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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

PIONEERING SPACE RESEARCH IN THE USSR AND MATHEMATICAL MODELING OF LARGE PROBLEMS OF RADIATION TRANSFER

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This review is to remind scientists of the older generation of some memorable historical pages and of many famous researchers, teachers and colleagues. For the younger researchers and foreign colleagues it will be useful to get to know about pioneer advancements of the Soviet scientists in the field of information and mathematical supply for cosmonautic problems on the eve of the space era. Main attention is paid to the scientific experiments conducted on the piloted space vehicles and the research teams who created the information and mathematical tools for the first space projects. The role of Mstislav Vsevolodovich Keldysh, the Major Theoretician of cosmonautics, is particulary emphasized. He determined for the most part the basic directions of development of space research and remote sensing of the Earth and planets that are shortly called remote sensing.

Космические исследования — это такая область фундаментальных и прикладных работ, которая не могла развиваться без использования ЭВМ. Освоение космического пространства послужило значительным фактором совершенствования ЭВМ и формирования новых научных направлений, связанных с математическим моделированием радиационного поля Земли, теорией переноса изображения, теорией видения, теорией обработки и распознавания образов и т. д. Информационно-математическое обеспечение — обязательная составная часть любого космического проекта. В настоящем обзоре основное внимание уделено проблемам информационно-математического обеспечения проблем космонавтики на заре космической эры. Речь идет о пионерских космических экспериментах по дистанционному зондированию Земли, которые проводились советскими космонавтами с пилотируемых космических кораблей. Отмечена важнейшая роль главного теоретика космонавтики академика Мстислава Всеволодовича Келдыша, который фактически определил основные направления становления и развития космических исследований и дистанционного аэрокосмического зондирования Земли как планеты, называемых в настоящее время во всем мире кратко «remote sensing».

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INTRODUCTION

The 20th century in history of the earthly civilization is that of the scientific and technical revolution, which is connected with three great discoveries:

— the penetration into the mysteries and the power of nuclear energy;
— the space exploration and input of man into space;
— the invention of computer and the creation of information technology.

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Two greatest epoch-making scientific projects — atomic and cosmic — have favoured the enormous progression of Soviet science, which could be competitive with world science in the 20th century. The solving of large problems on computer was demanded to realize the engineering and constructing projects in the first time. The foundations of the new technology were laid, which would be called «mathematical modeling» or «computer sciences».

It was not an accidental event that the Keldysh Institute participated in these projects, since M. V. Keldysh (10.02.1911–24.06.1978) was the Major Theoretician in cosmonautics, and his deputy A. N. Tikhonov (30.10.1906–07.10.1993) had already been much experienced in conducting the numerical experiments and the large problems solution in the framework of the atomic project having in mind that Andrei Nikolaevich conducted in 1948 the world's first numerical experiment to solve a large and complicated problem connected with modeling of a hydrogen bomb explosion for a layered model by A. D. Sakharov with a parallel process computation.

The development of informative-mathematical aspects of these projects has led to the golden age of the kinetic theory of the neutrons, charged particles, radiation of different nature transfer.

The progressive scientific community marked in 2007 the following three epochal jubilees, which are separated by the 50-year periods:

— the 150th anniversary of K. E. Tsiolkovsky's birth (05.09.1857–19.09.1935);

— the 100th anniversary of the S. P. Korolev's birth (12.01.1907–14.01.1966);

— the 50th anniversary of the launch of the first artificial Earth satellite (04.10.1957).

It should be noted that Soviet researchers were the first in the world history of science and technology to elaborate and construct the intercontinental rocket and implement launches of the artificial Earth satellites (AES), the piloted space vehicles (PSV) with cosmonauts on board and the long-lived orbital stations (LOS). These advancements in the Soviet Union (the USSR) were followed by specialists from the USA, France and other countries.

The first Soviet computers were put in the Institute for Applied Mathematics of the USSR Academy of Sciences. M. V. Keldysh established that Institute in 1953, was its first Director and worked as Major Theoretician on space exploration. Department «Kinetic Equation» was established in 1955, which took an active part in works on both projects. That Department was established by Professor E. S. Kuznetsov, who laid the foundation for the Mathematical Department of the Institute of Physics and Power Engineering (Obninsk) in 1952. Academician G. I. Marchuk had been the Chair of that Department after Professor E. S. Kuznetsov since 1955. Professor E. S. Kuznetsov was co-operating with Academician I. V. Kurchatov in the 1950s. Academician S. L. Sobolev was deputy of I. V. Kurchatov on computational works.

In the 1950s V. S. Vladimirov, E. S. Kuznetsov, and G. I. Marchuk were the chief specialists in the transfer theory, oriented to an atomic project and nuclear power. The present review is aimed at applications of the radiation transfer theory in space research projects.

In this paper a review is presented of the pioneering space research carried out under the leadership of M. V. Keldysh, K. Ya. Kondratyev, and G. I. Marchuk and with their direct participation. Some of these works concerned with radiation transfer in the atmosphere and remote sensing of the atmosphere and the Earth surface are cited. The pioneering achievements of scientists from the USSR and Russia in the field of exploration of the Earth from space are described. Fundamental and applied studies in atmospheric optics and other disciplines, connected with modeling of radiation transfer in the atmosphere — underlying surface and with analysis of data observed from space, have been shown in their development from the 1950s up to now. Mathematical modeling is considered as an indispensable tool of preparing and conducting space projects concerning studies of the Earth from space.

