

## MODELING TYPE Ia SUPERNOVAE AND QUARK NOVAE

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Compact astrophysical objects can change their structure and composition in combustion-like processes. In the case of white dwarfs, thermonuclear burning can even lead to an explosion as a Type Ia supernova.

We give a brief account of the physical concepts of combustion and of methods to model this phenomenon in large-scale numerical simulations. As examples for application we discuss thermonuclear explosions of white dwarfs and to hypothetical “quark novae” transforming neutron stars into strange quark stars.

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### 1. INTRODUCTION: COMBUSTION IN ASTROPHYSICAL COMPACT OBJECTS

Combustion is the burning of material in a single reaction or a sequence of reactions that produces heat and leads to the conversion of species. In certain situations, this process is restricted to a part of the total volume of the system and it propagates. Then, the phenomenon is called a *flame*. In contrast to terrestrial combustion, the astrophysical situation usually does not involve chemical reactions but rather results from nuclear physics processes. Thus, an “oxidizer” is not needed and it resembles the case of “premixed” combustion. Another important distinction of *astrophysical* combustion processes is that they proceed under extreme conditions — densities and temperatures are orders of magnitude higher than in terrestrial systems. Therefore, the state of matter is very different and the choice of the appropriate equation of state requires special consideration.

Two astrophysical situations that feature combustion-like processes are the nuclear burning of degenerate white dwarf matter leading to thermonuclear supernovae and the hypothetical burning of hadronic neutron stars into strange quark stars. Although the details of the microphysical conversion and transport

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processes are very different, a description on large scales (i.e., the numerical simulation of the process involving the entire stellar object) can be achieved by employing the same concepts of flame modeling. Of course, apart from these examples, combustion phenomena are observed in several other astrophysical objects, such as stars, novae, X-ray bursts, etc.

Thermonuclear burning in carbon/oxygen white dwarf material starts out with the fusion of  $^{12}\text{C}$  — a reaction that is extremely sensitive to temperature. In the regime of interest, the corresponding reaction rate scales with  $\sim T^{20}$ . Consequently, burning is restricted to a narrow region in space and proceeds in thin fronts, called flames or combustion waves. These possess an internal width on the order of millimetres to centimetres only [1], which is tiny compared to the scales of white dwarf stars. Therefore, it is justified to describe these thermonuclear flames as discontinuities separating the “fuel” from the “ashes”. On microscopic scales, however, the propagation of (deflagration) flames is driven by transport processes — in the situation considered here dominated by the thermal conduction of the electrons.

The conversion of nuclear matter into strange quark matter (SQM) starts out with the disintegration of nucleons on time scales of the strong interaction ( $t_{\text{strong}} \sim 10^{-24}$  s) and is followed by the conversion of d- into s-quarks on weak interaction timescales ( $t_{\text{weak}} \sim 10^{-8}$  s). The resulting flame speed ranges from about 0.2 to 4 percent of the speed of light and the flame width is typically on the order of a centimeter [2]; again much smaller than the radius of a neutron star. Therefore, modeling the conversion front as a discontinuity is appropriate here, too.

## 2. MODELING APPROACHES

As the Reynolds numbers of astrophysical flows are typically very large and the artificial viscosity of numerical hydro solvers dominates over any physical viscosity, a modeling ansatz in terms of the reactive Euler equations is appropriate. In combination with the appropriate jump conditions over the combustion wave, that is described in the discontinuity approximation, two regimes of propagation of the burning front can be identified: subsonic deflagrations and supersonic detonations (e.g., [3]). Here, we will focus exclusively on modeling deflagrations, although detonations may also play a significant role in astrophysical burning processes, in particular, in thermonuclear supernovae.

A numerical approach that treats combustion waves as discontinuities is the level-set technique [4]. It has been applied to simulations of thermonuclear supernova explosions and to the burning of neutron stars into strange quark stars (“quark novae”) [5].

Astrophysical flows often feature extremely high Reynolds numbers (typical values range from  $10^{10}$  to  $10^{14}$ ). It is therefore not surprising that the interaction

of deflagration flames with turbulent motions is an important aspect of modeling. In the events under consideration here, the interaction of the flame with turbulence is restricted over the most parts of the evolution to larger scales and turbulent eddies do not penetrate the internal flame structure. In the resulting *flamelet regime* of turbulent combustion, the entire flame structure is folded and wrinkled on large scales. This increases the flame surface area. Thus, the fuel consumption and energy production rates are enhanced and the flame accelerates. Due to the vast range of the involved spatial scales, this effect cannot be resolved in numerical simulations and has to be accounted for by appropriate models. One method that has been developed for thermonuclear supernova simulations and is employed in the simulations discussed here is based on a subgrid-scale turbulence model [3, 6]. With this model, the turbulent velocity fluctuations on the scale of the computational grid can be determined. According to [7], these set the effective speed of the flame front on large scales.

### 3. THERMONUCLEAR SUPERNOVAE

Type Ia supernovae (SNe Ia) are thought to result from thermonuclear explosions of white dwarf stars (for reviews, see [8, 9]). These objects are a key part of the cosmic cycle of matter — responsible for the production of the majority of iron in the Universe — and they are used as cosmic distance indicators. Based on the later measurements, the accelerated expansion of the Universe was discovered [10, 11].

