

# PHYSICS AROUND THE QCD (TRI)CRITICAL ENDPOINT AND NEW CHALLENGES FOR FEMTOSCOPY

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On the basis of exactly solvable models with the tricritical and critical endpoints, the physical mechanism of endpoints formation which is similar to the usual liquids is discussed. It is demonstrated that the necessary condition for the transformation of the 1st order deconfinement phase transition into the 2nd order phase transition at the (tri)critical endpoint is the vanishing of surface tension coefficient of large/heavy QGP bags. Using the novel model of the confinement phenomenon, it is argued that the physical reason for the cross-over appearance at low baryonic densities is the negative value of QGP bag surface tension coefficient. This implies the existence of highly nonspherical or, probably, even fractal surfaces of large and heavy bags at and above the cross-over, which, perhaps, can be observed via some correlations. The model with the tricritical endpoint predicts that at the deconfinement transition line the volume (mass) distribution of large (heavy) QGP bags acquires the power law form at the endpoint only, while in the model with the critical endpoint such a power law exists inside the mixed phase. The role of finite width of QGP bags is also discussed.

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## 1. WHAT IS MISSING IN THE STATISTICAL MODELS OF STRONGLY INTERACTING MATTER EQUATION OF STATE?

Almost 25 years ago, the first heavy-ion experiments started the searches for a new state of matter, the quark–gluon plasma (QGP). During this time there were made several nice discoveries, but the smoked gun of QGP creation is not found yet. Despite the claims that the Kink [1], the Strangeness Horn [2] and the Step [3] are the reliable signals of the onset of deconfinement [4], it is necessary to admit that, in fact, we do not exactly know what they really signal. Hence, from the present state of heavy ion physics I conclude that: (I) up to now our models are missing a few key elements which do not allow us to formulate some convincing signals of the deconfinement; (II) the low-energy programmes at RHIC (BNL), SPS(CERN), NICA (Dubna), and FAIR (GSI) will be hardly successful, even, if they «discover some irregularities», since without theoretical back up they will convince no one; (III) it is necessary to return to foundations of heavy-ion physics and start a systematic work to formulate and to account for the missing key elements.

One of the primary goals of low energy programmes mentioned above is the uncovering of the (tri)critical endpoint of the QCD phase diagram. Clearly, this task is even more hard than just the discovery of a new state of matter and, hence, I have serious doubts that it can be successful with the existing theoretical background. Indeed, tens of papers discuss possible signals of the QCD critical endpoint, but neither the physical reason of its existence nor the possible experimental consequences are under intense investigations! Furthermore,

from the very beginning it was clear that the systems studied in collisions of heavy ions at high energies are finite or even small, but up to now we have just a few general guesses on how to rigorously define the phase transition (PT) in finite systems and how the (tri)critical endpoint is modified in such systems.

Therefore, here I discuss the surface tension of quark gluon bags and their finite width which, as it is argued, are the missing two key elements. The role of surface tension of bags in generating the (tri)critical endpoint is discussed on the basis of exactly solvable statistical models with the tricritical (Model 1) [5,6] and critical (Model 2) [7,8] endpoints. Also the recent finding of the negative values of surface tension coefficient of bags [9] along with its role for the cross-over existence are discussed. In addition, I consider the finite width model of quark gluon bags [8,10,11] that sets some strict limitations on their experimental studies due to their very short lifetime. Analysis of novel physical phenomena associated with the vicinity of (tri)critical endpoint allows me to formulate the new challenges for femtoscopy whose study, I believe, is utterly necessary to qualitatively improve the present state of art.

The work is organized as follows. In the next section, the appearance of negative surface tension of QGP bags is discussed. Section 3 elucidates the role of surface tension at the (tri)critical endpoint. The new challenges for femtoscopy are formulated in Sec. 4.

