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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

SUPEREMBEDDING APPROACH TO Dp-BRANES, M-BRANES AND MULTIPLE D(0)-BRANE SYSTEMS

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We review the superembedding approach to M-branes and Dp-branes in its form based on the universal (D- and p-independent) *superembedding equation*, and its recent application in searching for supersymmetric and Lorentz covariant description of multiple Dp-brane systems. In particular, we present the structure of the multiple D0-brane equation as follows from our superembedding description and show that it describes the dielectric effect first noticed by Emparan and then by Myers. We also discuss briefly the relation with the boundary fermion approach by Howe, Lindström and Wulff.

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1. INTRODUCTION

Supersymmetric extended objects, super-p-branes [1–9], play a very important role in String/M-theory [10, 11] and its ADS/CFT applications [12, 13]. The ground states of D-dimensional super-p-branes (superstring for $p = 1$, supermembrane for $p = 2$) can be identified with the supersymmetric solutions of the corresponding supergravity theories [14]. The most interesting are the solutions of the maximal $D = 11$ supergravity and type II $D = 10$ supergravities appearing as low-energy limit of type II superstring theories. The p-brane dynamics can be described by supersymmetric actions [1, 2, 4, 6–9] or in the framework of superembedding approach [3, 5, 15–19].

In this contribution we give a review of superembedding approach to super-p-branes [3, 5, 15–21] in $D = 10$ and $D = 11$ superspaces and its recent application in search for the supersymmetric and Lorentz covariant (diffeomorphism invariant) description of the multiple brane systems [22]. In the part devoted to superembedding description of a single brane our emphasis will be on the superembedding description of Dirichlet super-p-branes (Dp-branes) (in contrast with the already existing review [19]). We begin by this case and then turn to the superembedding description of M2- and M5-brane.

The part devoted to multiple branes contains the results on multiple D0-brane system, which is to say multiple D-particles, which were briefly reported in [22]. We argue that to

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describe the multiple Dp-brane system, it is natural to try to put an additional $SU(N)$ gauge superform on the worldvolume of a single Dp-brane and impose a suitable set of superspace constraints on their (super)field strength two-form. If consistent, such a system provides, at least, an approximation of nearly coincident Dp-brane with the very low energy of the relative motion, but with the nonlinear («complete») Dirac–Born–Infeld description of the dynamics of the center of mass and of the $U(1)$ gauge field related to it. We show that such a consistent description, going beyond $U(N)$ Super-Yang–Mills (or Matrix model) approximation, does exist at least for the case of multiple D0-brane system [22]. We discuss the structure of the multiple D0-brane equation which follows from the superembedding approach and show that it possesses the dielectric effect first noticed by Emparan [23] and then by Myers [24].

Discussing the meaning of our results, we describe possible deformation of our basic equations and the relation with the boundary fermion approach by Howe, Lindström and Wulff [25, 26]. This latter approach does provide supersymmetric and covariant description of Dirichlet branes, but on the «classical» (or «minus one quantization») level in the sense that to arrive at the description of multiple brane system in terms of the variables corresponding to the standard single Dp-brane action [4,6,7] (usually considered as a classical or quasiclassical action) one has to perform a quantization of the boundary fermion sector.

1.1. D-Branes and Multiple D-Brane Systems. The first appearance of D-branes (Dirichlet p-branes) is dated by the late 1980s, when they were found as surfaces where the fundamental string can end [27–30]. Although in the first quantized string model they appeared as flat hyperplanes, it was clear that these surfaces must be dynamical in string theory. Indeed, as far as the open string theory contains closed string sector and this contains gravity in its quantum state spectrum, nondynamical surfaces cannot exist in string theory as the space-time itself is dynamical in it.

However, the special importance of D-branes for String/M-theory [10, 11] was widely appreciated in the middle 1990s, after it was discovered [31] that Dp-branes carry Ramond–Ramond (RR) charges, i.e., that they interact with the antisymmetric tensor gauge fields C_{p+1}, C_{p-1}, ... with respect to which the fundamental strings are neutral. In particular, this makes clear that Dp-branes are described by supersymmetric p-brane solutions of extended $\mathcal{N} = 2$ (type II) $D = 10$ supergravity, which had been found for any even/odd value of p in type IIA/IIB case and included a nonvanishing solution for C_{p+1} RR gauge field equations.

It was quickly appreciated that the low-energy dynamics of multiple Dp-brane system is described by the maximal supersymmetric $d = p + 1$ gauge theory with the gauge group $U(N)$ in the case of N D-branes [32]. The investigation of this limit was already quite productive [33]. In particular, it allowed one to formulate the conjecture of M(atrix) theory which states that the Matrix model [34], which can be considered as a theory of multiple D0-brane system, could provide a nonperturbative description of the M-theory.

The nonlinear supersymmetric action for a single Dp-brane was constructed in [35] for $p = 2$ and in [4, 6, 7] for general p [1]. It contains the nonlinear Dirac–Born–Infeld (DBI) term [32,35,37] and the Wess–Zumino (WZ) term describing the coupling to RR gauge fields

[1] The D-brane actions of [35] and [4, 6, 7] are complete up to terms containing the derivative of gauge field strength; in other words, they include nonlinear effects but contain contributions of lowest order in the derivatives of the field strength of the worldvolume gauge field only. Higher derivative corrections to these DBI+WZ actions are expected [36].

C_{p+1}, C_{p-1}, ... [38]. Notice that this explains why, e.g., the odd p Dp-branes cannot exist in type IIA case, where the supergravity multiplet contains only odd form gauge potential, C_{2n+1}, which can be coupled to even p super-p-branes (with odd dimension $d = p+1$ of the worldvolume W^{p+1}) through $\int_{W^{p+1}} \hat{C}_{p+1}$ (where hat implies pull-back of the differential form to W^{p+1}, see Subsecs. 1.3 and 2.1 for the notation).

Even before the actions for generic Dp-branes were constructed in [4, 6, 7], the supersymmetric equations of motion were derived in [3] by developing superembedding approach [15] for the case of Dp-branes. Notice that the same story happened to M5-brane: its equations of motion had been derived in [5] before the covariant and supersymmetric action was constructed in [8] and, independently, in [9].

As far as the nonlinear action for multiple D-brane systems is concerned, it was expected that this should be described by some non-Abelian generalization of the DBI plus WZ action. Tseytlin proposed using the symmetric trace prescription to construct the non-Abelian DBI action for the case of purely bosonic space-time filling D-brane [37, 39].

Although the search for a supersymmetric generalization of such a non-Abelian DBI action has not been successful, in 1999 Myers used it as a starting point and, applying a chain of dualities, derived the so-called «dielectric brane action» [24] which is widely accepted for the description of multiple D-brane system. This action, however, does not possess neither supersymmetry nor Lorentz symmetry. In spite of a number of attempts, its Lorentz covariant and/or supersymmetric generalizations are not known in general, although some progress was reached for the cases of low dimensions D, low-dimensional and low-co-dimensional branes [40, 41].

In [25, 26] a very interesting Lorentz covariant and supersymmetric description of D-branes is given in the framework of boundary fermion approach. It implies the extension of space-time/superspace by new fermionic coordinates of the type introduced in [43] as fields leaving at the end point of the open string. Upon quantization the boundary fermions of [43] are replaced by Dirac matrices and reproduce the Chan–Paton factors in the open string amplitudes. In the approach of [25, 26] one also has to quantize the boundary fermion sector to arrive at the description of multiple Dp-brane system similar to the standard description of single Dp-brane in [3, 4, 6, 7]. In this sense, the approach of [25, 26] can be called *minus one quantization* of Dp-brane. We will comment more on this approach in the concluding section of our review.

As far as the superembedding approach showed its efficiency in derivation of Dp-brane and M5-brane equations, it looks natural to apply it in the search for equations of motion for the multiple Dp-brane system. In this review we describe the results which this procedure gives for the simplest case of multiple D0-brane system [22] [1].

[1] Notice that the boundary fermion approach [25, 26] also uses a kind of superembedding formalism, but with embedding of a superspace with boundary fermion directions into space-time [25] or into the standard superspace [26]. Thus, for a sufficiently large N the number of fermionic directions of the worldvolume superspace exceeds 32, which is the fermionic dimension of the target type II superspace $\Sigma^{(10|16+16)}$. In this respect, the boundary fermion approach is similar to the superfield description of the NSR (Nevieu–Schwarz–Ramond) or spinning string, where the worldsheet superspace with two fermionic directions is embedded into space-time (zero fermionic directions). In this review we are dealing with the standard superembedding approach, in which the worldvolume superspace has twice less fermionic directions than the target superspace (16 versus 32 for 10-dimensional D-branes and 11-dimensional M-branes)

1.2. Contents. This review is organized as follows. After establishing our basic notation (Subsecs. 1.3 and 2.1), we begin (in Sec. 2) by describing the basic equations of the superembedding approach including the *superembedding equation* which essentially determines the dynamics of M-branes and D-branes (for a sufficiently large co-dimension $D - p > 4$).

In Sec. 3 we give a very brief review of superembedding approach to single Dp-branes for arbitrary p, with particular emphasis on D0-brane case. In Sec. 4 we describe more complicated cases of M2- and M5-brane where the construction of superembedding approach inevitably involves introduction of spinor moving frame variables (spinor harmonics) in addition to the moving frame variables. In Sec. 5 we first argue in favor of the idea to search for the description of multiple Dp-brane systems by trying to define a possible nonlinear generalization of the non-Abelian SYM multiplet by some set of constraints on the Dp-brane worldvolume superspace $\mathcal{W}^{(p+1|16)}$, the embedding of which in the type II target superspace $\Sigma^{(10|32)}$ is determined by the superembedding equation.

Then, turning to the case of multiple D0-brane, we propose the $d = 1$ $\mathcal{N} = 16$ SYM constraints which express its field strength in terms of nanoplet of $su(N)$ valued superfields $\mathbb{X}^i$ obeying a superembedding-like equation $D_\alpha \mathbb{X}^i = (\sigma^{0i}\Psi)_\alpha$. The leading component of this superfield, appearing in the expression for the dimension 1 (spinor–spinor) field strength of the $SU(N)$ gauge (super)fields, $G_{\alpha\beta} = \sigma^i_{\alpha\beta}\mathbb{X}^i$, describes the relative motion of N D0-brane constituents of the system. We show that our constraints lead to interacting supersymmetric equations of motion, which, in the case of flat target superspace, can also be obtained by dimensional reduction of a non-Abelian $D = 10$ Super-Yang–Mills (SYM) theory to $d = 1$ (the system which was used to define the Matrix model).

However, the superembedding approach is also able to produce multiple D0-brane equations in an arbitrary type IIA superspace supergravity background (and, to our best knowledge, it is not clear how to reproduce these equations just by SYM dimensional reduction). We analyze the general algebraic structure of the bosonic equations of motion for the multiple D0-brane in general type IIA supergravity background, which follow from our superembedding approach, and show that these describe the Emparan–Myers «dielectric brane» effect [23, 24] of polarization of multiple Dp-brane system by external higher form fluxes, i.e., show the coupling of multiple D0-brane system to the higher form gauge fields, which do not interact with a single D0-brane.

We conclude by discussion on our results, on possible generalizations of our approach and its relation with the boundary fermion approach by Howe, Lindström and Wulff [25, 26], and also on interesting directions for future study.

1.3. Basic Notations. *Target Superspaces of D-Branes and M-Branes.* We denote the local coordinates of $D = 11$ and type II $D = 10$ superspace by

$$Z^{\underline{M}} = (x^\mu, \theta^{\underline{\check{\alpha}}}), \quad \underline{\check{\alpha}} = 1, \ldots, 32, \quad \mu = 0, 1, \ldots, (D-1) \quad (D = 10, 11) \tag{1.1}$$

and supervielbein form by

$$E^{\underline{A}} := dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z) = (E^{\underline{a}}, \mathcal{E}^{\underline{\alpha}}), \quad \begin{cases} \underline{\alpha} = 1, \ldots, 32, \\ \underline{a} = 0, 1, \ldots, (D-1) \end{cases} \quad (D = 10, 11). \tag{1.2}$$

We find it convenient, following [42], to use different symbols for the D-component bosonic and for the 32-component fermionic supervielbein forms: $E^{\underline{a}} := dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{a}}(Z)$ and $\mathcal{E}^{\underline{\alpha}} := dZ^{\underline{M}} \mathcal{E}_{\underline{M}}{}^{\underline{\alpha}}(Z)$, respectively.

The supervielbein (1.2) describes supergravity when it obeys the set of superspace constraints [44–47] the most essential of which are collected in the expression for the bosonic torsion two-form

$$T^{\underline{a}} := DE^{\underline{a}} = -i\mathcal{E} \wedge \Gamma^{\underline{a}}\mathcal{E}, \quad \mathcal{E} \wedge \Gamma^{\underline{a}}\mathcal{E} := \mathcal{E}^{\underline{\alpha}} \wedge \Gamma^{\underline{a}}_{\underline{\alpha\beta}}\mathcal{E}^{\underline{\beta}}. \tag{1.3}$$

Here and below we write explicitly the exterior product symbol $\wedge$ [1]. In the 11D case $\Gamma^{\underline{a}}_{\underline{\alpha\beta}} = (\Gamma^{\underline{a}}C)_{\underline{\alpha\beta}} = \Gamma^{\underline{a}}_{\underline{\beta\alpha}}$, where $\Gamma^{\underline{a}} = (\Gamma^{\underline{a}})_{\underline{\alpha}}{}^{\underline{\beta}}$ is the 11D Dirac matrix and C is 11D charge conjugation matrix, which are imaginary in our mostly minus notation

$$\eta^{\underline{ab}} = \mathrm{diag}\ (+, -, \ldots, -). \tag{1.4}$$

For $D = 10$ type II cases it is convenient to split the fermionic supervielbein in two 16-component Majorana–Weyl spinor one-forms

$$\mathcal{E}^{\underline{\alpha}} = \begin{cases} (E^{\alpha 1}, E^2_{\alpha}) & \text{for type IIA}, \\ (E^{\alpha 1}, E^{\alpha 2}) & \text{for type IIB}, \end{cases} \tag{1.5}$$

In this notation the main supergravity constraints (1.3) read

$$T^{\underline{a}} := DE^{\underline{a}} = -i(E^1 \wedge \sigma^{\underline{a}}E^1 + E^2 \wedge \tilde{\sigma}^{\underline{a}}E^2) \quad \text{for type IIA}, \tag{1.6}$$

$$T^{\underline{a}} := DE^{\underline{a}} = -i(E^1 \wedge \sigma^{\underline{a}}E^1 + E^2 \wedge \sigma^{\underline{a}}E^2) \quad \text{for type IIB}, \tag{1.7}$$

where $\sigma^{\underline{a}} := \sigma^{\underline{a}}_{\alpha\beta} = \sigma^{\underline{a}}_{\beta\alpha}$ and $\tilde{\sigma}_{\underline{a}} := \tilde{\sigma}^{\alpha\beta}_{\underline{a}} = \tilde{\sigma}^{\beta\alpha}_{\underline{a}}$ are $D = 10$ Pauli matrices which obey

$$\sigma^{\underline{a}}\tilde{\sigma}^{\underline{b}} + \sigma^{\underline{b}}\tilde{\sigma}^{\underline{a}} = 2\eta^{\underline{ab}} = \mathrm{diag}\ (+, -, \ldots, -), \quad \sigma_{\underline{a}\alpha(\beta}\sigma^{\underline{a}}{}_{\gamma\delta)} \equiv 0, \quad \tilde{\sigma}^{\underline{a}\alpha(\beta}\tilde{\sigma}_{\underline{a}}{}^{\gamma\delta)} \equiv 0. \tag{1.8}$$

2. SUPEREMBEDDING EQUATION AS A BASIS OF SUPEREMBEDDING APPROACH TO D-BRANES AND M-BRANES

Following the so-called STV approach to superparticles and superstrings [48, 49] [2], the superembedding approach [3, 5, 15, 16, 19–21] describes the dynamics of super-p-brane in terms of embedding of a *worldvolume superspace* into the *target superspace*.

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[1] The exterior product of a q-form Ω_q and a p-form Ω_p has the property $\Omega_q \wedge \Omega_p = (-1)^{pq}\Omega_p \wedge \Omega_q$ if at least one of two differential forms is bosonic; when both are fermionic, an additional (-1) multiplier appears in the r.h.s. The exterior derivative acts on the products of the forms «from the right»: $d(\Omega_q \wedge \Omega_p) = \Omega_q \wedge d\Omega_p + (-1)^p d\Omega_q \wedge \Omega_p$. In particular, $T^{\underline{a}} := DE^{\underline{a}} = dZ^{\underline{M}} \wedge DE_{\underline{M}}{}^{\underline{a}}(Z)$, so that Eq. (1.3) implies $D_{\underline{M}}E_{\underline{N}}{}^{\underline{a}}(Z) - (-1)^{\epsilon(\underline{M})\cdot\epsilon(\underline{N})}D_{\underline{N}}E_{\underline{M}}{}^{\underline{a}}(Z) = +2i(-1)^{\epsilon(\underline{N})}\mathcal{E}_{\underline{M}}\Gamma^{\underline{a}}\mathcal{E}_{\underline{N}}$, where $\epsilon(\underline{M})$ is the Grassmann parity of $Z^{\underline{M}}$, $\epsilon(\underline{a}) = 0$, $\epsilon(\underline{\alpha}) = 1$.

[2] STV abbreviates the family names of Dmitri Sorokin, Vladimir Tkach and Dmitri Volkov, the authors of [48]. This approach to description of Brink–Schwarz superparticles and Green–Schwarz superstring was also called «twistor-like». See [19] for the review and more references and [50, 51] for related studies of the connection between Brink–Schwarz and spinning superparticles aimed to relate spinning (NSR) string and Green–Schwarz superstring already at the classical level. This line was further continued in [52].

