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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

AXIAL ANOMALY, QUARK–HADRON DUALITY AND TRANSITION FORM FACTORS

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We study the transition form factors of pseudoscalar mesons by means of anomaly sum rule — an exact relation which is a consequence of dispersive representation of axial anomaly. This sum rule (derived for the octet channel) combined with the quark–hadron duality allows us to relate the transition form factors of η and η' mesons. The notion of quark–hadron duality in connection with our approach is discussed and comparison with recent experimental data is done.

Изучены переходные формфакторы псевдоскалярных мезонов с помощью аномального правила сумм — точного соотношения, следующего из дисперсионного представления аксиальной аномалии. Это правило сумм (полученное для октетного канала) вместе с гипотезой кварк-адронной дуальности позволило получить связь между переходными формфакторами η- и η'-мезонов. Обсуждается понятие кварк-адронной дуальности в связи с предложенным подходом, а также проводится сравнение с новыми экспериментальными данными.

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INTRODUCTION

One of the first manifestations of axial anomaly [1] in particle physics was discovered in two-photon decays of pseudoscalar mesons. The dispersive approach to axial anomaly [2] extended the applicability of axial anomaly to the case of virtual photons and allowed one to derive the so-called anomaly sum rule (ASR) [3, 4]. This exact sum rule proved to be a useful tool for studying the processes of photon–meson transitions, e.g., $\gamma\gamma^* \to \pi^0(\eta, \eta')$ [5], which attracted a lot of interest [6] due to recent experimental data on η, η' transition form factors [7].

In this paper, we study the ASR in the octet channel, where the η and η' mesons make the main contributions and the mixing of them is significant.

1. ANOMALY SUM RULE AND QUARK–HADRON DUALITY

Let us briefly remind what is the anomaly sum rule derived for the octet channel of axial current (for details, see [4, 5]). The VVA triangle graph correlator

$$T_{\alpha\mu\nu}(k, q) = \int d^4x \, d^4y \, \mathrm{e}^{(ikx+iqy)} \langle 0|T\{J_{\alpha 5}(0)J_\mu(x)J_\nu(y)\}|0\rangle \tag{1}$$

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contains axial current $J^{(8)}_{\alpha 5} = 1/\sqrt{6}(\bar{u}\gamma_\alpha\gamma_5 u + \bar{d}\gamma_\alpha\gamma_5 d - 2\bar{s}\gamma_\alpha\gamma_5 s)$ and two vector currents $J_\mu = (e_u\bar{u}\gamma_\mu u + e_d\bar{d}\gamma_\mu d + e_s\bar{s}\gamma_\mu s)$; k, q are momenta of photons. This correlator can be written as a tensor decomposition with the Lorentz invariant coefficients $F_j = F_j(k^2, q^2, p^2; m^2)$, $p \equiv k + q$, $j = 1, \ldots, 6$.

We are interested in the case of one real and one virtual photon ($Q^2 = -q^2 > 0$). Then, for the invariant amplitude $F_3 - F_6$ the ASR can be obtained [4]:

$$\int_{4m^2}^{\infty} A_{3a}(t; q^2, m^2)\, dt = \frac{1}{2\pi\sqrt{6}}, \tag{2}$$

where $A_{3a} = (1/2)\,\mathrm{Im}\,(F_3 - F_6)$.

The ASR (2) is an exact relation, i.e., the integral has neither perturbative [8] nor nonperturbative (as it is expected from 't Hooft's principle) corrections. Another important property of this relation is that it holds for an arbitrary quark mass m and for any q^2.

Saturating the l.h.s. of the three-point correlation function (1) with the resonances and singling out their contributions to ASR (2), we get the sum of resonances with appropriate quantum numbers:

$$f^8_\eta F_\eta + f^8_{\eta'\gamma} F_{\eta'\gamma} + (\text{«other resonances»}) = \int_{4m^2}^{\infty} A_{3a}(t; q^2, m^2)\, dt = \frac{1}{2\pi\sqrt{6}}. \tag{3}$$

Here the form factors $F_{M\gamma}$ of transitions $\gamma\gamma^* \to M$ ($M = \eta, \eta'$) and the coupling (decay) constants f^a_M are defined by the matrix elements:

$$\int d^4x\, \mathrm{e}^{ikx} \langle M(p)|T\{J_\mu(x)J_\nu(0)\}|0\rangle = \epsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma F_{M\gamma}, \langle 0|J^{(a)}_{\alpha 5}(0)|M(p)\rangle = ip_\alpha f^a_M. \tag{4}$$

The terms denoted as «other resonances» can be replaced by the integral $\int_{s_0}^{\infty} A_{3a}(t; q^2, m^2)\, dt$ (continuum contribution), where s_0 is the continuum threshold in the local quark–hadron duality approach. Usually s_0 can be determined from the two-point QCD sum rules analysis, but, in the case of the octet channel the value of s_0 is not well calculated. However, in our approach s_0 can be treated as a free parameter and determined from the ASR itself in the large Q^2 limit.

