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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

HIGGS MODELS

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This lecture presented at the Baikal Summer School on Physics of Elementary Particles and Astrophysics in 2011 is devoted to the Higgs mechanism of the electroweak symmetry breaking within the Standard Model and in some models beyond it.

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1. HIGGS SECTOR OF THE STANDARD MODEL

The Standard Model (SM) is a gauge theory of electromagnetic, weak and strong interactions of fundamental matter fields: quarks and leptons. It has a few cornerstone concepts and leads to a huge amount of predictions, vast majority of which have been clearly confirmed in experiment.

All fundamental forces of the SM follow from the powerful *gauge principle*: the matter fields have internal degrees of freedom, but the physical observables are invariant under local transformations in this internal space performed. The word «local» means here that these transformations are performed at each space-time point independently. It contrasts with the global transformation, which is applied simultaneously to the fields at all space-time points. The group of internal transformations generating a given force is called the gauge group, and the gauge group of the SM is $SU(3)_c \times SU(2)_L \times U(1)_Y$.

Within SM, the electromagnetic and the weak interactions become two faces of the same coin: the electroweak interaction with the group $SU(2)_L \times U(1)_Y$, while the $SU(3)_c$ group corresponds to the strong color force. Although the Lagrangian of the SM is electroweak-symmetric, this symmetry is not manifest in our world, and therefore it must be broken. The Higgs mechanism, which postulates existence of fundamental scalar fields, is the most popular construction that naturally leads to the electroweak symmetry breaking (EWSB) keeping the theory renormalizable. Depending on the exact content of the scalar sector, one speaks of the minimal Higgs mechanism (the one used in the pure Standard Model) or non-minimal Higgs mechanisms. In the first half of these lectures we outline the minimal Higgs mechanism, while in the second half we'll briefly mention some non-minimal Higgs models. For further introductory reading on these subjects, one can refer to books [1, 2], lectures (e.g., [3]) and reviews (e.g., [4]).

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1.1. Abelian Higgs Model. In this pedagogical introduction, we start with the so-called Abelian Higgs model, which is basically the spontaneously broken quantum electrodynamics (QED). It illustrates the main idea how the masses of the gauge bosons and chiral fermions can arise but avoids the mathematical complications of the non-Abelian gauge theories.

Consider the QED Lagrangian:

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(iD_\mu\gamma^\mu - m)\psi, \tag{1}$$

where $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The field $\psi(x)$ describes the (Dirac) electron with charge -1. This theory has the $U(1)$ gauge symmetry: the Lagrangian is invariant under the local phase rotations of the electron field and simultaneous shifts of the gauge (that is, «compensating») field:

$$\psi \to {\rm e}^{-i\alpha(x)}\psi, \tag{2}$$

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x). \tag{3}$$

Quantization of the electromagnetic field A_μ leads to the massless photon.

Suppose we want to construct the QED with a massive photon. Adding the mass term by hand

$$-\mathcal{L}_{\rm mass} = \frac{m_A^2}{2}A_\mu A^\mu \tag{4}$$

does not work because this term is not gauge invariant. The (Abelian) Higgs mechanism is the way out in this situation. In this model we introduce a complex scalar field Φ with charge q which feels the gauge interaction and couples to itself:

$$\mathcal{L} = \mathcal{L}_{\rm QED} + (D_\mu\Phi)^*(D^\mu\Phi) - V(\Phi), \tag{5}$$

where $D_\mu = \partial_\mu - iqA_\mu$ when it acts on the Higgs field and the potential is

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4, \quad \mu^2,\ \lambda > 0. \tag{6}$$

Let us parametrize the complex field Φ as

$$\Phi = \frac{1}{\sqrt{2}}\phi(x)\,{\rm e}^{i\xi(x)}, \tag{7}$$

where $\phi(x)$ and $\xi(x)$ are real scalar fields. The scalar potential then simplifies to

$$V(\Phi) \to V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4. \tag{8}$$

The ground state of the model (the vacuum) corresponds to

$$\phi(x) = \phi_0 \equiv \sqrt{\frac{\mu^2}{\lambda}}. \tag{9}$$

Note that the second field, $\xi(x)$, disappears from the scalar potential. However, it is still present in the kinetic part of the scalar Lagrangian $|D_\mu\Phi|^2$:

$$D_\mu\Phi = \frac{1}{\sqrt{2}}\left[\partial_\mu\phi + i(\partial_\mu\xi - qA_\mu)\phi\right]{\rm e}^{i\xi(x)}. \tag{10}$$

But since we had the gauge symmetry in the model, we have a possibility to *gauge away* the field $\xi(x)$ by choosing $\alpha(x) = e\xi(x)/q$. In this case the field $\xi(x)$ disappears from the Lagrangian completely:

$$|D_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} q^2 \phi^2 A_\mu A^\mu. \tag{11}$$

Note however that this term also generates the quartic interaction $\phi^2(x) A^2(x)$.