2.1. Worldvolume Superspaces $W^{(p+1|16)}$**.** The target superspaces of Dp-branes (M-branes) were described in Subsec. 1.3. Their worldvolume superspaces $\mathcal{W}^{(p+1|16)}$ have $d = p+1$ bosonic and 16 fermionic dimensions. We denote the local coordinates of $\mathcal{W}^{(p+1|16)}$ by

$$\zeta^{\mathcal{M}} = (\xi^m, \eta^{\check{\alpha}}), \quad m = 0, 1, \ldots, p, \quad \check{\alpha} = 1, \ldots, 16. \tag{2.1}$$

The embedding of $\mathcal{W}^{(p+1|16)}$ into the $D=10$ type II ($D=11$) target superspace $\Sigma^{(10|16+16)}$ ($\Sigma^{(11|32)}$) can be described in terms of coordinate functions $\hat{Z}^{\underline{M}}(\zeta) = (\hat{x}^{\underline{m}}(\zeta), \hat{\theta}^{\underline{\check{\alpha}}}(\zeta))$,

$$W^{(p+1|16)} \in \Sigma^{(D|32)}: \quad \boxed{Z^{\underline{M}} = \hat{Z}^{\underline{M}}(\zeta)} = (\hat{x}^{\underline{m}}(\zeta), \hat{\theta}^{\underline{\check{\alpha}}}(\zeta)), \tag{2.2}$$

$D = 10, 11$, $\underline{m} = 0, 1, \ldots, (D-1)$, $\underline{\check{\alpha}} = 1, \ldots, 32$.

2.2. The Superembedding Equation. A particular beauty of the superembedding approach consists in that, for all known superbranes, the embedding of the worldvolume superspace into the target superspace is characterized by a universal equation which is called the *superembedding equation*. This geometrical equation (the name «geometrodynamic equation» was used in [49]) restricts the coordinate functions $\hat{Z}^{\underline{M}}(\zeta)$ and, in some cases, completely determines the dynamics of superbrane.

To write the most general form of this superembedding equation let us denote the supervielbein of $W^{(p+1|16)}$ by

$$e^A = d\zeta^{\mathcal{M}} e_{\mathcal{M}}{}^A(\zeta) = (e^a, e^\alpha), \quad a = 0, 1, \ldots, p, \quad \alpha = 1, \ldots, 16, \tag{2.3}$$

and write the general decomposition of the pull-back of the supervielbein $E^{\underline{A}}(Z)$ of target superspace, Eq. (1.2), to $\mathcal{W}^{(p+1|16)}$, $\hat{E}^{\underline{A}} := E^{\underline{A}}(\hat{Z})$, on this basis,

$$\hat{E}^{\underline{A}} := E^{\underline{A}}(\hat{Z}) = d\hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(\hat{Z}) = e^b \hat{E}_b^{\underline{A}} + e^\alpha \hat{E}_\alpha{}^{\underline{A}}. \tag{2.4}$$

Notice that the coincidence of the notation α, β for the 10D Majorana–Weyl spinor indices of the chiral supervielbein forms of the target type II superspace ($E^{1,2}$ in (1.5)) and for the indices enumerating the fermionic supervielbein of the worldvolume superspace is not occasional and is acceptable because, among the $D=10$ objects, we will discuss D-branes but not fundamental strings (F1-branes). We will comment on this more in the next section. In Sec. 4 devoted to M-brane we change the notation and substitute a multiindex αq for α in Eqs. (2.3), (2.4).

The superembedding equation states that the bosonic supervielbein form has zero projection on the worldvolume fermionic supervielbein form. This is to say, it reads

$$\boxed{\hat{E}_\alpha{}^{\underline{a}} := \nabla_\alpha \hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{a}}(\hat{Z}) = 0}, \quad \nabla_\alpha := e_\alpha^{\mathcal{M}}(\zeta)\partial_{\mathcal{M}}, \quad \zeta^{\mathcal{M}} = (\xi^m, \eta^{\check{\alpha}}). \tag{2.5}$$

It can be also presented in an equivalent form of

$$\hat{E}^i := \hat{E}^{\underline{a}} u_{\underline{a}}{}^i = 0, \tag{2.6}$$

where $u_{\underline{a}}^i = u_{\underline{a}}^i(\zeta)$ are $(D-p-1)$ space-like, mutually orthogonal and normalized D-vector superfields,

$$u_{\underline{a}}^i u^{\underline{a}j} = -\delta^{ij}. \tag{2.7}$$

Equation (2.6) means that $u_{\underline{a}}{}^i$ are orthogonal to the worldvolume superspace. We can complete their set till a complete *moving frame* by adding $d = (p+1)$ mutually orthogonal and normalized D-vector superfields $u_{\underline{a}}^b = u_{\underline{a}}^b(\zeta)$, which are tangential to the worldvolume superspace,

$$u_{\underline{a}}^a u^{\underline{a}i} = 0, \quad u_{\underline{a}}^a \eta^{\underline{ab}} u_{\underline{b}}^b = \eta^{ab}, \quad \begin{cases} a, b = 0, 1, \ldots, p, \\ \underline{a}, \underline{b} = 0, 1, \ldots, (D-1). \end{cases} \tag{2.8}$$

Their contraction with the pull-back $\hat{E}^{\underline{a}}$ of the target superspace bosonic supervielbein $E^{\underline{a}}$ provides us with a set of $d = (p+1)$ linearly independent nonvanishing one-forms, which can be used as bosonic supervielbein of the worldvolume superspace,

$$\hat{E}^a := \hat{E}^{\underline{b}} u_{\underline{b}}^a = e^a. \tag{2.9}$$

This e^a refers to as (super)vielbein form induced by the (super)embedding. Considered together, Eqs. (2.6) and (2.9) imply

$$\hat{E}^{\underline{a}} = e^b u_b^{\underline{a}}. \tag{2.10}$$

This is one more equivalent form of the superembedding equation. Indeed, Eqs. (2.9) and (2.6) can be obtained contracting (2.10) with $u_{\underline{a}}{}^a$ and $u_{\underline{a}}{}^i$, respectively. On the other hand, decomposing (2.10) on the worldvolume supervielbein, one arrives at the original form (2.5) of the superembedding equation. As a by-product on this way one derives the expression for the moving frame vectors $u_{\underline{b}}{}^a(\zeta)$ in terms of the (linear combination of the) bosonic derivatives of the coordinate functions,

$$u_b{}^{\underline{a}} = \hat{E}_b{}^{\underline{a}} := D_b \hat{Z}^{\underline{M}}(\zeta) E_{\underline{M}}{}^{\underline{a}}(\hat{Z}(\zeta)). \tag{2.11}$$

To obtain the consequences of the superembedding equation one can study its integrability (self-consistency) conditions

$$0 = D\hat{E}^i = \hat{T}^{\underline{a}} u_{\underline{b}}{}^i + e^b \wedge u_b^{\underline{a}} D u_{\underline{a}}{}^i = -i\hat{\mathcal{E}} \wedge \Gamma^{\underline{a}} \hat{\mathcal{E}} u_{\underline{b}}{}^i + e^b \wedge u_b^{\underline{a}} D u_{\underline{a}}{}^i. \tag{2.12}$$

To this end, one has to define the $SO(1, D-1)$ and $SO(D-p-1)$ connection, $\Omega^{bc} = -\Omega^{cb} = d\zeta^M \Omega_M^{bc}$ and $\Omega^{ij} = -\Omega^{ji} = d\zeta^M \Omega_M^{ij}$, entering the $SO(1, D-1) \times SO(1,p) \times SO(D-p-1)$ covariant derivatives

$$Du_{\underline{a}}{}^b := du_{\underline{a}}{}^b + \omega_{\underline{a}}{}^{\underline{b}} u_{\underline{b}}{}^b + u_{\underline{a}}{}^c \Omega_c{}^b \text{ and } Du_{\underline{a}}{}^i := du_{\underline{a}}{}^i + \omega_{\underline{a}}{}^{\underline{b}} u_{\underline{b}}{}^i + u_{\underline{a}}{}^j \Omega^{ji} \tag{2.13}$$

acting on the moving frame superfields and superforms in (2.12).

2.3. Moving Frame and Induced Connection on $W^{(p+1|16)}$. Notice that the orthogonality and normalization conditions for the moving frame vectors $u_{\underline{a}}{}^b$ and $u_{\underline{a}}{}^j$ imply that the $D \times D$ matrix U composed of their components, which we call *moving frame matrix*, is pseudo-orthogonal ($U\eta U^T = \eta$), i.e., Lorentz group valued

$$U_{\underline{a}}^{(\underline{b})} := \left(u_{\underline{a}}^b, u_{\underline{a}}^{\,i}\right) \in SO(1, D-1). \tag{2.14}$$

These moving frame vectors (also called Lorentz harmonics, see [53] as well as [15, 19, 20] and refs. therein) can be used to construct the $SO(1,p)$ and $SO(9-p)$ connections on the worldvolume superspace. In the case of flat target superspace, these would be given by the corresponding Cartan forms $u^{\underline{c}a}du_{\underline{c}}{}^{b}$ and $u^{\underline{a}i}du_{\underline{a}}^{j}$. In the case of curved target superspace, one has to use the pull-back of the spin connection to make the definition $SO(1,9)$ covariant. It is convenient to write the definition of the connections implicitly, using the $SO(1,D-1)\times SO(1,p)\times SO(D-p-1)$ covariant derivatives action on the moving frame vector, Eqs. (2.13),

$$Du_{\underline{b}}{}^{a} = u_{\underline{b}}{}^{i}\Omega^{ai}, \quad Du_{\underline{b}}{}^{i} = u_{\underline{b}a}\Omega^{ai}. \tag{2.15}$$

Both equations in (2.15) involve the one-form Ω^{ai}. This generalizes the $\dfrac{SO(1,9)}{SO(1,p)\otimes SO(8-p)}$ covariant Cartan form and obeys the generalized Peterson–Codazzi equations

$$D\Omega^{ai} = \hat{\mathbb{R}}^{ai}, \quad \hat{\mathbb{R}}^{ai} := (u\hat{\mathbb{R}}u)^{ai} := \hat{\mathbb{R}}^{\underline{cb}}u_{\underline{c}}^{a}u_{\underline{b}}^{i}, \tag{2.16}$$

where $\hat{\mathbb{R}}^{\underline{cb}}$ is the pull-back of the curvature of the corresponding type II target superspace. The curvatures of the induced $SO(1,p)$ and $SO(9-p)$ connections, $r^{ab} = -r^{ba}$ and $\mathbb{G}^{ij} = -\mathbb{G}^{ji}$, are defined, as usually, by Ricci identities, e.g., $DDu_{\underline{b}}{}^{a} =: \hat{\mathbb{R}}_{\underline{b}}{}^{\underline{c}}u_{\underline{c}}{}^{a} - u_{\underline{b}}{}^{b}r_{b}{}^{a}$, $DDu_{\underline{b}}{}^{i} =: \hat{\mathbb{R}}_{\underline{b}}{}^{\underline{c}}u_{\underline{c}}{}^{i} + u_{\underline{b}}{}^{j}\mathbb{G}^{ji}$. Using (2.15) and (2.16), one finds the following generalizations of the Gauss and Ricci equations (see [15]):

$$r^{ab} = (u\mathbb{R}u)^{ab} + \Omega^{ai}\wedge\Omega^{bi}, \quad \mathbb{G}^{ij} = (u\mathbb{R}u)^{ij} - \Omega_{a}{}^{i}\wedge\Omega^{aj}. \tag{2.17}$$

Now we can further specify the integrability condition (2.18) for the superembedding equation (2.6):

$$0 = D\hat{E}^{i} = -i\hat{\mathcal{E}}\wedge\Gamma^{\underline{b}}\hat{\mathcal{E}}\; u_{\underline{b}}{}^{i} + e_{b}\wedge\Omega^{bi}. \tag{2.18}$$

Decomposing Ω^{bi} on the worldvolume supervielbein, $\Omega^{bi} = e^{\alpha}\Omega_{\alpha}{}^{bi} + e^{b}\Omega_{a}{}^{bi}$, we see that (2.18) involves only antisymmetric part $\Omega_{[a\,b]}{}^{i}$ of the bosonic coefficient, while its symmetric part,

$$\Omega_{(a\,b)}{}^{i} = K_{a\,b}{}^{i} := -D_{(a}\hat{E}_{b)}{}^{\underline{c}}\; u_{\underline{c}}{}^{i}, \tag{2.19}$$

remains free at this stage. The last equality in (2.19) is derived using Eq. (2.11). $K_{a\,b}{}^{i}$ can be recognized as the (superfield generalization of the) second fundamental form of the worldvolume superspace considered as a surface in the target superspace. Then, the generalized Cartan form (one-form) gives a superform generalization of the second fundamental form $K_{ab}{}^{i}$.[1]

To move further we have to impose one more conventional constraint to determine the fermionic supervielbein form of the worldvolume superspace e^{α}. This latter, although excluded from the decomposition of the pull-back of the bosonic supervielbein by the superembedding equation (2.5), does enter the decomposition of the fermionic supervielbein $\mathcal{E}^{\underline{\alpha}} = e^{\beta}V_{\beta}{}^{\underline{\alpha}} + e^{a}\psi_{a}^{\underline{\alpha}}$, which is involved in the self-consistency condition (2.12) and also in the expression for the torsion two-form of the induced geometry of the worldvolume superspace,

$$De^{a} = -i\hat{\mathcal{E}}\wedge\Gamma^{\underline{b}}\hat{\mathcal{E}}u_{\underline{b}}{}^{a}. \tag{2.20}$$

[1]This is 0-form, but has a natural bosonic one-form representation as $e^{a}K_{ab}{}^{i}$. The term «second fundamental form» does not refer to *differential forms*, usually associated with antisymmetric tensors; it is from the language of the classical surface theory where the term «first fundamental form» refers to the metric. See refs. in [15].

The fermionic supervielbein form e^{α} of the worldvolume superspace $W^{(p+1|16)}$ can also be induced by superembedding. At this stage, when studying the case of M-branes and fundamental string, one has to introduce one more notion: the spinor moving frame variables or spinorial Lorentz harmonics [15] (used before in studying superparticles [54, 55] and twistor-like spinor moving frame action for superstrings and super-p-branes [56]). These objects, which are used to relate the worldvolume superspace fermionic supervielbein with the pull-back of its target superspace counterpart, $e^{\alpha} = \mathcal{E}^{\underline{\beta}} V_{\underline{\beta}}{}^{\alpha}$, will be discussed in Sec. 4 devoted to the superembedding approach to M-branes.

Surprisingly, the case of Dp-brane happens to be simpler in the sense that one can escape the necessity to introduce the notion of spinor moving frame, at least at this stage. This is why we begin a more concrete part of our review of the superembedding approach from the case of D-branes.

3. SUPEREMBEDDING APPROACH TO Dp-BRANES

The superembedding approach to Dp-branes was used to describe their dynamics in [3], where the superembedding equation was shown to produce their equations of motion some months before the generic nonlinear DBI+WZ action was found in [4, 6, 7] [1]. It was further studied in [16], where, in particular, the explicit form of the Dp-brane fermionic equations was derived for the first time (for the particular case of D4-brane these might be extracted from the M5-brane fermionic equations which were presented before in [5]). See [17, 18, 20] and references in [21, 57] for further development.

As already noticed, the basic equation of the superembedding approach to Dp-brane is the superembedding equation (2.5) equivalent to (2.6). All the formulae of Sec. 2 are valid for this case, so that we will continue specifying the fermionic supervielbein forms of the Dp-brane worldvolume superspace and using it to extract the consequences of the superembedding equation.

3.1. Fermionic Supervielbein Induced by Superembedding and the First Consequence of the Superembedding Equation. When describing Dp-branes, it is convenient to identify e^{α} with the pull-back to $W^{(p+1|16)}$ of, say, the first of two target space fermionic supervielbein forms

$$e^{\alpha} = \hat{E}^{\alpha 1}. \tag{3.1}$$

Then, the general decomposition of the second fermionic supervielbein form reads

$$\begin{cases} \hat{E}^2_{\alpha} = e^{\beta} h_{\beta\alpha} + e^{a} \chi_{a\alpha} \text{ for IIA case,} \\ \hat{E}^{\alpha 2} = e^{\beta} h_{\beta}{}^{\alpha} + e^{a} \chi^{\alpha}_{a} \text{ for IIB case.} \end{cases} \tag{3.2}$$

To resume,

$$\hat{\mathcal{E}}^{\underline{\alpha}} = (e^{\alpha}, e^{\beta} h_{\beta\alpha} + e^{a} \chi_{a\alpha}) \text{ for IIA case,} \tag{3.3}$$

$$\hat{\mathcal{E}}^{\underline{\alpha}} = (e^{\alpha}, e^{\beta} h_{\beta}{}^{\alpha} + e^{a} \chi_{a}{}^{\alpha}) \text{ for IIB case.} \tag{3.4}$$

---

[1] For the particular case of D2-brane the action had been found earlier in [35] by applying the $d = 3$ scalar-vector duality to the M2-brane action [1].

Now we are ready to find the first nontrivial consequence of the superembedding equation. Looking at the self-consistency conditions (2.18) for superembedding equation (2.6), we notice that the second term does not contribute to the lowest dimensional (dim 2, i.e., $\propto e^{\beta} \wedge e^{\alpha}$) component of this differential form equation. Thus, substituting (3.3) or (3.4) into Eq. (2.18) we find

$$h\tilde{\sigma}^{\underline{b}} h^T u_{\underline{b}}{}^i = -\sigma^{\underline{b}} u_{\underline{b}}{}^i \text{ for type IIA,} \tag{3.5}$$

$$h\sigma^{\underline{b}} h^T u_{\underline{b}}{}^i = -\sigma^{\underline{b}} u_{\underline{b}}{}^i \text{ for type IIB.} \tag{3.6}$$

We can continue by studying the higher dimensional components of Eq. (2.18) and also of the (conventional) equations for the fermionic supervielbein (3.2). On this way one finds, in particular, that the field strength F_{ab} of the worldvolume gauge field is related to the spin-tensor h in the decomposition (3.2). However, it is technically much simpler, using the knowledge on the very existence of the worldvolume gauge field, to introduce its superform counterpart on the worldvolume superspace, to restrict it by a suitable set of constraints and study their self-consistency conditions.

