

# ENERGY–MOMENTUM CONSERVATION EFFECTS ON TWO-PARTICLE CORRELATION FUNCTIONS

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Two-particle correlations are used to extract information about the characteristic size of the system in proton–proton and heavy-ion collisions. The size of the system can be extracted from the Bose–Einstein quantum mechanical effect for identical particles. However, there are also long-range correlations that shift the baseline of the correlation function from the expected flat behavior. A possible source of these correlations is the conservation of energy and momentum, especially for small systems, where the energy available for particle production is limited. A new technique, first used by the STAR collaboration, of quantifying these long-range correlations using energy–momentum conservation considerations is presented in this talk. Using Monte Carlo simulations of proton–proton collisions at 900 GeV, it is shown that the baseline of the two-particle correlation function can be described using this technique.

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## INTRODUCTION

Femtoscopia is a powerful technique that allows us to «look» directly into the interaction region in particle collisions. It was first used in proton–antiproton collisions by G. Goldhaber, S. Goldhaber, Lee and Pais in 1969 [1], but it was developed by Hanbury-Brown and Twiss in 1956 to measure the angular size of stars [2] and is therefore also known as HBT. With this technique, the size and lifetime of the emission region can be measured to study the shape and evolution of its components. The measured radii and their dependence on pair momentum and multiplicity provide information about the level of interaction of the created particles, and can be used in particular to determine the freezeout volume in heavy-ion and proton–proton collisions.

The HBT effect arises from the Bose–Einstein quantum-mechanical effect for identical particles. However, any given pair of particles can be correlated through other effects, like resonance decays, jets and mini-jets, or phase-space constraints from energy–momentum conservation. In this paper the latter are studied, and it is organized as follows: a brief introduction to the femtoscopic techniques is given in Sec. 1, the energy–momentum correlations are explained in Sec. 2, and their application to correlation functions are shown in Sec. 3.

## 1. FEMTOSCOPY

When dealing with two identical particles in quantum mechanics it is necessary to describe them with a symmetrized two-particle wavefunction:

$$\Psi(p_1, r_1, p_2, r_2) \sim e^{i(p_1 \cdot r_1 + p_2 \cdot r_2)} + e^{i(p_1 \cdot r_2 + p_2 \cdot r_1)}. \quad (1)$$

Wavefunction symmetrization leads to an enhanced probability of measuring pairs of particles with small momentum difference:

$$|\Psi^2| \sim 1 + \cos((p_1 - p_2) \cdot (r_1 - r_2)) = 1 + \cos(Q \cdot \Delta r), \quad (2)$$

where  $Q = p_1 - p_2$  is the momentum difference and  $\Delta r = r_1 - r_2$  is the relative emission point of the particles, see Fig. 1.

Pairs with small relative momentum  $Q$  will be observed with higher rates because identical bosonic particles are likely to be found in the same state. This gives rise to a peak at low momentum difference, and its width is inversely proportional to the size of the emission region. In particular, for a Gaussian source  $\rho(r) \sim e^{-r^2/2R^2}$  one obtains

$$C_{\text{th}}(Q) = 1 + \lambda e^{-R^2 Q^2}, \quad (3)$$

where  $\lambda$  is the correlation strength. The experimental correlation function is defined as the ratio of the two-particle and single-particle distributions:

$$C_{\text{exp}}(Q) = \frac{f(p_1, p_2)}{f(p_1)f(p_2)}. \quad (4)$$

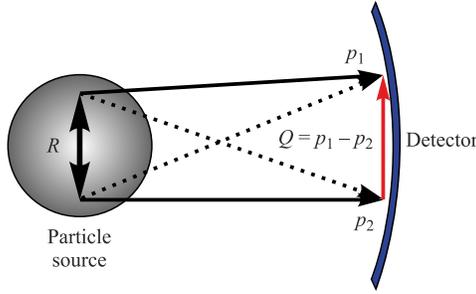


Fig. 1. HBT effect

Equation (3) assumes the only correlation between particles is bosonic. The ALICE collaboration measured Bose–Einstein correlations of pions at 900 GeV [4], and it was observed that the extracted radii depend on what type of baseline is used in the fitting procedure. In this paper we are concerned about studying a possible source of non-femtoscopic correlations that can affect the measured radii, namely Energy–Momentum Conservation Induced Correlations (EMCICs).

