma' + \mathbf{z}_\perp), \quad (31)$$

is determined as follows:

$$D_{\mu\nu}(z) = g_{\mu\nu} \partial^2 D_1(z^2) - \partial_\mu \partial_\nu D_2(z^2) = \\ = g_{\mu\nu}(2\omega \partial_u + 4z^2 \partial_u^2) D_1(z^2) - (2 g_{\mu\nu} \partial_u + 4 z_\mu z_\nu \partial_u^2) D_2(z^2), \quad (32)$$

where $u \equiv z^2$ and $z = v\sigma + \mathbf{z}_\perp - \bar{v}\sigma'$. Dimension regularization provides IR finiteness under $\omega = 4 - 2\varepsilon$, $\varepsilon < 0$. We will also use the Laplace transform of the functions $D_{1,2}$ and their derivatives $D^{(l)}_{1,2} \equiv \partial^l_u D_{1,2}(u)$ in Euclidean space:

$$D^{(l)}_{1,2} = (-1)^l \int\limits_0^\infty d\alpha \, \alpha^l \, \mathrm{e}^{-\alpha z_\perp^2} \tilde{D}_{1,2}(\alpha). \quad (33)$$

Equation (30) can be split up into four contributions:

$$\tilde{P}^{(1)}(\mathbf{z}_\perp; v, \bar{v}) = g^2 C_F I, \quad I = I_1 + I_{1'} + I_2 + I_{2'}, \quad C_F = \frac{N_c^2 - 1}{2N_c}, \tag{34}$$

and

$$\begin{aligned}
I_1 &= v_\mu \bar{v}_\nu \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \, 2\omega g^{\mu\nu} \partial_u \, D_1(z^2), \\
I_{1'} &= v_\mu \bar{v}_\nu \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \, 4 g^{\mu\nu} z^2 \partial_u^2 D_1(z^2), \\
I_2 &= -v_\mu \bar{v}_\nu \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \, 2 g^{\mu\nu} \partial_u \, D_2(z^2), \\
I_{2'} &= -v_\mu \bar{v}_\nu \int_{-\infty}^{\infty} d\sigma \int_{-\infty}^{\infty} d\sigma' \, 4 z^\mu z^\nu \partial_u^2 D_2(z^2).
\end{aligned} \tag{35}$$

Evaluating the line integrals (35), we use the representation

$$\mathrm{e}^{-\alpha z^2} = \mathrm{e}^{-\alpha L^2 \left(\sigma + \cos(\Delta\phi)\sigma'\right)^2} \, \mathrm{e}^{-\alpha L^2 \sigma'^2 \sin^2(\Delta\phi)} \, \mathrm{e}^{-\alpha b_\perp^2}, \tag{36}$$

where $\Delta\phi = (\phi - \bar{\phi})/2$.

After a straightforward, but tedious calculation (see for technical details [39]), we finally obtain

$$\tilde{P}^{(1)}(\mathbf{z}_\perp; v, \bar{v}) = g^2 C_F 2\pi \frac{\cos\Delta\phi}{|\sin\Delta\phi|} \left[(\omega - 2) D_1(\mathbf{z}_\perp^2) + 2\mathbf{z}_\perp^2 D_1'(\mathbf{z}_\perp^2) \right]. \tag{37}$$

Equation (43) is our main result for the skewed probability distribution in Euclidean space–time in the leading nontrivial order. Let us note that the contribution of the second term in the two-gluon correlator, D_2, falls out, which is indeed required by gauge invariance.

To establish the connection of this function with the physical jet quenching parameter, it is instructive to rewrite the angular factor in the covariant form

$$K(v, \bar{v}) \equiv \frac{\cos\Delta\phi}{|\sin\Delta\phi|} = \frac{(v \cdot \bar{v})}{\sqrt{v^2 \bar{v}^2 - (v \cdot \bar{v})^2}}, \tag{38}$$

which allows us to study in detail the transition to the light-cone Minkowskian layout.

In Minkowski space–time, for each pair of time-like vectors v and $\bar{v}$ traveling in the opposite time direction, a rest frame can be found in which they are parameterized as follows:

$$\begin{aligned} v &= L\left(\gamma_1, -\beta_1\gamma_1, \mathbf{0}_\perp\right) = L\left(\cosh\frac{\psi_1}{2}, -\sinh\frac{\psi_1}{2}, \mathbf{0}_\perp\right), \\ \bar{v} &= -L\left(\gamma_2, -\beta_2\gamma_2, \mathbf{0}_\perp\right) = -L\left(\cosh\frac{\psi_2}{2}, -\sinh\frac{\psi_2}{2}, \mathbf{0}_\perp\right). \end{aligned} \tag{39}$$

Evaluated in these two vectors, the function $K(v, \bar{v})$ reads:

$$K(v, \bar{v}) = \frac{v \cdot \bar{v}}{\sqrt{v^2\bar{v}^2 - (v \cdot \bar{v})^2}} = -i\frac{\cosh\left(\dfrac{\psi_1 - \psi_2}{2}\right)}{\left|\sinh\left(\dfrac{\psi_1 - \psi_2}{2}\right)\right|}. \tag{40}$$

In the case we are interested in, i.e., the case $\psi_1 = \psi_2$, this expression is singular. Thus, in the limit of v and $\bar{v}$ lying in the opposite space–time direction, the parameterization (40) of $K(v, \bar{v})$ is ill-defined.