## 2. SURFACE TENSION OF QUARK GLUON BAGS

The role of surface tension for QGP was discussed long ago [12,13], however, up to recently its importance for the existence of the QCD (tri)critical endpoint was not recognized. In nuclear and cluster physics the importance of the surface tension for the properties of endpoint is known from a number of exactly solvable cluster models with the 1st order PT which describe the critical endpoint properties very well. These are the Fisher droplet model (FDM) [14,15] and the simplified version of the statistical multifragmentation model (SMM) [16]. Both these models are built on the assumptions that the difference of the bulk part (or the volume dependent part) of free energy of two phases disappears at phase equilibrium and that, in addition, the difference of the surface part (or the surface tension) of free energy vanishes at the critical point. Note that such a mechanism of the critical endpoint generation is typical for ordinary liquids [14,17]. According to the contemporary paradigm at the deconfinement region, the QGP is a strongly interacting liquid [18], but two major questions are: what is the value of its surface tension and how can we measure it?

Very recently it was possible to find out the relation between the string tension  $\sigma_{\text{str}}(T)$  of the unbreakable color tube of length  $L$  and radius  $R \ll L$  which connects the static quark–antiquark pair and the surface tension coefficient  $\sigma_{\text{surf}}(T)$  of this tube [9]:

$$\sigma_{\text{surf}}(T) = \frac{\sigma_{\text{str}}(T)}{2\pi R} + \frac{1}{2} p_v(T)R, \quad (1)$$

where  $p_v(T)$  is the bulk pressure inside the tube. Equation (1) was derived by equating the free energies of confining string and the free energy of elongated cylindrical bag [9]. In fact, in deriving (1) we match an ensemble of all string shapes of fixed  $L$  to a mean elongated cylinder, which according to the original Fisher idea [14] and the results of the Hills and Dales Model (HDM) [19,20] represents a sum of all surface deformations of a given bag. Equation (1) allows one to determine the  $T$  dependence of bag surface tension coefficient, if  $R(T)$ ,  $\sigma_{\text{str}}(T)$ ,

and  $p_v(T)$  are known. Therefore, Eq. (1) opens a principal possibility to determine the bag surface tension for any temperature directly from the lattice QCD simulations. Also it allows us to estimate the surface tension at  $T = 0$ . Thus, taking the typical value of the bag model pressure which is used in hadronic spectroscopy  $p_v(T = 0) = -(0.25)^4 \text{ GeV}^4$  and inserting into (1) the lattice QCD values  $R = 0.5 \text{ fm}$  and  $\sigma_{\text{str}}(T = 0) = (0.42)^2 \text{ GeV}^2$  [21], one finds  $\sigma_{\text{surf}}(T = 0) = (0.2229 \text{ GeV})^3 + 0.5 p_v R \approx (0.183 \text{ GeV})^3 \approx 157.4 \text{ MeV} \cdot \text{fm}^{-2}$  [9].

The above results allow one to study the bag surface tension near the cross-over to QGP. The lattice QCD data indicate that near the deconfinement, i.e., for  $T \rightarrow T_{\text{dec}} - 0$ , the tube radius diverges  $R \rightarrow \infty$  and the string tension vanishes as [21]

$$\sigma_{\text{str}}(T) R^k \rightarrow \omega_k > 0, \quad (2)$$

with  $k = 2$ . However, one can extend a range for the power  $k > 0$  to study more general case. The value of constant  $\omega_k > 0$  is not of crucial importance here because my main interest is in the qualitative analysis.

Consider first the case of zero baryonic chemical potential, i.e.,  $\mu = 0$ . Then, using Eqs. (1) and (2) for  $L \gg R$  one can calculate the total bag pressure as

$$p_{\text{tot}} = p_v(T) - \frac{\sigma_{\text{surf}}(T)}{R} \equiv \frac{\sigma_{\text{surf}}(T)}{R} - \frac{\sigma_{\text{str}}}{\pi R^2} \rightarrow \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{1/k} \left[ \sigma_{\text{surf}} - \frac{\omega_k}{\pi} \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{\frac{k+1}{k}} \right], \quad (3)$$

which at fixed value of  $\mu = 0$  can be considered as the usual equation of state of a single variable  $T$ . Then the total entropy density of such a bag is

$$s_{\text{tot}} = \frac{\partial p_{\text{tot}}}{\partial T} \rightarrow \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{1/k} \frac{\partial \sigma_{\text{str}}}{\partial T} \frac{\sigma_{\text{surf}}}{k \sigma_{\text{str}}} + \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{1/k} \frac{\partial \sigma_{\text{surf}}}{\partial T} - \frac{k+2}{\pi k} \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{2/k} \frac{\partial \sigma_{\text{str}}}{\partial T}. \quad (4)$$