If the vacuum corresponds to the homogeneous field $\langle \phi \rangle = \phi_0$, then for perturbative field theory one should expand the field near this point: $\phi(x) = \phi_0 + h(x)$. This expansion generates the following Higgs Lagrangian:

$$-\mathcal{L}_{\rm Higgs} = \lambda \phi_0^2 h^2 + \lambda \phi_0 h^3 + \frac{\lambda}{4} h^4, \tag{12}$$

which gives the mass of the physical Higgs boson $m_h^2 = 2\lambda\phi_0^2 = 2\mu^2$ as well as the cubic and quartic self-interaction terms. The same expansion in (11) generates the mass of the «photon», $m_A = |q|\phi_0$ as well as the $hA_\mu A^\mu$ and $hhA_\mu A^\mu$ interactions.

Let us now take step back and see what role the gauge symmetry played in this mechanism.

If the original Lagrangian (1) had no gauge symmetry (no covariant derivative in the interaction term), there would be no freedom to «gauge away» the field $\xi(x)$. We would still have the global $U(1)$-symmetry with constant phase shift α. We could still spontaneously break the global $U(1)$-symmetry by selecting the real vacuum expectation value ϕ_0, but we would not be able to make $\Phi(x)$ real everywhere. In this case the small fluctuations of the Higgs field around the vacuum can be parametrized as

$$\Phi(x) \approx \frac{1}{\sqrt{2}}[\phi_0 + h(x) + ig(x)], \quad g(x) \equiv \phi_0 \xi(x). \tag{13}$$

We see that the field $g(x)$ is a dynamical field because it does not disappear from the kinetic term:

$$|\partial_\mu \Phi|^2 = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(\partial_\mu g)^2 + \ldots \tag{14}$$

but it is still absent in the potential. Therefore, the mass of the quanta of the field $g(x)$ is zero, and this is the Goldstone boson. In fact, such massless bosons appear always as a result of spontaneous breaking of a global continuous symmetry, and this statement is the essence of the Goldstone theorem.

After this overlook we can formulate how the Higgs mechanism works: in a gauge theory, the Goldstone boson disappears and re-emerges as a longitudinal degree of freedom of the gauge boson, which becomes massive.

Let us now consider briefly the situation in the fermionic sector. In the toy model we constructed, we used the Dirac spinor ψ to describe the electron field. Because of that it was possible to introduce the fermion mass by hand: $m\bar{\psi}\psi$ is $U(1)$-gauge invariant. Alternatively, we can write this term via chiral fermionic fields, ψ_L and ψ_R:

$$m\bar{\psi}\psi = m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L), \tag{15}$$

and we see that this term in fact mixes ψ_L and ψ_R. This mixing does not cause any problem in a theory with vector-like interaction because both chiral fields have the same quantum numbers. But in the full SM it will not be the case. So, in order to mimic the SM case, let us assume that there is no mixing of ψ_L and ψ_R in the Lagrangian and show how the Higgs mechanism solves the problem of the fermion mass. To do this, we introduce the Yukawa interaction between scalars and the fermions:

$$\mathcal{L}_f = y_\psi \bar{\psi}_L \Phi \psi_R + \text{h.c.}, \tag{16}$$

where y_ψ is a dimensionless coupling constant and the Higgs field must carry the quantum numbers balancing the quantum numbers of ψ_L and ψ_R. After the spontaneous breaking of the $U(1)$-symmetry, this term becomes

$$\mathcal{L}_f = m_\psi \bar{\psi}_L \psi_R + \frac{\sqrt{2} m_\psi}{\phi_0} \bar{\psi}_L \psi_R h + \text{h.c.}, \tag{17}$$

where the fermion gains mass $m_\psi = y_\psi \phi_0/\sqrt{2}$ and, in addition, couples to the Higgs boson with a coefficient proportional to its mass. This is one of the hallmark features of the Higgs mechanism.

1.2. Electroweak Interactions. The electroweak interactions of the SM are based on the non-Abelian gauge group $SU(2)_L \times U(1)_Y$. The three gauge bosons $A_\mu^{(i)}$, $i = 1, 2, 3$, for the $SU(2)_L$ group and one gauge boson B_μ for $U(1)_Y$ are all massless. The left and right chiral fermions interact with the gauge bosons in different ways. The left fermions ψ_L are postulated to be $SU(2)$-doublets, for example, $L = (\nu_L, e_L)^T$, and therefore interact with $A_\mu^{(i)}$, while the right fermions ψ_R are singlets and do not feel the $SU(2)$-interactions. The Higgs field, by construction, mixes the left and right chiral fermions ($\bar{\psi}_L \Phi \psi_R$); therefore, it must be an $SU(2)$-doublet itself: $\Phi = (\phi^+, \phi^0)^T$.

With all these fields, the electroweak Lagrangian (written for simplicity for a single-lepton generation made of an electron and neutrino) takes the form

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2} G^i_{\mu\nu} G^{\mu\nu,i} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) + \\ &+ i\bar{L} D_\mu \gamma^\mu L + i\bar{e}_R D_\mu \gamma^\mu e_R + i\bar{\nu}_R D_\mu \gamma^\mu \nu_R - f_e(\bar{L}\Phi e_R + \bar{e}_R \tilde{\Phi} L), \end{aligned}$$

where in the last term $\tilde{\Phi} = i\sigma_2 \Phi^*$. The covariant derivative can be written generically as

$$D_\mu = \partial_\mu - igT^i A^i_\mu - ig' \frac{Y}{2} B_\mu,$$

where g and g' are gauge coupling constants, T^i are $SU(2)$ generators ($T^i = \sigma^i/2$) and Y is a quantum number called the hypercharge. For a given gauge group representation it can be calculated as twice the average electric charge of the particles in this representation. Finally, tensors $G^i_{\mu\nu}$ and $F_{\mu\nu}$ are the field strengths for the $SU(2)$ and $U(1)$ groups, respectively.

