

LOCALIZATION OF THE SFT INSPIRED NONLOCAL LINEAR MODELS AND EXACT SOLUTIONS

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A general class of gravitational models driven by a nonlocal scalar field with a linear or quadratic potential is considered. We study the action with an arbitrary analytic function $\mathcal{F}(\square_g)$, which has both simple and double roots. The way of localization of nonlocal Einstein equations is generalized on models with linear potentials. Exact solutions in the Friedmann–Robertson–Walker and Bianchi I metrics are presented.

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INTRODUCTION

Recently a wide class of nonlocal cosmological models based on the string field theory (SFT) (for details see reviews [1]) and the p -adic string theory [2] emerges and attracts a lot of attention [3–21]. Due to the presence of phantom excitations, nonlocal models are of interest for the present cosmology. Generally speaking, models that violate the null energy condition (NEC) have ghosts, and therefore are unstable and physically unacceptable. Phantom fields look harmful to the theory and a local model with a phantom scalar field is not acceptable from the general point of view. Models with higher derivative terms produce well-known problems with quantum instability [22, 23]. An idea that could solve the problems is that terms with high-order derivatives can be treated as corrections valued only at small energies below the physical cut-off [24, 25]. This approach implies the possibility to construct a UV completion of the theory and requires detailed analysis.

Note that the possibility of the existence of dark energy with $w_{\text{DE}} < -1$ is not excluded experimentally. Contemporary cosmological observational data [26] strongly support that the present Universe exhibits an accelerated expansion providing thereby an evidence for a dominating dark energy component with the state parameter

$$w_{\text{DE}} = -1.0 \pm 0.2. \quad (1)$$

The present cosmological observations do not exclude an evolving parameter w_{DE} . Moreover, the recent analysis of the observation data indicates that the varying in time dark energy

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with the state parameter w_{DE} , which crosses the cosmological constant barrier, gives a better fit than a cosmological constant [27] (see also [28, 29] and references therein).

To obtain a stable model with $w_{\text{DE}} < -1$, one should construct the effective theory with the NEC violation from the fundamental theory, which is stable and admits quantization. From this point of view, the NEC violation might be a property of a model that approximates the fundamental theory and describes some particular features of the fundamental theory. With the lack of quantum gravity, we can just trust string theory or deal with an effective theory admitting the UV completion. It can be considered as a hint towards the SFT inspired cosmological models (details about the string cosmology see in reviews [30]). Note, also, that not only the string inspired cosmological models obey nonlocality [31].

In the flat space-time, nonlocal equations are actively investigated as well [32–35]. Note that differential equations of infinite order were began to study long time ago [36, 37].

The purpose of this paper is to study gravitational models with a nonlocal scalar field. We consider a general form of nonlocal action for the scalar field with a quadratic or linear potential, keeping the main ingredient, the analytic function $\mathcal{F}(\square_g)$, which in fact produces the nonlocality in question, almost unrestricted.

The possible way to find solutions of the Einstein equations with a quadratic potential of the nonlocal scalar field, is to reduce them to a system of Einstein equations describing many noninteracting local scalar fields [7, 14] (see also [18, 20]). Some of the obtained local scalar fields are normal and other of them are phantom ones. In this paper we generalize the algorithm of localization, proposed in [14, 20], on the case of a linear potential. Note that the ways of localization in the case of a linear potential and in the case of a quadratic potential with a linear term, considered in [16], are different.

The paper is organized as follows. In Sec. 1 we describe nonlocal cosmological models. In Sec. 2 we propose the algorithm to find particular solutions of the nonlocal Einstein equations, solving only local ones, and prove the self-consistence of it. Any solution for the obtained system of differential equations is a particular solution for the initial nonlocal Einstein equations. Exact solutions in the Friedmann–Robertson–Walker and Bianchi I metrics are presented in Sec. 3. In Conclusion we summarize the obtained results and propose directions for further investigations.

1. MODEL SETUP

The four-dimensional action with a quadratic or linear potential, motivated by the string field theory, has been studied in [7, 8, 14–16, 18, 20, 21]. Such a model appears as a linearization of the SFT inspired model in the neighborhood of an extremum of the potential (see [18] for details). For linear models, solving the nonlocal equations using the technique, proposed in [14], is completely equivalent to solving the equations using the diffusion-like partial differential equations [16]. By linearizing a nonlinear model about a particular field value, one is able to specify initial data for nonlinear models, which one then evolves into the full nonlinear regime using the diffusion-like equation [16].

In this paper we study nonlocal cosmological models with a quadratic potential, in other words, a linear nonlocal model, which can be described by the following action:

$$S = \int d^4x \sqrt{-g} \alpha' \left(\frac{R}{16\pi G_N} + \frac{1}{g_o^2} \left(\frac{1}{2} \phi \mathcal{F}(\square_g) \phi - V(\phi) \right) - \Lambda \right), \quad (2)$$

where G_N is the Newtonian constant: $8\pi G_N = 1/M_P^2$, where M_P is the Planck mass; α' is the string length squared; g_o is the string coupling constant. We use the signature $(-, +, +, +)$, $g_{\mu\nu}$ is the metric tensor, R is the scalar curvature, Λ is the cosmological constant. The potential is an arbitrary quadratic polynomial: $V(\phi) = C_2\phi^2 + C_1\phi + C_0$. The Beltrami–Laplace operator \square_g is applied to scalar functions and can be written as follows:

$$\square_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu. \quad (3)$$

The function \mathcal{F} is assumed to be an analytic function, therefore, one can represent it by the convergent series expansion:

$$\mathcal{F}(\square_g) = \sum_{n=0}^{\infty} f_n \square_g^n. \quad (4)$$

The function \mathcal{F} may have infinitely many roots manifestly producing thereby the nonlocality [13, 18]. This model has been studied in [7, 18] with an additional condition that all roots of the function \mathcal{F} are simple. In this paper we consider double roots as well. To clarify the interest to consider the case of double roots let us study a trivial example with

$$\mathcal{F}(\square_g) = (\square_g - J_1)(\square_g - J_2). \quad (5)$$

In the Minkowski space-time for ϕ , depending only on time, we obtain the following equation of motion:

$$(\partial_t^2 - J_1)(\partial_t^2 - J_2)\phi(t) = 0. \quad (6)$$

This fourth-order differential equation is equivalent to the following system of two second-order equations:

$$(\partial_t^2 - J_1)\xi(t) = 0, \quad (\partial_t^2 - J_2)\phi(t) = \xi(t). \quad (7)$$

The first equation has the general solution

$$\xi(t) = B_1 e^{\sqrt{J_1}t} + B_2 e^{-\sqrt{J_1}t}, \quad (8)$$

where B_1 and B_2 are arbitrary constants. So, we get the following second-order equation in ϕ

$$(\partial_t^2 - J_2)\phi(t) = B_1 e^{\sqrt{J_1}t} + B_2 e^{-\sqrt{J_1}t}. \quad (9)$$

In the nonresonance case (two simple roots J_1 and J_2) we get

$$\phi(t) = B_1 e^{\sqrt{J_1}t} + B_2 e^{-\sqrt{J_1}t} + B_3 e^{\sqrt{J_2}t} + B_4 e^{-\sqrt{J_2}t}, \quad (10)$$

whereas in the resonance case (one double root $J_2 = J_1$) the general solution is

$$\phi(t) = B_1 e^{\sqrt{J_1}t} + B_2 e^{-\sqrt{J_1}t} + B_3 t e^{\sqrt{J_1}t} + B_4 t e^{-\sqrt{J_1}t}, \quad (11)$$

where B_k are arbitrary constants. This trivial example shows that behavior of solutions in the cases of one double root and two simple roots are essentially different and one cannot approximate double roots by two simple roots, which are at a very small distance. Resonance phenomena are important and actively studied in various domains of physics.

2. ALGORITHM OF LOCALIZATION

2.1. Einstein Equations. From action (2) we obtain the following equations:

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu} - \Lambda g_{\mu\nu}), \quad (12)$$

$$\mathcal{F}(\square_g)\phi = \frac{dV}{d\phi}, \quad (13)$$

where $G_{\mu\nu}$ is the Einstein tensor.

The energy-momentum (stress) tensor $T_{\mu\nu}$, which is calculated by the standard formula

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}, \quad (14)$$

can be presented in the following form:

$$T_{\mu\nu} = \frac{1}{g_o^2} \left(E_{\mu\nu} + E_{\nu\mu} - g_{\mu\nu} (g^{\rho\sigma} E_{\rho\sigma} + W) \right), \quad (15)$$

where

$$E_{\mu\nu} \equiv \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \square_g^l \phi \partial_\nu \square_g^{n-1-l} \phi, \quad (16)$$

$$W \equiv \frac{1}{2} \sum_{n=2}^{\infty} f_n \sum_{l=1}^{n-1} \square_g^l \phi \square_g^{n-l} \phi - \frac{f_0}{2} \phi^2 + V(\phi). \quad (17)$$

In the case of the zero potential $V(\phi) = 0$, using the equation

$$\mathcal{F}(\square_g)\phi = 0, \quad \iff \quad f_0 \phi = - \sum_{n=1}^{\infty} f_n \square_g^n \phi, \quad (18)$$

one can obtain that W for $V(\phi) = 0$ is equal to

$$W_0 = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \square_g^l \phi \square_g^{n-l} \phi. \quad (19)$$

The formula for energy-momentum tensor with W_0 has been proposed in [7] (see also [18]).

The main idea of finding the solutions to the equations of motion is to start with equation (13) for $V(\phi) = 0$ and to solve it, assuming the function ϕ is an eigenfunction of the Beltrami–Laplace operator \square_g . If $\square_g \phi = J\phi$, then such a function ϕ is a solution to (13) if and only if

$$\mathcal{F}(J) = 0. \quad (20)$$

The latter condition is known as the *characteristic* equation. Note that values of roots of $\mathcal{F}(J)$ do not depend on the metric. In this paper we show how the case of an arbitrary quadratic potential $V(\phi)$ can be analyzed with the help of roots of the function $\mathcal{F}(J)$.

Let us denote simple roots of \mathcal{F} as J_i and double roots of \mathcal{F} as \tilde{J}_k . We seek a particular solution of equation (13) in the following form:

$$\phi_0 = \sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k, \quad (21)$$

where

$$(\square_g - J_i)\phi_i = 0, \quad (\square_g - \tilde{J}_k)^2 \tilde{\phi}_k = 0. \quad (22)$$

The fourth-order differential equation $(\square_g - \tilde{J}_k)(\square_g - \tilde{J}_k)\tilde{\phi}_k = 0$ is equivalent to the following system of the second-order equations:

$$(\square_g - \tilde{J}_k)\tilde{\phi}_k = \varphi_k, \quad (\square_g - \tilde{J}_k)\varphi_k = 0. \quad (23)$$

Without loss of generality we assume that for any i_1 and $i_2 \neq i_1$ conditions $J_{i_1} \neq J_{i_2}$ and $\tilde{J}_{i_1} \neq \tilde{J}_{i_2}$ are satisfied.

