ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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MOTT–ANDERSON FREEZE-OUT AND THE STRANGE MATTER "HORN"

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We discuss the $\sqrt{s}$-dependence of the K^+/π^+ ratio in heavy-ion collisions (the "horn" effect) within a Mott–Anderson localization model for chemical freeze-out. The different response of pion and kaon radii to the hot and dense hadronic medium results in different freeze-out conditions. We demonstrate within a simple model that this circumstance enhances the "horn" effect relative to statistical models with universal chemical freeze-out.

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1. MOTT–ANDERSON FREEZE-OUT FOR PIONS AND KAONS

We would like to investigate whether the Mott–Anderson model scenario [1, 2] would predict different chemical freeze-out conditions for kaons as compared to pions. According to this model, the chemical freeze-out of hadrons from an expanding and cooling fireball created in the course of an ultra-relativistic heavy-ion collision is based on the strong medium dependence of hadronic radii which govern hadron–hadron cross sections via the geometrical Povh–Hüfner law $\sigma_{hh'} = \lambda\langle r_h^2\rangle\langle r_{h'}^2\rangle$ [3]. The (inverse) collision time $\tau^{-1}_{\mathrm{coll},h} = \sum\limits_{h'}\sigma_{hh'}n_{h'}$ determines the relaxation of hadron species h towards its chemical equilibrium by reactive collisions with hadrons h' having a number density $n_{h'}$. Due to the reduction of the chiral condensate in hot and dense hadronic matter the hadron radii swell as a precursor effect for the Mott–Anderson delocalization of hadron wave functions in the course of the chiral restoration transition. This behaviour has been quantified for the pion within the NJL model [4] as

$$\langle r_\pi^2\rangle_{T,\mu} \simeq \frac{3}{4\pi^2}f_\pi^{-2}(T,\mu) = \frac{3M_\pi^2}{4\pi^2 m_q}\left|\langle\bar{q}q\rangle_{T,\mu}\right|^{-1}. \tag{1}$$

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In a hadronizing quark–gluon plasma the medium dependence of the chiral condensate has the form [1, 2, 5]

$$\langle \bar{q}q \rangle_{T,\mu} = \langle \bar{q}q \rangle_{T,\mu}^{\rm MF} + \sum_{h=M,B} \frac{\sigma_q^h}{m_q} n_h(T,\mu), \tag{2}$$

where the first term stands for the quark mean-field contribution to the chiral condensate and the second one comes from the correlated quarks in hadrons with the scalar density

$$n_h(T,\mu) = \frac{d_h}{2\pi^2} \int\limits_0^\infty dk k^2 \frac{m_h}{E_h} \frac{1}{{\rm e}^{(E_h-\mu_h)/T} \mp 1}. \tag{3}$$

Here $h = M, B$ stands for mesons and baryons with the minus (plus) sign for mesons (baryons) in Eq. (3). The hadron sigma terms are defined as $\sigma_f^h = m_f(\partial m_h/\partial m_f)$ for the quark flavors $f = u, d, s, \ldots$ [5]. The general expression (2) can also be applied for the strange quark condensate by replacing $q \leftrightarrow s$.

As shown in [1], already a schematic resonance gas model consisting of $d_\pi = 8$ pionic and $d_N = 20$ nucleonic degrees of freedom gives an excellent description of the universal freeze-out line in the $T - \mu_B$ plane. This curve has been given the parametric form [6]

$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4, \tag{4}$$

$$\mu_B(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}, \tag{5}$$

where $\sqrt{s}$ is the collision energy in the nucleon–nucleon center-of-mass system. We will identify this curve with the Mott–Anderson freeze-out line for pions.