The spontaneous symmetry breaking proceeds essentially as before. The Higgs potential $V(\Phi)$ is constructed in such a way that its vacuum expectation value (v.e.v.) of the lower (=neutral) component of the Higgs doublet is non-zero and can be taken real:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{18}$$

This v.e.v. has definite (and non-zero!) values of hypercharge $Y = 1$ and of the third component of the weak isospin: $T^3 = -1/2$. Colloquially, one can say that the vacuum does not conserve any of the electroweak charges separately. But the combination $Q \equiv T^3 + Y/2$ is zero for $\langle\Phi\rangle$, which means that this quantity is conserved. Therefore, one says that electroweak symmetry group $SU(2)_L \times U(1)_Y$ is broken not completely, but down to the $U(1)_Q$. Later we will see that this quantum number Q is the electric charge (in the units of $|e|$).

Let us now see what happens to the gauge sector of the model. When (18) is inserted into the Higgs kinetic term, we get

$$D_\mu \Phi = \left(\partial_\mu - \frac{i}{2} g \sigma^i A^i_\mu - \frac{i}{2} g' B_\mu\right)\Phi \to$$

$$\to \frac{i}{2\sqrt{2}}(gA^3_\mu - g'B_\mu)\begin{pmatrix}0\\ v\end{pmatrix} - \frac{i}{2\sqrt{2}} g(A^1_\mu - iA^2_\mu)\begin{pmatrix}v\\ 0\end{pmatrix}, \quad (19)$$

$$|D_\mu\Phi|^2 \to \frac{v^2}{8}(gA^3_\mu - g'B_\mu)^2 + \frac{g^2v^2}{8}(A^1_\mu - iA^2_\mu)^2 = \frac{\bar{g}^2 v^2}{8} Z_\mu Z^\mu + \frac{g^2v^2}{4} W^-_\mu W^{+\mu}.$$

Here we introduced $\bar{g}^2 \equiv g^2 + g'^2$ and the new fields which correspond to the weak vector bosons:

$$Z_\mu = \frac{gA^3_\mu - g'B_\mu}{\bar{g}}, \quad W^\pm_\mu = \frac{A^1_\mu + iA^2_\mu}{\sqrt{2}}. \quad (20)$$

The key feature is that they are massive:

$$m_Z = \frac{\bar{g}v}{2}, \quad m_W = \frac{gv}{2}. \quad (21)$$

Note that the diagonalization of the mass matrix was achieved by rotating between the fields A^3_μ and B_μ to produce Z_μ in (20) and the orthogonal combination $A_\mu = (g'A^3_\mu + gB_\mu)/\bar{g}$, which is identified with the photon. The angle of the rotation is called the Weinberg angle θ_W. It is defined in terms of coupling constants $g/\bar{g} = \cos\theta_W \equiv c_W$, $g'/\bar{g} = \sin\theta_W \equiv s_W$. It is also shown below that $gg'/\bar{g} = |e|$, the elementary electric charge. Note also that the photon field does not appear in (19), which means that the photon remains massless.

The electroweak model based on the Higgs doublet leads to an important prediction: $m_W/m_Z = \cos\theta_W$. This relation would be different in non-doublet versions of the Higgs mechanism or in many other theories of the electroweak symmetry breaking that goes significantly beyond the SM. Its experimental verification is, therefore, an important check of the model and, at the same time, a possible method to search for New Physics. This relation has been confirmed in experiment with a very high accuracy.

It remains to show how the usual vector-like QED appears from the chiral electroweak theory. For this, we need to check the coefficient in front of the electron–photon interaction. This part of the Lagrangian is

$$\frac{i}{2}(Y_d - 1)\,\bar{g} c_W s_W \bar{e}_L \gamma^\mu A_\mu e_L + \frac{i}{2} Y_S \bar{g} c_W s_W \bar{e}_R \gamma^\mu A_\mu e_R. \quad (22)$$

The hypercharge for the electron–neutrino left doublet $Y_d = -1$, while for the right electron singlet it is $Y_s = -2$. Therefore, both terms in this interaction have equal coefficients, and they can be grouped into

$$iQ|e|\bar{e}\gamma^\mu A_\mu e, \quad Q=-1, \quad |e| = \bar{g}c_W s_W = \frac{gg'}{\bar{g}}. \tag{23}$$

Let us finally see what happens to the Higgs fields. A doublet of complex fields has four degrees of freedom, and the excitations around the minimum point (18) can be represented as

$$\Phi = \langle\Phi\rangle + \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ h + i\phi_3\end{pmatrix}. \tag{24}$$

Just as it was with the field $\xi(x)$ in the Abelian Higgs model, here three degrees of freedom, namely, ϕ_i, $i=1,2,3$, correspond to the flat directions of the potential and can be «gauged away». The only remaining physical scalar boson is the Higgs boson h. In the words of the Goldstone theorem, three disappearing massless scalars re-appear as the longitudinal degrees of freedom of the $W^\pm$ and Z bosons.