3.2. Constraints for the Worldvolume Gauge Field. The constraints for the worldvolume gauge (super)field strength of the Dp-brane can be written as

$$F_2 := dA - \hat{B}_2 = \frac{1}{2} e^b \wedge e^a F_{ab}, \tag{3.7}$$

where $\hat{B}_2$ is the pull-back to the worldvolume superspace $W^{(p+1|16)}$ of the type IIB NS-NS superform potential B_2. The field strength of this is restricted by the constraints which can be collected in the following differential form expressions:

$$H_3 := dB_2 = -iE^{\underline{a}} \wedge (E^1 \wedge \sigma_{\underline{a}} E^1 - E^2 \wedge \tilde{\sigma}_{\underline{a}} E^2) + \\ + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} H_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \text{ for type IIA,} \tag{3.8}$$

$$H_3 := dB_2 = -iE^{\underline{a}} \wedge (E^1 \wedge \sigma_{\underline{a}} E^1 - E^2 \wedge \sigma_{\underline{a}} E^2) + \\ + \frac{1}{3!} E^{\underline{c}_3} \wedge E^{\underline{c}_2} \wedge E^{\underline{c}_1} H_{\underline{c}_1 \underline{c}_2 \underline{c}_3} \text{ for type IIB.} \tag{3.9}$$

The lowest dimensional of the nontrivial components of the Bianchi identity

$$dF_2 = -\hat{H}_3 \tag{3.10}$$

is $\propto e^{\gamma} \wedge e^{\beta} \wedge e^{a}$, this is to say of dim 2. It implies

$$\begin{aligned} h\sigma^{\underline{b}} h^T u_{\underline{b}}{}^a &= \sigma^{\underline{b}} u_{\underline{b}}{}^c k_c{}^a \text{ for IIB,} \\ h\tilde{\sigma}^{\underline{b}} h^T u_{\underline{b}}{}^a &= \sigma^{\underline{b}} u_{\underline{b}}{}^c k_c{}^a \text{ for IIA,} \end{aligned} \qquad k_a{}^b := (\eta + F)_{ac} (\eta - F)^{-1 cb}. \tag{3.11}$$

Notice that this equation relates the spin-tensor h, appearing in the decomposition of the pull-back of the fermionic supervielbein form (3.2), and the bosonic gauge field strength tensor superfield $F_{ab} = -F_{ba}$. One can easily check that the matrix k, constructed from F_{ab} as in (3.11), is $SO(1,p)$ group valued, i.e., it obeys $k\eta k^T = \eta$ [18, 57],

$$k = (\eta + F)(\eta - F)^{-1} \in SO(1,9). \tag{3.12}$$

Further study shows that the system of superembedding equation plus the worldvolume gauge field constraints (3.7) always contains the dynamical equations among their consequences (and for $p \leqslant 5$ Dp-branes [17] the superembedding equation along suffices for this purposes). However, the details of derivation are p-dependent. As an example, below we will give some details for the case of D0-brane which will be then used in Sec. 5. But before let us discuss a toy example: D(-1)-brane or D-instanton. What can one obtain from the superembedding approach in this case?

3.3. A Toy Example: D-Instanton (D(-1)-Brane). For instanton the dimension of the bosonic body of the worldvolume superspace is zero, $d = p+1 = 0$, so that this superspace is purely fermionic $W^{(0|16)}$. Its co-tangent superspace basis contains the fermionic supervielbein e^{α} only, all the space-time directions are orthogonal to the worldvolume superspace, so that the moving frame matrix is not needed. Hence, the superembedding equation for D-instanton reads

$$\hat{E}^{\underline{b}} = 0. \tag{3.13}$$

The fermionic supervielbein of the worldvolume superspace e^{α} can be identified with the pull-back $\hat{E}^{\alpha 1}$ of $E^{\alpha 1}$, and the general decomposition of the pull-back $\hat{E}^{\alpha 2}$ of $E^{\alpha 2}$ reads $\hat{E}^{\alpha 2} = e^{\beta} h_{\beta}{}^{\alpha}$. The self-consistency conditions for the superembedding equation imply vanishing of the pull-back of the target space bosonic torsion, $0 = \hat{T}^{\underline{a}} = D\hat{E}^{\underline{b}} = -ie^{\alpha} \wedge e^{\beta}(\sigma^{\underline{a}} + h\sigma^{\underline{a}}h^T)_{\alpha\beta}$. This results in equation

$$h\sigma^{\underline{a}}h^T = -\sigma^{\underline{a}}, \tag{3.14}$$

which *does not* have solution in the case of real h. However, there is an imaginary solution,

$$h_{\alpha}{}^{\beta} = i\delta_{\alpha}{}^{\beta}. \tag{3.15}$$

It implies that $\hat{E}^{\alpha 2} = i\hat{E}^{\alpha 1}$ and, hence, as far as

$$\hat{E}^{\alpha 1} = -i\hat{E}^{\alpha 2} = e^{\alpha}, \hat{E}^{\alpha 1} + i\hat{E}^{\alpha 2} = 0, \tag{3.16}$$

that both tangent superspace and worldvolume superspace fermionic supervielbeins are complex. This is in agreement with the well-known fact that D-instanton implies Wick rotation, i.e., exists only in the Euclidean version of the type IIB theory, where the real 16-component Weyl spinor is inevitably complex (versus the existence of real Majorana–Weyl spinor in the case of Lorentz $1+9$ signature).

This seems to be the only result one can get from superembedding description of D-instanton. It is not surprising as far as D-instanton has no dynamics: it is frozen to a point of Euclidean space-time (which is expressed by the statement that it is (-1)-brane).

3.4. D0-Brane in Superembedding Approach. In the case of D0-brane, this is to say D-particle, there are nine space-like directions orthogonal to the worldline, and the tangent to the worldline gives time-like directions, so that the corresponding set of moving frame vectors $(u^0_{\underline{a}}, u^i_{\underline{a}})$ obeys

$$u^0_{\underline{a}} u^{\underline{a}0} = 1, \quad u^i_{\underline{a}} u^{\underline{a}0} = 0. \quad u^i_{\underline{a}} u^{\underline{a}j} = -\delta^{ij}. \tag{3.17}$$

The worldvolume superspace $W^{(1|16)}$ has only one bosonic direction $e^a \mapsto e^0$, and the superembedding equation (2.10) (equivalent to (2.5)) reads

$$\hat{E}^{\underline{a}} = e^0 u_0^{\underline{a}}. \tag{3.18}$$

The expression (3.1) for the pull-backs of the fermionic supervielbein form simplifies to

$$\hat{E}^{\alpha 1} = e^\alpha, \tag{3.19}$$

$$\hat{E}_\alpha{}^2 = e^\beta h_{\beta\alpha} + e^0 \chi_\alpha. \tag{3.20}$$

It is convenient to write the self-consistency conditions (3.5) for the superembedding equation (3.18) in the form of

$$h\tilde{\sigma}^i h^T = -\sigma^i, \tag{3.21}$$

using the simplified notation

$$\sigma^0_{\alpha\beta} := \sigma^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^0, \quad \sigma^i_{\alpha\beta} := \sigma^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^i. \tag{3.22}$$

These are suggestive as far as the matrices (3.22) and $\tilde{\sigma}^0_{\alpha\beta} := \tilde{\sigma}^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^0$, $\tilde{\sigma}^i_{\alpha\beta} := \tilde{\sigma}^{\underline{b}}_{\alpha\beta} u_{\underline{b}}{}^i$ do possess the algebraic properties of $D = 10$ Pauli matrices. However, one should keep in mind that they are not constant matrices but rather obey

$$D\sigma^0_{\alpha\beta} = \sigma^i_{\alpha\beta}\Omega^i, \quad D\sigma^i_{\alpha\beta} = \sigma^0_{\alpha\beta}\Omega^i, \tag{3.23}$$

where Ω^i is the generalized Cartan form defined in (2.15). In this notation the general solution of Eq. (3.21) reads

$$h_{\alpha\beta} = \sigma^0_{\alpha\beta}. \tag{3.24}$$

This is the place to comment on the worldvolume gauge field constraints for the D0-brane case (worldline gauge field). For $p = 0$ the r.h.s. of Eq. (3.7) clearly vanishes, so that the constraints read

$$F_2 := dA - \hat{B}_2 = 0 \tag{3.25}$$

and the Bianchi identities (3.10) simplify to $\hat{H}_3 = 0$. Their only nontrivial consequence reads

$$h\tilde{\sigma}^0 h^T = \sigma^0. \tag{3.26}$$

Equation (3.26) is satisfied identically by the general solution (3.24) of Eq. (3.21). This shows that the gauge field constraints in the case of D0-brane are dependent, which is in agreement with the known statement that the superembedding equation alone is sufficient to describe dynamics in this case. On the other hand, to arrive at the equations of motion in a simpler way, it is convenient to impose the gauge field constraints (3.25) on the field strength of the worldvolume gauge field. Indeed, it is evident without any calculation that the general solution of Eqs. (3.21) and (3.26) is given by (3.24).

Another consequence of the self-consistency conditions for the superembedding equation (3.18) is that the generalized Cartan form Ω^i in (3.23) is expressed by

$$\Omega^i = e^0 K^i - 2ie^\beta(\sigma^0\tilde{\sigma}^i\chi)_\beta \tag{3.27}$$

in terms of fermionic superfield $\chi_\alpha = \hat{E}_{0\alpha}{}^2$ and bosonic superfield

$$K^i := -u^i_{\underline{a}} D_0 \hat{E}^{\underline{a}}_0, \quad \hat{E}^{\underline{a}}_0 := \nabla_0 \hat{Z}^{\underline{M}} E_{\underline{M}}{}^{\underline{a}}(\hat{Z}). \tag{3.28}$$

This latter is the superfield generalization of the mean curvatures of the particle worldline in target space. The generalized Cartan form (3.27) gives the superform generalization of this mean curvature for the case of D0-brane in type IIA superspace. It contains K^i as a dim 1 and the fermionic $\chi_\alpha = \hat{E}_{0\alpha}{}^2$ superfield as a dim 1/2 component; in this sense, χ_α is the superpartner of K^i. The bosonic and fermionic equations, which can be now obtained from the self-consistency condition for the fermionic equation (3.20), are formulated in terms of these superfields.

In flat target superspace the equations of motion imply vanishing of both χ_α and K^i,

$$\chi_\alpha := \hat{E}_{0\,\alpha}^{\;\;2} = 0, \quad K^i := -u^i_{\underline{a}} D_{0} \hat{E}_{0}{}^{\underline{a}} = 0. \tag{3.29}$$

In general type IIA supergravity background the fermionic equations of motion acquire the r.h.s.

$$\chi_\alpha := \hat{E}_{0\,\alpha}^{\;\;2} = \Lambda_\alpha \tag{3.30}$$

defined by

$$\Lambda_\alpha := (\hat{\Lambda}_1 - \hat{\Lambda}_2 \sigma^0)_\alpha, \tag{3.31}$$

where $\hat{\Lambda}_{1\alpha}$ and $\hat{\Lambda}_2{}^\beta$ are the pull-backs of the Grassmann derivatives of the dilaton superfield, and

$$\Lambda_{\alpha 1} := \frac{i}{2}(D_{\alpha 1}\Phi), \quad \Lambda_2^\alpha := \frac{i}{2}\,(D_2^\alpha\,\Phi) \tag{3.32}$$

are the pull-backs of the Grassmann derivatives of the dilaton superfield. The origin of the r.h.s. in Eq. (3.30) is nonvanishing fermionic torsion of the target type IIA superspace [47]

$$\begin{aligned} T^{\alpha 1} &= -2iE^{\alpha 1} \wedge E^{\beta 1}\Lambda_{\beta 1} + iE^1 \sigma^{\underline{a}} \wedge E^1\, \tilde{\sigma}_{\underline{a}}^{\alpha\beta} \Lambda_{\beta 1} + \propto E^{\underline{b}}, \\ T^2_\alpha &= -2iE^2_\alpha \wedge E^2_\beta \Lambda_2^\beta + iE^2 \tilde{\sigma}_{\underline{a}} \wedge E^2\, \sigma^{\underline{a}}_{\alpha\beta} \Lambda_2^\beta + \propto E^{\underline{b}}. \end{aligned} \tag{3.33}$$

The bosonic equation for D0-brane in general supergravity background reads

$$\begin{aligned} K^i := -u^i_{\underline{a}} D_0 \hat{E}^{\underline{a}}_0 = \frac{1}{16}\tilde{\sigma}^{i\alpha\beta}(t_{\alpha\beta} - D_\alpha \Lambda_\beta) + \frac{7i}{8}(\hat{\Lambda}_2 \sigma^{0i} \hat{\Lambda}_1) = \\ = \mathrm{e}^{\hat{\Phi}} \hat{R}^{0i} + \widehat{D^i \Phi} + \mathcal{O}\,(\text{fermi}^2), \end{aligned} \tag{3.34}$$

where

$$t_{\alpha\beta} = \left(\widehat{T}_{\alpha 1\, \underline{a}\beta}{}^2 + \sigma^0_{\alpha\gamma} \widehat{T}^\gamma_2{}_{\underline{a}\beta}{}^2 - \widehat{T}_{\alpha 1 \underline{a}}{}^{\delta 1} \sigma^0_{\delta\beta} - \sigma^0_{\alpha\gamma} \widehat{T}^\gamma_2{}_{\underline{a}}{}^{\delta 1} \sigma^0_{\delta\beta} \right) u^{\underline{a}\,0}. \tag{3.35}$$

To arrive at the second line of Eq. (3.34), written explicitly up to the fermionic contributions, one has to use the explicit form of the dim 1 target space torsion spin-tensors, entering (3.35), and of the derivatives of fermionic superfield $D_\alpha \Lambda_\beta$ which can be found in Appendix B.

4. M-BRANES IN THE SUPEREMBEDDING APPROACH

The basic superembedding equation describing the dynamics of M2- and M5-branes has the same form as (2.5), or equivalent to (2.6). However, in these cases the fermionic supervielbein $\mathcal{E}^{\underline{\alpha}}$ is in the minimal 32-component $D = 11$ Majorana spinor representation, so that the trick we used in the case of Dp-branes does not work and the relation between $\hat{\mathcal{E}}^{\underline{\alpha}}$ and the worldvolume superspace fermionic supervielbein form $e^{\alpha q}$ is now more complicated.

Notice that, when studying 11D M-branes (and also fundamental strings in $D = 10$), it is convenient to denote the fermionic supervielbein of the worldvolume superspace $W^{(p+1|32)}$ by $e^{\alpha q}$,

$$e^{\alpha} \text{ of Secs. } 2, 3 \text{ and } 5 \longleftrightarrow e^{\alpha q} \text{ of this sec.} \quad \text{with} \quad \begin{cases} \alpha = 1, \ldots, s_p, \\ s_p := \dim\,(\mathrm{Spin}(1,p)), \\ q = 1, \ldots, \dfrac{16}{s_p}, \end{cases} \tag{4.1}$$

i.e., to split the 16-valued (multi)index of this fermionic one-form on the Spin $(1,p)$ index α ($\alpha = 1,2$ for M2- and $\alpha = 1,2,3,4$ for M5-brane) and the Spin $(D-p-1)$ index q ($q = 1, \ldots, 8$ for M2- and $q = 1, \ldots, 4$ for M5-brane).

The fermionic supervielbein $e^{\beta p}$ induced by superembedding can be defined in terms of the pull-back $\hat{\mathcal{E}}^{\underline{\alpha}}$ of the $D = 11$ targets superspace fermionic supervielbein $\mathcal{E}^{\underline{\alpha}}$ with the use of 16×32 matrix $v_{\underline{\alpha}}{}^{\beta p}$ of rank 16,

$$e^{\beta p} = \hat{\mathcal{E}}^{\underline{\alpha}} v_{\underline{\alpha}}{}^{\beta p}. \tag{4.2}$$

The simplest choice of $v_{\underline{\alpha}}{}^{\beta p}$ to be a 32×16 block of unity matrix clearly breaks $SO(1,10)$ Lorentz symmetry (at least down to $SO(1,9)$, in which case we arrive at equation equivalent to (3.1)). To preserve the 11D Lorentz symmetry we have to assume that $v_{\underline{\alpha}}{}^{\beta p}$ is a 32×16 matrix superfield. It is convenient to consider it as a 32×16 block of a Spin $(1,10)$ group valued 32×32 matrix superfield

$$V_{\underline{\beta}}^{(\underline{\alpha})} = \left(v_{\underline{\beta}}^{\alpha q}, v_{\underline{\beta}\alpha\dot{q}} \right) \in \mathrm{Spin}\,(1,10), \begin{cases} \alpha, \beta = 1, 2, \\ q = 1, \ldots, 8 \text{ for M2-brane}, \end{cases} \tag{4.3}$$

$$V_{\underline{\beta}}^{(\underline{\alpha})} = \left(v_{\underline{\beta}}^{\alpha q}, v_{\underline{\beta}}{}^{q}_{\alpha} \right) \in \mathrm{Spin}\,(1,10), \begin{cases} \alpha, \beta = 1, 2, 3, 4, \\ q = 1, 2, 3, 4 \text{ for M5-brane}. \end{cases} \tag{4.4}$$

These *spinor moving frame superfields* (also called spinor Lorentz harmonics [54–56]) describe the spinor representation of the same $SO(1,10)$ Lorentz rotation, the vector representation of which is described by the moving frame variables (2.14) and, hence, carry the same local degrees of freedom as the moving frame vectors [1].