The main problem with modeling the physical processes leading to SNe Ia is the unknown nature of the progenitor system. To reach an explosive state, the white dwarf has to interact with a binary companion, but the kind of companion star and the details of the interaction process are unclear. For a long time, it was assumed that the companion is a normal star and that the white dwarf accretes matter from it until reaching the Chandrasekhar mass limit of  $\sim 1.4M_{\odot}$  (a prominent example of an explosion model resulting from this scenario is W7 [12]). Although this model has been successful in explaining some key characteristics of SNe Ia, recent observations and theoretical results cast some doubt on this scenario as being the only explanation for normal SNe Ia. Promising alternatives involve detonations in sub-Chandrasekhar mass white dwarfs [13], possibly initiated by violent mergers of two white dwarfs [14–17] or double detonations [18–20].

A prompt detonation in Chandrasekhar-mass white dwarf cannot explain SNe Ia because the star in hydrostatic equilibrium is entirely converted to iron by the supersonic detonation wave. If pre-expanded by a preceding subsonic deflagration phase [21–23], the results resemble observed SNe Ia [24], but there remain some problems in detail [25].

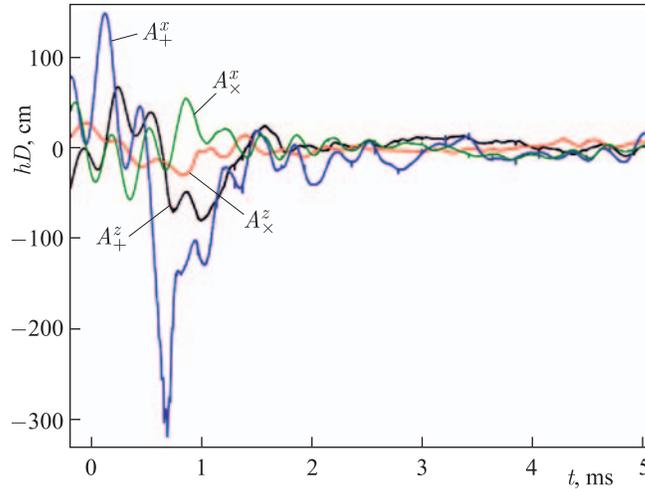
Pure deflagrations in Chandrasekhar-mass carbon–oxygen white dwarfs are subject to buoyancy instabilities when ignited in the center of the star. Conse-

quently, they interact with self-produced turbulence and accelerate. The result, however, even assuming favorable conditions for the ignition, falls short of explaining normal SNe Ia in terms of brightness (e.g., [26]) and spectral properties. Recent simulations [16,27] have shown that sparsely ignited, asymmetric deflagrations leave behind a bound remnant and lead to events that resemble a certain subclass of SNe Ia, the so-called 2002cx-like objects [16].

#### 4. QUARK NOVAE

The hypothetical conversion of a neutron star into a strange quark star as a “quark nova” is extensively discussed in the literature (e.g., [2,28–33]). As long as the process is exothermic, it can be modeled as a combustion process.

With the numerical approaches summarized in Sec.2, simulations of the process on scales of the neutron star have been performed [5]. Modifications of the simulation code that was originally developed for SNe Ia include an appropriate equation of state for hadronic matter [34,35] as well as for SQM (the MIT bag model for finite temperatures [36]) and an effective relativistic gravitational potential based on the Tolman–Oppenheimer–Volkoff equation [37]. The default initial star is set up as a nonrotating cold ( $T = 100$  keV) isothermal neutron star with a central density of  $10^{15}$  g · cm<sup>-3</sup>, a mass of  $1.4M_{\odot}$ , and a radius of 11 km. It was relaxed onto a three-dimensional computational grid with artificial friction. The conversion to SQM was initialized by a spherical seed of radius  $10^5$  cm with sinusoidal perturbation at the center of the star. Simulations of the burning process were performed for different values of the bag constant, variants of the



Color online. Filtered gravitational wave amplitudes from a 3D full-star quark nova simulation with the bag constant  $B^{1/4} = 147$  MeV (from [38])

Lattimer–Swesty equation of state (LS180, LS220), different neutron star masses, and varying numerical resolution.

The simulations demonstrate [5] that only the core of the stellar configuration is converted into SQM by a combustion process; the outer layers stay in the hadronic phase and the combustion wave does not reach the surface of the star because the conditions for exothermicity of the process are not fulfilled here. The numerically converged results show that the mass of the newly formed SQM core depends on the bag constant, the mass of the neutron star, and the choice of the equation of state for the hadronic material.

In contrast to SN Ia modeling, however, no direct comparison with observations is possible for quark nova simulations. To improve on this, gravitational wave signals have been extracted from the models. An example is shown in figure. We caution that due to numerical noise in the simulations that was filtered out from the plotted data, the shown signal is only a rough estimate. However, it still indicates that galactic events are possibly detectable with Advanced LIGO. The corresponding neutrino signal has been calculated from thermodynamic trajectories of the three-dimensional simulations [39].

## CONCLUSIONS

The concept of combustion fronts applies to various astrophysical phenomena. Often, these fronts are modeled in large-scale simulations as thin fronts or even discontinuities. Techniques to describe them numerically have been devised. The interaction of these fronts with turbulence requires special attention and is dynamically relevant.

Such models have been successfully applied to simulations of SNe Ia. They also facilitated the first three-dimensional full-star simulations of turbulent burning of neutron stars into strange quark stars. The later application is a proof of concept and refinements are certainly possible in future simulations.

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