1.3. Constraints on the Higgs Sector. Higgs phenomenology in the Standard Model (notably, production and decay channels) is a huge subject on its own. We will not touch on it in these lectures. However, we will mention two purely theoretical restrictions on properties of the SM Higgs sector.

The Higgs potential of the Standard Model contains one parameter that is not constrained by the model itself: the mass of the Higgs boson (or, alternatively, the quartic coupling λ of the Higgs self-interaction). Direct searches at the high-energy colliders, LEP, Tevatron and now the LHC, have not yet revealed the Higgs boson (although by the time these lectures have been printed, the LHC might have already announced the first hints of the Higgs boson). However, the structure of the electroweak theory itself forbids the Higgs boson to be too light or too heavy.

One of the important constraints is called *perturbative unitarity*. Consider a high-energy two-particle elastic scattering process. Unitarity of the scattering matrix, which encodes the conservation of the flux, implies that the scattering amplitude cannot be arbitrarily large. Using quantum mechanical notation, we write the scattering amplitude as the partial wave expansion:

$$\mathcal{A} = 16\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) a_\ell, \quad a_\ell = \frac{\mathrm{e}^{2i\delta_\ell} - 1}{2i}. \tag{25}$$

It is clear that Re $a_\ell \leqslant 1/2$. Therefore, if any tree-level calculation gives the partial wave $|a_\ell| > 1/2$ for any ℓ it means that very strong higher order corrections are to be expected. The tree-level (= lowest-order term of the perturbative expansion) calculation is then unreliable, and the perturbative treatment becomes simply misleading. The dynamics of scattering is completely modified with respect to the naive perturbative expectation. So, we need to check that the tree-level scattering amplitudes do not shoot up in the EW theory.

The most dangerous is scattering of longitudinally polarized W and Z bosons, e.g., $W_L^+ W_L^- \to W_L^+ W_L^-$. The reason is that the longitudinal polarization vector of a massive vector boson behaves like $e_L^\mu = (p, 0, 0, E)/M$. As a result, the scattering amplitude, which contains sufficiently many e_L's, rises with energy and, at some energy, it definitely overshoots the perturbative unitarity constraint.

The Higgs mechanism tames this rise. The amplitude stays finite even at $s \to \infty$, $\mathcal{A} = -2m_h^2/v^2$, but it can still be large. Requiring that $|\mathcal{A}| < 8\pi$ gives

$$m_h < 2\sqrt{\pi}v \approx 870 \text{ GeV}.$$

A more accurate coupled-channel analysis lowers this bound to 710 GeV. One can therefore conclude that if the LHC does not find the Higgs boson with mass up to 700 GeV, then there is definitely some physics beyond the Standard Model. Even if there are no other particles, the *dynamics* of the electroweak sector is very different from the expectations of the SM.

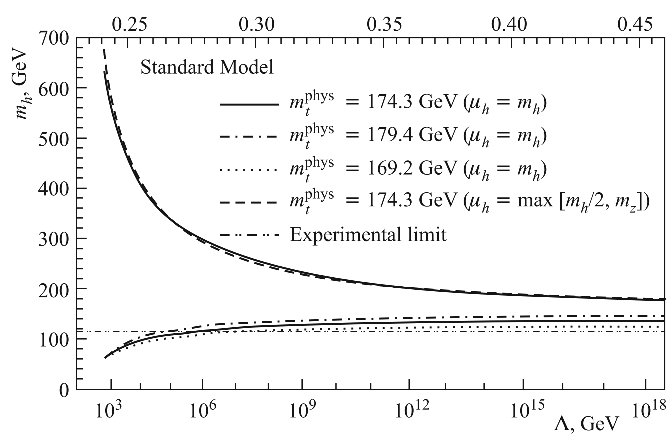
Even stronger constraints arise if one tries to understand the ultraviolet (small-distance) behavior of the theory. It is known in quantum field theory that loop corrections usually induce renormalization of the bare quantities introduced in the Lagrangian. The physical observables, such as masses and charges, etc. change with the observation scale. For example, if one studies quantities in collision of highly virtual particles with virtuality Q^2 (which corresponds to distances $\sim 1/Q$), then all quantities will (slowly) change with the Q^2 rise.

In the Higgs sector, the loop corrections affect the value of the quartic Higgs self-interaction parameter λ. An explicit calculation shows that if λ is sufficiently high (the Higgs boson is heavy) at the scale of $Q \sim 1$ TeV, then it will grow further with Q^2 rise. This evolution (known as the renormalization-group flow) is approximately described by the following equation:

$$\frac{d\lambda}{d\log Q^2} \approx \frac{3\lambda^2}{2\pi}. \tag{26}$$

So, sooner or later λ will overshoot the perturbativity constraint, and the theory must be modified. On the other hand, if λ is too small (the Higgs boson is too light), then the top-quark loops will drive λ down. Then, at certain scale λ becomes negative, which means that the potential is not bounded from below anymore. This is known as the *vacuum stability* constraint.

