

## UPPER CRITICAL FIELD AND (NON)SUPERCONDUCTIVITY OF MAGNETARS

*M. Sinha*<sup>1,2</sup>, *A. Sedrakian*<sup>1,\*</sup>

<sup>1</sup> Institute for Theoretical Physics, J. W. Goethe-University, Frankfurt am Main, Germany

<sup>2</sup> Indian Institute of Technology Rajasthan, Ratanada, Jodhpur, India

We construct equilibrium models of compact stars using a realistic equation of state and obtain the density range occupied by the proton superconductor in strong  $B$ -fields. We do so by combining the density profiles of our models with microscopic calculations of proton pairing gaps and the critical unpairing field  $H_{c2}$  above which the proton type-II superconductivity is destroyed. We find that magnetars with interior homogeneous field within the range  $0.1 \leq B_{16} \leq 2$ , where  $B_{16} = B/10^{16}$  G, are partially superconducting, whereas those with  $B_{16} > 2$  are void of superconductivity. We briefly discuss the neutrino emissivity and superfluid dynamics of magnetars in the light of their (non)superconductivity.

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### INTRODUCTION

The discoveries of the soft  $\gamma$ -ray repeaters and anomalous X-ray pulsars during the last decades and their identification with a new subclass of compact objects — magnetars — focus considerable attention on the properties of dense matter in strong magnetic fields. The inferred magnetic fields on the surfaces of these objects are of the order of  $B_{s16} \sim 0.01$ – $0.1$ . The magnitudes of interior fields in magnetars are not known, but have been frequently conjectured to be larger than their surface fields. Self-gravitating magnetic equilibria containing type-II superconductors were studied in the low-field limit, where the  $B$ -field is not much larger than the critical field  $H_{c1} \simeq 10^{13}$  G beyond which the creation of quantum flux tubes in protonic superconductor is energetically favorable [1–3]. These fields are well below the characteristic fields of magnetars. Nonsuperconducting self-gravitating equilibria were studied in the strong field regime [4, 5] and, as we argue below, are appropriate for describing magnetars with fields large enough ( $B_{16} > 2$ ) to destroy superconductivity.

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\*E-mail: sedrakian@th.physik.uni-frankfurt.de

Flux-tube arrays exist in type-II superconductors for  $B$ -fields up to the second critical field  $H_{c2}$  at which their normal cores touch each other and superconductivity vanishes. The microscopic parameters of the proton superconductor as well as the coherence length and the magnetic penetration depth depend on the local density and temperature of the protonic fluid. They have been computed previously in the zero-temperature limit [6]. An elementary estimate of the  $H_{c2}$  field follows from the observation that the coherence between the members of a Cooper pair will be lost when the Larmor radius of protons becomes of the order of coherence length  $\xi_p$  of Cooper pairs. This field is given by

$$H_{c2} = \frac{\Phi_0}{2\pi\xi_p^2}, \quad (1)$$

where  $\Phi_0 = \pi\hbar c/e$  is the flux quantum.

In this contribution, we construct equilibrium models of compact stars using a realistic equation of state and compute the density range occupied by the proton superconductor by combining the density profiles of our models with microscopic calculations of proton pairing gaps, which provide us with the density dependence of the local upper critical field  $H_{c2}$  required to destroy the proton superconductivity. We then go on to discuss the implications of the unpairing effect for neutrino cooling of magnetars and their superfluid dynamics.

## MODELING MAGNETARS

Our first step is to construct a model of a magnetar on the basis of a realistic equation of state (EoS) of dense nuclear matter. We assume that the magnetar's interior contains conventional nuclear matter composed of neutrons, protons, and electrons in  $\beta$ -equilibrium. As the underlying EoS, we take the one derived by [7], where the interaction between the nucleons is modelled in terms of the Argonne AV18 two-body interaction combined with a phenomenological three-body interaction. We also constructed models based on relativistic density functional theory [8] and found results similar to the previous EoS. The maximum masses of sequences built from these EOS are above the current observational lower limit  $2M_\odot$  on the maximum mass of any compact star. The EoS of [7] has a maximum mass  $2.67M_\odot$ .