1. TO THE HISTORY OF ATMOSPHERIC AND OPTICAL RESEARCH FROM SPACE

The role of the Earth sciences space research is currently enhanced as this is a particular discipline that must unite efforts of various specialists and enable all of them to use a general language of space research. The relevant consideration of the information and mathematical basis of the Earth sciences space research gets important to combine these interdisciplinary studies.

The Earth sciences space studies serve to obtain operative information about natural and man-made disasters of an anthropogenic and natural origination together with space monitoring of the environmental global changes including the environmental disasters with slowing down action.

Development and elaboration of a new mathematical tool is needed to solve direct and inverse problems of the radiation transfer theory in natural media to be realized on high-productive supercomputer as a follow-on of the international cooperation on air-space global monitoring of the Earth and of the international global project concerning studies of the evolution of the Earth, climate and natural disasters (about 50 countries signed an international Agreement late in 2004).

And how it started? Who laid down the basis of remote sensing and obtained the first results? The key role in the history of the native space rocket techniques belongs to Sergei Pavlovich Korolev who was a distinguished leader in the field of the Soviet cosmonautics. Mstislav Vsevolodovich Keldysh played a significant role, too. He was the Major Theoretician in cosmonautics and was really an organizer of the mathematical school that provided solutions to many practical problems of the rockets dynamics, ballistics and navigation of flights into space as well as laid down the basis of remote sensing of the Earth from space.

The first meeting on the artificial satellite of the Earth was held in February 1954 in Keldysh's cabinet (the Memorial museum-cabinet of Academician M. V. Keldysh since 1981). A letter to the Central Committee of the Communist Party of the Soviet Union and to the Council of Ministers was prepared in 1954 by M. V. Keldysh, S. P. Korolev and M. K. Tikhonravov with the proposal to create and launch an artificial Earth satellite. Decision on the Baikonur cosmodrome construction was made on 12 February 1955.

Space research was first spoken about in 1955. M. V. Keldysh was the main organizer in space research. A letter from the USSR Academy of Sciences was sent in accordance with his instruction in the summer of 1955 to the scientists of different specialities with the only question: *How may space be used?* There were many different opinions and proposals. To convince the USSR leader in the necessity to discover space and the space satellites and ships launches, M. V. Keldysh selected *the two major problems: intelligence service and observations of the Earth.* Many science and technology institutions were formed around the problems. A letter with the space research program was directed from the USSR Academy of Sciences to the Central Committee of the Communist Party of the Soviet Union and the

Council of Ministers in November 1955. A special commission was organized in January 1956 on the «D» object that was headed by M. V. Keldysh. S. P. Korolev and M. K. Tikhonravov were his deputies. The first satellite had the «PS-1» code that meant «the first simple satellite». «Ship-satellite» was called a satellite with cosmonaut on board by S. P. Korolev.

A ceremonial meeting was held on 17 September 1957 in the Column Hall of the Unions House, dedicated to the 100th anniversary of K. E. Tsiolkovsky's birth, who was the founder of the jet propulsion theory and who had a dream about interplanetary flights and space exploration. S. P. Korolev, Corresponding Member of the USSR Academy of Sciences, made a presentation regarding a practical significance of K. E. Tsiolkovsky's science and technology proposals to develop the rocket technology and the artificial Earth satellite launches. The first in the history of the mankind artificial Earth satellite was just launched in a pair of weeks on 4 October 1957.

«The three K» meeting (I. V. Kurchatov, M. V. Keldysh, and S. P. Korolev) took place after the first launches of the unpiloted space vehicles and further the atomic and space projects developed in parallel. The meeting was considered as «a wedding» and a giant strategic problem was set up of a «forestall of any rocket start from space» within the project of the «rocket-nuclear shield» that is a major deterrent factor of global wars up to date. Military and civil, science and technology and research projects have been conducted as mutually additional. Special Intergovernmental Science and Technology Councils have been created with many sections, where researchers and constructors from the academia Institutes and branches-organizations played a main role to decision making in space flight programs, information generalization and space observational data exchange.

In the USSR the first artificial «Sputnik» was launched into space on 4 October 1957 and Yuri Alexeevich Gagarin was the first cosmonaut, who made the first flight around the Earth on 12 April 1961. The launch of the first artificial satellite of the «Cosmos» series on 16 March 1962 set out the ground for the implementation of a complex scientific programme for optical investigations of the circumterrestrial space and the Earth itself.

The radiation field of the «atmosphere – Earth surface» system is an indispensable component of the Earth's ecosystem and climate. Radiation characteristics of the system are, in turn, carriers of information about the environment condition, atmosphere, cloudiness, hydrometeors and miscellaneous outbreaks of pollutants (as a result of anthropogenic hazards, military actions, forest and steppe fires, volcano eruptions, etc.).