Summarizing these two restrictions, one can state that if we insist that the Standard Model remains valid (and not modified!) up to the energy scale Λ, the Higgs boson cannot be too light nor too heavy. It is also obvious that the larger Λ, the narrower is the allowed region of the Higgs boson masses. This region is shown in figure. When discussing this



Perturbative unitarity (upper curves) and vacuum stability (lower curves) constraints on the mass of the Standard Model Higgs boson plot, physicists often mention the «nightmare scenario»: if it turns out experimentally that $130 < m_h < 180$ GeV and no indication of physics beyond the Standard Model is found, then it will mean that the SM can work even up to the Planck scale. In other words, we would have no clue on where and what the New Physics is. Certainly, this would be a depressing situation.

1.4. What Is Spontaneously Broken in EWSB? There is a subtlety in the nature of electroweak symmetry breaking that is often omitted in discussions of the Higgs mechanism. This subtlety concerns the question: what exactly is broken spontaneously when we speak of EWSB?

In 1975 Elitzur proved a theorem [5] which might sound paradoxical after what we learned in the previous sections: *it is impossible to spontaneously break a gauge symmetry*. Only global symmetries can be broken spontaneously. In fact, there is no contradiction here. If we look back, for example, at the Abelian Higgs model, we see that the gauge symmetry was *removed by hand* at that very moment when we adjusted phase $\alpha(x)$ to eliminate the field $\xi(x)$. After that, the spontaneous breaking happened only for the phase of the vacuum expectation value; that is, only the global $U(1)$-symmetry was actually broken spontaneously.

This observation implies that electroweak symmetry breaking is not really an unavoidable physical phenomenon that accompanies transition from the «EW-symmetric phase» to the «Higgs phase». We should be able to reformulate the electroweak theory in a way that does not refer explicitly to breaking of the gauge symmetry.

Consider again the Abelian Higgs model and write down terms quadratic in fields $h(x)$, $g(x)$ and $A_\mu(x)$ in the Higgs phase (with non-zero v.e.v. ϕ_0):

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \frac{q^2\phi_0^2}{2}\left(A_\mu - \frac{1}{q\phi_0}\partial_\mu g\right)^2. \quad (27)$$

Let us introduce the *gauge invariant field* B_μ:

$$B_\mu \equiv A_\mu - \frac{1}{q\phi_0}\partial_\mu g, \quad F_{\mu\nu}^{(A)} = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}^{(B)}. \quad (28)$$

Now the quadratic Lagrangian

$$\mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \frac{q^2\phi_0^2}{2}B_\mu B^\mu \quad (29)$$

contains only those degrees of freedom which are explicitly $U(1)$-gauge invariant. We see that there is simply no freedom of gauge transformation left in (29). The issue of (spontaneous) breaking of gauge symmetry becomes redundant, because there is no gauge symmetry anymore. More details about this remarkable phenomenon can be found in [2, 6].

2. SOME FEATURES OF NON-MINIMAL HIGGS SECTORS

When physicists discuss physics beyond the Standard Model, they can actually mean different things. They can mean, for example, models with new particles (for instance, with an extra generation of heavy fermions) but with the same minimal Higgs mechanism as in the SM. Or they can consider the same (matter) particle content as in the SM but with a more complicated (non-minimal) Higgs sector. In this section we will consider some variations of the latter possibility. For a broader review on various non-minimal Higgs mechanisms, see [4, 7].

2.1. Two-Higgs-Doublet Model. The two-Higgs-doublet model (2HDM) consists in introducing not one but two Higgs doublets with identical quantum numbers: $\phi_i = (\phi_i^+, \phi_i^0)^T$, $i = 1, 2$. These doublets interact with the gauge bosons, $|D_\mu \phi_i|^2$, with fermions via the Yukawa-type interactions, and they self-interact via the Higgs potential constructed from $(\phi_i^\dagger \phi_j)$. This model was introduced in 1973 and turned out to be quite rich, so that it is still actively explored. Two main physics motivations of 2HDM are its resemblance to the scalar sector of the MSSM (minimal supersymmetric extension of the Standard Model), and the possibility to induce CP-violation purely within the Higgs sector. This model is also attractive for cosmologists, as it can easily generate dark matter candidates in the Higgs sector and multiple strong thermal phase transitions. For a detailed account of theoretical and phenomenological aspects of 2HDM, see [8].

The main distinctive phenomenological feature of this model is presence of several physical Higgs bosons. The basic counting shows that two doublets have 8 degrees of freedom, and after EWSB there remains five physics scalars: three neutral h_1, h_2, h_3 and two charged, $H^\pm$. Often the neutral Higgs bosons come with a definite CP-parity, and in this case one speaks of two scalars, h and H, and one pseudoscalar, A.

In order to see explicitly how the EWSB proceeds in this model, let us consider a simple yet illustrative version of the 2HDM potential:

$$V = -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) + \lambda(\phi_1^\dagger\phi_1)^2 + \lambda(\phi_2^\dagger\phi_2)^2 + \\ + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) - \frac{\lambda_5}{2}\left[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2\right].$$

