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## CHARGED PION POLARIZABILITY IN THE NONLOCAL QUARK MODEL OF NAMBU–JONA-LASINIO TYPE

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The polarizability of a charged pion is estimated in the framework of the nonlocal chiral quark model of the Nambu–Jona-Lasinio type. Nonlocality is described by quark form factors of the Gaussian type. It is shown that the polarizability in this model is very sensitive to the form of nonlocality and choice of the model parameters.

Поляризуемость заряженного пиона оценена в рамках нелокальной киральной кварковой модели типа Намбу–Иона-Лазинио. Нелокальность описывается кварковыми формфакторами гауссового типа. Показано, что поляризуемость в этой модели очень чувствительна к типу нелокальности и модельным параметрам.

Recently, interest in direct determination of the pion polarizability in Primakoff scattering has been renewed. Now a new experiment is being prepared at CERN by the COMPASS collaboration where it is expected to obtain the statistics factor 6000 higher than in the previous experiment performed by the IHEP and JINR group in Protvino [1] at the beginning of the 1980s. However, the technical possibilities of this experiment allow one to obtain only very rough estimations of the pion polarizability with large error bars

$$\alpha_\pi = -\beta_\pi = 8.54 \pm 1.76 \pm 1.51, \quad (1)$$

where  $\alpha_\pi$  and  $\beta_\pi$  are electric and magnetic polarizabilities of the charged pion. Hereafter we express the polarizability in the units of  $10^{-42}$  cm<sup>3</sup>. The new experiment gives an opportunity to obtain a value of the pion polarizability with good accuracy [2].

As regards the theoretical aspect of this problem for the past years, there are a lot of estimations of this quantity made by many authors in different theoretical models; in particular, one of the authors of this article (M. K. V.) together with V. N. Pervushin obtained estimations of the pion polarizability in the framework of the nonlinear chiral model in 1975 [3]. After that, analogous calculations were performed by him in the quark linear sigma model of Nambu–Jona-Lasinio (NJL) type [4, 5]. The estimation obtained in the above-mentioned works corresponds to the experimental result (1).

In this short note, we want to return to this question and estimate the pion polarizability in the nonlocal model of the NJL type [6], where nonlocality is described by quark form factors of the Gaussian type.

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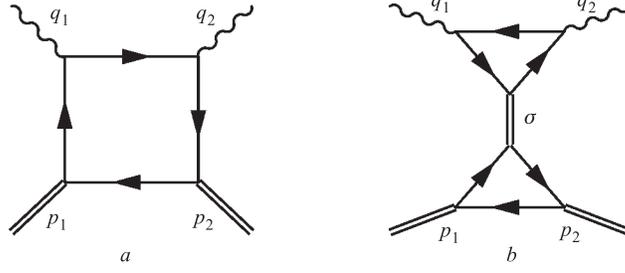


Fig. 1. Diagrams describing charge pion polarizability: a) box diagrams; b)  $\sigma$  pole diagrams

As is shown in [5], in the local NJL model the main contribution to the pion polarizability in the leading order of  $1/N_c$  expansion stems from two types of diagrams: the diagrams with light intermediate  $\sigma$  meson and the box diagrams (see Fig. 1). The contribution from the diagrams with other resonances (heavier scalars, vectors and axial vectors) is smaller than 3% and can be neglected in the present consideration.

The contribution of the box diagrams to the structure part of the Compton effect takes the form

$$A_{(a)}^{\mu\nu} = -\frac{\alpha}{9\pi f_\pi^2} (g^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu),$$

where  $q_1, q_2$  are the momenta of the incoming (outgoing) photons;  $\alpha = 1/137$  is the fine structure constant;  $f_\pi = 93$  MeV is the weak pion decay constant. The contribution of the  $\sigma$  pole diagram can be factorized into three parts:

$$A_{(b)}^{\mu\nu} = A_{\sigma\pi\pi} D_\sigma A_{\sigma\gamma\gamma}^{\mu\nu},$$

where the  $\sigma\pi\pi$  vertex is  $A_{\sigma\pi\pi} = 4mgZ = 4m^2 Z^{1/2}/f_\pi$ ; the  $\sigma$ -meson propagator is  $D_\sigma \approx 1/M_\sigma^2$ , and the  $\sigma\gamma\gamma$  vertex is [7]

$$A_{\sigma\gamma\gamma}^{\mu\nu} = \alpha C_{\sigma\gamma\gamma} (g^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu),$$

