

# OSCILLATING HBT RADII AND THE TIME EVOLUTION OF THE SOURCE — $\sqrt{s_{NN}} = 200$ GeV Au + Au DATA ANALYZED WITH AZIMUTHALLY SENSITIVE BUDA–LUND HYDRO MODEL <sup>1</sup>

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Identified particle spectra of pions, kaons and (anti)protons, and elliptic flow and azimuthal dependence of Bose–Einstein or HBT correlations of identified pions in  $\sqrt{s_{NN}} = 200$  GeV Au + Au collisions are analyzed simultaneously using an ellipsoidally symmetric generalization of the Buda–Lund hydrodynamical model. The transverse flow is found to be faster in the reaction plane than out of plane, which results in a reaction zone that gets slightly more elongated in-plane than out of plane.

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## INTRODUCTION

Important information about the properties of extremely hot strongly interacting matter comes from the observation of azimuthal anisotropies in non-central ultra-relativistic nuclear collisions. The second-order Fourier component of azimuthal hadron distributions is connected with the azimuthal dependence of transverse collective expansion velocity of the bulk matter [1,2]. That is in turn determined by the differences of the initial pressure gradients in the two perpendicular transverse directions, as well as by the initial geometry, the initial velocity and temperature distributions of the fireball, and the equation of state [3,4]. The anisotropic shape of the fireball measured with the help of correlation femtoscopy [5] at the instant of final decoupling of hadrons bears information about the total lifespan of the hot matter: with time the originally out-of-reaction-plane shape becomes more and more round and may even become in-plane extended [6]. Unfortunately, in determining the elliptic flow and azimuthally sensitive correlation radii individually two effects — spatial and flow anisotropy — are entangled. For example, the same elliptic flow can be generated with varying flow anisotropy strength if the spatial anisotropy is adjusted appropriately [7].

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In general, the precise way of the interplay between the two anisotropies is model-dependent. It has been studied and shown to be different within the Buda–Lund model [11] than in the Blast Wave model [7].

Here we report the results of the Buda–Lund hydro model analysis of azimuthally sensitive Hanbury Brown–Twiss (HBT) radii, using data from non-central heavy-ion collisions at RHIC. Note that the original, axially symmetric version of the Buda–Lund model described successfully data from central Au + Au collisions at RHIC, as measured by BRAHMS, PHENIX, PHOBOS, and STAR collaborations, including identified particle spectra and transverse mass-dependent HBT radii as well as the pseudorapidity distributions of charged particles as first presented in [8,9].

The Buda–Lund model formalism for non-central collisions, including elliptic flow and azimuthal angle dependence of HBT radii has first been proposed in [11]. The model is defined with the help of its emission function. In order to take into account the effects of resonance decays, it uses the core-halo model [12]. This ellipsoidal extension of the Buda–Lund model was shown before to describe well the transverse mass and the pseudorapidity dependence of elliptic flow parameter  $v_2$  of identified particles at various energies and centralities in [10].

In the present study, we improve on earlier versions of the Buda–Lund model, by scrutinizing the various components using azimuthally sensitive HBT data. Eventually we utilize a model that includes as a special case of T. S. Biró’s axially symmetric and accelerationless exact solution of relativistic hydrodynamics [13], in contrast to the original, earlier variant [11], which was based on an ellipsoidally symmetric, but also non-accelerating exact solution of relativistic hydrodynamics, given by [14]. Similarly to [15], we utilize here an improved calculation, using the binary source formalism, to obtain the observables by using two saddle points instead of only one. This results in an oscillating pre-factor in front of the Gaussian in the two-pion correlation function that we take into account for the formulae of the HBT radii. Details of the model and the evaluation of the observables from it are presented in [22].

Azimuthally sensitive HBT radii were also considered recently in cascade models, e.g., in the fast Monte-Carlo model of [17], or in the Hadronic Resonance Cascade [18].

Data analysis of correlation HBT radii performed earlier with the Blast Wave model indicates that the fireball at the freeze-out is elongated slightly out of the reaction plane [19]; i.e., spatial deformation is similar as in the initial state given by the overlap function. This is also supported by the theoretical results from hydrodynamic simulations [6,20] and URQMD [21]. It sets limitations on the total lifespan. From all previous analyses it seems, however, that the final-state anisotropy has an interesting non-monotonous dependence on collision energy with a minimum at the SPS energies [21]. In our analysis of the *same* data with a *different* model we observe for the first time at RHIC an in-plane elongation of the fireball at freeze-out.