## 2. ENERGY–MOMENTUM CONSERVATION INDUCED CORRELATIONS

The phase space available for particle production is constrained by energy–momentum conservation, therefore creating a correlation among particles. It has been shown by Chajecski and Lisa [5], that these correlations can be quantified by applying energy–momentum constraints on the single-particle distributions. This results in an expression for the energy–momentum correlation of two particles:

$$C_{\text{EMCIC}}(Q) = 1 - \frac{1}{N} \left( 2 \frac{\mathbf{p}_{T1} \cdot \mathbf{p}_{T2}}{\langle p_T^2 \rangle} + \frac{p_{z1} \cdot p_{z2}}{\langle p_z^2 \rangle} + \frac{(E_1 - \langle E \rangle)(E_2 - \langle E \rangle)}{\langle E^2 \rangle - \langle E \rangle^2} \right), \quad (5)$$

where  $N$  is the total event multiplicity. When femtoscopic correlations are present, the total correlation function can be written as

$$C(Q) = \Phi_{\text{femto}}(Q) \times C_{\text{EMCIC}}(Q), \quad (6)$$

where  $\Phi_{\text{femto}}$  is the femtoscopic or Bose–Einstein correlation. This expression implies that if all the quantities in Eq. (5) could be measured, the EMCICs would be completely characterized and it would be possible to remove their effect from the experimental correlation function. However, in experiment  $N$ ,  $\langle p_T^2 \rangle$ ,  $\langle p_z^2 \rangle$ ,  $\langle E \rangle$  and  $\langle E^2 \rangle$  cannot be measured, simply because not every particle is detected. Parametrizing the unknown quantities yields an equation that can be used to fit experimental data, as has been done in STAR for 200-GeV  $p + p$  collisions [6] and is being studied in ALICE for 900-GeV  $p + p$  collisions [7]:

$$C(Q) = 1 - M_1 \overline{\{\mathbf{p}_{T_1} \cdot \mathbf{p}_{T_2}\}} - M_2 \overline{\{p_{z_1} \cdot p_{z_2}\}} - M_3 \overline{\{E_1 \cdot E_2\}} + M_4 \overline{\{E_1 + E_2\}} - \frac{M_4^2}{M_3}. \quad (7)$$

The notation  $\overline{\{X\}}$  represents histograms of two-particle quantities that can be measured in experiment and are binned at the same time as the numerator and denominator of the correlation function in Eq. (4). The  $M_i$  parameters are related to the quantities that are not directly measurable in the following way:

$$M_1 = \frac{2}{N \langle p_T^2 \rangle}, \quad M_2 = \frac{1}{N \langle p_z^2 \rangle}, \quad (8)$$

$$M_3 = \frac{1}{N(\langle E^2 \rangle - \langle E \rangle^2)}, \quad M_4 = \frac{\langle E \rangle}{N(\langle E^2 \rangle - \langle E \rangle^2)}. \quad (9)$$

After a fit has been performed on the data, the physical quantities can be calculated using an additional equation for the energy of a characteristic particle with mass  $m_*$  in the system:

$$\langle E^2 \rangle = \langle p_T^2 \rangle + \langle p_z^2 \rangle + m_*^2. \quad (10)$$

The total multiplicity of the event is then calculated as

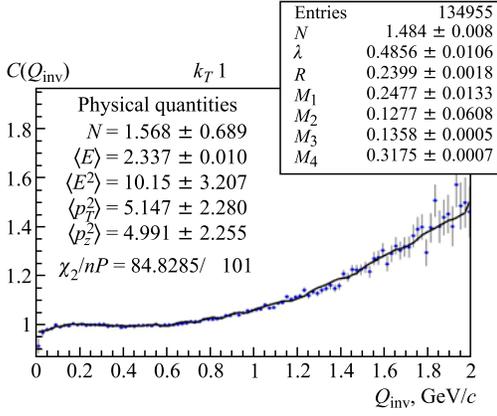
$$N = \frac{M_3^{-1} - 2M_1^{-1} - M_2^{-1}}{m_*^2 - (M_4/M_3)^2}. \quad (11)$$

The physical quantities can be used as a consistency check to tell if the fit results are reasonable.