Moreover, one has to be careful in using the definition (38) $K(v, \bar{v})$ in the light-cone case. To illustrate this, take the parameterization (39) of v and $\bar{v}$ in the limit of infinite rapidity:

$$\begin{aligned} v_{\rm LC} &= \frac{L}{2}\left(\mathrm{e}^{\psi_1/2}, -\mathrm{e}^{\psi_1/2}, \mathbf{0}_\perp\right), \\ \bar{v}_{\rm LC} &= -\frac{L}{2}\left(\mathrm{e}^{\psi_2/2}, -\mathrm{e}^{\psi_2/2}, \mathbf{0}_\perp\right), \\ v_{\rm LC}^2 &= \bar{v}_{\rm LC}^2 = 0. \end{aligned} \tag{41}$$

In the light-cone limit $\psi_1, \psi_2 \to \infty$, formula (40) is clearly ill-defined as well. However, one cannot simply insert (41) into definition (38), since that would yield:

$$K\left(v \cdot \bar{v}\right)_{\rm LC} = -\frac{L^2}{4}\frac{\mathrm{e}^{\psi_1/2}\mathrm{e}^{\psi_2/2} - \mathrm{e}^{\psi_1/2}\mathrm{e}^{\psi_2/2}}{\sqrt{-\dfrac{L^4}{16}\left(\mathrm{e}^{\psi_1/2}\mathrm{e}^{\psi_2/2} - \mathrm{e}^{\psi_1/2}\mathrm{e}^{\psi_2/2}\right)^2}} = \frac{0}{0}.$$

Thus, although we have a covariant definition (38) of the angular dependence $K(v, \bar{v})$, we are facing problems both in the evaluation of $K(v, \bar{v})$ for v and $\bar{v}$ opposite vectors in Minkowski space–time, and in the evaluation of $K(v, \bar{v})$ for vectors on the light cone.

Let us consider a straightforward solution of this problem, which requires just being more careful when taking the light-cone limit. Indeed, writing

$$v^* = \lim_{\epsilon\to 0} \frac{L}{2}\left(\mathrm{e}^{\psi_1/2}+\epsilon, -\mathrm{e}^{\psi_1/2}+\epsilon, \mathbf{0}_\perp\right),$$
$$\bar{v}^* = \lim_{\delta\to 0} -\frac{L}{2}\left(\mathrm{e}^{\psi_2/2}+\delta, -\mathrm{e}^{\psi_2/2}+\delta, \mathbf{0}_\perp\right),$$

the definition of $K(v,\bar{v})$ can be readily used, yielding:

$$K\left(v^*,\bar{v}^*\right) = \lim_{\epsilon,\delta\to 0}\left(-\frac{L^2}{4}\frac{\epsilon\,\mathrm{e}^{\psi_2/2}+\delta\,\mathrm{e}^{\psi_1/2}}{\sqrt{-\frac{L^4}{16}\left(\epsilon\,\mathrm{e}^{\psi_2/2}+\delta\,\mathrm{e}^{\psi_1/2}\right)^2}}\right) = i. \qquad (42)$$

This method suggests, however, setting the primordial partons off-mass-shell. An even more straightforward method is to place *only one vector on the light cone*, while the other vector remains time-like (that is, we define the *skewed* layout), for example:

$$v \to v_{\mathrm{LC}} = \frac{L}{2}\left(\mathrm{e}^{\psi_1/2}, -\mathrm{e}^{\psi_1/2}, \mathbf{0}_\perp\right),$$
$$\bar{v} = -L\left(\cosh\frac{\psi_2}{2}, -\sinh\frac{\psi_2}{2}, \mathbf{0}_\perp\right).$$

In that case, using the definition of $K(v_{\mathrm{LC}},\bar{v})$ yields (since $v_{\mathrm{LC}}^2 = 0$):

$$K(v_{\mathrm{LC}},\bar{v}) = \frac{v_{\mathrm{LC}}\cdot\bar{v}}{\sqrt{v_{\mathrm{LC}}^2\bar{v}^2-\left(v_{\mathrm{LC}}\cdot\bar{v}\right)^2}} = i.$$

The last equation suggests that the angular multiplier $K(v,\bar{v})$ is invariant in the skewed layout, so that one can readily put the second vector on the light cone. After a straightforward, but tedious calculation (see for technical details [39]), we finally obtain

$$\tilde{P}^{(1)}(\mathbf{z}_\perp; v,\bar{v}) = g^2 C_F 2\pi \frac{\cos\Delta\phi}{|\sin\Delta\phi|}\left[(\omega-2)D_1(\mathbf{z}_\perp^2;\lambda)+2\mathbf{z}_\perp^2 D_1'(\mathbf{z}_\perp^2;\lambda)\right], \quad (43)$$

where we take into account the IR-cutoff λ in D_1, which might be IR-singular. In contrast, the UV-finiteness is guaranteed by the effective UV-cutoff $\mathbf{z}_\perp$. Equation (43) is our main result for the skewed probability distribution in Euclidean space–time in the leading nontrivial order. Let us note that the contribution of the second term in the two-gluon correlator, D_2, falls out, which is indeed required by gauge invariance.

CONCLUSIONS

We discussed several examples of the quantum gauge-invariant correlation functions — hadronic and vacuum matrix elements — which include the complicated sets of the Wilson lines lying on or near the light cone. It is shown that the explicit path-dependence is the price one has to pay for the gauge invariance of the nonlocal correlators. The dependence of the geometrical layout shows up in the renormalization-group properties of the transverse-momentum-dependent parton distribution functions, defines their rapidity evolution and affects the universality of the TMD PDFs defined within the QCD factorization approach to the semi-inclusive hadronic processes. Another important example of the manifestation of the path-dependence is given by the Wilson lines correlator in the definition of the jet quenching parameter. In that case, the geometrical setup delivers unavoidable singularity in the angular-dependent factor, for which reason one must follow a specific procedure of the transition from Euclidean to Minkowski space–time in order to go around this problem. Therefore, the path-dependence in nonlocal operators in quantum field theory is not only of a significant theoretical interest, but yields direct consequences in the hadronic phenomenology.

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