In order to apply the model to kaon freeze-out, we have to consider the in-medium kaon radius which in analogy to (1) reads

$$\langle r_K^2 \rangle_{T,\mu} \simeq \frac{3}{4\pi^2} f_K^{-2}(T,\mu) = \frac{3M_K^2}{\pi^2(m_q + m_s)} \left| \langle \bar{q}q \rangle_{T,\mu} + \langle \bar{s}s \rangle_{T,\mu} \right|^{-1}. \tag{6}$$

Now we are in the position to understand why the Mott–Anderson model predicts different freeze-out lines for pions and kaons. While the pion radius directly responds to the medium dependence of the light quark condensate which gets modified already due to the presence of pions themselves as the lightest species in the system, the kaon radius depends on the strange quark condensate (see Eq. (2) for $q \leftrightarrow s$) which is more inert to the medium since its modification requires that strangeness-carrying species be abundant. The latter, however, are suppressed relative to light hadrons by their larger mass so that a possible approximation reads

$$\langle \bar{s}s \rangle_{T,\mu} \approx \langle \bar{s}s \rangle^{\rm MF}_{T,\mu}. \tag{7}$$

The numerical results for the pion and kaon freeze-out lines are shown in Fig. 4, *b* below.

2. THE KAON FREEZE-OUT LINE FROM K^+/π^+ DATA

2.1. Pion and Strangeness Chemical Potentials. To address the strange matter "horn", we have to consider the ratio of yields of positively charged kaons to positively charged pions, denoted as $K^+/\pi^+ = n_{K^+}/n_{\pi^+}$. Within the thermal statistical model, the number densities of charged pions and kaons are given by

$$n_{\pi^+} = n_{\pi^-} = \int\limits_0^\infty \frac{dp}{2\pi^2} p^2 \frac{1}{{\rm e}^{(\sqrt{p^2+m_\pi^2}-\mu_\pi)/T} - 1}, \tag{8}$$

$$n_{K^\pm} = \int\limits_0^\infty \frac{dp}{2\pi^2} p^2 \frac{1}{{\rm e}^{(\sqrt{p^2+m_K^2}\mp(\mu-\mu_s))/T} - 1}, \tag{9}$$

where $\mu = \mu_B/3$ is the light quark chemical potential. For the strange quark chemical potential μ_s which has to assure vanishing net strangeness, we take $\mu_s = \varepsilon\mu$, being guided by [7] to adopt a proportionality to μ.

Forming the ratio $K^+/\pi^+ = n_{K^+}/n_{\pi^+}$, one can now attempt a preliminary comparison with data and explore the role of the two parameters μ_π and ε, see Fig. 1.

The chemical potential μ_π for pions was introduced as a phenomenological parameter to describe the low-momentum enhancement of the pion distribution observed at CERN SPS [8], with a value of the order of the pion mass signalling the onset of pion condensation. Recently, the parameter μ_π has been rediscovered as a possibility to solve the proton-to-pion puzzle at the LHC [9]. Here it has been given the meaning of parameter characterizing the nonequilibrium nature of the pion distribution as imprinted in their transverse momentum spectra. Note that the role of a pion chemical potential in the context of condensation phenomena has been pointed out, e.g., in [10]. An elucidation of the emergence of this parameter from an underlying nonequilibrium description of pion production in heavy-ion collisions is, to best of our knowledge, still missing.

2.2. Parametrizing Different Kaon and Pion Freeze-Out. In the next step, we want to explore the possibility that the kaon freeze-out points in the $T-\mu_B$ plane do not coincide with those for the pions which are assumed to lie on the