$$C_{\sigma\gamma\gamma} = \frac{10Z^{-1/2}}{9\pi f_\pi}.$$

Here  $m = 280$  MeV is the constituent quark mass after taking into account the  $\pi - a_1$  mixing [5];  $g = mZ^{-1/2}/f_\pi$  is the  $\sigma$ -meson coupling constant; factor  $Z$  is  $Z = (1 - 6m^2/M_{a_1}^2)^{-1} \approx 1.4$  ( $M_{a_1} = 1.26$  GeV is the mass of the  $a_1$  meson);  $M_\sigma$  is the  $\sigma$ -meson mass  $M_\sigma = \sqrt{4m^2 + M_\pi^2} \approx 2m$ , and  $M_\pi$  is the pion mass. One can express the amplitude of the process as

$$A^{\mu\nu} = A_{(a)}^{\mu\nu} + A_{(b)}^{\mu\nu} = C (g^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu). \quad (2)$$

As a result, the pion polarizability takes the form [7, 8]

$$\alpha_\pi = -\beta_\pi = \frac{C}{2M_\pi} = \frac{5\alpha}{9\pi M_\pi f_\pi^2} \left(1 - \frac{1}{10}\right) = 7.3. \quad (3)$$

One can see that the polarizability does not depend on the model parameters.

Let us emphasize that the contribution of the box diagrams makes up only 10% of the contribution of the  $\sigma$  pole diagrams<sup>1</sup>.

Our calculation shows that in the nonlocal model the choice of the form factor  $f(p)$  and the model parameters have a greater influence on the pion polarizability than taking into account the box diagrams. Therefore, in this note we restrict ourselves to only the estimations of the contribution of the  $\sigma$  pole diagrams.

In the nonlocal model, the quark mass depends on momentum. In [6], the following representation for the quark propagator is proposed:

$$\frac{m^2(p)}{m^2(p) + p^2} = \exp(-p^2/\Lambda^2), \quad (4)$$

where  $\Lambda$  is the parameter of nonlocality. As a result, the pole part of the quark propagator has no singularities on the whole real axis, which leads to the quark confinement. The quark mass function takes the form [6]

$$m^2(p) = \left( \frac{p^2}{\exp(p^2/\Lambda^2) - 1} \right). \quad (5)$$

From this equation one can see that  $m(0) = \Lambda$ . Then, from the condition that the weak pion decay constant equals 93 MeV, it follows that  $m(0) = \Lambda = 340$  MeV. For the  $\sigma\pi\pi$  vertex and the  $\sigma$ -meson mass we have  $A_{\sigma\pi\pi} = 1.57$  GeV,  $M_\sigma = 420$  MeV [6].

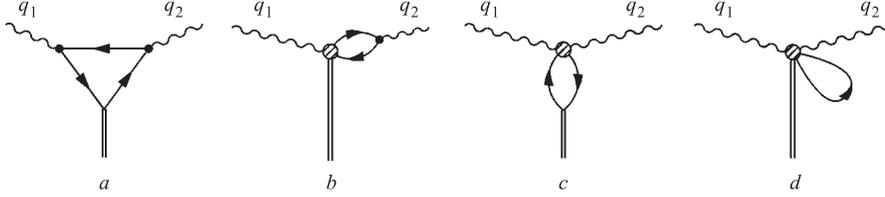


Fig. 2. Diagrams describing vertex  $\sigma\gamma\gamma$

The vertex  $\sigma\gamma\gamma$  is more complicated. Due to the  $\mathcal{P}$  exp factor in the action [6,9], there are additional nonlocal photon vertices. The reason for appearing of these vertices is the momentum dependence of the quark mass and the meson-quark vertices. The technique of obtaining these vertices can be found in [9,10]. As a result, there are four types of diagrams describing the process  $\sigma\gamma\gamma$  (Fig. 2), only the sum of them is gauge-invariant. As a consequence, we have<sup>2</sup>

$$C_{\sigma\gamma\gamma} = 0.86 \text{ GeV}^{-1},$$

and pion polarizability is

$$\alpha_\pi = 1.5. \quad (6)$$

<sup>1</sup>It is worth noting that a similar situation with relative contributions from the  $\sigma$  pole and box diagrams takes place in nonlocal model (see, e.g., [8]).

<sup>2</sup>It is easy to see that  $A_{\sigma\pi\pi}$ ,  $C_{\sigma\gamma\gamma}$  noticeably decrease in comparison with the local NJL model (see also Table 1). We would like to emphasize that a similar situation takes place in another nonlocal model with form factors of the Gaussian type [12].