The role of remote sensing Earth's sciences has now been greatly increasing as the discipline that enables one to combine efforts of various specialists and to allow them to «speak together» in «a joint language» of satellite observations (Kozoderov et al., 1998; 2000). The remote sensing Earth's sciences discipline fosters applications of the operative information about the natural disasters and the environmental problems of natural and anthropogenic origination by monitoring of the Earth from space.

Information and mathematical principles of remote sensing Earth's sciences are getting more and more important since they serve to link interdisciplinary studies regarding the information content assessment for remotely sensed and ground-based data, analysis and interpretation techniques of airspace images, understanding of predictability problem of global and regional change using temporal data set analysis of satellite data, optimization and efficiency research of observation systems for various applications (Kondratyev et al., 1985; 1992). Imbedded by interests of international cooperation in the field of airspace global monitoring of the Earth, an international global project on Earth evolution research, climate and dangerous phenomena, an elaboration of new mathematical procedures is required to solve direct and inverse problems of radiation transfer theory in natural media to be realized on high-productive multiprocessor supercomputers (Sushkevich, 2005).

The information and mathematical as well as theoretical and calculation studies were conducted in the USSR by the following three leading communities of scientists (Sushkevich, 1997; Sushkevich and Maksakova, 1999a; 1999b; 1999c):

— under K. Ya. Kondratyev and V. V. Sobolev in Leningrad;

— under M. V. Keldysh, E. S. Kuznetsov and A. M. Obukhov in Moscow;

— under G. I. Marchuk and G. A. Mikhailov in Novosibirsk.

Kirill Yakovlevich Kondratyev had taken the Chair of Atmospheric Physics at the Physical Faculty of the State University in Leningrad since 1957. He had published monographs concerning radiation heat exchange (Kondratyev et al., 1954; 1956) by that time. K. Ya. Kondratyev had been one of the leading heads and organizers of space research and the first space and ground-based experiments since the first years of space exploration and formation of observation systems and remote sensing as well as satellite meteorology (Kondratyev et al., 1985; 1992).

E. S. Kuznetsov, A. M. Obukhov, K. Ya. Kondratyev, and G. I. Marchuk interacted in the 1940s and 1950s in the Geophysical Institute of the USSR Academy of Sciences. O. Yu. Shmidt established that Institute. K. Ya. Kondratyev and G. I. Marchuk had been not only leaders in space research, but also friends since their student times in Leningrad University. K. Ya. Kondratyev had cooperated for many years with the laboratory headed by G. A. Mikhailov in Novosibirsk (Kondratyev et al., 1977), who also graduated from Leningrad University. In fact, most scientists who dealt with space exploration graduated either from the Lomonosov Moscow State University or from the Leningrad State University.

The author of this paper started to work in the Keldysh Institute after graduation from the Physical Faculty of the Lomonosov Moscow State University and was personally acquainted and cooperated with all the listed scientists in the period of 1961–1965.

For thousands of years mankind has studied the stars and the planets of the solar system by visual and later by photographic and photo-electric observations. But the Earth planet had remained uncovered by this research by the 1950s. Only reflected light from the Moon enabled the integral radiation of the Earth to be estimated at that time. Wide opportunities for exploring the radiation characteristics of our planet emerged as a result of the construction and development of rockets and space vehicles.

Experience in the space programme of the USSR confirmed that prospects were real for piloted space vehicles, long-lived orbital stations, automated inter-planet stations (AIS), spacecraft vehicles (CV), and artificial Earth satellites for studies of the natural media and the natural resources from space. The first «Zarya» module (Russia) of the First International Space Station (ISS–MKS), the present and future space laboratory, was launched on 20 November 1998.

Optical studies, based on the radiation transfer, were an important part of the first space research programmes. These studies were represented by visual observations, photometric and spectral exploration of twilight and daylight atmosphere to investigate vertical profiles of optically active components (aerosol, ozone, gas admixtures), analysis of reflection spectra of various types of natural formations on the Earth surface as well as estimation of the influence of the atmosphere on spectral radiances and contrasts of the natural targets during their observations from space. The analysis of spectra of natural formations from space (spectral radiances, coefficients of intensity of spectral radiation, spectral contrasts) revealed a principal opportunity to solve fundamental and applied problems of remote sensing Earth's sciences. PSV and LOS piloted by cosmonauts played a great role in the achievements of Soviet space research (Lazarev et al., 1979; 1987; Malkevich, 1973).

K. Ya. Kondratyev was active in the supporting the development of piloted flights to carry out the scientific experiments from space and played an important role in research elaboration on orbital stations.

Yu. A. Gagarin's flight around the Earth on board the PSV «Vostok» during 108 minutes on 12 April 1961 was the first outlook on the Earth from space, i.e., these were the first visual observations of the Earth from space.

G. S. Titov's flight aboard the PSV «Vostok-2» (August 1961) as well as A. G. Nikolaev's and P. R. Popovich's flights aboard the PSV «Vostok-3» (August 1962) extended the perception of the possibilities of visual observations. In the beginning of his second circuit on «Vostok-2» around the Earth on 6 August 1961, G. S. Titov was the first person who conducted film survey from space.