The Spin group, the double covering of the Lorentz group SO, is defined by the conditions of the preservation of the gamma matrices. Hence, the above-mentioned relation between

[1]These moving frame vectors can be identified with derivatives of the coordinate functions (see Eq. (2.11)), so that one can either state that they are auxiliary fields which do not bring new *dynamical* degrees of freedom, or, equivalently, say that they carry some «momentum» part of degrees of freedom; in other words, they are counterparts of momentum variable p in the first-order formulation of the particle mechanics $S = \int d\tau\, p\dot{q} - \int d\tau\, e(p^2 - m^2)/2$.

vector and spinor moving frame variables (vector and spinor Lorentz harmonics) of Eqs. (2.14) and (4.3) (or (4.4)) is given by

$$V\Gamma^{(\underline{a})}V^T = \Gamma^{\underline{b}}U_{\underline{b}}{}^{(\underline{a})} \Rightarrow \begin{cases} V\Gamma^a V^T = \Gamma^{\underline{b}}u_{\underline{b}}{}^a, \\ V\Gamma^i V^T = \Gamma^{\underline{b}}u_{\underline{b}}{}^i, \end{cases} \tag{4.5}$$

or, equivalently, by

$$V^T\tilde{\Gamma}^{\underline{a}}V = \tilde{\Gamma}^{(\underline{b})}U_{(\underline{b})}{}^{\underline{a}} = \tilde{\Gamma}^b u_b{}^{\underline{a}} - \tilde{\Gamma}^i u^{i\underline{a}}. \tag{4.6}$$

In the dimensions where the charge conjugation matrix C exists, including the cases of $D = 11$ we are interested in here (but not in $D = 10$ $\mathcal{N} = 1$ and type IIB cases), the condition of its conservation should be also listed among the defining relations of the spinorial moving frame variables,

$$VCV^T = C, \quad V^T C^{-1} V = C^{-1}. \tag{4.7}$$

These relations imply that the inverse spinor moving frame matrix V^{-1},

$$V^{-1}{}_{(\underline{\alpha})}{}^{\underline{\beta}} \equiv V_{(\underline{\alpha})}{}^{\underline{\beta}} := \left(iv_{\alpha q}{}^{\underline{\beta}}, iv_{\dot{q}}^{\alpha\underline{\beta}}\right) \text{ for M2-brane}, \tag{4.8}$$

$$V^{-1}{}_{(\underline{\alpha})}{}^{\underline{\beta}} \equiv V_{(\underline{\alpha})}{}^{\underline{\beta}} := \left(v_{\alpha q}{}^{\underline{\beta}}, v_{q}^{\alpha\underline{\beta}}\right) \text{ for M5-brane}, \tag{4.9}$$

obeying

$$V_{(\underline{\alpha})}^{\underline{\gamma}}V_{\underline{\gamma}}^{(\underline{\beta})} = \delta_{(\underline{\alpha})}^{(\underline{\beta})} = \begin{cases} \text{diag }(\delta_\alpha{}^\beta\delta_q{}^p, \delta^\alpha{}_\beta\delta_{\dot{q}}{}^{\dot{p}}) \text{ for M2-brane}, \\ \text{diag }(\delta_\alpha{}^\beta\delta_q{}^p, \delta^\alpha{}_\beta\delta_q{}^p) \text{ for M5-brane} \end{cases} \tag{4.10}$$

can be explicitly constructed from the original harmonic matrix (4.3) or (4.4), $V^{-1} = CV^TC^{-1}$. In the case of M2- and M5-branes the components of the inverse matrices (4.8) and (4.9) are defined by

$$v_{\alpha q}{}^{\underline{\alpha}} = C^{\underline{\alpha}\underline{\delta}}\epsilon_{\alpha\beta}v_{\underline{\delta}}{}^{\beta q}, \qquad v_{\dot{q}}^{\alpha\underline{\alpha}} = C^{\underline{\alpha}\underline{\delta}}\epsilon^{\alpha\beta}v_{\underline{\delta}\,\beta\dot{q}} \text{ for M2-brane}, \tag{4.11}$$

$$v_{\alpha q}{}^{\underline{\alpha}} = iC^{\underline{\alpha}\underline{\delta}}C_{qp}v_{\underline{\delta}\,\beta}^{p}, \qquad v_{q}^{\alpha\underline{\alpha}} = iC^{\underline{\alpha}\underline{\delta}}C_{qp}v_{\underline{\delta}}^{\alpha p} \text{ for M5-brane}, \tag{4.12}$$

where $C^{\underline{\alpha}\underline{\delta}}$ and C_{qp} are the $D = 11 = 1 + 10$ and $d = 5 = 5 + 0$ charge conjugation matrices; see Appendix A for more details on our notation. Notice that we found it more convenient to introduce $i = \sqrt{-1}$ in the definition of the inverse moving frame matrix components (4.3) for the case of M2-brane, while in the case of M5-brane we introduced it in the relation between the components of the inverse and the original moving frame matrices (4.12). The latter choice looks more natural, while the former is explained by that in the case of $p = 2$ there exists the $SL(2,\mathbb{R}) = \text{Spin}\,(1,2)$ invariant antisymmetric tensor $\epsilon^{\alpha\beta} = i\sigma^2$ and its inverse $\epsilon_{\alpha\beta} = -i\sigma^2$, which can be used to rise and to lower the $SL(2,\mathbb{R})$ ($SO(1,2)$ spinorial) indices; then, the use of notation similar to the one accepted for M5-brane case might produce a confusion.

When the charge conjugation matrix does not exist (like in the $D = 10$ $\mathcal{N} = 1$ case involving the Majorana–Weyl spinor representation), the inverse spinor moving frame variables are defined just by the constraint $V^{-1}V = I$ (Eq. (4.10)), i.e., its dependence on the original harmonics remains implicit.

As the spinor moving frame variables (spinor harmonics) (4.3) (or (4.4)) carry the same local degrees of freedom as the vector harmonics (moving frame variables) (2.14), their derivatives are expressed through the same generalized Cartan forms (2.15). To find this one just notice that the Lorentz group $SO(1, D-1)$ and its doubly covered Spin $(1, D-1)$ are locally isomorphic. Then, isomorphic is the co-tangent and tangent space to these groups, spin $(1, D-1) \approx so(1, D-1)$. In the case of $SO(1, D-1)$, the latter has the natural basis described by the generalized Cartan forms $\Omega^{(\underline{a})(\underline{b})} = u_{\underline{c}}{}^{(\underline{a})} D^L u^{\underline{c}(\underline{b})}$, where D^L is the Lorentz covariant derivative constructed with the use of target superspace spin connection, $D^L u_{\underline{a}}^{(\underline{b})} = du_{\underline{c}}^{(\underline{b})} + \omega_{\underline{a}}{}^{\underline{c}} u_{\underline{c}}^{(\underline{b})}$. The isomorphism of spin $(1, D-1)$ and $so(1, D-1)$ algebras is described by the following universal (D-independent) relation between the generalized Cartan forms of Spin $(1, D-1)$ and of $SO(1, D-1)$:

$$V^{-1} D^L V = \frac{1}{4} \Omega^{(\underline{a})(\underline{b})} \Gamma_{(\underline{a})(\underline{b})} := \frac{1}{4} (U^{-1} D^L U)^{(\underline{a})(\underline{b})} \Gamma_{(\underline{a})(\underline{b})} = \\ = \frac{1}{4} \Omega^{ab} \Gamma_{ab} + \frac{1}{4} \Omega^{ij} \Gamma^{ij} - \frac{1}{2} \Omega^{ai} \Gamma_a \Gamma^i, \quad (4.13)$$

where $D^L V = dV - (1/4) \omega^{\underline{ab}} \Gamma_{\underline{ab}} V$.

In superembedding approach it is convenient to consider the spinor moving frame variables as homogeneous coordinates of the coset $\frac{\text{Spin}\,(1, D-1)}{\text{Spin}\,(1, p) \otimes \text{Spin}\,(D-p-1)}$, using the natural Spin $(1, p) \otimes$Spin $(D-p-1)$ gauge symmetry of the embedding of the worldvolume superspace as an identification relation. In practical terms this implies that it is convenient to rewrite Eq. (4.13) in terms of Spin $(1, D-1) \otimes$ Spin $(1, p) \otimes$ Spin $(D-p-1)$-covariant derivative D:

$$DV := dV - \frac{1}{4} \omega^{\underline{ab}} \Gamma_{\underline{ab}} V - \frac{1}{4} V \Gamma_{ab} \Omega^{ab} - \frac{1}{4} V \Gamma^{ij} \Omega^{ij} = -\frac{1}{2} \Omega^{ai} V \Gamma_a \Gamma^i. \quad (4.14)$$

To specify further the above equations, one needs to use explicitly an $SO(1, p) \times SO(D-p-1)$ invariant representation for the Γ-matrices

$$\Gamma^{(\underline{a})} = (\Gamma^a, \Gamma^i), \quad (4.15)$$

so that the further details are p-dependent and will be discussed in the case-by-case manner. The representation convenient for the study of M2- and M5-branes and useful relations for corresponding spinor moving frame variables can be found in Appendix A.

To conclude the general description of the spinor moving frame variables, let us notice that their use is also inevitable when constructing superembedding approach to fundamental string [15] (see [58] for recent review and elaboration of a specific case of type IIB superstring in $AdS_5 \otimes S^5$ background).

4.1. Superembedding Description of M2-Brane (Also Known as $D = 11$ Supermembrane). In this section we will show how the dynamical M2-brane equations follow from the superembedding equation (2.6) (equivalent to (2.5)) [15],

$$\hat{E}^i := \hat{E}^{\underline{a}} u_{\underline{a}}{}^i = 0. \quad (4.16)$$

We have tried to make this section «closed», so that it can be read independently; this explains some repetitions of the statement of the previous sections.

The geometry of the worldvolume superspace is induced by superembedding. This implies, in particular, that its bosonic supervielbein form and $SO(1, 2) \otimes SO(8)$ connection are defined by (2.9) and (2.15). The fermionic supervielbein of the M2-brane worldvolume superspace $\mathcal{W}^{(3|16)}$ can be identified with, say, $\hat{E}^{\alpha}_{q} = \hat{\mathcal{E}}^{\underline{\beta}} v_{\underline{\beta}q}{}^{\alpha}$. Then,

$$\hat{E}^{\alpha}_{q} := \hat{\mathcal{E}}^{\underline{\beta}} v_{\underline{\beta}q}{}^{\alpha} = e^{\alpha q}, \quad \hat{E}_{\beta\dot{q}} := \hat{\mathcal{E}}^{\underline{\beta}} v_{\underline{\beta}\beta\dot{q}} = e^{\alpha q} h_{\alpha q\,\beta\dot{q}} + e^{b}\chi_{b\,\beta\dot{q}}. \tag{4.17}$$

With such conventional constraints, the lowest dimensional (dim 0) spin-tensorial component of the integrability condition for superembedding equation, Eq. (2.18), reads $\gamma^{i}_{q\dot{q}} h_{\beta p\,\alpha\dot{q}} + \gamma^{i}_{p\dot{q}} h_{\alpha q\,\beta\dot{q}} = 0$. The solution of this equation is trivial, $h_{\alpha q\,\dot{q}}{}^{\beta} = 0$, so that Eqs. (4.17) simplify to [15,59]

$$\hat{E}^{\alpha}_{q} = e^{\alpha q}, \quad \hat{E}_{\beta\dot{q}} = e^{b}\chi_{b\,\beta\dot{q}}. \tag{4.18}$$

Using Eqs. (4.18), the tangent superspace torsion constraints (1.3), the conventional constraints resumed in the first equation of (2.15) and the superembedding equation (4.16), one finds that the bosonic torsion of the worldvolume superspace reads

$$De^{a} = 2ie^{\alpha q} \wedge e^{\beta q}\gamma^{b}_{\alpha\beta} - i\, e^{b} \wedge e^{c}\chi_{b}\gamma^{a}\chi_{c}. \tag{4.19}$$

Now, the dim 1/2 $\propto e^{b} \wedge e^{\alpha q}$ component of Eq. (2.18) expresses the spinorial component of Cartan form Ω^{ai}, $\Omega^{ai}_{\alpha q} = 2i\gamma^{i}_{q\dot{p}}\chi^{a}_{\alpha\dot{p}}$; the dim 1 $\propto e^{b} \wedge e^{c}$ component implies $\Omega_{[a\,b]}{}^{i} = 0$, which means that the pure bosonic component of Ω^{ai} is symmetric, $\Omega_{b\,a}{}^{i} = \Omega_{(a\,b)}{}^{i} := u_{(a}{}^{\underline{c}} D_{b)} u_{\underline{c}}{}^{i} = -D_{(a} u_{b)}{}^{\underline{c}}\; u_{\underline{c}}{}^{i}$ and coincides with the (superfield generalization of the) second fundamental form of the worldvolume superspace considered as a surface in the target superspace, Eq. (2.19).

To resume, the dim 1/2 and 1 components of the integrability conditions (2.18) for the superembedding equation (4.16) give us the expression for the generalized Cartan form Ω^{ai} in terms of the second fundamental form $K_{b}{}^{ai}$ of Eq. (2.19), and in terms of the fermionic superfield $\chi_{b\dot{q}}{}^{\beta} = \hat{E}^{\underline{\alpha}}_{b}\, v_{\underline{\alpha}\dot{q}}{}^{\beta} := D_{b}\hat{Z}^{\underline{M}}\,\mathcal{E}_{\underline{M}}{}^{\underline{\alpha}}(\hat{Z}) v_{\underline{\alpha}\dot{q}}{}^{\beta}$, which, in this sense, is a superpartner of the second fundamental form,

$$\begin{aligned} &\Omega^{ai} = 2ie^{\alpha q}\gamma^{i}_{q\dot{p}}\chi^{a}{}_{\alpha\dot{p}} + e_{b}K^{ab\,i}, \quad K_{a\,b}{}^{i} := -D_{(a}\hat{E}_{b)}{}^{\underline{c}} u_{\underline{c}}{}^{i}, \\ &\chi_{b\dot{q}}{}^{\beta} = \hat{E}^{\underline{\alpha}}_{b}\, v_{\underline{\alpha}\dot{q}}{}^{\beta} := D_{b}\hat{Z}^{\underline{M}}\,\mathcal{E}_{\underline{M}}{}^{\underline{\alpha}}(\hat{Z})\, v_{\underline{\alpha}\dot{q}}{}^{\beta}. \end{aligned} \tag{4.20}$$

Now we turn to the self-consistency conditions for the second equation in (4.18). It reads

$$\begin{aligned} 0 = D(\hat{E}_{\alpha\dot{q}} - e^{b}\chi_{b\,\alpha\dot{q}}) = \hat{T}^{\underline{\alpha}} v_{\underline{\alpha}\,\alpha\dot{q}} - \frac{1}{2}e^{\beta p} \wedge \Omega^{ai}\,\gamma_{a\,\alpha\beta}\gamma^{i}_{p\dot{q}} + ie^{\beta p} \wedge e^{\gamma p}\gamma^{b}_{\beta\gamma}\chi_{b\alpha\dot{q}} + \\ + ie^{b} \wedge e^{c}\chi_{b}\gamma^{a}\chi_{c}\chi_{a\beta\dot{q}} - e^{b} \wedge D\chi_{b\beta\dot{q}}, \end{aligned} \tag{4.21}$$

where we have used the expression for the bosonic torsion of the worldvolume superspace (4.19), as well as the expression for the derivative of the spinorial harmonic,

$$Dv_{\underline{\alpha}\alpha\dot{q}} = -\frac{1}{2}\Omega^{ai}\, v_{\underline{\alpha}}{}^{\beta p}\gamma^{i}_{p\dot{q}}\gamma_{a\alpha\beta}, \tag{4.22}$$

which appears as of the rectangular blocks of Eq. (4.14).

Taking into account expression (4.20) for Ω^{ai}, one finds that the lowest dimensional $\propto e^{\beta p} \wedge e^{\gamma p'}$ component of Eq. (4.21) reads $-i\gamma^{i}_{p\dot{q}}\gamma^{i}_{p'\dot{p}}\gamma^{a}_{\alpha\beta}\chi_{a\gamma\dot{p}} - i\gamma^{i}_{p'\dot{q}}\gamma^{i}_{p\dot{p}}\gamma^{a}_{\alpha\gamma}\chi_{a\beta\dot{p}} +$

$2i\delta_{pp'}\chi_{a\,\alpha\dot{q}} = 0$. The only consequence of this equation is that $\gamma_{a\alpha\beta}\chi^{a}_{\gamma\dot{p}} - \gamma_{a\alpha\gamma}\chi^{a}_{\beta\dot{p}} = 0$, which is an equivalent form of the fermionic equations

$$\tilde{\gamma}^{a\alpha\beta}\chi_{a\beta\dot{q}} := \tilde{\gamma}^{a\alpha\beta}\hat{E}_a{}^{\underline{\alpha}}v_{\underline{\alpha}\beta\dot{q}} = 0. \tag{4.23}$$

Then, the dim 1 $\propto e^b \wedge e^{\beta p}$ component of Eq. (4.21) is $0 = D_{\beta p}\chi_{b\alpha\dot{q}} + v_{\beta p}{}^{\underline{\beta}}\hat{T}_{\underline{\beta a}}{}^{\underline{\gamma}}v_{\underline{\gamma}\ \alpha\dot{q}}u_b{}^{\underline{a}} + (1/2)\gamma^{i}_{p\dot{q}}\gamma^{a}_{\alpha\beta}K_{ab}{}^{i}$. Contracting this equation with $\tilde{\gamma}^{b\gamma\alpha}$, one finds

$$\gamma^{i}_{p\dot{q}}K_a{}^{ai}\delta_\beta{}^\gamma = -2v_{\beta p}{}^{\underline{\alpha}}\hat{T}_{\underline{\alpha a}}{}^{\underline{\delta}}v_{\underline{\delta}\ \alpha\dot{q}}\tilde{\gamma}^{b\alpha\gamma}u_b{}^{\underline{a}} - 2D_{\beta p}(\tilde{\gamma}^a\chi_{a\dot{q}})^\gamma. \tag{4.24}$$

The last term vanishes due to the fermionic equation of motion (4.23), so that

$$\gamma^{i}_{p\dot{q}}K_a{}^{ai}\delta_\beta{}^\gamma = -2v_{\beta p}{}^{\underline{\alpha}}\hat{T}_{\underline{\alpha a}}{}^{\underline{\delta}}v_{\underline{\delta}\ \alpha\dot{q}}\tilde{\gamma}^{b\alpha\gamma}u_b{}^{\underline{a}}. \tag{4.25}$$

The bosonic equations of motion are obtained by contracting this equation with $1/16\gamma^{i}_{p\dot{q}}\delta_\gamma{}^\beta$. It reads

$$K_a{}^{ai} := -D^a\hat{E}_a{}^{\underline{b}}\,u_{\underline{b}}{}^{i} = -\frac{1}{8}v_{\beta p}{}^{\underline{\alpha}}\gamma^{i}_{p\dot{q}}\tilde{\gamma}^{b\beta\alpha}v_{\underline{\delta}\ \alpha\dot{q}}u_b{}^{\underline{a}}\hat{T}_{\underline{\alpha a}}{}^{\underline{\delta}}. \tag{4.26}$$