The mechanical stability of the cylindrical bag means an equality of the total bag pressure, Eq. (3), to the outer pressure, but the thermodynamic stability requires positive value for the entropy density (4). Models 1 and 2 predict [5, 7] that everywhere at the cross-over line, except for the (tri)critical endpoint, the surface tension coefficient  $\sigma_{\text{surf}}$  is nonzero and its derivative  $\partial \sigma_{\text{surf}} / \partial T$  is finite at  $T \rightarrow T_{\text{dec}} - 0$ . Remembering this, from Eq. (4) one finds that its first term on the right-hand side is dominant, since  $\sigma_{\text{str}} \rightarrow 0$  and hence

$$s_{\text{tot}} \rightarrow \frac{1}{k \sigma_{\text{str}}} \left[ \frac{\sigma_{\text{str}}}{\omega_k} \right]^{1/k} \frac{\partial \sigma_{\text{str}}}{\partial T} \sigma_{\text{surf}} > 0, \quad (5)$$

which requires that at  $T \rightarrow T_{\text{dec}} - 0$  the surface tension coefficient must be negative  $\sigma_{\text{surf}}(T_{\text{dec}}) < 0$ , since the string melts in this limit, i.e.,  $\partial \sigma_{\text{str}} / \partial T < 0$ . Actually, this result is not surprising since the calculations of surface partitions for physical clusters [19, 20] and the models of quark gluon bag with surface tension [5, 7] also predict that at low baryonic densities the deconfining PT is transformed into a cross-over just because the surface tension coefficient of large bags becomes negative in this region (see also the next section). Equation (5) clearly shows that the color string model shares the possibility of negative values of bag surface tension coefficient available in the cross-over region. It is necessary to stress that negative value of the surface tension coefficient  $\sigma_{\text{surf}}(T)$  for temperatures above  $T_{\text{dec}}$  does not mean anything wrong. Fisher argued first [14] that the surface tension coefficient consists

of energy and entropy parts which have opposite signs [14]. Therefore,  $\sigma_{\text{surf}}(T) < 0$  does not mean that the surface energy changes the sign, but it rather means that the surface entropy, i.e., the logarithm of the degeneracy of bags of a fixed volume, simply exceeds its surface energy over  $T$ . In other words, the number of nonspherical bags of a fixed volume becomes so large that the Boltzmann exponent, which accounts for the energy «costs» of these bags, cannot suppress them anymore as rigorously was shown within the HDM [19,20]. The above results are valid for the baryonic chemical potential values which are smaller than that one of the (tri)critical endpoint, i.e., for  $\mu \leq \mu_{\text{cep}}$  [9].

Analysis of Eq. (5) [9] shows also that for  $k > 1$  the entropy density of cylindrical bag can develop a singularity at vanishing string tension even at finite  $L$ . Such a surprising conclusion can be naturally explained by the appearance of fractal string surfaces [9]. Their appearance at the cross-over temperature can be easily understood within the present model, if one recalls that only at this temperature the fractal surfaces can emerge at no energy costs due to zero total pressure.

From the discussion above the **first challenge for the femtoscopy** can be formulated as follows: *to study the emission from highly nonspherical bags with complicated and even the fractal surfaces in order to find the indicator which is able to distinguish the case of positive, zero, and negative surface tension coefficient.* As I argue in the next section, the line  $\sigma_{\text{surf}}(T, \mu) = 0$  plays an important role as the boundary separating two different physics.