In the core of a neutron star, protons pair in the  $^1S_0$  channel because of their relatively low density. If the matter is nearly isospin symmetric at high densities, there is also the possibility of  $^3D_2$  pairing in neutron-proton matter [9]. We adopt the  $S$ -wave gap in proton matter from [7] which, consistent with the underlying EoS, is based on the Argonne interaction supplemented by a three-body force. With this input we solved the Oppenheimer-Volkoff equations to

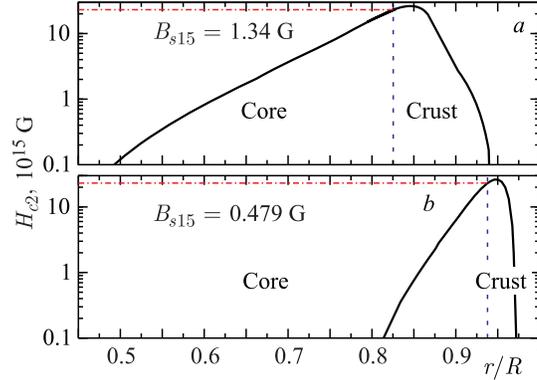


Fig. 1. Dependence of the critical magnetic field  $H_{c2}$  on the normalized internal radius  $r/R$  of the star for  $1.4M_{\odot}$ ,  $R = 13.85$  km star (a) and maximum mass  $2.67M_{\odot}$ ,  $R = 11.99$  km star (b). The crust-core interface (vertical dashed line) is located at  $R_{\text{core}} = 11.43$  km for the  $1.4M_{\odot}$  star and at  $R_{\text{core}} = 11.25$  km for the  $2.67M_{\odot}$  one. The maximal value of  $H_{c2}$  in each model is attained at the crust-core interface and is indicated by the horizontal dash-dotted line; the values of the corresponding surface fields are shown in the plot

obtain configuration of the given gravitational mass. In the following, we will examine two models of neutron stars, one with “canonical”  $1.4M_{\odot}$  mass and another with the maximum mass  $M = 2.67M_{\odot}$ .

Because of the density dependence of the microscopic parameters, notably the gap function, the critical fields  $H_{c1}$  and  $H_{c2}$  are density-dependent, which translates into dependence of these fields on the radius of the star. We relate the total nucleonic density to the proton fraction using fits provided in [7]. The density dependence of  $H_{c2}$  is shown in Fig. 1 for two models of neutron stars with canonical ( $1.4M_{\odot}$ ) and maximal ( $2.67M_{\odot}$ ) gravitational masses. It is seen that the maximal value of  $H_{c2}$ , which is about  $2B_{16}$ , is attained at the crust-core interface, and its value drops approximately linearly with decreasing radius. The field at the crust-core boundary  $H_b$  can be related to the surface field  $B_s$  of the star by the relation  $B_s \simeq \alpha_B H_b$  [3], where  $\alpha_B = \epsilon_b/3$ , and  $\epsilon_b R$  is the thickness of the crust,  $R$  being the radius of the star. Assuming the outer boundary of the core at  $0.5n_0$ , where  $n_0$  is the nuclear saturation density, we find  $\alpha_B = 0.058$  for the  $1.4M_{\odot}$  model and  $\alpha_B = 0.021$  for the  $2.67M_{\odot}$  model. The maximal value of  $H_{c2} \simeq 2B_{16}$  is attained at the crust-core boundary, and corresponding surface fields are  $B_{s16} = 0.134$  for the  $1.4M_{\odot}$  model and  $B_{s16} = 0.0479$  for the maximum mass  $2.67M_{\odot}$  model. Having established the limiting surface field at which superconductivity vanishes completely, we now consider decreasing it down to values characteristic for ordinary neutron stars. Figure 2 shows the extend of superconducting phase for different  $B_s$ , where we related the surface