The fact that other irreducible parts of the r.h.s. of Eq. (4.25) vanish, i.e., that $v_{\beta p}{}^{\underline{\alpha}}\hat{T}_{\underline{\alpha a}}{}^{\underline{\delta}}v_{\underline{\delta}\alpha\dot{q}}\tilde{\gamma}^{b\alpha\gamma}u_b{}^{\underline{a}} \propto \gamma^{i}_{p\dot{q}}\delta_\beta{}^\gamma$, might contain a nontrivial information on the geometry of the $D = 11$ superspace supergravity background. One can check that this is satisfied identically for

$$T_{\underline{\beta a}}{}^{\underline{\gamma}} = -\frac{i}{144}\left(F^{\underline{c_1c_2c_3c_4}}\Gamma_{\underline{ac_1c_2c_3c_4}} + 8F_{\underline{ac_1c_2c_3}}\Gamma^{\underline{c_1c_2c_3}}\right)_{\underline{\beta}}{}^{\underline{\gamma}}, \tag{4.27}$$

which follows from the standard superspace constraints of $D = 11$ supergravity [44, 45] by studying the Bianchi identities. Using (4.27), one can obtain the more specific form of the (superfield) bosonic equations of the M2-brane: Eq. (4.26) is equivalent to

$$K_a{}^{ai} = \frac{1}{3}\,F^i{}_{abc}\varepsilon^{abc}, \tag{4.28}$$

where

$$F^i{}_{abc} := F_{\underline{abcd}}(\hat{Z})u^{\underline{a}i}u_b{}^{\underline{b}}u_c{}^{\underline{c}}u_d{}^{\underline{d}}. \tag{4.29}$$

To make a contact with standard formulation of the supermembrane [1], let us notice that, on the bosonic worldvolume, ignoring fermions, and writing equations in terms of the induced metric ($g_{mn} = e_m{}^a e_{an} = \hat{E}_m{}^{\underline{a}}\hat{E}_{n\underline{a}}$), one finds that $D^a\hat{E}_a{}^{\underline{b}} = D_m(\sqrt{|g|}g^{mn}\hat{E}_n{}^{\underline{b}})$, where D_m is the $SO(1,9)$ covariant derivative on the worldvolume. Hence, Eq. (4.28) coincides in this case with the standard supermembrane equation

$$D_m(\sqrt{|g|}g^{mn}\hat{E}_n{}^{\underline{b}}) = -\frac{1}{3}\eta^{\underline{ba}}\,F_{\underline{a}bcd}\varepsilon^{bcd}, \quad F_{\underline{a}\ abc} := F_{\underline{abcd}}\hat{E}_b{}^{\underline{b}}\hat{E}_c{}^{\underline{c}}\hat{E}_d{}^{\underline{d}} \tag{4.30}$$

contracted with the orthogonal harmonics $u_{\underline{a}}{}^{i}(K_a{}^{ai} := -D^a\hat{E}_a{}^{\underline{b}}u_{\underline{b}}{}^{i} = D_m(\sqrt{|g|}g^{mn}\hat{E}_n{}^{\underline{b}})u_{\underline{b}}{}^{i})$. The projection of the supermembrane equation onto the vector harmonics $u_{\underline{b}}{}^{a}$, tangential to the worldvolume, $D^a\hat{E}_a{}^{\underline{b}}u_{\underline{b}}{}^{b} = \ldots$, can be shown to be satisfied identically. This is the Noether identity reflecting the reparametrization invariance of the supermembrane (action and of the) equations of motion. Thus, Eq. (4.28) is equivalent to the standard supermembrane equation, Eq. (4.30) modulo fermionic contributions.

Coming back, let us stress that Eq. (4.24) gives us the interrelation between the fermionic and the bosonic equations, Eqs. (4.23) and (4.26), of supermembrane in general $D = 11$ superfield supergravity background. It shows that the bosonic equation of motion of the M2-brane can be obtained as a second component in the decomposition of the superfield generalization of the fermionic equation of motion on the Grassmann coordinate.

4.2. M5-Brane in Superembedding Approach. The dynamics of M5-brane is also fixed by the superembedding equation (2.5) [5] equivalent to (2.6),

$$\hat{E}^i := \hat{E}^{\underline{a}} u_{\underline{a}}{}^i = 0. \tag{4.31}$$

The bosonic supervielbein of the worldvolume superspace is defined by (2.9) and the worldvolume superspace $SO(1,5)$ and $SO(5)$ connections — by (2.15). The fermionic supervielbein of the M2-brane worldvolume superspace $\mathcal{W}^{(6|16)}$ can be identified with, say, $\hat{E}^{\alpha q} := \hat{\mathcal{E}}^{\underline{\beta}} v_{\underline{\beta}}{}^{\alpha q}$. Then,

$$\hat{E}^{\alpha q} := \hat{\mathcal{E}}^{\underline{\beta}} v_{\underline{\beta}}{}^{\alpha q} = e^{\alpha q}, \tag{4.32}$$

$$\hat{E}^{q}_{\beta} := \hat{\mathcal{E}}^{\underline{\alpha}} v_{\underline{\alpha}\beta}{}^{q} = e^{\alpha q} h_{\alpha\beta} + e^b \chi_{b\,\beta}{}^q. \tag{4.33}$$

To be more precise, the general decomposition of the second projection of the pull-back of the target superspace fermionic supervielbein $\mathcal{E}^{\underline{\alpha}}$ reads $\hat{\mathcal{E}}^{\underline{\alpha}} v_{\underline{\alpha}\beta}{}^q = e^{\alpha p} h_{\alpha p\;\beta}{}^q + e^b \chi_{b\,\beta}{}^q$. However, as the further study shows anyway that $h_{\alpha p\;\beta}{}^q = h_{\alpha\beta}\delta_p{}^q$, we have allowed ourselves to make a shortcut substituting this expression in Eq. (4.33) from the very beginning.

Equations (4.32) and (4.33) can be collected in

$$\hat{\mathcal{E}}^{\underline{\alpha}} = e^{\beta q} V_{\beta q}{}^{\underline{\alpha}}(h) + e^a \chi_{a\beta}{}^p v_p{}^{\beta\underline{\alpha}}, \quad V_{\beta p}{}^{\underline{\alpha}}(h) := v_{\beta p}{}^{\underline{\alpha}} + h_{\beta\gamma} v_p{}^{\gamma\underline{\alpha}}. \tag{4.34}$$

For the discussion below it is useful to notice that the «deformed harmonics» $V_{\beta p}{}^{\underline{\alpha}}(h) := v_{\beta p}{}^{\underline{\alpha}} + h_{\beta\gamma} v_p{}^{\gamma\underline{\alpha}}$ obeys (see Appendix A, Subsec. A2 for our notation Γ-matrices representation and γ-matrices properties)

$$u_a{}^{\underline{a}} u_b{}^{\underline{b}} V_{\beta p}{}^{\underline{\alpha}}(h) \Gamma_{\underline{ab}\;\underline{\alpha\delta}} V_{\beta p}{}^{\underline{\delta}}(h) = 2i(\gamma_{ab}h)_{[\alpha\beta]} C_{qp}. \tag{4.35}$$

The lowest dimensional ($\propto e^{\alpha q} \wedge e^{\beta p}$) component of the integrability conditions for the superembedding equation, Eq. (2.18), results in $h_{\alpha\beta} = h_{\beta\alpha}$. As in $d = 6$ the basis of symmetric spin tensor matrix is provided by $\gamma^{abc}_{\alpha\beta}$ (notice that $\gamma^a_{\alpha\beta} = -\gamma^a_{\alpha\beta} = (1/2)\epsilon_{\alpha\beta\gamma\delta}\tilde{\gamma}^{a\gamma\delta}$ and $\gamma^{(a}\tilde{\gamma}^{b)} = \eta^{ab}$; see [66] and Appendix A, Subsec. A2 for more detail), so that

$$h_{\alpha\beta} = \frac{1}{3!} h_{abc} \gamma^{abc}_{\alpha\beta}. \tag{4.36}$$

As far as $\gamma^{abc}_{\alpha\beta}$ is anti-self-dual, $\gamma^{abc}_{\alpha\beta} = -(1/3!)\epsilon^{abcdef}\gamma_{def\,\alpha\beta}$, the antisymmetric tensor h_{abc} in (4.36) is self-dual,

$$h_{abc} = \frac{1}{3!} \epsilon_{abcdef} h^{def}. \tag{4.37}$$

An important property of the symmetric spin-tensor $h_{\alpha\beta}$ is (cf. (3.11))

$$h\tilde{\gamma}^a h = \gamma^b k_b{}^a, \quad k_b{}^a = -2h_{bcd}h^{cda}. \tag{4.38}$$

One easily obtains this taking into account that, as a consequence of the self-duality (4.37), the contraction of h_{abc} with $\tilde{\gamma}^{\alpha\beta}_{abc} = +(1/3!)\epsilon_{abcdef}\tilde{\gamma}^{def\,\alpha\beta}$ vanishes. Then, $h\tilde{\gamma}^a h = h^{abc}(\tilde{\gamma}_{bc}h)$ from which one easily arrives at (4.38).

The appearance of a third-rank antisymmetric self-dual tensor reflects the fact that the linearized spectrum of the M5-brane includes the chiral two-form potential [67], i.e., the two-form $6d$ gauge field with the self-dual three-form field strength. Beyond the linear approximation, one finds that the gauge field strength tensor obeys a nonlinear generalization of the self-duality condition [5, 8, 9].

The dim 3/2 and dim 2 components of the integrability condition Eq. (2.18) determine the generalized Cartan form to be

$$\Omega^{ai} = 2e^{\alpha q}\gamma^i_{qp}\chi_\alpha{}^{ap} + e_b K^{ab\,i}, \tag{4.39}$$

where $K^{ab\,i} = K^{ab\,i}$ is the second fundamental form defined as in Eq. (2.19) and $\gamma^i_{qp} = -\gamma^i_{qp} = (1/2)\epsilon_{qprs}\tilde{\gamma}^{i\,rs} = -(\tilde{\gamma}^{i\,qp})^*$ are the $SO(5)$ Klebsh–Gordan coefficients (see Appendix A, Subsec. A2 for their properties).

The bosonic torsion of the worldvolume geometry induced by superembedding reads

$$De^a = -ie^{\alpha q}\wedge e^{\beta p}C_{qp}\gamma^b_{\alpha\beta}\, m_b{}^a \; + 2ie^b\wedge e^{\alpha q}C_{qp}(h\tilde{\gamma}^a\chi_b{}^p)_\alpha + ie^c\wedge e^b\psi^q_b\tilde{\gamma}^a\psi^p_c C_{qp}, \tag{4.40}$$

where [5, 60]

$$m_a{}^b = \delta_a{}^b + k_a{}^b = \delta_a{}^b - 2h_{acd}h^{bcd}. \tag{4.41}$$

Generically, this matrix is invertible (and not k of (4.38); cf. Eq. (3.11) in the case of Dp-branes).

Now we could pass to studying the self-consistency condition for the fermionic one-form equation (4.33),

$$\begin{aligned} 0 = D(\hat{E}_\beta{}^q - e^{\alpha q}h_{\alpha\beta} - e^b\chi_{b\,\beta}{}^q) = \hat{T}^{\underline{\alpha}}v_{\underline{\alpha}\beta}{}^q - \frac{i}{2}e^{\alpha p}\wedge\Omega^{ai}\gamma_{a\alpha\beta}(\gamma^i C)_p{}^q - \\ - e^{\alpha q}\wedge Dh_{\alpha\beta} - e^b\wedge D\chi_{b\,\beta}{}^q - De^{\alpha q}h_{\alpha\beta} - De^b\chi_{b\,\beta}{}^q, \end{aligned} \tag{4.42}$$

and obtain all the dynamical equations from this. In the second equality of (4.42) we have used the second of the following two spinorial counterparts of Eqs. (2.15),

$$Dv_{\underline{\alpha}}{}^{\alpha q} = \frac{i}{2}\Omega^{ai}v_{\underline{\alpha}\beta}{}^p\tilde{\gamma}^{\beta\alpha}_a(\gamma^i C)_p{}^q, \quad Dv_{\underline{\alpha}\alpha}{}^q = -\frac{i}{2}\Omega^{ai}v^{\beta p}_{\underline{\alpha}}\tilde{\gamma}_{a\,\beta\alpha}(\gamma^i C)_p{}^q, \tag{4.43}$$

while the first one has to be used in calculation of fermionic torsion. Clearly, neither this nor Eq. (4.42) as a whole looks simple in general type II supergravity background.

However, the study may be simplified essentially if we use the presence of the above-mentioned two-form gauge field on the M5 worldvolume, generalize it to the superform b_2 on the worldvolume superspace, impose the constraints on its generalized field strength and study the corresponding Bianchi identities. This is the counterpart of imposing the gauge field constraints on the worldvolume superspace of Dp-branes which we discussed in Sec. 3.

The constraints on the three-form field strength [5] can be written in the form

$$H_3 := db_2 - \hat{C}_3 = \frac{1}{3!}e^c\wedge e^b\wedge e^a H_{abc}, \tag{4.44}$$

where $\hat{C}_3$ is the pull-back to $\mathcal{W}^{(6|16)}$ of the three-form gauge potential of the superspace 11D supergravity, the field strength of which obeys the constraints

$$\mathcal{F}_4 := dC_3 = \frac{1}{4} E^{\underline{b}} \wedge E^{\underline{a}} \wedge \mathcal{E} \wedge \Gamma_{\underline{ab}} \mathcal{E} + \frac{1}{4!} E^{\underline{d}} \wedge \ldots \wedge E^{\underline{a}} F_{\underline{abcd}}. \tag{4.45}$$

The Bianchi identities

$$dH_3 = -\mathcal{F}_4 \tag{4.46}$$

result in the relation between the tensor field strength H_{abc} and self-dual tensor h_{abc} of Eqs. (4.36), (4.37) [5,60]

$$m_a{}^d H_{bcd} = h_{abc} = \frac{1}{3!} \epsilon_{abcdef} h^{def} \tag{4.47}$$

as well as

$$D_{\alpha q} H_{abc} = -6iC_{qp} (h \tilde{\gamma}^d \psi_{[a}) H_{bc]d}, \tag{4.48}$$

$$D_{[a} H_{bcd]} = -3iC_{qp} (\psi_{[a} \tilde{\gamma}^e \psi_b) H_{cd]e} + \frac{1}{4} u_a{}^{\underline{a}} \ldots u_d{}^{\underline{d}} F_{\underline{abcd}}(\hat{Z}). \tag{4.49}$$

Clearly, Eq. (4.47), in the derivation of which one uses identity (4.35), provides a nonlinear generalization of the self-duality equation and, hence, implies dynamical equations of motion for the two-form gauge field b_2. (To convince that this is the case, it is sufficient to note that the standard self-duality implies that the linearized two-form gauge field equations of motion in $d = 6$ are satisfied.)

The above relatively simple derivation of the nonlinear self-duality equation (4.47) gives one more example of the usefulness of introducing the worldvolume superspace gauge potentials and studying the corresponding Bianchi identities for their constrained field strengths. The details on derivation of the dynamical equations for the M5-brane coordinate functions from the superembedding description can be found in the original articles [5, 60, 61] and in the review [19]. The proof of their equivalence to the equations of motion derived from the worldvolume action [8,9] is the subject of [60,61].

5. MULTIPLE D0-BRANE EQUATIONS FROM SUPEREMBEDDING APPROACH

It is the usual expectation that the action for a system of N Dp-branes will essentially be a nonlinear generalization of the $U(N)$ SYM action. In particular, the (purely bosonic and not Lorentz invariant) Myers action [24] is of this type. Then, the equations of motion which should follow from a hypothetical supersymmetric and Lorentz covariant generalization (or modification) of this action are expected to contain the $SU(N)$ SYM equations ($U(N) = SU(N) \times U(1)$), while the center-of-mass motion is expected to be described by a usual type of coordinate functions $\hat{Z}^{\underline{M}}(\xi)$ and by related equations for the $U(1)$ gauge fields (presumably coupled to the $SU(N)$ equations). Notice that the center-of-mass equations of motion (and equations for $U(1)$ gauge fields which are expected to be involved in the center-of-mass supermultiplet) are expected to be quite close to the equations for a single Dp-brane, but with the single brane tension (mass) T replaced by NT. In this section we review, following [22], the application of the superembedding approach in search for such supersymmetric equations.

5.1. Non-Abelian $\mathcal{N} = 16$, $d = 1$ SYM Constraints on D0-Brane. In [22] the worldvolume superspace of multiple D0-brane system was assumed to obey the same superembedding equation (2.6) as in the case of single D-brane.

To motivate this, let us notice that the superembedding equation is pure geometrical. It states, in its form of (2.5), that the pull-back of the target space bosonic vielbein to the worldvolume superspace $\mathcal{W}^{(1|16)}$ does not have projections on the fermionic vielbein of $\mathcal{W}^{(1|16)}$. Hence, it is natural to assume that the center-of-mass motion of the system of multiple D0-brane will also obey the superembedding equations.

Of course this is not a proof. But the universality of the superembedding equation, which is valid for all extended objects studied till now in their maximal worldvolume superspace formulations, and the difficulties one arrives at in any attempt to modify to try to impose it, following [22], at least as an approximation (see concluding Sec. 6 for more discussion on this).

As far as the superembedding equation puts the $p < 6$ Dp-brane models on the mass shell, our superembedding approach to $p < 6$ NDp-brane model predicts that the center-of-mass motion will be described by the motion of single brane with tension NT. Then, in the light of the above-stated, and taking in mind that a good low-energy approximation to mutiple Dp-brane is given by maximally supersymmetric $d = p + 1$ $U(N)$ SYM action, the only possibility to describe the multiple D0-brane system in the framework of superembedding approach seems to consider a *non-Abelian SU(N) gauge field supermultiplet* on the D0-brane worldvolume superspace $W^{(1|16)}$. (See [21] for more discussion on a similar issue in the context of searching for hypothetical Q7-branes [68].)

This can be defined by an $su(n)$ valued non-Abelian gauge potential one-form $A = e^0 A_0 + e^\alpha A_\alpha$ with the field strength

$$G_2 = dA - A \wedge A = \frac{1}{2} e^\alpha \wedge e^\beta G_{\alpha\beta} + e^0 \wedge e^\beta G_{\beta 0}, \tag{5.1}$$

which obeys the Bianchi identities

$$DG_2 = dG_2 - G_2 \wedge A + A \wedge G_2 \equiv 0 \ . \tag{5.2}$$

As in the Abelian case discussed in Sec. 3, to get a nontrivial consequences for the structure of the field strengths $G_{\alpha\beta}$, $G_{\beta 0}$ form Bianchi identities, one has to impose constraints. A natural possibility is

$$G_{\alpha\beta} = i\sigma^i_{\alpha\beta} \mathbb{X}^i, \tag{5.3}$$

with some $su(N)$ valued $SO(9)$ vector superfield $\mathbb{X}^i$. (See Subsec. 5.5 for discussion on possible modification of this constraint). The Bianchi identities (5.2) are satisfied if $\mathbb{X}^i$ obeys

$$D_\alpha \mathbb{X}^i = 4i(\sigma^0 \tilde{\sigma}^i)_\alpha{}^\beta \Psi_\beta \tag{5.4}$$

and $G_{\alpha 0} = i\Psi_\alpha + (i/2)(\sigma^{0i}\Lambda)_\alpha \mathbb{X}^i$. It is natural to call (5.4) *superembedding-like equation* as it gives a matrix $SU(N)$ gauge invariant generalization of the gauge fixed form of the linearized superembedding equation (2.5) (this reads $D_\alpha X^i = \propto (\sigma^0 \tilde{\sigma}^i (\Theta^2 - \Theta^1))_\alpha$, see [3]).