### 3. THE ROLE OF SURFACE TENSION AT THE (TRI)CRITICAL ENDPOINT

It is well known [5, 7, 24] that the most convenient way to study the phase structure of statistical models similar to FDM and SMM is to use the isobaric partition for analyzing its rightmost singularities. The isobaric partition is the Laplace transform image of the grand canonical one  $Z(V, T, \mu)$ :

$$\hat{Z}(s, T, \mu) \equiv \int_0^\infty dV e^{-sV} Z(V, T, \mu) = \frac{1}{[s - F(s, T, \mu)]}, \quad (6)$$

with  $F(s, T, \mu)$  containing the discrete  $F_H$  and continuous  $F_Q$  volume spectra of the bags [5]

$$F(s, T, \mu) \equiv F_H(s, T, \mu) + F_Q(s, T, \mu) = \sum_{j=1}^n g_j \exp\left(\frac{\mu}{T} b_j - v_j s\right) \phi_j(T) + \quad (7)$$

$$+ u(T) \int_{V_0}^\infty \frac{dv}{v^\tau} \exp[(s_Q(T, \mu) - s)v - \Sigma(T, \mu)v^\kappa]. \quad (8)$$

$u(T)$  and  $s_Q(T, \mu)$  are continuous and, at least, double differentiable functions of their arguments. The particle number density of bags with mass  $m_k$ , eigen volume  $v_k$ , baryon

charge  $b_k$ , and degeneracy  $g_k$  is given by  $\phi_k(T) \equiv \frac{g_k}{2\pi^2} \int_0^\infty p^2 dp \exp\left[-\frac{\sqrt{p^2 + m_k^2}}{T}\right] =$

$$g_k \frac{m_k^2 T}{2\pi^2} K_2\left(\frac{m_k}{T}\right).$$

The continuous part of the spectrum (8) generalizes the exponential mass spectrum introduced by Hagedorn [22] and it can be derived either within the MIT bag model [23] or, in more general fashion, within the finite width model of QGP bags [10,24]. The term  $e^{-sv}$  accounts for the hard-core repulsion of the Van der Waals type in (8).  $\Sigma(T, \mu) = \sigma_{\text{surf}}(T, \mu)/T$  denotes the reduced surface tension coefficient which has the form

$$\Sigma(T, \mu) = \begin{cases} \Sigma^- > 0, & T \rightarrow T_{\Sigma}(\mu) - 0, \\ 0, & T = T_{\Sigma}(\mu), \\ \Sigma^+ < 0, & T \rightarrow T_{\Sigma}(\mu) + 0. \end{cases} \quad (9)$$

Such a simple surface free energy parameterization in (8) is based on the original Fisher idea [14] which allows one to account for the surface free energy by considering a mean bag of volume  $v$  and surface extent  $v^{\kappa}$ . The power  $\kappa < 1$  inherent in bag effective surface is a constant which, in principle, may differ from the usual FDM and SMM value  $2/3$  [5, 7]. From (9) one can see that, in contrast to FDM and SMM, the precise disappearance of  $\Sigma(T, \mu)$  above the critical endpoint is not required.

The Model 1 corresponds to Fisher parameter  $1 < \tau \leq 2$  and continuous values of function  $\Sigma(T, \mu)$  and its first derivatives. Under these conditions and for a reasonable choice of other parameters, Model 1 has the 1st order deconfinement PT and the second order PT at the line  $\Sigma(T, \mu) = 0$  for  $\mu \geq \mu_{\text{cep}}$  and for temperatures larger than the temperature of the deconfinement PT  $T_{\text{dec}}(\mu)$ . As one can see from Eq. (8), the volume distribution of large QGP bags has the power law  $1/v^{\tau}$  at the line  $\Sigma(T, \mu) = 0$  [5, 6] for  $\mu \geq \mu_{\text{cep}}$ . This model allows one to naturally interpret the possible states on quark gluon matter. Thus, the state, in which a single QGP bag of infinite size dominates, is similar to the usual liquids and, hence, it can be called the *quark gluon liquid*. It is located for  $\mu > \mu_{\text{cep}}$  and temperatures satisfying the inequalities  $T_{\text{dec}}(\mu) \leq T < T_{\Sigma}(\mu)$ . On the other hand, the state existing at temperatures above  $T_{\Sigma}(\mu)$  consists of the bags of finite mean size with highly nonspherical surfaces due to  $\Sigma(T, \mu) < 0$  and, hence, it is *QGP in its traditional sense*.