This presentation is a conference contribution which is based on a more detailed manuscript [22], where we described the azimuthally sensitive Buda–Lund model fully and detailed the analytic formulas that were obtained from it and were fitted simultaneously to the single particle spectra of identified particles, the elliptic flow and the azimuthal angle-dependent HBT radii of identified pions.

## 1. asBUDA–LUND HYDRO COMPARED TO DATA

Observables like spectra, elliptic flow or Bose–Einstein correlation functions have been calculated analytically from the azimuthally sensitive Buda–Lund hydrodynamic model

(or as Buda–Lund hydro in a shortened form), using a double saddle-point approximation in the integration, as detailed in [22, 26]. We have determined [22] the best values of the model parameters by fitting these analytic, parametrically given expressions for the observables to experimental data with the help of the CERN Minuit fitting package. This is possible given that the Buda–Lund hydro model relies on exact but parametric solutions of (relativistic) hydrodynamics in certain limiting cases when these solutions are known, and interpolates between these solutions in other cases. So dynamics is mapped to time evolution of model parameters, and hadronic observables are sensitive only to the values of these model parameters around the time of freeze-out, as was explicitly demonstrated recently in [25]. However, penetrating probes, for example, the direct photon spectra, are known to carry explicit information about the equation of state through the cooling history of the continuous radiation of these penetrating probes, for example, direct photons. This property of the model was used recently to determine the equation-of-state parameters of the strongly interacting quark–gluon plasma in 200-GeV Au + Au collisions in [23, 24].

Data from 20–30% centrality class of 200A GeV Au + Au collisions provided by PHENIX [32, 33] and STAR [34, 35] were used in this analysis. The fits were performed simultaneously to azimuthally integrated transverse mass spectra of positive and negative pions, kaons, and (anti)protons [32], the transverse momentum dependence of the elliptic flow parameter  $v_2$  of pions [34] and to the HBT radii due to pion correlations as functions of transverse mass and the azimuthal angle [35]. The results are plotted in Figs. 1–3.

The interpretation of the model parameters is summarized in Table 1. The two radii  $R_{sx}$  and  $R_{sy}$  correspond to «thermal surface» sizes, corresponding to distances where the temperature drops to half of its central value, to  $T_0/2$ , while parameter  $T_e$  corresponds to the temperature of the center after most of the particle emission is over (cooling due to evaporation and expansion). Sudden emission corresponds to the  $T_e = T_0$ , and  $\Delta\tau \rightarrow 0$  limit. Also note that we use  $\mu_B$ , baryochemical potential, calculated from the chemical potential of protons and antiprotons as  $\mu_B = 1/2 (\mu_{0,p} - \mu_{0,\bar{p}})$ , see Table 1. The flow profile is linear in both transverse directions, but the Hubble constant is direction-dependent, denoted by  $H_x$  and  $H_y$  in the reaction plane and out of the reaction plane, respectively.

In Table 2, we present the model parameters obtained from simultaneous fits to the data sets. For comparison, results are shown from our earlier analysis of 0–30% centrality collisions [36], too, which was performed with a previous version of the model corresponding to the axially symmetric limit of the current ellipsoidally generalized Buda–Lund hydrodynamic model.

The general observation is that the Buda–Lund model parameters describing the source of non-central reactions are usually slightly smaller than those of more central collisions. However, the changes are usually within 2 standard deviations: therefore, the above statement is based on the tendency of the parameters, and on some lower energy results not shown here but presented in [36], too. For example, the central temperature in these particular non-central reactions is below that of the more central ones. Also, the transverse geometrical radii at the mean emission time are considerably smaller compared to the more central values. Moreover, the geometric shape evolution due to the asymmetric particle transverse flow in plane ( $x$ ) and out of plane ( $y$ ) directions results in a source more elongated in-plane. Due to the smaller longitudinal source size, the parameter corresponding to the formation of hydrodynamic phase is about 10% smaller than that in more central collisions,  $\tau_0(20\text{--}30\%) = (5.4 \pm 0.1) \text{ fm}/c$ . The elongation in longitudinal direction is similarly smaller,  $\Delta\eta(20\text{--}30\%) = 2.5 \pm 0.3$ .