5.2. Multiple D0-Brane Equations of Motion from $d = 1$ $\mathcal{N} = 16$ SYM Constraints. Flat Target Superspace. Let us, for simplicity, consider the case of flat target type IIA superspace, in which, on the mass shell of D0-brane, $\Omega^i = 0$, so that $\sigma^0_{\alpha\beta}$ and $\sigma^i_{\alpha\beta}$ are covariant constants,

$D\sigma^0_{\alpha\beta} = 0 = D\sigma^i_{\alpha\beta}$. In this case, the integrability conditions ($D_{(\beta}D_{\alpha)}\mathbb{X}^i = \ldots$) for Eq. (5.4) result in

$$D_\alpha \Psi_\beta = -\frac{1}{2}\sigma^i_{\alpha\beta} D_0 \mathbb{X}^i + \frac{1}{16}\sigma^{0ij}_{\alpha\beta}[\mathbb{X}^i, \mathbb{X}^j] \tag{5.5}$$

and the integrability conditions for Eq. (5.5) result in $1d$ Dirac equation of the form [1]

$$D_0 \Psi_\beta + \frac{1}{4}[(\sigma^{0j}\Psi)_\beta, \mathbb{X}^j] = 0. \tag{5.6}$$

Applying the Grassmann covariant derivative D_α to the fermionic equation (5.6), one derives, after some algebra, the following set of equations:

$$D_0 D_0 \mathbb{X}^i - \frac{1}{32}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] + \frac{i}{8}\{\Psi_\alpha, \Psi_\beta\}\tilde{\sigma}^{i\alpha\beta} = 0, \tag{5.7}$$

$$[D_0 \mathbb{X}^i, \mathbb{X}^i] - 4i\{\Psi_\alpha, \Psi_\beta\}\,\tilde{\sigma}^{0\alpha\beta} = 0. \tag{5.8}$$

Equation (5.7) is a candidate bosonic equation of motion of multiple D0-brane system. Equation (5.8) has the meaning of Gauss low which appears in gauge theories as an equation of motion for the time component of gauge potential (which usually plays the role of Lagrange multiplier).

5.3. Relation to $D = 10$ SYM and M(atrix) Model. The appearance of the counterpart of Gauss low (5.8), characteristic of gauge theory, is not occasional. The point is that our equations appear to be the $D = 10$ SYM equations dimensionally reduced to $d = 1$. The reason is that our constraints (5.3) for $d = 1$, $\mathcal{N} = 16$ SYM multiplet can be obtained as a result of dimensional reduction of $D = 10$ supersymmetric gauge theory. Indeed, the standard $D = 10$ SYM constraints imply vanishing of spinor–spinor component of the field strength,

$$\mathbb{F}_{\alpha\beta} := 2\mathbb{D}_{(\alpha}\mathbb{A}_{\beta)} + \{\mathbb{A}_\alpha, \mathbb{A}_\beta\} - 2i\sigma^{\underline{a}}_{\alpha\beta}\mathbb{A}_{\underline{a}} = 0. \tag{5.9}$$

Assuming independence of fields on the nine-spacial coordinate, one finds that spacial components $\mathbb{A}_i$ of the ten-dimensional field strength are covariant and can be treated as scalar fields

$$\mathbb{A}_i = \mathbb{X}^i/2. \tag{5.10}$$

Then, the minimal covariant field strength for $d = 1$ SYM can be defined as $G_{\alpha\beta} := 2\mathbb{D}_{(\alpha}\mathbb{A}_{\beta)} + \{\mathbb{A}_\alpha, \mathbb{A}_\beta\} - 2i\sigma^{\underline{0}}_{\alpha\beta}\mathbb{A}_{\underline{0}}$ and, due to the original $D = 10$ SYM constraints (5.18), this is equal to $i\sigma^i\mathbb{X}^i$, as in Eq. (5.3),

$$G_{\alpha\beta} := 2\mathbb{D}_{(\alpha}\mathbb{A}_{\beta)} + \{\mathbb{A}_\alpha, \mathbb{A}_\beta\} - 2i\sigma^{\underline{0}}_{\alpha\beta}\mathbb{A}_{\underline{0}} = i\sigma^i_{\alpha\beta}\mathbb{X}^i. \tag{5.11}$$

The above observation is important, in particular, because it indicates the relation with Matrix model [34]. Indeed, this is described by the Lagrangian obtained by dimensional reduction of the $D = 10$ SYM down to $d = 1$ [34]. Actually, the $d = 1$ dimensional reduction of the $U(N)$ $D = 10$ SYM was the first model used to describe D0-brane dynamics

[1] An important check on consistency is that the irreducible $\propto \sigma_{\underline{a}_1\cdots\underline{a}_5}$ part of this integrability conditions is satisfied identically; its $\propto \sigma^0$ part gives (5.6), while $\propto \sigma^i$ part gives (5.6) times σ^{0i}.

in [33] even before the nonlinear DBI+WZ action for super-Dp-branes was constructed in [4, 6,7]. Our superembedding approach description [22] differs from the above-mentioned $U(N)$ SYM approximation by that it uses the $SU(N)$ SYM to describe the relative motion of the constituent branes, while the $U(1)$ gauge field entering the multiplet describing the motion of the center of mass obeys the nonlinear Born–Infeld type equations; also the coordinate function describing the embedding of worldline superspace into the target superspace obeys the nonlinear equations. Even if such a manifestly supersymmetric and Lorentz covariant description appeared to be only approximate, this would be a wider applicable approximation the use of which might be productive.

In the light of identification (5.10) it becomes clear that the superembedding-like equation for the $SU(N)$-valued superfield $\mathbb{X}^i$ (5.4) comes from the consequence $\mathbb{F}_{\alpha\underline{a}} = 2i(\sigma_{\underline{a}}\tilde{\Psi})_\alpha$ (with Ψ defined by $\tilde{\Psi} =: \tilde{\sigma}^0\Psi$) of the constraints (5.18) and thus provides the *general* solution of the Bianchi identity

$$I_{\alpha\beta\gamma} := D_{(\alpha}G_{\beta\gamma)} + t_{(\alpha\beta}{}^{\delta}G_{\gamma)\delta} + 4i\sigma^0_{(\alpha\beta}G_{\gamma)0} = 0 \qquad (5.12)$$

in the presence of these constraints.

To resume, for the multiple D0-brane system in flat target type IIA superspace the worldline superspace $\mathcal{W}^{(1|16)}$ is flat and our superembedding approach results in equations which are equivalent to the ones obtained as a result of dimensional reduction of $D = 10$ SYM. However, it can also be used to describe the multiple D0-brane system in curved supergravity background, where the way through 10D SYM dimensional reduction is obscure.

5.4. Multiple D0-Branes in Curved Type IIA Background. Polarization by External Fluxes. In the case of worldvolume superspace of D0-brane moving in curved target type IIA superspace, the calculations become more complex due to the presence of bosonic and fermionic background superfields. For instance, instead of (5.5), one finds

$$D_\alpha\Psi_\beta = -\frac{1}{2}\sigma^i_{\alpha\beta} + \frac{1}{16}\sigma^{0ij}_{\alpha\beta}[\mathbb{X}^i, \mathbb{X}^j] + \hat{\Lambda}_{1\epsilon}\Psi_\delta\Sigma_1{}^{\epsilon\delta}{}_{\alpha\beta} + (\hat{\Lambda}_2\sigma^0)_\epsilon\Psi_\delta\Sigma_2{}^{\epsilon\delta}{}_{\alpha\beta} \qquad (5.13)$$

with spin-tensors $\Sigma_{1,2}{}^{\epsilon\delta}{}_{\alpha\beta}$ possessing the properties $\sigma^{\underline{ab}}{}_\delta{}^\alpha\Sigma_{1,2}{}^{\epsilon\delta}{}_{\alpha\beta} \propto \sigma^{\underline{ab}}{}_\beta{}^\epsilon$ and $D_\gamma\Sigma_{1,2}{}^{\epsilon\delta}{}_{\alpha\beta} \propto \Lambda$. We will not need an explicit form of these (we leave this and other details for future publication) as our main interest here will be in the algebraic structure of the bosonic equations of motion (see Appendix C for the structure of fermionic equations). Up to the fermionic bilinears proportional to the fermionic background fields these bosonic equations read

$$D_0D_0\mathbb{X}^i - \frac{1}{32}[[\mathbb{X}^i, \mathbb{X}^j], \mathbb{X}^j] + \frac{i}{8}\{\Psi_\alpha, \Psi_\beta\}\,\tilde{\sigma}^{i\alpha\beta} =$$
$$= D_0\mathbb{X}^j\,\mathbb{F}^{j,i} + \frac{1}{16}[\mathbb{X}^j, \mathbb{X}^k]\mathbb{G}^{jk,i} + \mathcal{O}(\hat{\Lambda}_{1,2}\cdot\Psi) + \mathcal{O}(\hat{\Lambda}_{1,2}\cdot\hat{\Lambda}_{1,2}). \qquad (5.14)$$

The $SO(9)$ tensor coefficients $\mathbb{F}^{j,i}$ and $\mathbb{G}^{jk,i}$ in the r.h.s. of (5.14) are expressed in terms of the NS–NS and RR fluxes by

$$\mathbb{F}^{j,i} = q_0\widehat{D_0\Phi}\delta^{ij} + p_1\hat{R}^{ij} + q_2\hat{H}^{0ij}, \qquad (5.15)$$

$$\mathbb{G}^{jk,i} = p_0\delta^{i[j}\widehat{D^{k]}\Phi} + q_1\delta^{i[j}\hat{R}^{k]0} + p_2\hat{H}^{ijk} + q_3\hat{R}^{0ijk}. \qquad (5.16)$$

Here $q_{0,1,2,3}$ and $p_{0,1,2}$ are constant coefficients characterizing couplings to dilaton as well as to electric and magnetic fields strength of the one-form, two-form and three-form gauge fields; the RR field strength is defined by $R_{2n+2} = dC_{2n+1} - C_{2n-1} \wedge H_3$ and the three-form field strength of the NS–NS two-form gauge field is simply $H_3 = dB_2$.

Notice that the center-of-mass motion is factored out and is described by the single D0-brane equations (3.34),

$$K^i := D_0 D_0 \hat{X}^i + \ldots = \mathrm{e}^{\hat{\Phi}} \hat{R}^{0i} + \widehat{D^i \Phi} + \mathcal{O}\ (\mathrm{fermi}^2), \tag{5.17}$$

where $\hat{X}^i := \hat{Z}^M E_M^{\underline{a}}(\hat{Z}) u_{\underline{a}}{}^i = \hat{X}^{\underline{a}} u_{\underline{a}}{}^i + \ldots$ Comparing Eq. (5.17) with Eq. (5.14) we see that the multiple D0-branes, as described by this equation, acquire interaction with higher form «electric» and «magnetic» fields $\hat{H}^{0ij} := H_{\underline{abc}}(\hat{Z}) u^{\underline{a}0} u^{\underline{b}i} u^{\underline{c}j}$, $H^{ijk} := H_{\underline{abc}}(\hat{Z}) u^{\underline{a}i} u^{\underline{b}j} u^{\underline{c}k}$, $\hat{R}^{0ijk} := R_{\underline{abcd}}(\hat{Z}) u^{\underline{a}0} u^{\underline{b}i} u^{\underline{c}j} u^{\underline{d}k}$. As one D0-brane does not interact with these backgrounds, one may say that the multiple D0-brane system is «polarized» by the external fluxes such that the interaction with higher brane gauge fields is induced, much in the same way as neutral dielectric is polarized and, due to this polarization, interacts with electric field. This is the famous «dielectric brane» effect first observed by Emparan [23] and then by Myers in his purely bosonic nonlinear action [24].

5.5. Possible Deformation of the Constraints and Superembedding Equations. The relation of our description of multiple D0-brane system with the dimensional reduction of $SU(N)$ SYM model suggests a possible existence of modifications of our $d = 1$ $\mathcal{N} = 16$ SYM constraints (5.3). What one can certainly state is that such a modification exists for the case of multiple D0-brane system in flat target superspace.

Indeed, according to [62,63] the most general deformation of the $D = 10$ SYM constraints by contributions of the fields of SYM supermultiplet at the order $(\alpha')^2$ reads [1]

$$\mathbb{F}_{\alpha\beta} = \beta(\sigma^{\underline{a}}\tilde{\Psi})_\alpha (\sigma^{\underline{b}}\tilde{\Psi})_\beta \mathbb{F}_{\underline{ab}}, \tag{5.18}$$

where β is a constant proportional to the second power of the Regge slop parameter, $\beta \propto (\alpha')^2$, and $\tilde{\Psi}$ is the basic superfield strength of the $D = 10$ SYM multiplet. This appeared in the equation $\mathbb{F}_{\alpha\underline{a}} = 2i(\sigma_{\underline{a}}\tilde{\Psi})_\alpha$, which follows from the standard SYM constraints $\mathbb{F}_{\alpha\beta} = 0$. Of course, when the dim 1 constraint becomes (5.18), the dim 3/2 equation also gets modified by $\propto \beta$ contributions, $\mathbb{F}_{\alpha\underline{a}} = 2i(\sigma_{\underline{a}}\tilde{\Psi})_\alpha + \mathcal{O}(\beta)$.

The dimensional reduction of the deformed SYM theory characterized by the constraints (5.18) implies the following constraints for the minimal field strength of the dimensional reduced $d = 1$ theory:

$$G_{\alpha\beta} = i\sigma^i_{\alpha\beta}\mathbb{X}^i - \beta\Psi_{(\alpha}(\sigma^{0i}\Psi)_{\beta)} D_0 \mathbb{X}^i + \frac{\beta}{4}(\sigma^{0i}\Psi)_\alpha (\sigma^{0j}\Psi)_\beta [\mathbb{X}^i, \mathbb{X}^j], \tag{5.19}$$

$$\Psi = \sigma^0 \tilde{\Psi}. \tag{5.20}$$

This can be used now as a constraint for $d = 1$, $\mathcal{N} = 16$ SYM model leaving on the worldline of a D0-brane moving in flat targets superspace (as such a superspace is flat). This, in its turn, implies the following modification of the superembedding-like equation (which

[1] The author thanks Linus Wulff for useful discussions on the SYM deformations.

can be obtained by dimensional reduction of the consequence $\mathbb{F}_{\alpha\underline{a}} = 2i(\sigma_{\underline{a}}\tilde{\Psi})_\alpha + \mathcal{O}(\beta)$ of the modified constraint (5.18)):

$$D_\alpha \mathbb{X}^i = 4i(\sigma^{0i}\Psi)_\alpha + 3i\beta\tilde{\sigma}^{i\beta\gamma} D_{(\alpha} \left(\Psi_\beta (\sigma^{0i}\Psi)_{\gamma)} D_0 \mathbb{X}^i + \frac{1}{4}(\sigma^{0i}\Psi)_\beta (\sigma^{0j}\Psi)_{\gamma)} [\mathbb{X}^i, \mathbb{X}^j] \right). \quad (5.21)$$

Even leaving aside the question of whether a counterpart of such a modification can be found for the case of D0-brane worldvolume moving in an arbitrary curved type IIA supergravity background, one sees that these constraints are too complex. It is very hard to deal with them, at least without the use of a computer programme (see [64] for an efficient use of computer programmes in superfield calculations).

Then, even if our formulation of superembedding approach to multiple D-brane system based on superembedding and superembedding-like equation as well as on the constraints (5.3) is approximate, it promises to be an efficient approximation to study such systems. Following [22], we have proved that such an approach exists and is consistent in the case of multiple D-particle (D0-brane) system. An important problem is to understand whether it can be extended to type IIB multiple D-strings (D1-branes), D-membrane (D2-brane) and higher Dp-brane systems.

6. CONCLUSION AND DISCUSSIONS

In this contribution we review superembedding approach to D-branes and M-branes [3, 5, 15] as well as its recent application [22] to searching for the covariant and supersymmetric equation for multiple D-brane systems.

We begin by general review of the superembedding approach to Dp-brane, which happens to be simpler because, at least on the level of details of the present contribution, it does not require introducing the spinor moving frame variables (see [57] where one can see the stage on which the introduction of these variables is hardly possible without breaking the Lorentz invariance). Then, we review superembedding approach to M2- and M5-branes, where the spinor moving frame variables do play essential role. In our review of superembedding description of D- and M-branes we put an emphasis first on the universality of the superembedding equation which, for the most interesting cases of M2-, M5- and Dp-branes with $p < 6$, specifies completely not only the worldvolume superspace geometry but also the dynamics of the brane. We also stressed the usefulness of introducing the worldvolume superspace gauge forms corresponding to the worldvolume gauge fields and studying the Bianchi identities for their constrained field strength. This is inevitable for Dp-branes with $p > 5$, but also very convenient for the branes the dynamics of which is completely specified by the superembedding equation. The superfield description of the worldvolume gauge fields for a single D-brane (and chiral two-form gauge field of M5-brane) suggests trying to describe a multiple Dp-brane system by putting an additional non-Abelian $SU(N)$ gauge supermultiplet, described by a set of worldvolume superspace constraints, on the worldvolume superspace of a single Dp-brane.

In Sec. 5 we, following [22], apply superembedding approach to search for the multiple D0-brane equations on this line. We show that for the case of arbitrary (on-shell) type II supergravity background the dynamical equations obtained from the superembedding approach describe the coupling of multiple D0-branes to the higher NS-NS and RR fluxes (H^{0ij}, H^{ijk} and R^{0ijk}). Thus, our equations of motion show the «polarization» of multiple D0-brane system which generates charges characteristic for higher D-brane. This is the content of the so-called «dielectric brane effect» [23, 24] characteristic for the (purely bosonic) Myers action [24]. Further study of these equations and of possible restrictions which they might put on the pull-back of background fluxes to the worldline is an interesting problem for future study.