We search for the minimum in the form

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 \end{pmatrix}.$$

It is useful to introduce the following notation: $v^2 = v_1^2 + v_2^2 = (246\ \text{GeV})^2$, $v_1 = v\cos\beta$, $v_2 = v\sin\beta$, $\tan\beta = v_2/v_1$. In order to minimize the potential, we first set derivatives to zero:

$$\frac{\partial V}{\partial v_1} = v_1\left[-\mu_1^2 + \lambda v_1^2 + \lambda_{345}v_2^2\right] = 0,$$
$$\frac{\partial V}{\partial v_2} = v_2\left[-\mu_2^2 + \lambda v_2^2 + \lambda_{345}v_1^2\right] = 0,$$

where $\lambda_{345} \equiv \lambda_3 - \lambda_4 - \lambda_5$. The second derivatives are

$$\frac{\partial^2 V}{\partial v_1^2} = [\cdots]_1 + 2\lambda v_1^2, \quad \frac{\partial^2 V}{\partial v_2^2} = [\cdots]_2 + 2\lambda v_2^2, \quad \frac{\partial^2 V}{\partial v_1 \partial v_2} = 2\lambda_{345}v_1v_2, \tag{30}$$

where $[\cdots]_1$ and $[\cdots]_2$ denote the expressions in brackets in the previous equations.

Now note that depending on the parameters there exist several possibilities (several phases): EW-symmetric, when $v_1 = v_2 = 0$, «partially broken», when $v_1 \neq 0$, $v_2 = 0$

or vice versa, and «fully broken», when v_1, $v_2 \neq 0$. Explicit conditions on μ_1^2, μ_2^2 and λ's that separate these phases can be easily found.

Let us pick up the last phase ($\tan\beta \neq 0, \infty$). Expanding the Higgs fields around the minimum

$$\phi_1 = \begin{pmatrix} w_1^+ \\ \dfrac{1}{\sqrt{2}}(v_1 + h_1 + i\eta_1) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} w_2^+ \\ \dfrac{1}{\sqrt{2}}(v_2 + h_2 + i\eta_2) \end{pmatrix}$$

and substituting them in the potential give the following quadratic terms:

$$\begin{aligned} V_2 = (\lambda_4 + \lambda_5)(v_2 w_1^- - v_1 w_2^-)(v_2 w_1^+ - v_1 w_2^+) + \\ + \lambda_5 (v_2\eta_1 - v_1\eta_2)^2 + \lambda v_1^2 h_1^2 + \lambda v_2^2 h_2^2 + 2\lambda_{345} v_1 v_2 h_1 h_2, \\ = m_{H^\pm}^2 H^- H^+ + \frac{1}{2} M_A^2 A^2 + \frac{1}{2} m_h^2 h^2 + \frac{1}{2} m_H^2 H^2, \end{aligned}$$

where

$$\begin{aligned} H^+ = \sin\beta\, w_1^+ - \cos\beta\, w_2^+, \quad m_{H^\pm}^2 = (\lambda_4 + \lambda_5) v^2, \\ A = \sin\beta\, \eta_1 - \cos\beta\, \eta_2, \quad m_A^2 = \lambda_5 v^2, \end{aligned}$$

while h and H are light and heavy linear combinations of h_1 and h_2. Note that the three Goldstone fields $G^\pm = \cos\beta\, w_1^\pm + \sin\beta\, w_2^\pm$ and $G^0 = \cos\beta\, \eta_1 + \sin\beta\, \eta_2$ disappear in this quadratic term.

As for the fermionic sector, there are several ways to couple two Higgs doublets to fermions. For example, only one Higgs doublet (e.g., ϕ_2) couples to all fermions (this model is known as Type I model); or ϕ_2 couples to all up-type fermions, ϕ_1 couples to down-type fermions (Type II), etc. The constant in each Yukawa term is adjusted to give the right mass to the fermions, but the coupling to the Higgs bosons can be enhanced or suppressed with respect to the SM. This deviation from the SM patterns arises because v.e.v.'s (v_1, v_2) and the physical neutral Higgses (h, H) are not aligned. These deviations lead to important phenomenological properties of all these Higgs bosons.

Let us mention several phenomena that can arise in specific variants of the 2HDM. First, the model we considered above is CP-conserving. The neutral Higgs bosons have definite CP-parity and do not mix with each other. However, it is possible to construct a potential that would induce spontaneous CP-violation in the Higgs sector. Namely, although all the coefficients in the potential are real, the v.e.v. have a non-zero relative phase: $v_1 = |v_1|$, $v_2 = |v_2| \exp(i\xi)$. As a result, the neutral Higgs mass matrix contains non-diagonal terms that mix scalars with pseudoscalars, so that the physical Higgs bosons do not have definite CP-parity. Such CP-violating 2HDM leads to additional contributions to many CP-sensitive observables.

Another important feature of a typical 2HDM is the decoupling limit. It refers to a situation when all the Higgs bosons (H, A, $H^\pm$) except for one are very heavy, while the lightest Higgs boson h lies in the «standard» region of 100–200 GeV. In this case it turns out that the properties of h usually resemble very much the properties of the Standard Model Higgs boson. This is a rather unfortunate situation because if an SM-like Higgs boson is observed at the LHC, it would be difficult to distinguish the true SM from the models like 2HDM in the decoupling limit.

It is possible that one of the Higgs fields gets zero v.e.v. but gains mass after EWSB. This Higgs boson cannot then decay into gauge bosons or other Higgs particles. If it also decouples from fermions, it becomes stable and can be a natural dark matter candidate. Such models known as «Inert doublet models» have received much attention [9].