**Table 1. Theoretical values of pion polarizability**

Model	$m(0)$ , MeV	$\Lambda$ , MeV	$M_\sigma$ , MeV	$A_{\sigma\pi\pi}$ , GeV	$C_{\sigma\gamma\gamma}$ , GeV <sup>-1</sup>	$\alpha_\pi$ , sigma pole, 10 <sup>-42</sup> cm <sup>3</sup>	$\alpha_\pi$ , sum rules, 10 <sup>-42</sup> cm <sup>3</sup>
[6]	340	340	420	1.57	0.86	1.5	2.0
[11]	340	920	421	1.57	1.16	2.0	2.7
[11]	300	1085	398	1.47	1.75	3.3	3.6
[11]	280	1187	384	1.39	2.1	3.9	5.0
[5]	280	1250	577	4	3.2	8.1	

Let us notice that the calculations of the pion polarizability in a similar nonlocal model of the NJL type with quark form factors of the Gaussian type is performed in [11] by using the chiral sum rule method. Those form factors lead to the following momenta dependence of quark mass function:

$$m(p) = m_0 \exp(-2p^2/\Lambda^2).$$

In contrast to the previous model, here the only one condition is used  $f_\pi = 93$  MeV for fixing two main model parameters  $m_0$ ,  $\Lambda$ . This keeps some freedom in choosing model parameters. Particularly, in [11] the model parameters  $m_0 = 300$  MeV,  $\Lambda = 1.085$  GeV are used. As a result, the values of the pion polarizability obtained by the sum rule method is  $\alpha_\pi = 3.6$ . On the other hand, we can carry out calculations of the pion polarizability using the  $\sigma$  pole diagrams. We have

$$\begin{aligned} A_{\sigma\pi\pi} &= 1.47 \text{ GeV}, \\ M_\sigma &= 398 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 1.75 \text{ GeV}^{-1}, \end{aligned} \tag{7}$$

and, as a result, we obtain  $\alpha_\pi = 3.3$ . It is worth noting that if  $m_0$  is equal to 340 MeV, as in model [6] (see Eq. (5)), the model quantities equal

$$\begin{aligned} \Lambda &= 0.92 \text{ GeV}, \\ A_{\sigma\pi\pi} &= 1.57 \text{ GeV}, \\ M_\sigma &= 421 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 1.16 \text{ GeV}^{-1}, \end{aligned} \tag{8}$$

the polarizability of the pion is  $\alpha_\pi = 2.0$  (sum rule estimation is  $\alpha_\pi = 2.7$ ). Taking  $m_0 = 280$  MeV, as in the local NJL model, one can obtain<sup>1</sup>

$$\begin{aligned} \Lambda &= 1.187 \text{ GeV}, \\ A_{\sigma\pi\pi} &= 1.39 \text{ GeV}, \\ M_\sigma &= 384 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 2.1 \text{ GeV}^{-1}, \\ \alpha_\pi &= 3.9 (\alpha_\pi = 5.0 \text{ from the sum rules}). \end{aligned} \tag{9}$$

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<sup>1</sup>Note that in the local NJL model,  $\Lambda$  equals 1.25 GeV [5].

As we can see, in the nonlocal model there is a strong dependence of the pion polarizability on the form of nonlocality. We summarize theoretical results in Table 1.

The experimental values of pion polarizability are given in Table 2.

To conclude, we consider the estimation of the pion polarizability in the framework of the local and nonlocal models of the NJL type. It is shown that the pion polarizability in the nonlocal model is noticeably smaller than that in the local NJL model, and is very sensitive to the form of nonlocality and choice of the model parameters. Planned experiments must give more accurate values of the pion polarizability. We hope that these data will allow us to choose a more realistic version of the nonlocal model.

**Table 2. Experimental values of the pion polarizability: [1] — measurement via Primakoff scattering; [13] — deduced from the processes  $\gamma\gamma \rightarrow \pi\pi$ ; [14, 15] — obtained from the sum rules for vector and axial-vector correlation functions in  $\tau$  decays**

$\alpha_{\pi}^{\text{exp}}$	Experiment
$8.54 \pm 1.76 \pm 1.51$	[1]
$3.3 \pm 0.6$	[13]
$3.31 \pm 0.45$	[14]
$3.4 \pm 1.11$	[15]

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