V. F. Bykovsky, on board the PSV «Vostok-5», and V. V. Tereshkova, on board the PSV «Vostok-6» (June 1963), made the first photos of daylight and twilight horizons of the Earth. In fact, this was the first scientific experiment from space (Rozenberg and Nikolaeva-Tereshkova, 1965b). The onset took place of the instrumental studies of optically active components of the atmosphere from a PSV (Rozenberg et al., 1971a; 1971b; 1980). G. V. Rozenberg conducted theoretical accounting for the experiments (Rozenberg, 1942; 1963; 1965a). T. A. Sushkevich carried out the mathematical modeling.

Spectrographic experiments started from the PSV «Soyuz-5» (B. V. Volynov, E. V. Khrunov, January 1969) under K. Ya. Kondratyev's leadership. The world's first spectra of radiation of the atmosphere and the Earth surface in visual spectral region were obtained at that time. Photographic and spectrographic survey of the Sun rise enabled one to simultaneously receive the synergetic information about the spatial and spectral structure of solar radiation and that of the Earth's atmosphere and, in particular, about the aerosols and ozone layers.

A combined experiment was conducted from the PSV «Soyuz-7» (V. N. Volkov, V. V. Gorbatko, October 1969) under K. Ya. Kondratyev's leadership, concerning the photographic engineering of separate plots of the USSR area from the aircrafts and from space to study the effects of the atmospheric transfer function on the results of the optical observations from space. Another experiment from the PSV «Soyuz-9» (A. G. Nikolaev, V. I. Sevastyanov, June 1970) was conducted for meteorological forecasting purposes.

The first spectrographic measurements of the Earth surface parallel with survey in several different spectral bands took place from the PSV «Soyuz-12» (V. G. Lazarev, O. G. Makarov, September 1973) and further from the PSV «Soyuz-13» (P. I. Klimuk, V. V. Lebedev, December 1973).

A. V. Filipchenko and N. N. Rukavishnikov from the PSV «Soyuz-16» (December 1974) were the first who conducted photo survey of the Earth surface and atmosphere in polarized light on the route nearly 30 thousand km (Sushkevich et al., 1984; 1990; 2005).

Optical studies were conducted by A. A. Leonov and V. N. Kubasov in accordance with the «Soyuz–Apollo» programme from the PSV «Soyuz-19» (July 1975) and modeled by T. A. Sushkevich (see Rozenberg et al., 1980). The development of science and technology methods and equipment to study the Earth surface from space and its geological and geographical characterization took place from the PSV «Soyuz-22» (V. F. Bykovsky, V. V. Aksenov, September 1976). The first multi-scale full radiation experiment was conducted by using the relevant data together with data obtained from aircrafts.

V. V. Kovalenok and V. V. Ryumin, the Soviet cosmonauts, completed the scientific space programme from the PSV «Soyuz-25» (October 1977).

The programme of the visual and instrumental optical observations of the Earth was greatly extended after the launch of the first LOS «Salyut», in April 1971. The first docking of the PSV «Soyuz-10» (V. A. Shatalov, A. S. Eliseev, N. N. Rukavishnikov) and LOS «Salyut» occurred on 24 April 1971. Starting from the LOS «Salyut-3» (June 1974) and on all further LOS «Salyut-4» (December 1974), LOS «Salyut-5» (June 1976), LOS «Salyut-6» (September 1977), LOS «Salyut-7» (April 1982), «Mir» (1986), the programme of space observations of the Earth was continued.

The first complex international experiment «Kursk-1985» was carried out in July 1985 with the simultaneous observations from the LOS «Salyut-7», artificial satellites, aircraft and helicopter campaign (Kondratyev et al., 1985; 1992; Kozoderov et al., 1998; 2000; Curran et al., 1990). The stages of planning, development and achievements of the listed optical experiments from space were published in many monographs, proceedings and thematic books.

2. MATHEMATICAL MODELING AND SPACE PROJECTS

Space research is such an area of the fundamental and applied studies that could not develop from the first steps of its origin without the use of computers. Space exploration has served as a significant factor in the improvement of computers and the formation of new research directions connected with mathematical modeling of the radiation field of the Earth, image transfer theory, vision theory, the theory of data processing and pattern recognition, etc. To provide information and mathematical tools is seen as a due composite part of any space project.

The radiation field of the «atmosphere – Earth surface (land, ocean)» system is an indispensable component of the biosphere, ecosystems and climate of the Earth, on the one hand. The radiation characteristics of the system, on the other hand, are carriers of information about the environment condition, the atmosphere, cloudiness, ocean, anthropogenic disasters, military actions, forest and steppe fires, volcanic explosions, as well as the industry and transport infrastructure.

Radiation, registered by various instruments, is a major source of remote sensing information about the composition and physical properties of the atmosphere and the surface of different planets. The external flux of short-wave solar radiation (in ultraviolet, visible and near infrared spectral bands), as well as the thermal long-wave radiation (in infrared and microwave bands), is the major source of the relevant passive observation systems.