Yet another possibility that can exist in 2HDM is the so-called «charge-breaking vacuum». It arises when the v.e.v.'s of the two doublets have the form

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} u \\ v_2 \end{pmatrix}$$

with some non-zero u. In this case the $SU(2)\times U(1)$ gauge group breaks down completely, the photon also becomes massive, and the electric charge is not conserved anymore (neutral and charged Higgs bosons mix). This type of vacuum, of course, does not correspond to what we see today, but it is possible that the early Universe evolved through such an exotic phase when it was cooling down and passed through the electroweak scale [10]. The charge-breaking and charge-restoring phase transitions might have been sufficiently violent to leave some traces in the observable Universe.

We close this discussion by the remark about 2HDM with the most general scalar sector. When we introduce a model (for example, 2HDM), it is natural to consider it in its most general formulation and deduce the full list of phenomenological possibilities that can occur there. In the case of 2HDM, the most general scalar potential with all quadratic and quartic interactions between ϕ_1 and ϕ_2 contains 14 terms, each carrying its own free parameter. It turns out that one cannot minimize this potential explicitly with straightforward algebra.

However, in the last few years a number of basis-invariant tools have been developed that yield a lot of information on the properties of the minima of the potential without explicitly finding them. The results include the number and coexistence of minima, the symmetries which can be encoded in the potential and their breaking patterns, and even the full phase diagram of the model in the tree-level approximation.

It is also interesting to pursue the idea of «Higgs generations» further and consider a model with N Higgs doublets. Examples include Weinberg's 3HDM [11], Adler's 6HDM [12], the private Higgs model by Porto and Zee [13] in which each fermion couples to its own Higgs doublet, etc.

One of the motivations to consider such multi-doublet models is to alleviate the fermion mass hierarchy problem: the fact that the fermion masses (and therefore the dimensionless Higgs-fermion coupling constants) span many orders of magnitude without any obvious reason. It turns out, indeed, that in certain multi-doublet models there appears a natural hierarchy among the v.e.v.'s of several doublets, which can then lead, also quite naturally, to very different fermionic masses. These models are more difficult to analyze than 2HDM, but they are still studied by many groups.

2.2. Extra Singlets. The Higgs fields do not have to be only doublets, but can transform under any representation of the $SU(2)\times U(1)$ gauge group: singlets, triplets, etc. The only requirement is that they contain a neutral component, which will acquire the vacuum expectation value. The experimental data, and especially the relation $m_W/m_Z = \cos\theta_W$ which is tested in experiment with much accuracy, indicate though that the largest Higgs v.e.v. must come from a doublet. However, additional Higgs bosons can come in any representation.

In the simplest model of this type, one introduces neutral singlet field σ in addition to the usual Higgs doublet ϕ. Below we give a sketchy description of one such model introduced in [14] which bears a picturesque name «Higgs portal into Hidden valley».

The main assumption of this model is that there exists a «hidden valley» in the particle physics landscape: a whole sector of new matter and new gauge fields which interact with each other, but which are «blind» to the interactions of the Standard Model. These new particles, of course, are not directly observable because our detectors are made of «normal» particles. Our matter and gauge fields, in turn, do not participate in the new interactions.

Suppose we want to describe both sectors of the reality in a single renormalizable quantum field theoretic model. We write the Lagrangian as

$$\mathcal{L} = \mathcal{L}_{\rm our} + \mathcal{L}_{\rm hidden} + \mathcal{L}_{\rm link}, \tag{31}$$

where $\mathcal{L}_{\rm link}$ describes the interaction terms that links our world with the hidden valley. It turns out that if we insist on the renormalizability of the interaction term, we have very little freedom in constructing $\mathcal{L}_{\rm link}$: apart from gravity and kinetic mixing, the only interaction can occur via the quadratic Higgs term. Indeed, a renormalizable theory must contain operators of dimension not higher than 4. All terms in the SM Lagrangian have dimension exactly equal to 4 (and therefore no additional field can be put there!) except for one: $-\mu^2(\phi^\dagger\phi)$. The operator here has dimension 2, and it is accompanied by a dimensional coupling μ^2. One can say that this is the only term in the SM that bears an «exterior» mass scale explicitly introduced in the model.

The idea of the «Higgs portal» consists in the assumption that μ is not a fixed mass scale but is the vacuum expectation value of the new «hidden» scalar σ:

$$\eta(\phi^\dagger\phi)\langle\sigma^2\rangle \equiv -\mu^2(\phi^\dagger\phi). \tag{32}$$

This new field σ must be an EW-singlet, but it can transform non-trivially under the «hidden» interactions, and this explains why only σ^2 term appears in the interaction. In a simplest model, such an interact term after EWSB leads to interactions between the usual Higgs boson h and «hidden» scalar h_σ:

$$\eta(\phi^\dagger\phi)\sigma^2 \to \eta(2vh + h^2)(2\langle\sigma\rangle h_\sigma + h_\sigma^2). \tag{33}$$

Since $\langle\sigma\rangle \neq 0$, there opens up the possibility of the direct mixing $h \leftrightarrow h_\sigma$ and of the decay $h^{(*)} \to h_\sigma h_\sigma$ with a very direct collider signature: $gg \to h^{(*)} \to h_\sigma(h_\sigma)$ with a subsequent decay into hidden valley particles (here $h^{(*)}$ denotes either the real or highly virtual Higgs boson). These particles are not directly seen by detectors, but the overall kinematical balance will allow us to observe a large missing transverse momentum. This simple model illustrates an important statement: the Higgs boson physics is interesting not only on its own but also as a window into possible New Physics.