In the case of flat tangent superspace, when the background fluxes vanish, the $d = 1\,\mathcal{N} = 16$ worldvolume superspace of D0-brane is flat and the dynamical equations for the relative motion of D0-brane «constituents», which follows from the superembedding approach, are those of the $D = 10\,SU(N)$ SYM dimensionally reduced down to $d = 1$. They, thus, essentially coincide with what had been used for the very low energy description of multiple D0-brane system [33] and with the Matrix model [34].

The purely bosonic limit of our equations is clearly simpler than the equations following from the Myers action [24]. It is tempting to propose that these simpler but covariant and manifestly supersymmetric equations, together with the single D0-brane equation describing the center-of-mass motion, actually give the «complete» description of the multiple D0-brane system [22]. Furthermore, as we have already stressed, these give the completely supersymmetric and Lorentz invariant description of the «dielectric brane effect». The advantage of this description is that it is supersymmetric and also Lorentz invariant, while the Myers proposal [24] does possess neither of these symmetries expected for a system of multiple Dp-branes [1].

However, the existence of the deformation of our equations for the case of multiple D0 in flat target type IIB superspace, which follows from the existence of the deformation of the 10D SYM equations in flat $D = 10\,\mathcal{N} = 1$ superspace, suggests allowing the possible existence of deformation of our equations. However, one sees that the deformed multiple D0 equations in flat target type IIB superspace, the explicit form of which is presently available, are very complicated and their use looks inefficient (at least without the using of computer programmes).

Then, even if approximate, our superembedding description based on superembedding and superembedding-like equation plus simplest gauge field constraints, might provide useful approximation of nearly-coincident multiple branes, which goes beyond the $U(N)$ SYM description as far as the fields related to the center-of-mass motion are allowed to be strong.

As we have mentioned in the text, a very interesting boundary fermion approach to the description of multiple Dp-branes was developed by Howe, Lindström and Wulff in [25, 26]. Presently the top-line result of this approach is the supersymmetric action possessing the kappa symmetry on the classical (or «minus one quantization») level, i.e., before quantizing boundary fermions [26]. However, the parameter of this κ symmetry depends on the boundary fermions which implies, as noticed already in [26], that quantization of boundary fermions

[1] Let us recall that the Myers action was (and is) motivated by that it is derived from T-duality. The starting point for the corresponding chain of duality transformations is the purely bosonic $D = 10$ non-Abelian Born–Infeld action based on the symmetric trace prescription [37] for the ordering of the SYM field strength operators. Notice, however, that supersymmetric generalization of this 10D symmetric trace BI action is not known.

should result in an action possessing a non-Abelian κ symmetry. The previous attempts to construct the models with non-Abelian κ symmetry gave negative results [65]. Actually, this requirement of non-Abelian κ symmetry comes from the fact that all the coordinate functions in the approach of [25, 26] depend on the boundary fermions so that, after quantization, all the coordinate functions become matrices and, to remove the extra unwanted $(p+1)$ bosonic and 16 fermionic components, one needs to have the reparametrization and κ symmetry with matrix parameters.

The problem with non-Abelian κ symmetry appears at $(\alpha')^4$ order [65]. Probably, the further development of the boundary fermion approach will help to resolve it. However, even if it were confirmed that the non-Abelian κ-symmetric DBI action is impossible to construct using the natural multibrane degrees of freedom, this would not imply that the approach of [26] is incorrect. It certainly provides a complete *classical* description of string theory with D-branes (or, better, «pre-classical», see below). However, the consequent quantization of such a model implies simultaneous quantization of both boundary fermions and coordinate functions. This would result in an appearance of not only the Dp-brane worldvolume fields, but also of the bulk supergravity fields and massive string state. A search for a Myers-like non-Abelian DBI-like action in this perspective is reformulated as a search for a way to quantize *only* the boundary fermions leaving the classical description of the branes by coordinate functions untouched. Even if it happened that such a description is impossible to realize in its complete form, this could not be treated as incorrectness of the boundary fermion approach [26], which gives a complete description of string theory Dp-branes, but on the «minus one quantized» level (considering the standard description of single Dp-brane to be classical).

In our more traditional, but probably approximate, superembedding approach description of multiple brane systems only the coordinate (super)fields corresponding to the center-of-mass motion are transformed by the target space Lorentz group transformations and 32-component target space supersymmetry. The relative motion of constituent branes is described by the $SU(N)$ SYM multiplet, involving in addition to $d=(p+1)$ dimensional gauge potentials, only $(9-p)$ $su(N)$-valued matrix scalars $\mathbb{X}^i$, the Grassmann derivative of which is expressed through the 16 fermionic $su(N)$-valued matrix spinors Ψ_α. The leading components of the superfields $\mathbb{X}^i$ and Ψ_α correspond to a *non-Abelian generalization of the static gauge coordinate functions*, so that neither non-Abelian reparametrization invariance nor non-Abelian κ symmetry is needed to reach the balance of degrees of freedom characteristic for a supersymmetric theory.

To conclude, the existence of supersymmetric deformations of the SYM constraints in flat target superspace suggests that our choice of basic equations, including the superembedding equation and the constraints on the worldvolume $SU(N)$ SYM field strength, might be not unique also for the case of curved worldvolume superspace of a D-brane moving in a nontrivial supergravity background. However, we hope that even in this case, an approximate description given by our superembedding approach, corresponding to a low energy of relative motion and of the non-Abelian gauge field corresponding to it, but unrestricted (in the framework of DBI approximation) nonlinear description of the $U(1)$ gauge fields and coordinate functions corresponding to the center-of-mass motion, can be useful in future development of the fields.

Such a description in the framework of superembedding approach has been shown to be allowed for $p=0$ case, i.e., for the multiple Dp-brane systems. An important problem is to check whether such a description is possible for higher branes. It is natural to begin with the simplest cases of multiple type IIB D1- and type IIA D2-brane systems. If the answer for the second case happens to be affirmative, one can also search for similar superembedding description for the nearly coincident M2-branes which, if exists, should be related with the Bagger–Lambert–Gustavsson [69] and Aharony–Bergman–Jafferis–Maldacena [70] models.

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Appendix A

CONVENIENT REPRESENTATIONS FOR 11D DIRAC MATRICES

A1. $SO(1,2)\otimes SO(8)$ **Invariant Representation for** $D=11$ **Dirac Matrices.** In the superembedding description for M2-brane we use the following $SO(1,2)\otimes SO(8)$ invariant representations for the eleven-dimensional gamma matrices and charge conjugation matrix:

$$(\Gamma^{\underline{a}})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{a},\Gamma^{i}\right), \quad a=0,1,2, \quad i=1,\ldots,8,$$
$$(\Gamma^{a})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{\underline{0}},\Gamma^{\underline{9}},\Gamma^{\underline{10}}\right) \equiv \left(\Gamma^{0},\Gamma^{1},\Gamma^{2}\right) = \begin{pmatrix} \gamma^{a\beta}_{\alpha}\delta_{qp} & 0 \\ 0 & -\gamma^{a\,\alpha}_{\beta}\delta_{\dot q\dot p}, \end{pmatrix}$$
$$(\Gamma^{i})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{1},\ldots,\Gamma^{8}\right) = \begin{pmatrix} 0 & -i\epsilon_{\alpha\beta}\gamma^{i}_{q\dot p} \\ -i\epsilon^{\alpha\beta}\tilde\gamma^{i}_{\dot q p} & \end{pmatrix}, \tag{A.1}$$
$$C^{\underline{\alpha\beta}} = -C^{\underline{\beta\alpha}} = \mathrm{diag}\left(i\epsilon^{\alpha\beta}\delta_{qp}, i\epsilon_{\alpha\beta}\delta_{\dot q\dot p}\right),$$
$$C_{\underline{\alpha\beta}} = \mathrm{diag}\left(-i\epsilon_{\alpha\beta}\delta_{qp}, -i\epsilon^{\alpha\beta}\delta_{\dot q\dot p}\right).$$

Here $\gamma^{a}{}_{\alpha}{}^{\beta}$ and $\gamma^{i}_{q\dot q}$ are the $SO(1,2)$ Dirac matrices and $SO(8)$ Pauli matrices (Klebsh–Gordan coefficients), respectively. Some of their useful properties are

$$\gamma^{a}_{\alpha\beta} := -i\gamma^{a}{}_{\alpha}{}^{\gamma}\epsilon_{\gamma\beta} = \gamma^{a}_{\beta\alpha} = \gamma^{a}_{(\alpha\beta)}, \quad \tilde\gamma^{\alpha\beta}_{a} := i\epsilon^{\alpha\gamma}\gamma^{a}{}_{\gamma}{}^{\beta} = \tilde\gamma^{(\alpha\beta)}_{a}, \quad \epsilon^{\alpha\gamma}\epsilon_{\gamma\beta} = \delta^{\alpha}_{\beta}, \tag{A.2}$$
$$\gamma^{ab} = -i\epsilon^{abc}\gamma_{c}, \quad \gamma^{a}_{\alpha\beta}\tilde\gamma^{\gamma\delta}_{a} = 2\delta_{(\alpha}{}^{\gamma}\delta_{\beta)}{}^{\delta},$$
$$\tilde\gamma^{i}_{\dot p q} := \gamma^{i}_{q\dot p}, \quad \gamma^{i}_{q\dot p}\gamma^{j}_{q\dot p} + \gamma^{j}_{q\dot p}\gamma^{i}_{q\dot p} = 2\delta^{ij}\delta_{qp}, \quad \gamma^{i}_{p\dot q}\gamma^{j}_{p\dot p} + \gamma^{j}_{p\dot q}\gamma^{i}_{p\dot p} = 2\delta^{ij}\delta_{\dot q\dot p}, \tag{A.3}$$
$$\gamma^{i}_{q\dot q}\gamma^{i}_{p\dot p} = \delta_{qp}\delta_{\dot q\dot p} + \frac{1}{4}\gamma^{ij}_{qp}\tilde\gamma^{ij}_{\dot q\dot p} \Rightarrow \gamma^{i}_{(q|\dot q}\gamma^{i}_{|p)\dot p} = \delta_{qp}\delta_{\dot q\dot p} = \gamma^{i}_{q(\dot q|}\gamma^{i}_{p|\dot p)}.$$

Notice that both 11D and $3d$ Dirac matrices are imaginary in our mostly minus signature conventions,

$$\eta^{\underline{ab}} = \mathrm{diag}(+,-,\ldots,-), \quad \eta^{ab} = \mathrm{diag}\,(+,-,-). \tag{A.4}$$

Now we are ready to specify relations (4.5) and (4.6) for the spinor moving frame variables adapted to the (super)embedding of the M2-brane worldvolume (superspace):

$$\delta_{qp}\tilde\gamma^{\alpha\beta}_{a}u_{\underline{a}}{}^{a} = v^{\alpha q}\tilde\Gamma_{\underline{a}}v^{\beta p} = -v^{\alpha q}\Gamma_{\underline{a}}v^{\beta p}, \tag{A.5}$$
$$\delta_{\dot q\dot p}\gamma_{a\alpha\beta}u^{a}_{\underline{a}} = v_{\alpha\dot q}\tilde\Gamma_{\underline{a}}v_{\beta\dot p} = -v_{\alpha\dot q}\Gamma_{\underline{a}}v_{\beta\dot p}, \tag{A.6}$$
$$\delta^{\alpha}_{\beta}\gamma^{i}_{q\dot p}u^{i}_{\underline{a}} = -v^{\alpha q}\tilde\Gamma_{\underline{a}}v_{\beta\dot p} = v^{\alpha q}\Gamma_{\underline{a}}v_{\beta\dot p}, \tag{A.7}$$

and

$$u_{\underline{b}}^{a}\Gamma^{\underline{b}}_{\underline{\alpha}\underline{\beta}} = v_{\underline{\alpha}}^{\alpha q}(\gamma_a)_{\alpha\beta}v_{\underline{\beta}}^{\beta q} + v_{\underline{\alpha}\alpha\dot{q}}(\gamma_a)_{\alpha\beta}v_{\underline{\beta}\beta\dot{q}}, \quad (A.8)$$

$$u_{\underline{b}}^{i}\Gamma^{\underline{b}}_{\underline{\alpha}\underline{\beta}} = -2v_{(\underline{\alpha}|}{}^{\alpha q}\gamma^{i}_{q\dot{q}}v_{|\underline{\beta})\alpha\dot{q}}. \quad (A.9)$$

An equivalent form for the set of relations (A.10), (A.11) is

$$u_{\underline{b}}^{a}\tilde{\Gamma}^{\underline{b}\underline{\alpha}\underline{\kappa}} = -v_{\alpha q}{}^{\underline{\alpha}}\tilde{\gamma}^{a\alpha\beta}v_{\beta q}{}^{\underline{\kappa}} - v^{\alpha\dot{q}\underline{\alpha}}\gamma^{a}_{\alpha\beta}v^{\beta\dot{q}\underline{\kappa}}, \quad (A.10)$$

$$u_{\underline{b}}{}^{i}\tilde{\Gamma}^{\underline{b}\,\underline{\alpha}\,\underline{\beta}} = 2v^{\alpha q(\underline{\alpha}|}\gamma^{i}_{q\dot{q}}v_{\alpha\dot{q}}{}^{|\underline{\beta})}. \quad (A.11)$$

Another useful equation is the following «unity decomposition»:

$$\delta_{\underline{\beta}}{}^{\underline{\alpha}} = i(v_{\underline{\beta}}{}^{\alpha q}v_{\alpha q}{}^{\underline{\alpha}} + v_{\underline{\beta}\alpha\dot{q}}v^{\underline{\alpha}\alpha\dot{q}}). \quad (A.12)$$

The difference of the contractions of the same rank 16 blocks, $v_{\underline{\beta}}{}^{\alpha q}v_{\alpha q}{}^{\underline{\alpha}} - v_{\underline{\beta}\alpha\dot{q}}v^{\underline{\alpha}\alpha\dot{q}}$, defines the matrix

$$\bar{\Gamma}^{\underline{\alpha}}{}_{\underline{\beta}} := \frac{i}{3!}\varepsilon_{abc}(\Gamma u^{a}u^{b}u^{c})^{\alpha}{}_{\beta} = \frac{i}{3!}\varepsilon_{abc}(v_{\alpha q}{}^{\underline{\alpha}}\tilde{\gamma}^{abc\,\alpha\beta}v_{\beta q}{}^{\underline{\gamma}} + v^{\alpha\dot{q}\underline{\alpha}}\gamma^{abc}_{\alpha\beta}v^{\beta\dot{q}\underline{\gamma}})C_{\underline{\gamma}\underline{\beta}} = = -i(v_{\underline{\beta}}{}^{\alpha q}v_{\alpha q}{}^{\underline{\alpha}} - v_{\underline{\beta}\alpha\dot{q}}v^{\underline{\alpha}\alpha\dot{q}}) \quad (A.13)$$

entering the κ-symmetry projector of the standard formulation of M2-brane [1].

A2. $SO(1,5)\otimes SO(5)$ **Invariant Representation for** $D = 11$ **Dirac Matrices.** In the superembedding description of M5-brane we use the following $SO(1,5)\otimes SO(5)$ invariant representations for the eleven-dimensional gamma matrices and charge conjugation matrix:

$$\begin{gathered}(\Gamma^{\underline{a}})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{a},\Gamma^{i}\right), \quad a=0,1,\ldots,5, \quad i=1,\ldots,5,\\ (\Gamma^{a})_{\underline{\alpha}}{}^{\underline{\beta}} = \begin{pmatrix} 0 & -i\gamma^{a}_{\alpha\beta}\delta_{q}{}^{p} \\ +i\tilde{\gamma}^{a\alpha\beta}\delta_{q}{}^{p} & \end{pmatrix},\\ (\Gamma^{i})_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\Gamma^{1},\ldots,\Gamma^{8}\right) = \begin{pmatrix} (\gamma^{i}C)_{q}{}^{p}\delta_{\alpha}{}^{\beta} & 0 \\ 0 & -(\gamma^{i}C)_{q}{}^{p}\delta^{\alpha}{}_{\beta}\end{pmatrix},\\ C^{\underline{\alpha}\underline{\beta}} = -C^{\underline{\beta}\underline{\alpha}} = \begin{pmatrix} 0 & -i\delta^{\alpha}{}_{\beta}C^{qp} \\ -i\delta_{\alpha}{}^{\beta}C^{qp} & 0\end{pmatrix},\\ C_{\underline{\alpha}\underline{\beta}} = \begin{pmatrix} 0 & i\delta_{\alpha}{}^{\beta}C^{qp} \\ i\delta^{\alpha}{}_{\beta}C^{qp} & 0\end{pmatrix}.\end{gathered} \quad (A.14)$$

The $SO(1,5)$ generalized Pauli matrices ($SU^{*}(4)$ Klebsh–Gordan coefficients) are antisymmetric $\gamma^{a}_{\alpha\beta} = -\gamma^{a}_{\beta\alpha} = \gamma^{a}_{[\alpha\beta]}$, $\tilde{\gamma}^{\alpha\beta}_{a} = -\tilde{\gamma}^{\beta\alpha}_{a} = \tilde{\gamma}^{[\alpha\beta]}_{a}$ and possess the following properties:

$$\begin{gathered}(\gamma^{(a}\tilde{\gamma}^{b)})_{\alpha}{}^{\beta} = \eta^{ab}\delta_{\alpha}{}^{\beta}, \quad \eta^{ab} = \mathrm{diag}\,(+,-,-,-,-,-), \quad \tilde{\gamma}^{\alpha\beta}_{a} = \frac{1}{2}\epsilon^{\alpha\beta\gamma\delta}\gamma_{a\gamma\delta},\\ \gamma^{a}_{\alpha\beta}\tilde{\gamma}^{\gamma\delta}_{a} = -4\delta_{[\alpha}{}^{\gamma}\delta_{\beta]}{}^{\delta}, \quad \gamma^{a}_{\alpha\beta}\gamma_{a\gamma\delta} = -2\epsilon_{\alpha\beta\gamma\delta}, \quad \gamma^{abcdef}{}_{\alpha}{}^{\beta} = \epsilon^{abcdef}\delta_{\alpha}{}^{\beta},\\ \gamma^{abc}{}_{\alpha\beta} = \gamma^{abc}{}_{(\alpha\beta)} = -\frac{1}{3!}\epsilon^{abcdef}\gamma_{def\alpha\beta}, \quad \tilde{\gamma}^{abc\alpha\beta} = \tilde{\gamma}^{abc(\alpha\beta)} = \frac{1}{3!}\epsilon^{abcdef}\gamma^{\alpha\beta}_{def}.\end{gathered} \quad (A.15)$$