The theory of radiation transfer is applicable for the appropriate mathematical description of the transmission, scattering and absorption of radiation in natural media. In 1989 scientists celebrated the 100th anniversary of the transfer equation that was formulated by O. D. Chwolson (Chwolson, 1889; Ivanov, 1994), a Russian Professor. The theory of radiation transfer was first applied mainly to the field of optics and astrophysics. Fundamental contribution to the transfer theory in the 20th century, as connected with space research problems, has been made by the following Soviet scientists: V. A. Ambartsumjan, V. V. Sobolev, V. V. Ivanov, I. N. Minin, O. I. Smokty, G. A. Mikhailov as well as by G. I. Marchuk,

G. V. Rozenberg, E. S. Kuznetsov and his successors E. M. Feigelson, L. M. Romanova, M. S. Malkevich, M. V. Maslennikov, T. A. Germogenova, M. G. Kuzmina, T. A. Sushkevich, S. A. Strelkov, etc.

We give some more details of the development of the spherical models of the radiation transfer in the system «atmosphere – Earth surface». Complications in carrying out space research and the realization of space projects were due to necessity to deal with the following interconnected problems («a closed circle»):

— to measure characteristics of the radiation field of the Earth, preliminary estimates of calculations of these characteristics were needed using the radiation transfer models with the multiple scattering and absorption of the solar radiation;

— to model radiation transfer in the «atmosphere – Earth surface» system, data about spatial and spectral distributions of optical and geophysical parameters, describing the interaction of the solar radiation with components of the Earth atmosphere and surface were also needed.

Mathematical Statement of the Problem. We consider the problem of radiation transfer in the Earth atmosphere in approach of a spherical shell, upon which an external parallel flux is incident. Our interest is the problem of calculation of the Earth radiation field in scale over all planet (various conditions of illumination, horizon, twilights, shadow, polar regions). Total $\Phi_\lambda(\mathbf{r},\mathbf{s})$ in point $A(\mathbf{r})$ having radius vector $\mathbf{r}$ in direction $\mathbf{s}$ is found as solution of general boundary-value problem (BVP) of transfer theory

$$K\Phi = F^{\text{in}}, \quad \Phi\,|_t = F^t, \quad \Phi\,|_b = \varepsilon R\Phi + F^b, \quad K \equiv D - S. \tag{1}$$

Transfer operator

$$D \equiv (\mathbf{s}, \text{grad}) + \sigma_{\text{tot}}(\mathbf{r}), \quad (\mathbf{s}, \nabla\Phi) =$$
$$= \cos\vartheta \frac{\partial\Phi}{\partial r} + \frac{\sin\vartheta\cos\varphi}{r}\frac{\partial\Phi}{\partial\psi} - \frac{\sin\vartheta}{r}\frac{\partial\Phi}{\partial\vartheta} + \frac{\sin\vartheta\sin\varphi}{r\sin\psi}\frac{\partial\Phi}{\partial\eta} - \frac{\sin\vartheta\sin\varphi\,\text{ctg}\,\psi}{r}\frac{\partial\Phi}{\partial\varphi},$$

collision integral — source function

$$B(\mathbf{r},\mathbf{s}) \equiv S\Phi = \sigma_{\text{sc}}(\mathbf{r}) \int\limits_{\Omega} \gamma(\mathbf{r},\mathbf{s},\mathbf{s}')\Phi(\mathbf{r},\mathbf{s}')\,d\mathbf{s}', \quad d\mathbf{s}' = \sin\vartheta' d\vartheta' d\varphi',$$

reflection operator — integral

$$[R\Phi](\mathbf{r}_b,\mathbf{s}) = \int\limits_{\Omega^-} q(\mathbf{r}_b,\mathbf{s},\mathbf{s}^-)\Phi(\mathbf{r}_b,\mathbf{s}^-)\,d\mathbf{s}^-, \quad \mathbf{s}\in\Omega^+.$$

At $R \equiv 0$ (or $\varepsilon = 0$) the first boundary-value problem

$$K\Phi_0 = F^{\text{in}}, \quad \Phi_0\,|_t = F^t, \quad \Phi_0\,|_b = F^b. \tag{2}$$

Solution of general BVP (1) is superposition $\Phi = \Phi_0 + \Phi_q$.

Problem to evaluate contribution of reflecting underlying surface

$$K\Phi_q = 0, \quad \Phi_q\,|_t = 0, \quad \Phi_q\,|_b = \varepsilon R\Phi_q + \varepsilon E, \quad E(\mathbf{r}_b,\mathbf{s}^+) \equiv R\Phi_0. \tag{3}$$

Solution of the first boundary-value problem

$$K\Phi = 0, \quad \Phi\,|_t = 0, \quad \Phi\,|_b = f(\mathbf{s}^h; r_\perp, \mathbf{s}), \quad r_\perp = (\psi, \eta) \in \Omega, \quad dr_\perp = \sin\psi\, d\psi\, d\eta, \tag{4}$$

being distribution, can be written in the form of linear functional — superposition integral

$$\Phi(\mathbf{s}^h; r, r_\perp, \mathbf{s}) \equiv (\Theta, f) \equiv \\ \equiv \frac{1}{2\pi}\int\limits_{\Omega^+} d\mathbf{s}_h^+ \frac{1}{4\pi}\int\limits_{\Omega} \Theta(\mathbf{s}_h^+; r, r_\perp - r'_\perp, \mathbf{s}) f(\mathbf{s}^h; r'_\perp, \mathbf{s}_h^+)\, \sin\psi'\, d\psi'\, d\eta', \tag{5}$$

its kernel is influence function $\Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s})$ — solution of the first BVP (Model 1)

$$K\Theta = 0, \quad \Theta\,|_t = 0, \quad \Theta\,|_b = f_\delta$$

with parameter $\mathbf{s}_h^+ \in \Omega^+$ and source $f_\delta(\mathbf{s}_h^+; r_\perp, \mathbf{s}) = \delta(r_\perp)\delta(\mathbf{s} - \mathbf{s}_h^+)$.