2.2. Zero Potential $V(\phi)$. It is convenient to consider the cases $C_1 = 0$ and $C_1 \neq 0$ separately. In this Subsection we consider the case of zero potential ($C_1 = 0$), the case of a linear potential is considered in the next Subsection.

Modifying values of f_0 and Λ , we can transform action (2) with the potential $V(\phi) = C_2\phi^2 + C_0$ to the action with zero potential. So, without loss of generality, we can put $C_2 = 0$ and $C_0 = 0$ and use the energy-momentum tensor for ϕ_0 , which has been calculated in [20]. It has been obtained that for any analytical function $\mathcal{F}(J)$, which has simple roots J_i and double roots \tilde{J}_k , and for any ϕ_0 given by (21), the energy-momentum tensor is

$$T_{\mu\nu}(\phi_0) = T_{\mu\nu}\left(\sum_{i=1}^{N_1} \phi_i + \sum_{k=1}^{N_2} \tilde{\phi}_k\right) = \sum_{i=1}^{N_1} T_{\mu\nu}(\phi_i) + \sum_{k=1}^{N_2} T_{\mu\nu}(\tilde{\phi}_k), \quad (24)$$

where all $T_{\mu\nu}$ are given by (15), and

$$E_{\mu\nu}(\phi_i) = \frac{\mathcal{F}'(J_i)}{2} \partial_\mu \phi_i \partial_\nu \phi_i, \quad W(\phi_i) = \frac{J_i \mathcal{F}'(J_i)}{2} \phi_i^2, \quad (25)$$

$$E_{\mu\nu}(\tilde{\phi}_k) = \frac{\mathcal{F}''(\tilde{J}_k)}{4} \left(\partial_\mu \tilde{\phi}_k \partial_\nu \varphi_k + \partial_\nu \tilde{\phi}_k \partial_\mu \varphi_k \right) + \frac{\mathcal{F}'''(\tilde{J}_k)}{12} \partial_\mu \varphi_k \partial_\nu \varphi_k, \quad (26)$$

$$W(\tilde{\phi}_k) = \frac{\tilde{J}_k \mathcal{F}''(\tilde{J}_k)}{2} \tilde{\phi}_k \varphi_k + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) \varphi_k^2. \quad (27)$$

The primes in (25)–(27) denote a derivative with respect to J : $\mathcal{F}' \equiv d\mathcal{F}/dJ$, $\mathcal{F}'' \equiv d^2\mathcal{F}/dJ^2$ and $\mathcal{F}''' \equiv d^3\mathcal{F}/dJ^3$. The result has been obtained for an arbitrary metric.

Considering the following local action:

$$S_{\text{loc}} = \int d^4x \sqrt{-g} \left(\frac{R}{16\pi G_N} - \Lambda \right) + \sum_{i=1}^{N_1} S_i + \sum_{k=1}^{N_2} \tilde{S}_k, \quad (28)$$

where

$$S_i = -\frac{1}{g_0^2} \int d^4x \sqrt{-g} \frac{\mathcal{F}'(J_i)}{2} (g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i + J_i \phi_i^2), \quad (29)$$

$$\begin{aligned} \tilde{S}_k = & -\frac{1}{g_o^2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} \left(\frac{\mathcal{F}''(\tilde{J}_k)}{4} (\partial_\mu \tilde{\phi}_k \partial_\nu \varphi_k + \partial_\nu \tilde{\phi}_k \partial_\mu \varphi_k) + \right. \right. \\ & \left. \left. + \frac{\mathcal{F}'''(\tilde{J}_k)}{12} \partial_\mu \varphi_k \partial_\nu \varphi_k \right) + \frac{\tilde{J}_k \mathcal{F}''(\tilde{J}_k)}{2} \tilde{\phi}_k \varphi_k + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) \varphi_k^2 \right), \quad (30) \end{aligned}$$

we can see that solutions of the Einstein equations and equations in ϕ_k , $\tilde{\phi}_k$, and φ_k , obtained from this action, solve the initial system of nonlocal equations (12) and (13). Thus, we obtain that one can find special solutions of nonlocal equations by solving the system of local (differential) equations.

To clarify physical interpretation of local fields $\tilde{\phi}_k$ and φ_k , we diagonalize the kinetic terms of these scalar fields in (28). Expressing $\tilde{\phi}_k$ and φ_k in terms of new fields ξ_k and χ_k :

$$\tilde{\phi}_k = \frac{1}{2\mathcal{F}''(\tilde{J}_k)} \left(\left(\mathcal{F}''(\tilde{J}_k) - \frac{1}{3} \mathcal{F}'''(\tilde{J}_k) \right) \xi_k - \left(\mathcal{F}''(\tilde{J}_k) + \frac{1}{3} \mathcal{F}'''(\tilde{J}_k) \right) \chi_k \right), \quad \varphi_k = \xi_k + \chi_k, \quad (31)$$

we obtain the corresponding \tilde{S}_k in the following form:

$$\begin{aligned} \tilde{S}_k = & -\frac{1}{g_o^2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} \frac{\mathcal{F}''(\tilde{J}_k)}{4} (\partial_\mu \xi_k \partial_\nu \xi_k - \partial_\nu \chi_k \partial_\mu \chi_k) + \right. \\ & \left. + \frac{\tilde{J}_k}{4} \left((\mathcal{F}''(\tilde{J}_k) - \frac{1}{3} \mathcal{F}'''(\tilde{J}_k)) \xi_k - (\mathcal{F}''(\tilde{J}_k) + \frac{1}{3} \mathcal{F}'''(\tilde{J}_k)) \chi_k \right) (\xi_k + \chi_k) + \right. \\ & \left. + \left(\frac{\tilde{J}_k \mathcal{F}'''(\tilde{J}_k)}{12} + \frac{\mathcal{F}''(\tilde{J}_k)}{4} \right) (\xi_k + \chi_k)^2 \right). \quad (32) \end{aligned}$$