Besides the inequality  $\tau > 2$ , Model 2 requires the fulfillment of several additional conditions [7, 8]. Thus, for  $\mu \geq \mu_{\text{cep}}$  the line  $\Sigma(T, \mu) = 0$  coincides with the deconfinement PT line in the  $\mu - T$ -plane, i.e.,  $T_{\text{dec}}(\mu) = T_{\Sigma}(\mu)$  for  $\mu \geq \mu_{\text{cep}}$ , and  $T$  derivative of the reduced surface tension coefficient has a discontinuity at the deconfinement PT line, i.e.,  $\partial\Sigma^+/\partial T \neq \partial\Sigma^-/\partial T$  at  $T = T_{\text{dec}}(\mu)$ . In this model, the *quark gluon liquid* can exist inside the mixed phase only [7], whereas in the  $\mu - T$ -plane the QGP exists everywhere above the line  $\Sigma(T, \mu) = 0$ . As a consequence, the volume distribution of large bags of Model 2 has the power law right at the mixed phase [7]. Obviously, the different location of the power law in volume distribution of large bags distinguishes models with the tricritical and with critical endpoints and it can serve as a clear experimental indicator to distinguish them. In Models 1 and 2 the volume of the QGP bag is proportional to its mass and, hence, one can also search for the power law in the mass distribution of heavy bags [24]. This property remains also valid within the more realistic statistical model which accounts for the finite width of the QGP bags [10]. Therefore, the ***second challenge for femtoscopy*** is *an elucidation of the power law of the volume (mass) distribution of large (heavy) bags from the available data*.

It is necessary to stress that the vast majority of statistical models is simply unrealistic since they do not account for the finite width of QGP bags. The latter is absolutely required in order to naturally explain [10,25] the huge existing deficit in the number of heavy hadronic resonances compared to the Hagedorn mass spectrum. Recently within the finite width

model [10,11] it was shown that even in a vacuum the mean width of a resonance of mass  $M$  behaves as  $\Gamma(M) \approx 400\text{--}600 [M/M_0]^{1/2}$  MeV (with  $M_0 \approx 2.5$  GeV), whereas in a media it increases with the temperature. These results not only naturally explain the existing deficit in the number of heavy hadronic resonances mentioned above, but also allow us to establish a novel view at the confinement problem. Thus, usually the confinement is understood as an impossibility to separate the color charges confined by the gluonic fields. The finite width of large/heavy QGP bags demonstrates another feature of the confinement – the large/heavy QGP bags are very unstable in the vacuum, i.e., they decay fast without the stabilizing external conditions. Clearly, the large width of QGP bags should affect the space-time evolution of quark gluon matter (liquid or plasma) created in the relativistic nuclear collisions. Therefore, in my mind, *the third challenge for femtoscopy is an investigation of the influence of finite width of quark gluon bags on their space-time evolution during the course of high-energy nuclear collision*. Some ideas on how to reach this goal are discussed in [26].

#### 4. NEW CHALLENGES FOR FEMTOSCOPY

In summary, the main challenges for femtoscopy are as follows:

I. *To study the emission from highly nonspherical (fractal) bags and to find the indicator which is able to distinguish the case of positive, zero, and negative surface tension coefficient.*

II. *To elucidate the power law of the volume (mass) distribution of large (heavy) bags from the available experimental data.*

III. *To study the influence of finite width of quark gluon bags on their space-time evolution during the course of high-energy nuclear collision.*

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