If source $f(r_\perp)$ is isotropic and horizontally nonhomogeneous, then the solution of the first BVP (4) is linear functional — convolution

$$\Phi(r, r_\perp, \mathbf{s}) = \mathcal{F}_c(f) \equiv (\Theta_c, f) \equiv \frac{1}{4\pi}\int\limits_{\Omega} \Theta_c(r, r_\perp - r'_\perp, \mathbf{s}) f(r'_\perp)\, \sin\psi'\, d\psi'\, d\eta'$$

with kernel — influence function

$$\Theta_c(r, r_\perp, \mathbf{s}) = \frac{1}{2\pi}\int\limits_{\Omega^+} \Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s})\, d\mathbf{s}_h^+,$$

which must satisfy BVP (4) with axial symmetry (Model 2)

$$K_c\Theta_b = 0, \quad \Theta_b\,|_t = 0, \quad \Theta_b\,|_b = \delta(r_\perp).$$

For anisotropic and horizontally uniform source $f(\mathbf{s}^h; \mathbf{s})$

$$\Phi(\mathbf{s}^h; r, \mathbf{s}) = \mathcal{F}_r(f) \equiv (\Theta_r, f) \equiv \frac{1}{2\pi}\int\limits_{\Omega^+} \Theta_r(\mathbf{s}_h^+; r, \mathbf{s}) f(\mathbf{s}^h; \mathbf{s}_h^+)\, d\mathbf{s}_h^+$$

with linear functional kernel

$$\Theta_r(\mathbf{s}_h^+; r, \mathbf{s}) = \frac{1}{4\pi}\int\limits_{\Omega} \Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s})\, \sin\psi\, d\psi\, d\eta.$$

Influence function Θ_r is solution of one-dimensional spherical first BVP with azimuth dependence (Model 3)

$$K_r\Theta_r = 0, \quad \Theta_r\,|_t = 0, \quad \Theta_r\,|_b = \delta(\mathbf{s} - \mathbf{s}_h^+).$$

At isotropic and horizontally homogeneous source the solution of first BVP (4)

$$\Phi(r, \mathbf{s}) = fW(r, \mathbf{s}), \quad f = \text{const},$$

is calculated by influence function

$$W(r,\mathbf{s}) = \frac{1}{2\pi}\int\limits_{\Omega^+} d\mathbf{s}_h^+ \frac{1}{4\pi}\int\limits_{\Omega} \Theta(\mathbf{s}_h^+; r, r_\perp, \mathbf{s}) \sin\psi\, d\psi\, d\eta =$$

$$= \frac{1}{4\pi}\int\limits_{\Omega} \Theta_c(r, r_\perp, \mathbf{s}) \sin\psi\, d\psi\, d\eta = \frac{1}{2\pi}\int\limits_{\Omega^+} \Theta_r(\mathbf{s}_h^+; r, \mathbf{s})\, d\mathbf{s}_h^+,$$

which is also called transport function taking account of multiple scattering and must be determined as solution of one-dimensional spherical first BVP (Model 4)

$$K_r W = 0, \quad W\,|_t = 0, \quad W\,|_b = 1.$$

On the basis of regular perturbation theory by means of series

$$\Phi_q(\mathbf{s}^h; \mathbf{r}, \mathbf{s}) = \sum_{k=1}^{\infty} \varepsilon^k \Phi_k,$$

general BVP (3) is reduced to recursive system of first BVP (4)

$$K\Phi_k = 0, \quad \Phi_k\,|_t = 0, \quad \Phi_k\,|_b = E_k \tag{6}$$

with source $E_k = R\Phi_{k-1}$ for $k \geqslant 2$, $E_1 = E$.

Operation describing single act of radiation interaction with boundary by influence function

$$[Gf](\mathbf{s}^h; \mathbf{r}_b, \mathbf{s}) \equiv R(\Theta, f) = \int\limits_{\Omega^-} q(\mathbf{r}_b, \mathbf{s}, \mathbf{s}^-)(\Theta, f)\, d\mathbf{s}^-.$$

Solutions of system first BVP (6) are found as linear functionals (5)

$$\Phi_1 = (\Theta, E), \quad \Phi_k = (\Theta, R\Phi_{k-1}) = (\Theta, G^{k-1}E).$$

Asymptotically exact solution of general BVP (3) is obtained as linear functional (5) — optical transfer operator

$$\Phi_q = (\Theta, Y),$$

where optical image «scenario» or underlying surface brightness

$$Y \equiv \sum_{k=0}^{\infty} G^k E = \sum_{k=0}^{\infty} R\Phi_k$$

is sum of Neumann series to multiplicity radiation reflection from boundary taking account of multiple scattering in medium.