It is easy to see that each \tilde{S}_k includes one phantom scalar field and one standard scalar field. So, in the case of one double root we obtain a quintom model. In the Minkowski space appearance of phantom fields in models, when $\mathcal{F}(\square)$ has a double root, has been obtained in [32]. If $\mathcal{F}(J)$ has simple real roots, then positive and negative values of $\mathcal{F}'(J_i)$ alternate, so we can obtain phantom fields and, in the case of two simple roots, a quintom model.

Remark 1. If $\mathcal{F}(J)$ has an infinity number of roots, then one nonlocal model corresponds to infinity number of different local models. In this case the initial nonlocal action (2) generates infinity number of local actions (28).

Remark 2. We should prove that the way of localization is self-consistent. To construct local action (28) we assume that equations (22) are satisfied. Therefore, the method of localization is correct only if these equations can be obtained from the local action S_{loc} . The straightforward calculations show that

$$\frac{\delta S_{\text{loc}}}{\delta \phi_i} = 0 \quad \Leftrightarrow \quad \square_g \phi_i = J_i \phi_i; \quad \frac{\delta S_{\text{loc}}}{\delta \tilde{\phi}_k} = 0 \quad \Leftrightarrow \quad \square_g \varphi_k = \tilde{J}_k \varphi_k. \quad (33)$$

Using (33) we obtain

$$\frac{\delta S_{\text{loc}}}{\delta \varphi_k} = 0 \quad \Leftrightarrow \quad \square_g \tilde{\phi}_k = \tilde{J}_k \tilde{\phi}_k + \varphi_k. \quad (34)$$

So, the way of localization is self-consistent in the case of $\mathcal{F}(J)$ with simple and double roots [20]. The self-consistence of similar approach for $\mathcal{F}(J)$ with only simple roots has been proven in [14, 18].

In spite of the above-mentioned equations, we obtain from S_{loc} the Einstein equation:

$$G_{\mu\nu} = 8\pi G_N (T_{\mu\nu}(\phi_0) - \Lambda g_{\mu\nu}), \quad (35)$$

where ϕ_0 is given by (21), and $T_{\mu\nu}(\phi_0)$ can be calculated by (24).

So, we obtained such systems of differential equations that any solutions of these systems are particular solutions of the initial nonlocal equations (12) and (13).

2.3. Linear Potential $V(\phi)$. Let us consider the model with action (2) in the case $C_1 \neq 0$. For the string field theory inspired form of $\mathcal{F}(\square)$, the case $f_0 \neq 0$ has been considered in [16]. In this case the effective potential: $-f_0\phi^2/2 + V(\phi) + \Lambda$, is a quadratic potential. Using the condition $f_0 \neq 0$, we boil down the case with an arbitrary C_1 to the case with $C_1 = 0$. Indeed, we work in a new scalar field

$$\chi = \phi - \frac{C_1}{f_0} \quad (36)$$

and get the energy-momentum tensor in the form (15) with

$$E_{\mu\nu} = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \partial_\mu \square_g^l \chi \partial_\nu \square_g^{n-1-l} \chi, \quad (37)$$

$$W = \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=1}^{n-1} \square_g^l \chi \square_g^{n-l} \chi - \frac{f_0}{2} \chi^2 + \frac{C_1^2}{2f_0}. \quad (38)$$

It is easy to see that

$$\mathcal{F}(\square)\phi = C_1 \iff \mathcal{F}(\square)\chi = 0. \quad (39)$$

The constant $C_1^2/(2f_0)$ can be considered as a part of the cosmological constant. Thus, in terms of χ we obtain a model without linear term and can conclude that at $f_0 \neq 0$ the adding of a linear term to the potential shifts the scalar field on the constant and changes the value of cosmological constant.

Let us consider the case $f_0 = 0$. In this case $J = 0$ is a root of the characteristic equation (20). It is easy to show, that the function

$$\tilde{\chi} = \phi_0 + \psi, \quad (40)$$

where ϕ_0 and ψ are solutions of the following equations:

$$\mathcal{F}(\square)\phi_0 = 0, \quad \square^m \psi = \frac{C_1}{f_m}, \quad (41)$$

m is the order of the root $J = 0$, satisfies

$$\mathcal{F}(\square)\tilde{\chi} = C_1. \quad (42)$$

The function ϕ_0 is given by (21), but the sum does not include ϕ_{i_0} , which corresponds to the root $J = 0$, because this function cannot be separated from ψ . We consider the cases

of $m = 1$ and $m = 2$. In the last case, when $J = 0$ is a double root, we denote the function ψ as $\tilde{\psi}$.