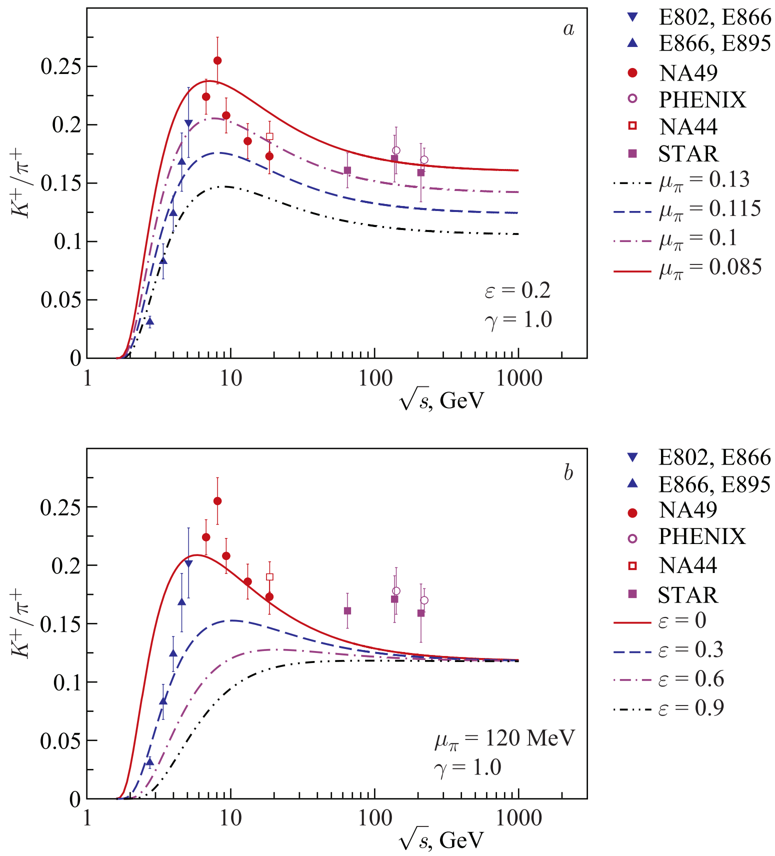

Fig. 1. The ratio $K^+/\pi^+ = n_{K+}/n_{\pi+}$ according to Eqs. (8), (9) compared with experimental data for varying nonequilibrium parameter μ_π (a) and varying strangeness chemical potential ε (b)

universal freeze-out curve parametrized by Eqs. (4) and (5). To this end, we make the ansatz

$$\mu_{\rm fo}^K(T) = \sqrt{\gamma}\ \mu_{\rm fo}^\pi(T), \tag{10}$$

where $\mu_{\rm fo}^\pi(T)$ is a solution of the biquadratic equation (4)

$$\mu_{\rm fo}^\pi(T) = \sqrt{\frac{b}{2c}\left(\sqrt{1 + \frac{4c}{b^2}(T_0 - T)} - 1\right)}. \tag{11}$$

We have denoted $T_0 = a$ in order to make clear the meaning of the parameter a as the freeze-out temperature at vanishing baryochemical potential. The para-