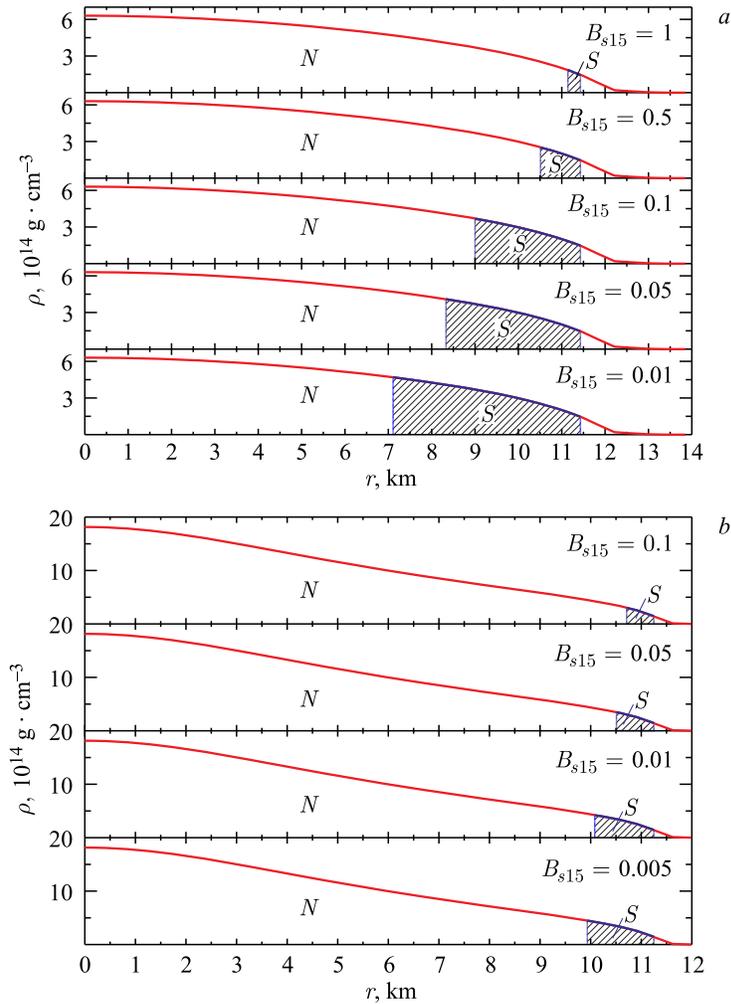


Fig. 2. Density profiles and the extent of superfluid phases (shaded areas) of  $1.4M_{\odot}$  star (upper panel) and  $2.67M_{\odot}$  star (lower panel) for different values of the surface field in units of  $10^{15}$  G. The normal and superconducting regions are labeled as  $N$  and  $S$

field to the crust-core boundary field according to the relations above. It is seen that with decreasing surface field, the superconducting volume increases and eventually reaches its volume in an ordinary neutron star. Hence, some magnetars may not have superconducting cores at all, while some may have, for moderate surface fields, a relatively small superconducting shell surrounding a core containing unpaired protons.

We conclude this section by stating our main observation: *intermediate-field magnetars with interior fields  $B_{16} \leq 2$  are partially nonsuperconducting due to the unpairing effect of the magnetic field on the proton superconductivity. Magnetars with interior fields  $B_{16} \geq 2$  are void of superconductivity.*

**Neutrino Emissivity and Heat Capacity.** The table summarizes the key neutrino emission processes in the cases of low and high magnetic fields. Strong magnetic fields lift the kinematical constraint on the direct Urca process, i.e., it can operate below the Urca threshold on proton fraction [10, 11]. As is well known, the proton and neutron pairing, which is characterized by the gaps  $\Delta_{n/p}$ , cuts the neutrino emission rates at low temperatures  $T \ll \Delta_{n/p}$  by an exponential factor  $\exp(-\Delta/T)$  for each neutron ( $n$ ) and proton ( $p$ ). If locally  $B > H_{c2}$ , the unpairing effect implies that this suppression is inoperative for protons, i.e., the suppression of the direct Urca process will be only due to the neutron pairing. Because  $\Delta_p \gg \Delta_n$ , the pairing induced suppression differs strongly from the one expected in the case of superconducting protons. In  $B$ -fields, an additional neutrino pair bremsstrahlung channel operates because of the paramagnetic splitting in the neutron and proton energies [12]. The unpairing effect implies that the direct bremsstrahlung process  $p \rightarrow p + \nu + \bar{\nu}$  will not be suppressed locally if  $B > H_{c2}$ , which, as in the case of the direct Urca process, will enhance neutrino emission compared to the low-field case. The third channel of neutrino emission is the pair-breaking (PB) neutrino emission from neutron and proton condensates [13–16]. The unpairing effect implies that the PB process for protons will be absent locally if  $B > H_{c2}$ .

**Allowed ( $\checkmark$ ) and forbidden ( $\times$ ) processes in the low- and high-field neutron stars. Here  $N \in n, p$  refers to nucleons;  $(NN)$ , to the Cooper pair**

Name	Process	Low- $B$	High- $B$
Direct Urca	$n \rightarrow p + e + \bar{\nu}_e$	$\times$	$\checkmark$
Direct bremsstrahlung	$N \rightarrow N + \nu_f + \bar{\nu}_f$	$\times$	$\checkmark$
PB neutrons	$n + n \rightarrow (nn) + \nu_f + \bar{\nu}_f$	$\checkmark$	$\checkmark$
PB protons	$p + p \rightarrow (pp) + \nu_f + \bar{\nu}_f$	$\checkmark$	$\times$