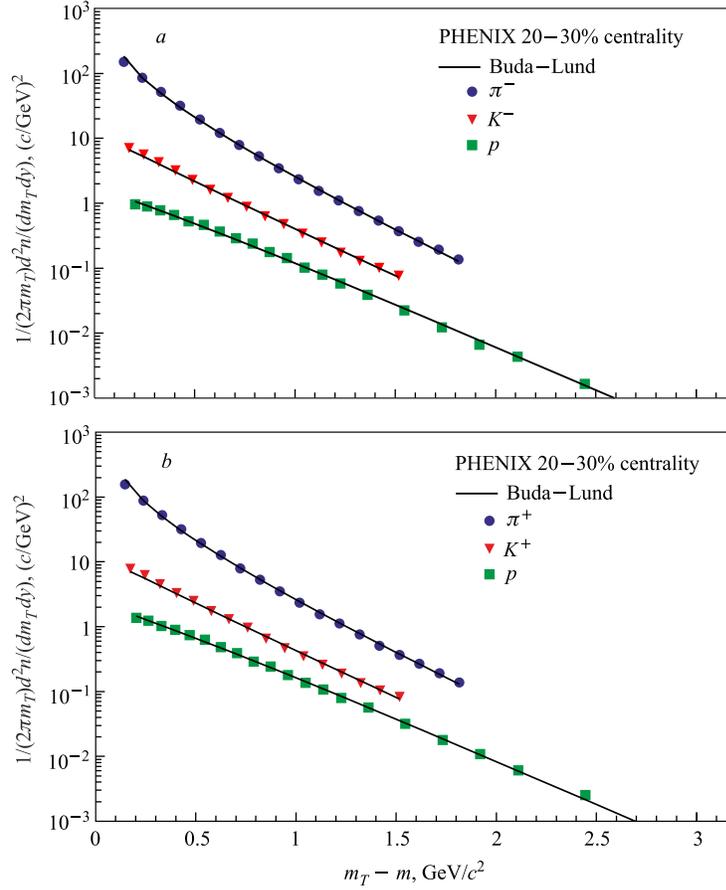


Fig. 1. Buda-Lund model fits to  $\sqrt{s_{NN}} = 200$  GeV Au + Au data of [32] on azimuthally integrated transverse momentum spectra of negatively (a) and positively (b) charged particles

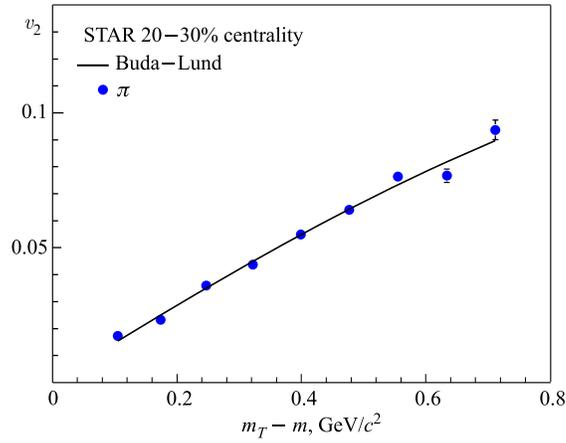


Fig. 2. Buda-Lund model fit to RHIC 200-GeV Au + Au data on  $v_2$  elliptic flow of pions, data from [34]