The «scenario» must satisfy the Fredholm equation of the second kind

$$Y = R(\Theta, Y) + E,$$

which is called equation of «near-ground image». The total radiation or «space image» is described by functional

$$\Phi = \Phi_0 + (\Theta, Y).$$

The approach to solve the general BVP for planar model is similar to the approach for solution of the general BVP of radiation transfer in spherical shell model.

Spherical multidimensional models of radiation transfer, in spite of their being complicated and vast in numerical realization on the first generations of computers in the USSR (M-20, BESM-4, BESM-6, AS-6 — the Soviet computers in the 1960s–1980s), had been exclusively actual due to the projection and the construction of rocket space systems, investigation of near and far space, the organization and the implementation of the space research and the space observations. Scientific and fundamental problems of the meteorology, the oceanography, the physics of the atmosphere, the investigation of natural resources, the remote sensing of the atmosphere, land surface, ocean, cloudiness, and hydrometeors had been developed in parallel with the listed problems.

Theoretical and calculation studies, while the projecting and realizing the first space vehicles as well as the first optical experiments in space, had been implemented by three leading communities of experts on mathematical modeling of the radiation transfer in natural media using computers (in Leningrad, Moscow, Novosibirsk — see above).

Groups in Leningrad had been working in Leningrad State University and Main Geophysical Observatory under the leadership of V. V. Sobolev and K. Ya. Kondratyev. Monographs by V. A. Ambartsumjan (Ambartsumjan, 1960) and V. V. Sobolev (Sobolev, 1956; 1972) were dedicated to the energy radiation transfer in the atmospheres of stars and planets. Both scientists had occupied the Astrophysics Chair of the Mathematical and Mechanical Faculty of Leningrad State University. Many successors of V. V. Sobolev had started their research on the theory of multiple scattering of radiation at the Astronomical Observatory of Leningrad University in the mid-1950s. The Leningrad school of these successors had been formed by the beginning of the 1960s. In particular, I. N. Minin was one of its representatives. During his work in the Main Geophysical Observatory (1954–1957), together with Prof. K. S. Shifrin, he solved a number of practical problems of atmospheric optics. Following K. Ya. Kondratyev's initiative, he was incorporated into studies connected with space exploration. The first publications were prepared by V. V. Sobolev and I. N. Minin, concerning approximate models of light scattering in a spherical atmosphere (Sobolev and Minin, 1962; 1963; 1964). The relevant results were imbedded into their separate monographs (Sobolev, 1972; Minin, 1988). V. V. Sobolev, I. N. Minin and O. I. Smokty elaborated the first combined plane and spherical model of the Earth's atmosphere in V. V. Sobolev's approach that included single scattering of radiation for any spherically symmetrical layer, while its multiple scattering was taken into account partly in a diffuse approximation for any plane layer. Oleg Ivanovich Smokty, one of the successors of K. Ya. Kondratyev and V. V. Sobolev, has been a leading scientist since the 1960s on theoretical and calculation studies on mathematical modeling or computer science as this discipline is called now (Smokty, 1967a; 1967b; 1967c; 1969; 1986; 1989).

A scientific school on the radiation transfer theory had been formed by the 1950s in Moscow by E. S. Kuznetsov's successors. E. S. Kuznetsov had dealt since 1925 with the sun and heat radiation transfer in the atmosphere and sea in the context of the problems of aviation, climate, weather forecast, meteorology, harvest, etc. (Kuznetsov, 2003, «Selected Scientific Works», edited by T. A. Sushkevich). He was editor of the publication in Russian in 1953 of the book «Radiation Energy Transfer» by S. Chandrasekhar. Scientists from the Keldysh Institute and from the Institute of Atmospheric Physics of the USSR Academy of Sciences (established in 1956), where A. M. Obukhov was the first director, were incorporated into this school. Theoretical and calculation radiation studies were conducted in the Kinetic Equations Department of the Keldysh Institute. M. V. Maslennikov, T. A. Germogenova, M. G. Kuzmina, T. A. Sushkevich (the youngest of E. S. Kuznetsov's successors) worked in this Department. The theoretical and experimental studies of the radiation transfer were conducted in G. V. Rozenberg Department of the Obukhov Institute, where M. S. Malkevich, E. M. Feigelson, L. M. Romanova and other E. S. Kuznetsov's successors had worked.

More than 40 years ago for the first time in the world, T. A. Sushkevich elaborated a model of solar radiation transfer in a spherical atmosphere of the Earth on a planetary scale (Sushkevich, 1966). This model gave a basis for obtaining pioneer results on the remote sensing of the aerosols and ozone layers as well as for solving many applied problems during the process of the commencing stage of the space exploration and as development progresses of the space research. These results were awarded by the Prize by the Government of the Russian Federation in the field of science and technology in 2002 for the work «Elaboration and Development of Methods and Technologies of Airspace Monitoring of Natural Media» (co-authors: V. G. Bondur, A. S. Victorov, A. M. Volkov, A. S. Isaev, V. V. Kozoderov, G. N. Korovin, L. A. Makridenko, V. A. Malinnikov, G. M. Polistchuk, V. I. Sukhikh, S. A. Ushakov, V. P. Savinykh, O. I. Smokty, T. A. Sushkevich). T. A. Sushkevich elaborated a deterministic (not stochastic) approach to modeling of the global radiation field of the Earth (Sushkevich, 1966; 2005).