### 3. BASELINE STUDY OF TWO-PION CORRELATION FUNCTIONS

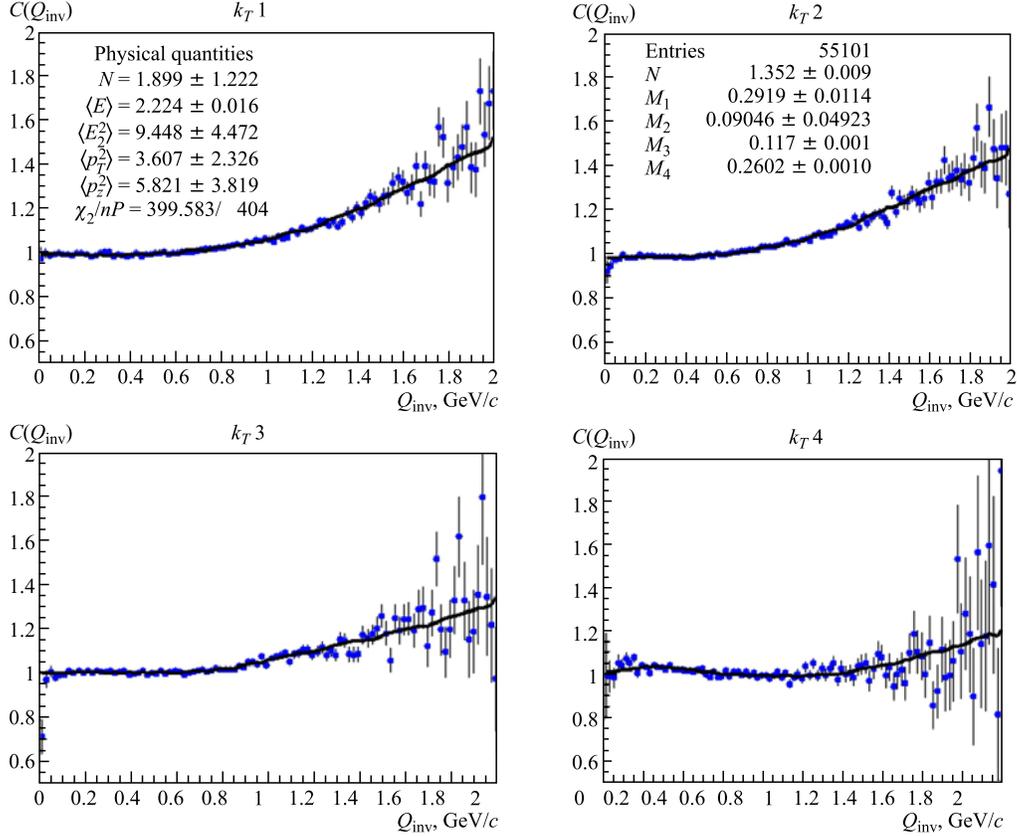
The shape of the baseline is studied using Monte Carlo simulations of proton–proton collisions at 900 GeV, with a data set consisting of 10M events. The data analysis used to obtain all the histograms mentioned above, is done using the AliFemto code. A transverse momentum cut is applied to ensure correct identification of pions in a detector like ALICE, i.e.,  $0.1 < p_T < 1.2$  GeV. The analysis also includes binning in four pair momentum bins  $k_T = 1/2|\mathbf{p}_{T_1} + \mathbf{p}_{T_2}|$ , whose edges are 0.1, 0.25, 0.4, 0.55, 1.0 GeV.

A fit to the  $Q_{\text{inv}}$  correlation function with no  $k_T$  binning can be seen in Fig. 2. The behavior at low  $Q_{\text{inv}}$  is from two track effects, known as merging and splitting. At large  $Q_{\text{inv}}$  the long-range correlation is very pronounced. The fitting function used is Eq. (6), including the Gaussian component to account for the underlying event at low  $Q_{\text{inv}}$ . The fit adjusts very well to the data; however, it was observed that the  $M_i$  values are not unique and depend on the initial values given to the parameters. This means that there are many local minima, which is expected from a one-dimensional function with 7 parameters. The physical quantities obtained

Fig. 2. EMCIC fit to the  $Q_{\text{inv}}$  correlation function

from the  $M_i$ , also shown in Fig. 2, indicate that the fit is in a physical region. The average energy  $\langle E \rangle = (2.34 \pm 0.01)$  GeV is the most significant here given the small error bar, and it is a reasonable value. The total event multiplicity is much lower than expected, and the average transverse momentum is quite high, but both have a large error bar.

A way to constrain the parameter space is to use the  $k_T$  binning mentioned above and fit all bins simultaneously with independent  $N, \lambda, R_{\text{inv}}$  but the same  $M_i$ . This is possible since the  $M_i$ 's are related to physical quantities which in this case are the same for all  $k_T$  bins. The results of this procedure are shown in Fig. 3. The fitting function adjusts

Fig. 3. EMCIC fit to the  $Q_{\text{inv}}$  correlation function simultaneously in 4  $k_T$  bins

very well in all four  $k_T$  bins, and it was observed that the convergence of the  $M_i$  values was more consistent when modifying initial values. As before, the average energy  $\langle E \rangle = (2.22 \pm 0.02)$  GeV is the most significant quantity, due to its small error bars, and it is again a reasonable value indicating a good fit.

#### 4. SUMMARY AND FUTURE WORK

A new technique to quantify and remove effects of energy–momentum conservation correlations from femtoscopic effects has been used here to study the baseline of the  $Q_{\text{inv}}$  correlation function. It was shown that the convergence of fits to the data is dependent on initial parameters, but it explains the long-range correlations from first principles. Future work on this topic includes using the same procedure to fit correlation functions in the longitudinal co-moving system LCMS and in spherical harmonics, where there should be more constraints on the parameters allowing the fit to converge easier.

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