To localize the Einstein equations, one should calculate the energy-momentum tensor for $\tilde{\chi}$:

$$T_{\mu\nu}(\tilde{\chi}) = T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi_0) + T_{\mu\nu}^{cr}(\psi, \phi_0). \quad (43)$$

Let us calculate

$$W(\tilde{\chi}) = \frac{1}{2} \sum_{n=2}^{\infty} f_n \sum_{l=1}^{n-1} \square_g^l \tilde{\chi} \square_g^{n-l} \tilde{\chi} + C_1 \tilde{\chi}. \quad (44)$$

To simplify notation, we choose $\phi_0 = \phi_i$, where J_i is a simple root, the generalization to an arbitrary ϕ_0 is straightforward. In the case of the simple root $J = 0$, we have $\square\psi = C_1/f_1$ and

$$W(\psi + \phi_i) = \frac{f_2 C_1^2}{2f_1^2} + W(\phi_i) + \sum_{n=2}^{\infty} \frac{C_1}{f_1} f_n J_i^{n-1} \phi_i + C_1(\psi + \phi_i). \quad (45)$$

Using

$$\sum_{n=2}^{\infty} \frac{C_1}{f_1} f_n J_i^{n-1} \phi_i = \frac{C_1}{f_1 J_i} \sum_{n=1}^{\infty} f_n J_i^n \phi_i - C_1 \phi_i = \left(\frac{C_1 \mathcal{F}(J_i)}{f_1 J_i} - C_1 \right) \phi_i = -C_1 \phi_i, \quad (46)$$

we obtain

$$W(\psi + \phi_i) = W(\psi) + W(\phi_i), \quad (47)$$

where $W(\phi_i)$ is given by (25), and

$$W(\psi) = C_1 \psi + \frac{f_2 C_1^2}{2f_1^2}. \quad (48)$$

Similar calculations give

$$E_{\mu\nu}(\tilde{\chi}) = E_{\mu\nu}(\psi) + E_{\mu\nu}(\phi_0), \quad (49)$$

where

$$E_{\mu\nu}(\psi) = \frac{1}{2} f_1 \partial_\mu \psi \partial_\nu \psi. \quad (50)$$

The function ϕ_0 is given by (21) and satisfies equation (13) with $C_1 = 0$, therefore, we use W_0 instead of W to calculate $T_{\mu\nu}(\phi_0)$ and obtain equality (24).

So, we get

$$T_{\mu\nu}^{cr}(\psi, \phi_0) = 0 \quad \text{and} \quad T_{\mu\nu}(\tilde{\chi}) = T_{\mu\nu}(\psi) + T_{\mu\nu}(\phi_0). \quad (51)$$

In the case of the double root $J = 0$, we have equation

$$\square^2 \tilde{\psi} = \frac{C_1}{f_2}, \quad \iff \quad \begin{cases} \square \tilde{\psi} = \tau, \\ \square \tau = \frac{C_1}{f_2}. \end{cases} \quad (52)$$

We obtain

$$T_{\mu\nu}(\tilde{\chi}) = T_{\mu\nu}(\tilde{\psi}) + T_{\mu\nu}(\phi_0), \quad (53)$$

$$E_{\mu\nu}(\tilde{\psi}) = \frac{1}{2} \left(f_2(\partial_\mu \tilde{\psi} \partial_\nu \tau + \partial_\nu \tilde{\psi} \partial_\mu \tau) + f_3 \partial_\mu \tau \partial_\nu \tau \right), \quad (54)$$

$$W(\tilde{\psi}) = \frac{f_2}{2} \tau^2 + C_1 \tilde{\psi} + \frac{f_3 C_1}{f_2} \tau. \quad (55)$$

The obtained formulae allow one to generalize the algorithm of localization, proposed in [20] to the case $C_1 \neq 0$. For an arbitrary quadratic potential $V(\phi) = C_2 \phi^2 + C_1 \phi + C_0$, there exists the following algorithm of localization:

- Find roots of the function $\mathcal{F}(J)$ and calculate orders of them.
- Select a finite number of simple and double roots.
- Change values of f_0 and Λ such that the potential takes the form $V(\phi) = C_1 \phi$.
- Construct the corresponding local action. In the case $C_1 = 0$, one should use formula (28). In the case $C_1 \neq 0$ and $f_0 \neq 0$, one should use (28) with the replacement of the scalar field ϕ by χ (formula (36)) and the corresponding modification of the cosmological constant. In the case $C_1 \neq 0$ and $f_0 = 0$, the local action is the sum of (28) and either

$$S_\psi = -\frac{1}{2g_o^2} \int d^4x \sqrt{-g} \left(f_1 g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + 2C_1 \psi + \frac{f_2 C_1^2}{f_1^2} \right),$$

in the case of simple root $J = 0$, or

$$S_{\tilde{\psi}} = \frac{-1}{2g_o^2} \int d^4x \sqrt{-g} \left(g^{\mu\nu} (f_2(\partial_\mu \tilde{\psi} \partial_\nu \tau + \partial_\nu \tilde{\psi} \partial_\mu \tau) + f_3 \partial_\mu \tau \partial_\nu \tau) + f_2 \tau^2 + 2C_1 \tilde{\psi} + \frac{f_3 C_1}{2f_2} \tau \right),$$

in the case of double root $J = 0$. Note that in the case $C_1 \neq 0$ and $f_0 = 0$, the local action (28) has no term, which corresponds to the root $J = 0$.

- Vary the obtained local action and get a system of the Einstein equations and equations of motion. The obtained system is a finite order system of differential equations, i.e., we get a local system.

- Seek solutions of the obtained local system.