This was an iterative characteristics method that enabled the realization on the Soviet computers of a global spherical model of the radiation field in the «atmosphere–Earth» system. The characteristics are the fundamental solutions of the transfer differential equation described of the photon's trajectories (Sushkevich et al., 1990; Sushkevich, 1966; 2005). A comparative analysis of the methods, which were used for the interpretation of the first space data, in particular, of spectrophotometer measurements of the horizon and background of the Earth as well as of space during sunrise/sunset, was conducted. The first algorithms of the characteristics method (with interpolation and without it) for a spherical heterogeneous atmosphere with axial symmetry were elaborated by T. A. Sushkevich in 1965–1966 (Sushkevich, 1966).

Partial cases (under significant restrictions on the structure of scattering and absorbing media as well as on illumination and observation conditions) of the radiation transfer integration in a single scattering approximation are described in the works of O. A. Avaste and O. I. Smokty. Later on and at present practically in all realizations of the spherical problem by the Monte-Carlo method the single scattering approximation is calculated by a method of the integration on characteristics which are coincident with trajectories of light beams (photons).

A laboratory for statistical modeling and Monte-Carlo methods was formed in 1964–1965 in the Computing Centre of the Siberian Branch of the USSR Academy of Sciences, Novosibirsk (now the Institute of Computational Mathematics and Mathematical Geophysics of the Siberian Branch of the Russian Academy of Sciences). G. I. Marchuk was the founder and the first director of that Centre and G. A. Mikhailov has been the Head of the Laboratory until now. The first algorithms of local calculations by the Monte-Carlo method (stochastic approach) for a spherical model of the Earth given by an inhomogeneous gas and aerosol shell, illuminated by an external parallel flux of solar beams, were conducted in the Computer Centre of the Siberian Branch of the USSR Academy of Sciences under the leadership of G. I. Marchuk and G. A. Mikhailov (Marchuk et al., 1967a; 1967b; 1968; 1976). The mathematical tool of conjugated equations proposed by G. I. Marchuk and developed by G. A. Mikhailov, M. A. Nazaraliev, V. S. Antyufeev, and R. A. Darbinyan had played an essential role in effectiveness of these algorithms. In fact, algorithms for the solution of direct and inverse problems of the radiation transfer theory were proposed for the first time in these studies for finding answers to major questions concerning the interpretation value of radiation information.

This was a starting point, i.e., the Monte-Carlo method for the first time in the world practice was applied to the modeling of solar radiation transfer in the atmosphere of the Earth. No person now exists who could be doubtful in effectiveness of the Monte-Carlo method, that has made practical benefits in using updated supercomputers with parallel architecture. The community of Soviet scientists represented by G. I. Marchuk (head of the work), G. I. Mikhailov, S. M. Ermakov, V. G. Zolotukhin, and N. N. Chentsov was awarded by the State Prize for a number of works on development and application of statistical modeling method to solve multidimensional problems of radiation transfer theory in 1979.

O. A. Avaste had elaborated an approximate approach to radiation transfer. V. V. Sobolev's method had been developed by L. G. Titarchuk. Doctor's dissertations by I. N. Minin, O. I. Smokty, G. A. Mikhailov, T. A. Sushkevich, L. G. Titarchuk, M. A. Nazaraliev, V. S. Antyufeev included the spherical models of radiation transfer in planetary atmospheres. T. A. Germogenova, M. V. Maslennikov, A. M. Obukhov, M. S. Malkevich, G. V. Rozenberg, A. B. Sandomirsky, A. I. Lazarev, E. O. Fedorova, V. P. Kozlov, V. N. Sergeevich, I. I. Koksharov, Ch. J. Villman, O. A. Avaste, V. E. Plyuta, G. M. Grechko and others took part in the study of the relevant problems and discussions of the results obtained.

The first attempts to solve the spherical problem were undertaken in the USA by Sekera and Lenoble (Lenoble et al., 1961; 1985; 1990). They proposed to use the sequential approximations method, corresponding to solution expansion on a small parameter taking a plane problem solution as a first approximation and a ratio of effective height of the homogeneous atmosphere to the Earth's radius as the small parameter. The method of the invariant imbedding based on the invariant principle (Lenoble, 1985; 1990) has remained on a theoretical level without its practical realization. A multi-flux approach was realized in the Dodecation Approach to Radiative Transfer (DART) tool in the USA (Lenoble, 1985; 1990). The research results of the International Working Group, which was organized by the Radiation Commission of the International Association on Meteorology and Atmospheric Sciences, are presented in a book by editor J. Lenoble (Lenoble, 1985; 1990 in Russian).

CONCLUSION

Space technologies of remote sensing of the Earth, its atmosphere, land surface and ocean have taken an international character in recent years and have got a particular interest for many applications. Their role and significance in the first space experiments, conducted under the leadership and with participation of K. Ya. Kondratyev, G. I. Marchuk and other Soviet scientists and their publications, which are learned by many scientists, is important to emphasize in the year of the 50th anniversary of the first artificial satellite launch. In fact, the basis of the updated techniques and the remote sensing means and also the methods of the numerical solution of the multidimensional boundary-value problem of radiation transfer was laid down by the Soviet scientists mentioned above.

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