Let us consider, more quantitatively, how the unpairing effect will change the neutrino luminosity of the magnetar's core. Consider first the limit  $B = 0$ . The superconducting gap defines a shell in the star with a volume  $V$ . At nonzero  $B$ , a part of this shell will become normal (see Fig. 2); we denote the volume of this normal shell as  $V_N$  and the remainder superconducting volume as  $V_S = V - V_N$ . The net luminosity of the proton superconducting shell at any  $B$  can be written as  $L = L_S + L_N$ , with  $L_{S/N} \equiv \bar{\epsilon}_{S/N} V_{S/N}$ , where for the sake of the argument we use volume averaged values of the emissivities  $\bar{\epsilon}_{S/N}$ . Assuming that superfluidity

suppresses the neutrino luminosity, its maximum is achieved when the entire shell is normal, i.e.,  $L_{\max} = \bar{\epsilon}_N V$ . For arbitrary  $B$ -field the luminosity normalized to  $L_{\max}$  will become

$$\mathcal{R} = \frac{L}{L_{\max}} = 1 - \frac{V_S}{V} \left( 1 - \frac{\bar{\epsilon}_S}{\bar{\epsilon}_N} \right) \simeq 1 - \frac{V_S}{V}, \quad (2)$$

where the last equality follows in the limit  $\bar{\epsilon}_S/\bar{\epsilon}_N \ll 1$ . Because the right-most approximate value of  $r$  depends only on the volumes of normal and superconducting shells, we can extract its dependence on the  $B_s$ -field. For the  $M = 1.4M_\odot$  model in Fig. 2, we find that  $\mathcal{R}(B_s/10^{15} \text{ G})$  has the following values:  $\mathcal{R}(0.05) = 0.19$ ,  $\mathcal{R}(0.1) = 0.33$ ,  $\mathcal{R}(0.5) = 0.7$ ,  $\mathcal{R}(1) = 0.9$ . Similarly, for the  $M = 2.67M_\odot$  model in Fig. 2, we find  $\mathcal{R}(0.01) = 0.1$ ,  $\mathcal{R}(0.05) = 0.39$ ,  $\mathcal{R}(0.1) = 0.56$ . It is seen that the increase of the magnetic field leads to an increase of neutrino luminosity if unpaired matter emits more neutrinos than the superconducting one.

Finally, we note that the unpairing effect will enhance the heat capacity of the magnetar's core and, therefore, the magnetar's thermal relaxation time-scale to a given temperature. In the regions where locally  $B > H_{c2}$ , nonsuperconducting protons will double the heat capacity, which is mainly provided by relativistic electrons. The heat capacity of neutrons forming  $S$ - and  $P$ -wave condensates is negligible.

**Reheating and Rotational Response.** The magnetic energy is the key source of internal heating for magnetars [17, 18]. Conversion of magnetic energy into thermal energy depends on the electrical conducting properties of the fluid core of the star. If protons are superconducting, the magnetic field is frozen in the flux tubes and can be changed only if these migrate out of the fluid core. If, however, protons are unpaired by the  $B$ -field, then the magnetic field decay produces heat on a time-scale [19]

$$\tau \propto \frac{4\pi R^2}{c^2} \sigma, \quad (3)$$

where  $\sigma$  is the field-free electrical conductivity,  $R$  is a characteristic scale of the magnetic field. For  $R \sim 1 \text{ km}$ ,  $\tau \sim 10^8 \text{ yr}$ . The decay time-scale for the transverse  $B$ -field is substantially reduced in strong fields, where the transverse conductivity scales as  $\sigma_\perp^{-1} \sim B^2$  [20].

The unpairing effect in magnetars has a profound effect on the superfluid dynamics of its core. In low-field neutron stars, the core dynamics is determined by the interaction of the neutron vortices with the protonic flux tubes and the electromagnetic interactions of electrons with this conglomerate. In contrast, nonsuperconducting protons will couple to the electron fluid on plasma time-scales, which are much shorter than the hydrodynamical time-scales. Therefore, the core of a magnetar is a two-fluid system with neutron condensate forming the superfluid component and the proton-plus-electron fluids forming the normal

component. The neutron superfluid will couple to the electron–proton plasma by scattering of protons off the neutron quasiparticles confined in the cores of vortices via the strong nuclear force. The relaxation time-scale for this process is of order in the range from several minutes (at the crust-core interface) to a few seconds (in the deep core) for magnetar periods of the order of 10 s and temperatures of the order of  $10^8$  K [21].

To summarize, we computed the density profiles of the critical field  $H_{c2}$  in realistic models of compact stars and showed that they are strongly density-dependent. This implies that magnetars with approximately homogeneous constant interior  $B$ -fields are either completely or partially nonsuperconducting provided these fields are by a factor of 10 to 15 larger than the observed surface fields of magnetars.

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