3. EXACT SOLUTIONS

3.1. Root of $\mathcal{F}(\square)$ in the Case of the SFT Inspired Models. The particular forms of $\mathcal{F}(\square_g)$ are inspired by the fermionic SFT and the most well understood process of tachyon condensation. Namely, starting with a nonsupersymmetric configuration, the tachyon of the fermionic string rolls down towards the nonperturbative minimum of the tachyon potential. This process represents the non-BPS brane decay according to Sen's conjecture (see [1] for details). From the point of view of the SFT, the whole picture is not yet known and only vacuum solutions were constructed. An effective field theory description explaining the rolling tachyon in contrary is known and numeric solutions describing the tachyon dynamics were obtained [35]. This effective field theory description does capture the nonlocality of the SFT. Linearizing the latter Lagrangian around the true vacuum one gets a model which is of main concern in the present paper. The SFT inspired form of the function $\mathcal{F}(\square_g)$, which has the nonlocal operator $\exp(\square_g)$ as a key ingredient:

$$\mathcal{F}_{\text{SFT}}(\square_g) = \xi^2 \square_g + 1 - c e^{2\square_g}, \quad (56)$$

where ξ is a real parameter and c is a positive constant, has been considered in [8, 14, 16]. The form of the term $(e^{\square_g \phi})^2$ is analogous to the form of the interaction term for the tachyon field in the SFT action.

The characteristic equation $\mathcal{F}_{\text{SFT}}(J) = 0$ has the following solutions:

$$J_n = -\frac{1}{2\xi^2} \left(2 + \xi^2 W_n \left(-\frac{2c}{\xi^2} e^{-2/\xi^2} \right) \right), \tag{57}$$

where n is an integer number, W_n is the n s branch of the Lambert function satisfying a relation $W(z) e^{W(z)} = z$. The Lambert function is a multivalued function, so $\mathcal{F}_{\text{SFT}}(J)$ has an infinite number of roots. Parameters ξ and c are real, therefore if J_n is a root of $\mathcal{F}_{\text{SFT}}(J)$, then the complex adjoined number J_n^* is a root as well.

If $J = \tilde{J}_0$ is a multiple root, then at this point $\mathcal{F}_{\text{SFT}}(J) = 0$ and $\mathcal{F}'_{\text{SFT}}(J) = 0$. These equations give that

$$\tilde{J}_0 = \frac{1}{2} - \frac{1}{\xi^2}, \tag{58}$$

hence the root \tilde{J}_0 is a real number. \tilde{J}_0 is a double root because:

$$\mathcal{F}''_{\text{SFT}}(\tilde{J}_0) = -4c e^{2\tilde{J}_0} \neq 0. \tag{59}$$

The function $\mathcal{F}_{\text{SFT}}(J)$ has a double root if and only if $c = (\xi^2/2) e^{(2/\xi^2-1)}$.

Roots of $\mathcal{F}_{\text{SFT}}(J)$ do not depend on metric. In the Minkowski space-time these roots have been studied in [8]. The function \mathcal{F}_{SFT} always has an infinity number of complex roots. Let us consider real roots of \mathcal{F}_{SFT} . There are three different cases:

- If $c < 1$, then for any values ξ the function $\mathcal{F}_{\text{SFT}}(J)$ has two simple real root: one is positive, another is negative.
- If $c = 1$, then $J = 0$ is a simple root at $\xi^2 \neq 2$. A positive root exists if and only if $\xi^2 > 2$. At $\xi^2 < 2$ a negative root exists. If $\xi^2 = 2$, then $J = 0$ is a double root.
- If $c > 1$, then $\mathcal{F}_{\text{SFT}}(J)$ has
 - two negative simple roots for $\xi^2 < \xi_1^2$,
 - a negative double root for $\xi^2 = \xi_1^2$,
 - no real roots for $\xi_2^2 > \xi^2 > \xi_1^2$,
 - a positive double root for $\xi^2 = \xi_2^2$,
 - two real positive roots for $\xi^2 > \xi_2^2$, where

$$\xi_1^2 = -\frac{2}{W_{-1}(-\exp(-1)/c)}, \quad \xi_2^2 = -\frac{2}{W_0(-\exp(-1)/c)}. \tag{60}$$

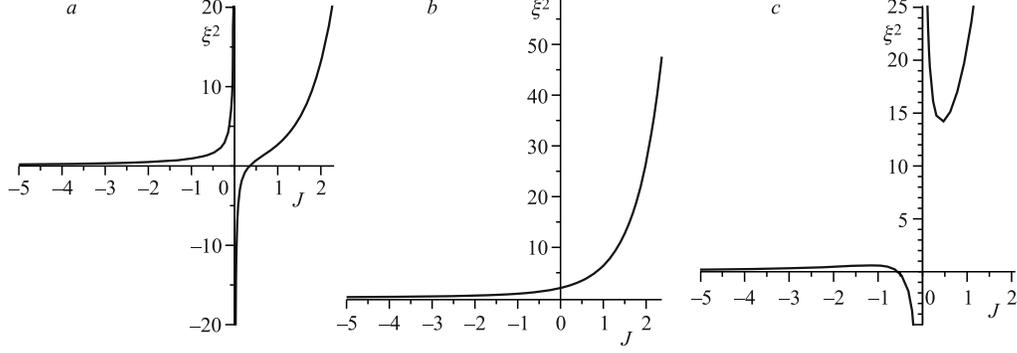
To illustrate the dependence of the parameter ξ^2 on real roots we introduce the function $g(J, c)$:

$$g(J, c) = \xi^2 = \frac{c e^{2J} - 1}{J}, \tag{61}$$

and plot $g(J, c)$ as a function of J at three different values of c (see the Figure).