Using the one-loop expression for continuum part of spectral function $A_{3a} = \dfrac{1}{2\pi\sqrt{6}} \times \dfrac{Q^2}{(s+Q^2)^2}$ from ASR (2) we finally come to

$$\pi f^8_\eta F_{\eta\gamma}(Q^2) + \pi f^8_{\eta'} F_{\eta'\gamma}(Q^2) = \frac{1}{2\pi\sqrt{6}} \frac{s_0}{Q^2 + s_0}. \tag{5}$$

Let us note, that in (3) we single out both η and η' mesons, while the rest of contributions are absorbed by the continuum. This is because the η' meson decays into two photons (since continuum contribution vanishes at $Q^2 = 0$), while the higher contributions are suppressed due to conservation of the axial current in the chiral limit. Let us also stress that the relation (5) is correct for all Q^2 due to the absence of corrections to A_{3a} [10] which allows one to utilize the above expression for different Q^2.

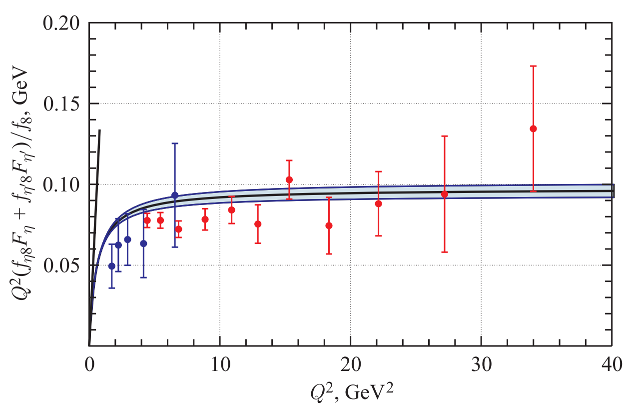
Relying on the prediction of QCD factorization [9] for the transition form factors at large Q^2,

$$Q^2 F^{as}_{M\gamma} = \frac{2}{\sqrt{6}}(f^8_M + 2\sqrt{2} f^0_M), \tag{6}$$

we can express s_0 in terms of decay constants f^a_M:

$$s_0 = 4\pi^2((f^8_\eta)^2 + (f^8_{\eta'})^2 + 2\sqrt{2}[f^8_\eta f^0_\eta + f^8_{\eta'} f^0_{\eta'}]). \tag{7}$$

Equation (5) with substituted s_0 from (7) relates the transition form factors $F_{M\gamma}$ and decay constants f^a_M *for arbitrary* Q^2. The decay constants can be related basing on particular mixing scheme. Here, we restrict ourselves to the simplest one with one mixing angle θ: $f^8_\eta = f_8 \cos\theta, f^8_{\eta'} = f_8 \sin\theta, f^0_\eta = -f_0 \sin\theta, f^0_{\eta'} = f_0 \cos\theta$. For this scheme $s_0 = 4\pi^2 f_8^2$ does not depend on f_0, while f_8 can be calculated from (5) in the limit $Q^2 = 0$ (η, η' decay widths are used in this case). $\theta = -16°$. The plot of the octet combination of the transition form factors (l.h.s. of Eq. (5) multiplied by Q^2) compared with the experimental data [7] is shown in the Figure.



The ASR (5) for one-angle mixing scheme, $\theta = -16°$. Filled stripe denotes the uncertainties originated from the experimental errors of meson decay widths and thus determination of f_8. Inclined line represents ASR at $Q^2 = 0$

We see, that the available data are described well, though they manifest a slight tendency to grow, resembling the isovector (π^0) channel, but at larger Q^2. This is a result of mixing in the octet channel — the form factors themselves $Q^2 F_{M\gamma}$ do not show such a kind of behaviour. Theoretically, this growth corresponds to a possible correction to continuum contribution [5].

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