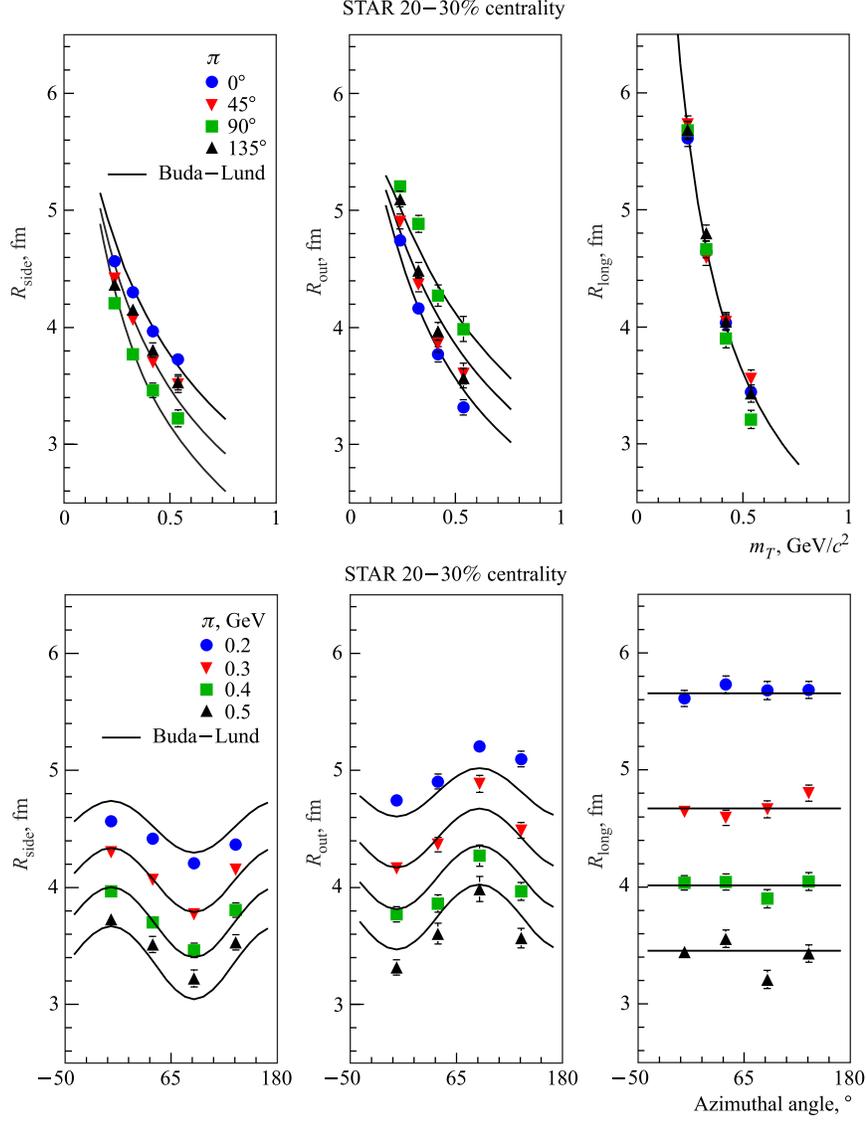


Fig. 3. Buda-Lund model fits to RHIC 200-GeV Au + Au data of [35] on HBT radii as functions of transverse mass for different azimuthal angles (top). The same data and the same fit are also shown as a function of the azimuthal angle for different values of the transverse mass (bottom)

In both cases the baryochemical potential is found to be small as compared to the proton mass. We emphasize again that the observations are based on all the fit results in [36].

Note that some of the azimuthally sensitive data have large systematic errors that affect the success of fits which we had to take into account. The reason for that is the difficulty of precise determination of the event reaction plane the data are relative to. Several methods are used by the experiments to overcome it and we mention those applied for the selected data.

**Table 1. Description of the parameters of the asBuda–Lund hydro model. The baryochemical potential is evaluated as  $\mu_B = 1/2 (\mu_{0,p} - \mu_{0,\bar{p}})$ . For details, see [22]**

Buda–Lund	Parameter description
$T_0$	Temperature in the center at $\tau_0$
$T_e$	Temperature in the center at $\tau_0 + \Delta\tau$
$\mu_B$	Baryochemical potential in the center at $\tau_0$
$R_x$	Geometrical size in direction $x$
$R_y$	Geometrical size in direction $y$
$R_{xs}$	Thermal size, where $T = T_0/2$ in direction $x$
$R_{ys}$	Thermal size, where $T = T_0/2$ in direction $y$
$H_x$	Hubble constant in direction $x$
$H_y$	Hubble constant in direction $y$
$\tau_0$	Mean freeze-out proper time
$\Delta\tau$	Distribution width in proper time $\tau$
$\Delta\eta$	Distribution width in space-time rapidity $\eta$
$\mu_B$	Baryochemical potential

**Table 2. Source parameters from simultaneous fits to PHENIX and STAR data of Au + Au collisions at  $\sqrt{s_{NN}} = 200$  GeV, as given in Figs. 1–3, obtained by the Buda–Lund model. For non-central data the value of  $\chi^2/\text{NDF}$  refers to fits with statistical errors only**