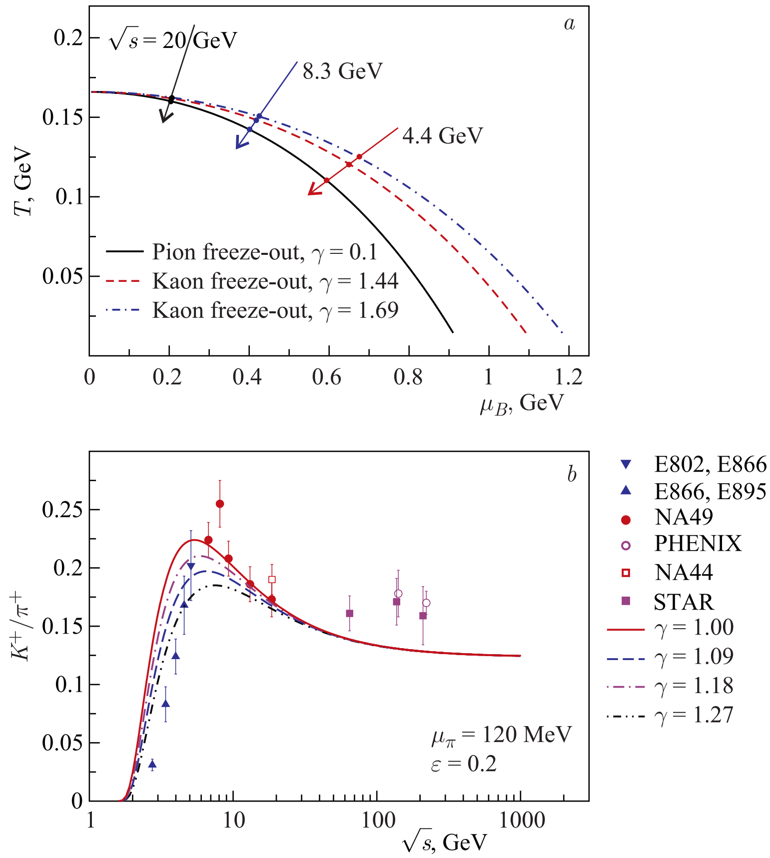

Fig. 2. *a*) The difference between the freeze-out lines for kaons $T_{\rm fo}^K(\mu)$ and for pions $T_{\rm fo}^\pi(\mu)$ (solid line) is being quantified by the parameter γ. Two lines with $\gamma > 1.0$ are shown for comparison. *b*) K^+/π^+ ratio as a function of $\sqrt{s}$ for different values of γ compared to the data (symbols)

meter $\gamma \geqslant 1$ reflects our expectation from the Mott–Anderson model of Sec. 1 that the freeze-out line for the kaons lies for a given temperature at larger μ_B-values than that for pions, see Fig. 2, *a*.

The assumption we make now is that the evolution of the system across the freeze-out lines follows lines of constant entropy $T/\mu = \kappa$ for a given $\sqrt{s}$, and

$$\kappa(\sqrt{s}) = \frac{T(\sqrt{s})}{\mu(\sqrt{s})} \tag{12}$$

can be found from the ratio of Eqs. (4) and (5). In order to determine for given $\sqrt{s}$ the freeze-out point for kaons, one finds first $\mu_{\rm fo}^K$ from solving the fourth-order

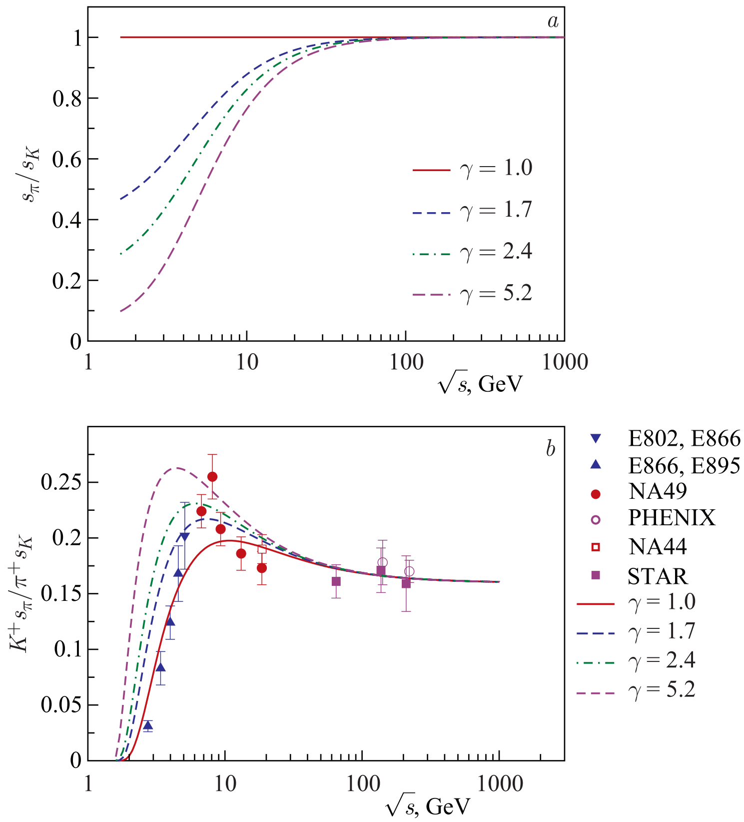

Fig. 3. *a*) Ratio of entropy densities as a measure for the change in the freeze-out volume between kaon and pion freeze-out. *b*) K^+/π^+ ratio for different values of γ including the volume expansion effect