Let us consider special values of ξ^2 and c , which have been obtained in the SFT inspired cosmological model. From the action for the tachyon in the SFT [38] the following equation has been obtained [39]:

$$(-\xi_0^2 \tilde{\alpha}^2 + 1) = 3 e^{-\tilde{\alpha}^2/4}, \tag{62}$$



The dependence of the function $g(J, c)$, which is equal to ξ^2 , on J at $c = 1/2$ (a), $c = 1$ (b) and $c = 3$ (c)

where

$$\xi_0^2 = -\frac{1}{4 \ln\left(\frac{4}{3\sqrt{3}}\right)} \approx 0.9556. \quad (63)$$

Substituting $J = -\tilde{\alpha}^2/8$, we obtain \mathcal{F}_{SFT} with $\xi_{\text{SFT}}^2 = 8\xi_0^2$ and $c = 3$. At $c = 3$ we obtain that $\xi_1^2 = 0.6080355395$ and $\xi_2^2 = 14.16157383$. Therefore, $\xi_2^2 > \xi_{\text{SFT}}^2 > \xi_1^2$, so there exists no real root at these values of parameters.

3.2. Exact Solution in the Friedmann–Robertson–Walker Metric. Let us consider the Einstein equations, which correspond to a real simple root J_1 in the Friedmann–Robertson–Walker metric [14]:

$$\begin{cases} 3H^2 = \frac{4\pi G\mathcal{F}'(J_1)}{g_o^2} (\dot{\phi}^2 + J_1\phi^2) + 8\pi G\Lambda, \\ \dot{H} = -\frac{4\pi G\mathcal{F}'(J_1)}{g_o^2} \dot{\phi}^2, \end{cases} \quad (64)$$

where a dot denotes a time derivative.

Exact real solutions of this system have been obtained in [9, 14]. In our notations these solutions are as follows:

At $J_1 > 0$,

$$\phi(t) = \pm \frac{\sqrt{3J_1g_o^2}}{6\pi G\mathcal{F}'(J_1)}(t - t_0), \quad H(t) = -\frac{J_1g_o^2}{6\pi G\mathcal{F}'(J_1)}(t - t_0), \quad (65)$$

where t_0 is an arbitrary constant. These solutions exist only at

$$\Lambda = -\frac{J_1g_o^2}{24G^2\pi^2\mathcal{F}'(J_1)}. \quad (66)$$

At $J_1 = 0$, summing the first and the second equations of (64), we obtain:

$$\dot{H} = 8\pi G\Lambda - 3H^2. \quad (67)$$

The type of solution depends on the sign of Λ :

- $\Lambda = 0$,

$$H(t) = -\frac{1}{3(t-t_0)}, \quad \phi(t) = \tilde{C}_1 \pm \frac{\sqrt{3}g_o}{\sqrt{\pi G\mathcal{F}'(0)}} \ln(t-t_0), \quad (68)$$

where t_0 and \tilde{C}_1 are arbitrary constants.

- If $\Lambda > 0$, then we obtain solutions:

$$H_1(t) = \frac{2\sqrt{6\pi G\Lambda}}{3} \tanh\left(2\sqrt{6\pi G\Lambda}(t-t_0)\right), \quad (69)$$

$$\phi_1(t) = \pm \sqrt{\frac{-g_o^2}{12\pi G\mathcal{F}'(0)}} \arctan\left(\sinh\left(2\sqrt{6\pi G\Lambda}(t-t_0)\right)\right) + \tilde{C}_2 \quad (70)$$

and

$$\tilde{H}_1(t) = \frac{2\sqrt{6\pi G\Lambda}}{3} \coth\left(2\sqrt{6\pi G\Lambda}(t-t_0)\right), \quad (71)$$

$$\tilde{\phi}_1(t) = \pm \sqrt{\frac{g_o^2}{12\pi G\mathcal{F}'(0)}} \ln\left(\tanh\left(\sqrt{6\pi G\Lambda}(t-t_0)\right)\right) + \tilde{C}_2, \quad (72)$$

hereafter t_0 and \tilde{C}_2 are arbitrary real constants.

- In the case $\Lambda < 0$, we obtain the following real solution:

$$H_2(t) = -\frac{2\sqrt{-6\pi G\Lambda}}{3} \tan\left(2\sqrt{-6\pi G\Lambda}(t-t_0)\right), \quad (73)$$

$$\phi_2(t) = \pm \sqrt{\frac{g_o^2}{12\pi G\mathcal{F}'(0)}} \operatorname{arctanh}\left(\sin\left(2\sqrt{-6\pi G\Lambda}(t-t_0)\right)\right) + \tilde{C}_2. \quad (74)$$

The stability of the exact solutions, obtained in the Friedmann–Robertson–Walker metric [14], has been analysed in [40].

3.3. Exact Solutions in the Bianchi I Metric. In Bianchi I metric with the interval

$$ds^2 = -dt^2 + a_1^2(t) dx_1^2 + a_2^2(t) dx_2^2 + a_3^2(t) dx_3^2, \quad (75)$$

the Einstein equations, which correspond to the simple root $J = 0$, have the following form:

$$H_1 H_2 + H_1 H_3 + H_2 H_3 = 8\pi G_N \left(\frac{\mathcal{F}'(0)}{2g_o^2} \dot{\phi}^2 + \Lambda \right), \quad (76)$$

$$\dot{H}_2 + H_2^2 + \dot{H}_3 + H_3^2 + H_2 H_3 = -8\pi G_N \left(\frac{\mathcal{F}'(0)}{2g_o^2} \dot{\phi}^2 - \Lambda \right), \quad (77)$$

$$\dot{H}_1 + H_1^2 + \dot{H}_2 + H_2^2 + H_1 H_2 = 8\pi G_N \left(\frac{\mathcal{F}'(0)}{2g_o^2} \dot{\phi}^2 - \Lambda \right), \quad (78)$$

$$\dot{H}_1 + H_1^2 + \dot{H}_3 + H_3^2 + H_1 H_3 = -8\pi G_N \left(\frac{\mathcal{F}'(0)}{2g_o^2} \dot{\phi}^2 - \Lambda \right), \quad (79)$$

where $H_k \equiv \dot{a}_k/a_k$, $k = 1, 2, 3$. Note that $\mathcal{F}'(0) \neq 0$.