Buda–Lund parameters	Au + Au 200 GeV central (0–30%)	Au + Au 200 GeV non-central (20–30%)
$T_0$ , MeV	$196 \pm 13$	$174 \pm 6$
$T_e$ , MeV	$117 \pm 11$	$130 \pm 6$
$\mu_B$ , MeV	$31 \pm 28$	$27 \pm 16$
$R_x$ , fm	$13.5 \pm 1.7$	$9.5 \pm 0.5$
$R_y$ , fm	$R_x$	$7.0 \pm 0.2$
$R_{sx}$ , fm	$12.4 \pm 1.6$	$12.8 \pm 0.8$
$R_{sy}$ , fm	$R_{sx}$	$16.9 \pm 1.6$
$H_x$	$0.119 \pm 0.020$	$0.158 \pm 0.002$
$H_y$	$H_x$	$0.118 \pm 0.002$
$\tau_0$ , fm/c	$5.8 \pm 0.3$	$5.4 \pm 0.1$
$\Delta\tau$ , fm/c	$0.9 \pm 1.2$	$2.5 \pm 0.2$
$\Delta\eta$	$3.1 \pm 0.1$	$2.5 \pm 0.3$
$\chi^2/\text{NDF}$	$114/208 = 0.55$	$269.4/152 = 1.77$

The data set we used for fitting  $v_2$  was calculated by the four-particle cumulants reaction plane determination method that is based on calculations of  $N$ -particle correlations and non-flow effects subtracted to first order when  $N$  is greater than 2. The higher  $N$  is the more precise the event plane determination is, as expected. STAR published two-particle cumulants  $v_2$  data in the same reference, too, but because of the visible deviations between the two kinds of data sets and with respect to the comments above we used  $v_2\{4\}$  data only. For further details, see [34].

In case of azimuthally sensitive correlation radii, STAR has cast about 10% possible systematic errors on the data on average. The most likely deviations were assumed to take effect on the «side» and «out» radii of transverse momentum of 0.2 GeV/ $c$ . The  $\chi^2/\text{NDF}$  for the full fit, including HBT radii with their statistical errors, is 269.6/152, which corresponds to a very low confidence level. But, when we tested our fits with the above-mentioned two radii of «side» and «out» of transverse momentum of 0.2 GeV/ $c$  shifting them within their systematic errors (about  $\pm 5\%$ ), we could achieve an acceptable 1% confidence level for the full simultaneous fit. Without the contribution of the HBT radii to  $\chi^2/\text{NDF}$  the confidence level is of an acceptable level of 5.1%.

Our results were compared also to results of [7,28] of the azimuthally sensitive extension of the Blast Wave model in the detailed write-up [22].

## 2. CONCLUSIONS AND OUTLOOK

The ellipsoidally symmetric generalization of the Buda–Lund hydrodynamic model compares favourably to the identified particle spectra and to the elliptic flow and azimuthally sensitive Bose–Einstein correlation radii of identified pions. From model fits to 20–30% central Au + Au collision data at  $\sqrt{s_{NN}} = 200$  GeV at mid-rapidity, the source parameters characterizing these non-central ultra-relativistic heavy-ion reactions were extracted.

The results of our analysis indicate that the central temperature  $T_0 = (174 \pm 6 \text{ (stat.)})$  MeV in the 20–30% centrality class is somewhat lower than that in more central collisions, where an earlier analysis found  $T_0 = (196 \pm 13 \text{ (stat.)})$  MeV. We have found that the transverse flow is stronger in the reaction plane than out of plane with Hubble constants  $H_x = 0.158 \pm 0.002$  and  $H_y = 0.118 \pm 0.002$ . The almond shape of the reaction zone initially elongated out of plane gets slightly elongated in the direction of the impact parameter by the time the particle emission rate reaches its maximum. The effect is reflected by the geometrical radii in the two perpendicular directions at that time,  $R_x(\text{in-plane}) = (9.5 \pm 0.5 \text{ (stat.)})$  fm,  $R_y(\text{out-of-plane}) = (7.0 \pm 0.2 \text{ (stat.)})$  fm. As far as we know, this study is the first one where an in-plane extended source has been reconstructed from simultaneous and reasonably successful hydro model fits to identified particle spectra, elliptic flow and azimuthally sensitive HBT data in 200A GeV Au + Au collisions at RHIC. It is a remarkable property of this hadronic final state that the ratio of the Hubble constants is approximately the same as the ratio of the geometric source sizes:  $H_x/H_y = R_x/R_y$  within the errors of the analysis.

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