2.3. Extra Triplets. As the last example, let us consider a model in which a Higgs doublet is accompanied by a triplet:

$$\phi(T=1/2) = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \xi(T=1) = \begin{pmatrix} \xi^{++} \\ \xi^+ \\ \xi^0 \end{pmatrix} \tag{34}$$

with the following interaction potential:

$$V = -m^2(\phi^\dagger\phi) - M^2(\xi^\dagger\xi) + \lambda_1(\phi^\dagger\phi)^2 + \lambda_2(\xi^\dagger\xi)^2 + \lambda_3(\phi^\dagger\phi)(\xi^\dagger\xi) + \\ + \mu\left[\xi^0\phi^0\phi^0 + \sqrt{2}\xi^-\phi^+\phi^0 + \xi^{--}\phi^+\phi^+\right]. \quad (35)$$

If we assume that parameter M^2 is very large, then both $\langle\phi^0\rangle = v$ and $\langle\xi^0\rangle = u$ are non-zero, but $u \approx \mu v^2/M^2$ is naturally small. This small vacuum expectation value can be used to generate neutrino masses without right-handed neutrinos (neutrino masses at the eV scale require $M \sim 10^{13}$ GeV). Simultaneously, the same model can also lead to the weak lepton number violation, which is a prerequisitive for the leptogenesis. An interaction term that performs this task is

$$f_{ij}\left[\xi^0\nu_i\nu_j + \xi^+(\nu_i l_j + l_i\nu_j)/\sqrt{2} + \xi^{++}l_i l_j\right] + \text{h.c.} \quad (36)$$

The violation of the lepton number can be traced from the fact that heavy triplet ξ^{++} can decay both into $\phi^+\phi^+$ with $L = 0$ and into $l_i^+ l_j^+$ with $L = -2$. (By the way, decay of a boson into two fermions instead of a fermion–antifermion pair is nothing strange. To give a simple example in hadronic physics, one can excite a deuteron so that it will break into a proton and a neutron, both of which are fermions.) A realistic version of this model was studied in [15] and it was shown that two such triplets plus a CP-violation naturally lead to leptogenesis required to explain the matter–antimatter asymmetry.

3. SUMMARY

Physics of the Higgs boson(s) within the Standard Model and, especially, beyond SM is a very hot topic in high-energy physics. It will become even hotter if the LHC finds (hopefully, soon) any hint of the non-SM electroweak symmetry breaking mechanism. Many variants of the electroweak symmetry breaking scenarios have been introduced and developed, and we covered only a very small number of them. Many other exciting models can be found in the reference list.

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REFERENCES

1. *Okun L. B.* Leptons and Quarks. M.: Nauka, 1990;
   Emelyanov V. M. The Standard Model and Its Extensions. M.: Fizmatlit, 2007.
2. *Rubakov V. A.* Classical Gauge Fields. M.: URSS, 1999.
3. *Wells J. D.* Lectures on Higgs Boson Physics in the Standard Model and Beyond. arXiv:0909.4541.
4. *Groghan K.* New Approaches to Electroweak Symmetry Breaking Mechanism // Uspekhi. 2007. V. 177. P. 3–42.

5. *Elitzur S.* Impossibility of Spontaneously Breaking Local Symmetries // Phys. Rev. D. 1975. V. 12. P. 3978–3982.
6. *Perez A., Sudarsky D.* On the Symmetry of the Vacuum in Theories with Spontaneous Symmetry Breaking // arXiv:0811.3181.
7. *Accomando E. et al*. Workshop on CP Studies and Non-Standard Higgs Physics. arXiv:hep-ph/0608079.
8. *Branco G. C. et al.* Theory and Phenomenology of Two-Higgs-Doublet Models. arXiv:1106.0034.
9. *Lopez Honorez L. et al.* The Inert Doublet Model: An Archetype for Dark Matter // JCAP. 2007. V. 0702. P. 028.
10. *Ivanov I. P.* Thermal Evolution of the Ground State of the Most General 2HDM // Acta Phys. Polon. B. 2009. V. 40. P. 2789–2807;
    Ginzburg I. F., Ivanov I. P., Kanishev K. A. The Evolution of Vacuum States and Phase Transitions in 2HDM during Cooling of Universe // Phys. Rev. D. 2010. V. 81. P. 085031.
11. *Weinberg S.* Gauge Theory of CP Nonconservation // Phys. Rev. Lett. 1976. V. 37. P. 657.
12. *Adler S. L.* Model for Particle Masses, Flavor Mixing, and CP Violation, based on Spontaneously Broken Discrete Chiral Symmetry as the Origin of Families // Phys. Rev. D. 1998. V. 59. P. 015012.
13. *Porto R. A., Zee A.* The Private Higgs // Phys. Lett. B. 2008. V. 666. P. 491–495.
14. *Patt B., Wilczek F.* Higgs-Field Portal into Hidden Sectors. arXiv:hep-ph/0605188.
15. *Ma E., Sarkar U.* Neutrino Masses and Leptogenesis with Heavy Higgs Triplets // Phys. Rev. Lett. 1998. V. 80. P. 5716–5719.