equation in μ

$$\mu^4 + \frac{b\gamma}{c}\mu^2 + \frac{\gamma^2\ \kappa(\sqrt{s})}{c}\mu - \frac{\gamma^2 T_0}{c} = 0, \tag{13}$$

and then inserts the solution in (12) to find the kaon freeze-out temperature

$$T_{\rm fo}^K(\sqrt{s}) = \kappa(\sqrt{s})\ \mu_{\rm fo}^K(\sqrt{s}). \tag{14}$$

As a result of this procedure, we obtain the ratio K^+/π^+ shown in Fig. 3, where kaon and pion densities are to be taken at their respective, differing freeze-out points which are related by a straight line in the $T - \mu_B$ plane. Varying $\sqrt{\gamma}$ in the range $1 \leqslant \sqrt{\gamma} \leqslant 1.3$, we observe that the slope of the K^+/π^+ ratio below the "horn" gets diminished, while the asymptotics at high $\sqrt{s}$ remains unaffected.

3. RESULTS AND DISCUSSION

In Fig. 4 we show in the panel *a* the result for the K^+/π^+ ratio with a pointwise fit of the function $\gamma(\sqrt{s})$ in order to describe the shape of the “horn” effect. This parametrizes the deviation of the kaon freeze-out line from that of the pions in the $T-\mu$ plane as shown in the panel *b* of that figure. For not too high values of $\mu < 0.7\ \mu_{\max}$ the comparison with the prediction from the Mott–Anderson model works very well. Beyond this value of the chemical potential, i.e., for energies below the peak position of the “horn” at $\sqrt{s} \sim 8$ GeV, both the simple assumption of neglecting strange hadrons in the formula for the strange conden-

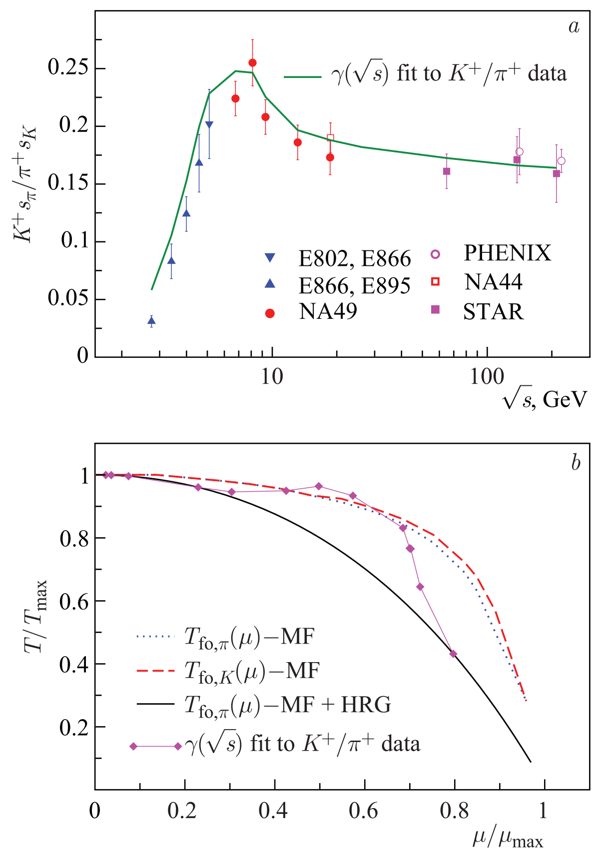

Fig. 4. *a*) Fit of $\gamma(\sqrt{s})$ (line) to the experimental K^+/π^+ ratio (symbols). *b*) Freeze-out lines of kaons and pions in the $T-\mu$ plane from the Mott–Anderson localization model compared with values extracted from the experimental K^+/π^+ ratio by parametrizing the $\gamma(\sqrt{s})$ of the toy model