Our goal is to find exact solutions to system (76)–(79). Of course, there exist isotropic solutions, which coincide with exact solutions in the Friedmann–Robertson–Walker metric. For those solutions $H_1(t) = H_2(t) = H_3(t)$. At the same time, exact anisotropic solutions do exist.

For $\Lambda = 0$, we obtain the following solution:

$$H_1(t) = \frac{\tilde{C}_2 + \tilde{C}_1 + 1}{\tilde{C}_2(t - t_0)}, \quad H_2(t) = -\frac{\tilde{C}_1}{\tilde{C}_2(t - t_0)}, \quad H_3(t) = -\frac{1}{\tilde{C}_2(t - t_0)}, \quad (80)$$

$$\phi(t) = \pm \frac{\sqrt{-\pi G_N \mathcal{F}'(0) (\tilde{C}_1 \tilde{C}_2 + \tilde{C}_1^2 + \tilde{C}_1 + \tilde{C}_2 + 1)}}{4\pi G_N g_o \mathcal{F}'(0) \tilde{C}_2} \ln \left(\tilde{C}_2(t - t_0)^2 + \tilde{C}_3 \right), \quad (81)$$

where \tilde{C}_1 , \tilde{C}_2 , \tilde{C}_3 , and t_0 are arbitrary constants.

For all $\mathcal{F}'(0) < 0$, we obtain that $\phi(t)$ is a real function at

$$\tilde{C}_1 \geq -1, \quad \tilde{C}_2 > 0 \quad \text{or} \quad \tilde{C}_1 < -1, \quad -\frac{\tilde{C}_1^2 + \tilde{C}_1 + 1}{\tilde{C}_1 + 1} > \tilde{C}_2 > 0. \quad (82)$$

For $\mathcal{F}'(0) > 0$, we obtain that $\phi(t)$ is a real function at

$$\tilde{C}_1 < -1, \quad \tilde{C}_2 > -\frac{\tilde{C}_1^2 + \tilde{C}_1 + 1}{\tilde{C}_1 + 1}.$$

Let us consider the case of positive $\Lambda = 1/8\pi G_N$. There exists not only the following isotropic solution:

$$H_1(t) = H_2(t) = H_3(t) = \frac{1}{\sqrt{3}} \tanh \left(\sqrt{3}(t - t_0) \right), \quad (83)$$

but also an anisotropic one

$$\begin{aligned} H_1(t) &= \frac{1}{\sqrt{3}} \tanh \left(\frac{\sqrt{3}}{2}(t - t_0) \right), \\ H_2(t) &= \frac{1}{\sqrt{3}} \coth \left(\frac{\sqrt{3}}{2}(t - t_0) \right), \\ H_3(t) &= \frac{1}{2\sqrt{3}} \left(\tanh \left(\frac{\sqrt{3}}{2}(t - t_0) \right) + \coth \left(\frac{\sqrt{3}}{2}(t - t_0) \right) \right). \end{aligned} \quad (84)$$

The corresponding scalar field is real at $\mathcal{F}'(0) > 0$ and is equal to

$$\tilde{\phi}(t) = \tilde{C}_4 \pm \frac{1}{3\sqrt{2\pi G g_o^2 \mathcal{F}'(0)}} \left(\ln(e^{\sqrt{3}(t-t_0)} + 1) - \ln(e^{\frac{\sqrt{3}}{2}(t-t_0)} - 1) - \ln(e^{\frac{\sqrt{3}}{2}(t-t_0)} + 1) \right),$$

where \tilde{C}_4 is an arbitrary real constant.

CONCLUSION

The main result of this paper is the generalization of the algorithm of localization on nonlocal models with linear potentials. This algorithm is proposed for an arbitrary analytic function $\mathcal{F}(\square_g)$, which has both simple and double roots. We have proved that the same functions solve the initial nonlocal Einstein equations and the obtained local Einstein equations. We have found the corresponding local actions and proved the self-consistence of our approach.

It is interesting to consider nonlocal models with an arbitrary analytic $\mathcal{F}(\square_g)$, without any restrictions on the order of roots. The consideration of simple and double roots allows us to make the conjecture that the existence of local actions, which correspond to a nonlocal action, does not depend on the order of $\mathcal{F}(\square_g)$ roots and the method of finding particular solutions of the nonlocal Einstein equations can be generalized on a nonlocal action with an arbitrary analytic $\mathcal{F}(\square_g)$.

In the case of simple roots, exact solutions in the Friedmann–Robertson–Walker metric have been found in [14] (their stability is considered in [40]). In this paper, we present exact solutions in Friedmann–Robertson–Walker and Bianchi I metrics. The algorithm of localization does not depend on metric, so it can be used to find solutions in other metrics. For example, the well-known Fisher solutions [41] (see also [42,43]), which are static spherically symmetric solutions for gravitational system with a massless scalar field, are solutions of the nonlocal Einstein equations (12), (13) for any $\mathcal{F}(J)$, which has a simple root $J_0 = 0$.

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