They are pseudoreal in the sense that the conjugate matrices $\gamma^{a*}_{\dot\alpha\dot\beta} := (\gamma^a_{\alpha\beta})^*$ are expressed through $\gamma^a_{\alpha\beta}$ with the use of matrix $B_\alpha{}^{\dot\alpha}$ [66] obeying $BB^* = -I$,

$$\begin{gathered}(B\gamma^{a*}B^T) := B_\alpha{}^{\dot\alpha}\gamma^{a*}_{\dot\alpha\dot\beta}B_\beta{}^{\dot\beta} = \gamma^a_{\alpha\beta},\\ (B^{*T}\tilde\gamma^{a*}B^*) := B^*{}_{\dot\alpha}{}^{\alpha}\tilde\gamma^{a*\dot\alpha\dot\beta}B_\beta{}^{\dot\beta} = \gamma^a_{\alpha\beta},\\ B_\alpha{}^{\dot\beta}B^*{}_{\dot\beta}{}^{\beta} = -\delta_\alpha{}^\beta.\end{gathered}\tag{A.16}$$

The properties of the $SO(5)$ Klebsh–Gordan coefficients (generalized Pauli matrices) and of the charge conjugation matrix are

$$\gamma^i_{qp} = -\gamma^i_{pq} = -(\tilde\gamma^{i\,qp})^* = \frac{1}{2}\epsilon_{qprs}\tilde\gamma^{i\,rs}, \quad i = 1,\ldots,5, \quad q,p,r,s = 1,\ldots,4, \tag{A.17}$$

$$\boxed{\gamma^i\tilde\gamma^j + \gamma^j\tilde\gamma^i = 2\delta^{ij}\delta_q{}^p}, \quad \gamma^i\tilde\gamma^j - \gamma^j\tilde\gamma^i =: 2\gamma^{ij}{}_q{}^p,$$

$$C_{qp} = -C_{pq} = -(C^{qp})^* = \frac{1}{2}\epsilon_{qprs}C^{rs}, \quad C_{qr}C^{rp} = \delta_q{}^p, \tag{A.18}$$

$$C\tilde\gamma^iC = -\gamma^i, \quad C\gamma^iC = -\tilde\gamma^i,$$

$$\gamma^i_{qp}\tilde\gamma^{irs} = -4\delta_{[q}{}^r\delta_{p]}{}^s - C_{qp}C^{rs}, \quad \gamma^i_{qp}\gamma^i_{rs} = -2\epsilon_{qprs} - C_{qp}C_{rs}. \tag{A.19}$$

These properties can be deduced from the properties of $SU(4)$ Klebsh–Gordan coefficients γ^I_{ji}, $\tilde\gamma^{I\,ij}$, $I = 1,\ldots,6$,

$$\gamma^I_{qp} = -\gamma^I_{pq} = -(\tilde\gamma^{Iqp})^* = \frac{1}{2}\epsilon_{pqrs}\tilde\gamma^{I\,rs}, \quad I = 1,\ldots,6, \quad p,q,r,s = 1,\ldots,4, \tag{A.20}$$

$$\boxed{\gamma^I\tilde\gamma^J + \gamma^J\tilde\gamma^I = 2\delta^{IJ}\delta_q{}^p}, \quad \gamma^I\tilde\gamma^J - \gamma^J\tilde\gamma^I =: 2\gamma^{IJ}{}_q{}^p, \tag{A.21}$$

$$\gamma^I_{qp}\tilde\gamma^{Irs} = -4\delta_{[q}{}^r\delta_{p]}{}^s, \quad \gamma^I_{qp}\,\gamma^I_{rs} = -2\epsilon_{qprs}, \tag{A.22}$$

after identification $\gamma^I_{qp} = (\gamma^i_{qp}, C_{qp})$, $i = 1,\ldots,5$.

Relations (4.5) and (4.6) for the spinor moving frame variables adapted to the (super)embedding of the M5-brane worldvolume superspace are given by

$$v^{\alpha q}\tilde\Gamma_{\underline{a}}v^{\beta p} = u_{\underline{a}}{}^b\tilde\gamma_b{}^{\alpha\beta}C^{qp}, \qquad v_{\alpha q}\Gamma_{\underline{a}}v_{\beta p} = u_{\underline{a}}{}^b\gamma_{b\alpha\beta}C_{qp}, \tag{A.23}$$

$$v_\alpha{}^q\tilde\Gamma_{\underline{a}}v_\beta{}^p = -u_{\underline{a}}{}^b\gamma_{b\alpha\beta}C_{qp}, \quad v_q{}^\alpha\Gamma_{\underline{a}}v_p{}^\beta = -u_{\underline{a}}{}^b\tilde\gamma_b{}^{\alpha\beta}C_{qp}, \tag{A.24}$$

$$v^{\alpha q}\tilde\Gamma_{\underline{a}}v^p_\beta = iu_{\underline{a}}{}^i\tilde\gamma^{iqp}\delta^\alpha{}_\beta, \qquad v_{\alpha q}\Gamma_{\underline{a}}v^\beta_p = -iu_{\underline{a}}{}^i\gamma^i{}_{qp}\delta_\alpha{}^\beta, \tag{A.25}$$

and

$$u^a_{\underline{b}}\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = v_{\underline{\alpha}}{}^{\alpha q}(\gamma_a)_{\alpha\beta}v_{\underline{\beta}}^{\beta p}C_{qp} - v_{\underline{\alpha}\alpha}{}^q\tilde\gamma^{\alpha\beta}_a v_{\underline{\beta}\beta}{}^pC_{qp}, \tag{A.26}$$

$$u_{\underline{b}}{}^i\Gamma^{\underline{b}}_{\underline{\alpha\beta}} = 2iv_{(\underline{\alpha}|}{}^{\alpha q}\gamma^i_{qp}v_{|\underline{\beta})\alpha}{}^p, \tag{A.27}$$

as well as

$$u^a_{\underline{b}}\tilde\Gamma^{\underline{b}\underline{\alpha}\underline{\beta}} = v_{\alpha q}{}^{\underline{\alpha}}\tilde\gamma^{a\alpha\beta}v_{\beta p}{}^{\underline{\beta}}C^{qp} - v_q{}^{\alpha\underline{\alpha}}\gamma^a_{\alpha\beta}v_p^{\beta\underline{\beta}}C^{qp}, \tag{A.28}$$

$$u^i_{\underline{b}}\tilde\Gamma^{\underline{b}\,\underline{\alpha}\,\underline{\beta}} = -2iv^{(\underline{\alpha}|}_{\alpha q}\tilde\gamma^{i\,qp}v_p{}^{\alpha|\underline{\beta})}. \tag{A.29}$$

The «unity decomposition» reads simply as

$$\delta_{\underline{\beta}}{}^{\underline{\alpha}} = v_{\underline{\beta}}{}^{\alpha q} v_{\alpha q}{}^{\underline{\alpha}} + v_{\underline{\beta}\alpha}{}^{q} v_q^{\alpha\underline{\alpha}}, \tag{A.30}$$

but the components of the inverse spinor moving frame matrix $v_{\alpha q}{}^{\underline{\alpha}}$ and $v_q^{\alpha\underline{\alpha}}$ are expressed through $v_{\underline{\beta}}{}^{\alpha q}$ and $v_{\underline{\beta}\alpha}{}^{q}$ by (4.12).

The derivatives of spinor moving frame variables are expressed through the generalized Cartan forms by

$$Dv_{\underline{\alpha}}{}^{\alpha q} = \frac{i}{2}\Omega^{ai} v_{\underline{\alpha}\beta}{}^{p} \tilde{\gamma}_a^{\beta\alpha} (\gamma^i C)_p{}^q, \qquad Dv_{\alpha q}{}^{\underline{\alpha}} = \frac{i}{2}\Omega^{ai} \gamma_{a\alpha\beta} (\gamma^i C)_q{}^p v_p^{\beta\underline{\alpha}}, \tag{A.31}$$

$$Dv_{\underline{\alpha}\alpha}{}^{q} = -\frac{i}{2}\Omega^{ai} v_{\underline{\alpha}}^{\beta p} \tilde{\gamma}_{a\beta\alpha} (\gamma^i C)_p{}^q, \qquad Dv_q{}^{\alpha\underline{\alpha}} = -\frac{i}{2}\Omega^{ai} \tilde{\gamma}_a^{\alpha\beta} (\gamma^i C)_q{}^p v_{\beta p}{}^{\underline{\alpha}}. \tag{A.32}$$

Appendix B

SOME DETAILS ON TYPE IIA SUPERGRAVITY SUPERSPACE

The type IIA superspace geometry was worked out in [47]. Here we present some equations in our present notation.

Fermionic torsion of general type IIA supergravity superspaces reads

$$\begin{aligned} T^{\alpha 1} &= \frac{1}{2} E^{\underline{b}} \wedge E^{\underline{a}} T_{\underline{ab}}{}^{\alpha 1} + E^{\underline{a}} \wedge \mathcal{E}^{\underline{\beta}} T_{\underline{\beta}\underline{a}}{}^{\alpha 1} - 2iE^{\alpha 1} \wedge E^{\beta 1}\Lambda_{\beta 1} + iE^1 \sigma^{\underline{a}} \wedge E^1 \tilde{\sigma}_{\underline{a}}^{\alpha\beta} \Lambda_{\beta 1}, \\ T_\alpha^2 &= \frac{1}{2} E^{\underline{b}} \wedge E^{\underline{a}} T_{\underline{ab}\alpha}{}^{2} + E^{\underline{a}} \wedge \mathcal{E}^{\underline{\beta}} T_{\underline{\beta}\underline{a}\alpha}{}^{2} - 2iE_\alpha^2 \wedge E_\beta^2 \Lambda_2^\beta + iE^2 \tilde{\sigma}_{\underline{a}} \wedge E^2 \sigma_{\alpha\beta}^{\underline{a}} \Lambda_2^\beta, \end{aligned} \tag{B.1}$$

where

$$\mathcal{E}^{\underline{\beta}} = (E^{\beta 1}, E_\beta{}^2), \quad \Lambda_{\beta 1} = \frac{i}{2} D_{\beta 1}\Phi, \quad \Lambda^\beta{}_2 = \frac{i}{2} D^\beta{}_2 \Phi, \tag{B.2}$$

and

$$T_{\beta 1\underline{a}}{}^{\gamma 1} = -\frac{1}{8} H_{\underline{abc}} (\sigma^{\underline{bc}})_\beta{}^\gamma = T^\gamma_{2\underline{a}\beta 1}, \tag{B.3}$$

$$\begin{aligned} T_{\beta 1\underline{a}\gamma}{}^{2} = \frac{e^\Phi}{8\cdot 2!} R_{\underline{bc}} (\sigma_{\underline{a}} \tilde{\sigma}^{\underline{bc}})_{\beta\gamma} - \frac{e^\Phi}{8\cdot 4!} R_{\underline{bcde}} (\sigma_{\underline{a}} \tilde{\sigma}^{\underline{bcde}})_{\beta\gamma} + \\ + \frac{i}{8} \Lambda_2 \sigma_{\underline{bc}} \Lambda_1 (\sigma_{\underline{a}} \tilde{\sigma}^{\underline{bc}})_{\beta\gamma} - \frac{3i}{16\cdot 4!} \Lambda_2 \sigma_{\underline{bcde}} \Lambda_1 (\sigma_{\underline{a}} \tilde{\sigma}^{\underline{bcde}})_{\beta\gamma}, \end{aligned} \tag{B.4}$$

$$\begin{aligned} T_{2\,\underline{a}}^{\beta}{}^{\gamma 1} = \frac{e^\Phi}{8\cdot 2!} R_{\underline{bc}} (\tilde{\sigma}_{\underline{a}} \sigma^{\underline{bc}})^{\beta\gamma} + \frac{e^\Phi}{8\cdot 4!} R_{\underline{bcde}} (\tilde{\sigma}_{\underline{a}} \sigma^{\underline{bcde}})^{\beta\gamma} + \\ + \frac{i}{8} \Lambda_2 \sigma_{\underline{bc}} \Lambda_1 (\tilde{\sigma}_{\underline{a}} \sigma^{\underline{bc}})^{\beta\gamma} + \frac{3i}{16\cdot 4!} \Lambda_2 \sigma_{\underline{bcde}} \Lambda_1 (\tilde{\sigma}_{\underline{a}} \sigma^{\underline{bcde}})^{\beta\gamma}. \end{aligned} \tag{B.5}$$

The Riemann curvature two-form of the type IIA superspace is expressed through the above dim 1 torsion components and through the dim 3/2 ones by the solution of the Bianchi identities,

$$\mathbb{R}^{\underline{ab}} := d\omega^{\underline{ab}} - \omega^{\underline{ac}} \wedge \omega_{\underline{c}}{}^{\underline{b}} = 2iE^{\alpha 1} \wedge E^{\beta 1} \sigma^{[\underline{a}}{}_{\gamma(\alpha} T_{\beta)1}{}^{\underline{b}]\gamma 1} + 2iE^2_\alpha \wedge E^2_\beta \tilde{\sigma}^{[\underline{a}\gamma(\alpha} T_2^{\beta)\underline{b}]}{}^2_\gamma +$$
$$+ 4iE^{\alpha 1} \wedge E^2_\beta \left(\sigma^{[\underline{a}}{}_{\gamma\alpha} T_2^{\beta\underline{b}]\gamma 1} + \tilde{\sigma}^{[\underline{a}|\gamma\beta} T_{\alpha 1}{}^{|\underline{b}]}{}^2_\gamma \right) +$$
$$+ E^{\underline{c}} \wedge E^{\alpha 1} \left(2iT_{\underline{c}}{}^{[\underline{a}|\beta 1} \sigma^{|\underline{b}]}{}_{\beta\alpha} - iT^{\underline{ab}\beta 1} \sigma_{\underline{c}\beta\alpha} \right) +$$
$$+ E^{\underline{c}} \wedge E^2_\alpha \left(2iT_{\underline{c}}{}^{[\underline{a}|}{}^2_\beta \tilde{\sigma}^{|\underline{b}]\beta\alpha} - iT^{\underline{ab}}_\beta{}^2 \sigma_{\underline{c}\beta\alpha} \right) + \frac{1}{2} E^{\underline{d}} \wedge E^{\underline{c}} \mathbb{R}_{\underline{cd}}{}^{\underline{ab}}. \quad \text{(B.6)}$$

The following equations are also useful:

$$D_{\alpha 1}\Lambda_{\beta 1} := \frac{i}{2} D_{\alpha 1} D_{\beta 1}\Phi = -\frac{1}{2}\sigma^{\underline{a}}_{\alpha\beta} D_{\underline{a}}\Phi + \frac{1}{4!}\left(H_{\underline{abc}} - \frac{i}{2}\Lambda_1 \tilde{\sigma}_{\underline{abc}}\Lambda_1 \right) \sigma^{\underline{abc}}_{\alpha\beta}, \quad \text{(B.7)}$$

$$D_2^\alpha \Lambda_2^\beta := \frac{i}{2} D_2^\alpha D_2^\beta \Phi = -\frac{1}{2}\tilde{\sigma}^{\underline{a}\alpha\beta} D_{\underline{a}}\Phi + \frac{1}{4!}\left(H_{\underline{abc}} + \frac{i}{2}\Lambda_2 \sigma_{\underline{abc}}\Lambda_2 \right) \tilde{\sigma}^{\underline{abc}\,\alpha\beta}, \quad \text{(B.8)}$$

$$D_2^\beta \Lambda_{\alpha 1} := \frac{i}{2} D_2^\beta D_{\alpha 1}\Phi = (t - i\Lambda_2\Lambda_1)\delta_\alpha{}^\beta + \frac{3}{16}\left(e^\Phi R_{\underline{ab}} + 2i\Lambda_2\sigma_{\underline{ab}}\Lambda_1 \right) \tilde{\sigma}^{\underline{ab}}{}_\alpha{}^\beta +$$
$$+ \frac{1}{8\cdot 4!}\left(e^\Phi R_{\underline{abcd}} + \frac{3i}{2}\Lambda_2 \sigma_{\underline{abcd}}\Lambda_1 \right) \tilde{\sigma}^{\underline{abcd}}{}_\alpha{}^\beta = -D_{\alpha 1}\Lambda_2^\beta := -\frac{i}{2} D_{\alpha 1} D_2^\beta \Phi. \quad \text{(B.9)}$$

Appendix C

STRUCTURE OF FERMIONIC EQUATIONS FOR MULTIPLE D0-BRANE SYSTEM IN THE PRESENCE OF FLUXES

The fermionic equations of motion which follow from our superembedding description of multiple D0-brane system in general type IIA supergravity background have the structure of [22]

$$\frac{7}{8}\left(D_0\Psi - \frac{1}{4}[\mathbb{X}^i, (\sigma^{0i}\Psi)] \right) = (e^{\hat{\Phi}}\hat{R}^{0i} + \widehat{D^i\Phi})(\sigma^{0i}\Psi) + \frac{1}{64}\hat{H}^{0ij}\sigma^{0k}\sigma^{ij}\sigma^{0k}\Psi -$$
$$- \frac{1}{64}\sigma^{0k}\left(-\frac{1}{2!}e^{\hat{\Phi}}\hat{R}_{\underline{bc}}\sigma^{\underline{bc}} - \frac{1}{4!}e^{\hat{\Phi}}\hat{R}_{\underline{bcde}}\sigma^{\underline{bcde}} \right)\sigma^{0k}\Psi + D_0\mathbb{X}^i\left(a_1\sigma^{0i}\hat{\Lambda}_1 + a_2\sigma^i\hat{\Lambda}_2 \right) +$$
$$+ \frac{1}{16}[\mathbb{X}^i, \mathbb{X}^j]\left(b_1\sigma^{ij}\hat{\Lambda}_1 - b_2\sigma^{0ij}\hat{\Lambda}_2 \right) + \mathcal{O}(\hat{\Lambda}_{1,2}\cdot\hat{\Lambda}_{1,2}\cdot\Psi) \quad \text{(C.1)}$$

with some constants $a_{1,2}$ and $b_{1,2}$.

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