sate in the Mott–Anderson model and the simplifications of the toy model for extracting the kaon freeze-out line break down. Among the issues to be included upon improvement are resonance decays, excluded volume effects, the canonical statistical suppression factor and an elucidation of the interrelation between the non-universal freeze-out discussed here and the strangeness enhancement factors used in alternative approaches [11–13]. Moreover, it will be interesting to study the relationship to other approaches to the "horn" effect like, e.g., the modified statistical model [14] or the "quarkyonic" explanation of [15]. Nevertheless, the first step gave a promising result and the necessary improvements to go further can clearly be identified.

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REFERENCES

1. *Blaschke D. B. et al.* Chiral Condensate and Chemical Freeze-Out // Phys. Part. Nucl. Lett. 2011. V. 8. P. 811.
2. *Blaschke D. et al.* Chiral Condensate and Mott–Anderson Freeze-Out // Few Body Syst. 2012. V. 53. P. 99.
3. *Povh B., Hüfner J.* Geometric Interpretation of Hadron–Proton Total Cross Sections and a Determination of Hadronic Radii // Phys. Rev. Lett. 1987. V. 58. P. 1612.
4. *Hippe H. J., Klevansky S. P.* Nambu–Jona-Lasinio Model Compared with Chiral Perturbation Theory: The Pion Radius in $SU(2)$ Revisited // Phys. Rev. C. 1995. V. 52. P. 2172.
5. *Jankowski J., Blaschke D., Spalinski M.* Chiral Condensate in Hadronic Matter // Phys. Rev. D. 2013. V. 87. P. 105018.
6. *Cleymans J. et al.* Comparison of Chemical Freeze-Out Criteria in Heavy-Ion Collisions // Phys. Rev. C. 2006. V. 73. P. 034905.
7. *Karsch F., Redlich K.* Probing Freeze-Out Conditions in Heavy Ion Collisions with Moments of Charge Fluctuations // Phys. Lett. B. 2011. V. 695. P. 136.
8. *Kataja M., Ruuskanen P. V.* Nonzero Chemical Potential and the Shape of the p_T Distribution of Hadrons in Heavy Ion Collisions // Phys. Lett. B. 1990. V. 243. P. 181.
9. *Begun V., Florkowski W., Rybczynski M.* Explanation of Hadron Transverse-Momentum Spectra in Heavy-Ion Collisions at $\sqrt{s_{NN}} = 2.76$ TeV within Chemical Non-Equilibrium Statistical Hadronization Model // Phys. Rev. C. 2014. V. 90. P. 014906.

10. *Turko L.* Quantum Gases with Internal Symmetry Revisited: Condensation of Hadronic Matter // Z. Phys. C. 1994. V. 61. P. 297.
11. *Becattini F. et al.* Hadronization and Hadronic Freeze-Out in Relativistic Nuclear Collisions // Phys. Rev. C. 2012. V. 85. P. 044921.
12. *Petran M., Rafelski J.* Universal Hadronization Condition in Heavy-Ion Collisions at $\sqrt{s_{NN}} = 62$ GeV and at $\sqrt{s_{NN}} = 2.76$ TeV // Phys. Rev. C. 2013. V. 88. P. 021901.
13. *Braun-Munzinger P. et al.* Hadron Production in Au–Au Collisions at RHIC // Phys. Lett. B. 2001. V. 518. P. 41.
14. *Sagun V. V. et al.* Strangeness Enhancement at the Hadronic Chemical Freeze-Out // Ukr. J. Phys. 2014. V. 59. P. 1043.
15. *Andronic A. et al.* Hadron Production in Ultra-Relativistic Nuclear Collisions: Quarkyonic Matter and a Triple Point in the Phase Diagram of QCD // Nucl. Phys. A. 2010. V. 837. P. 65.