

GRAND UNIFICATION IN THE MINIMAL LEFT–RIGHT SYMMETRIC EXTENSION OF THE STANDARD MODEL

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The simplest minimal left–right symmetric extension of the Standard Model is studied in the high energy limit. Some consequences of the Grand Unification hypothesis are explored assuming that the parity-breaking scale is the only relevant energy between the electroweak scale and the unification point. While the model is shown to be compatible with the observed neutrino phenomenology, the parity-breaking scale and the heavy boson masses are predicted to be above 10^7 TeV, which is beyond the reach of present-day experiments. Below that scale, only an almost sterile right-handed neutrino with a mass $M(\nu_R)$ of ≈ 100 TeV is allowed.

Простейшее минимальное лево-правосимметричное расширение Стандартной модели изучается в пределе высоких энергий. Также исследуются некоторые следствия гипотезы Великого объединения в предположении, что нарушение четности может происходить только при энергиях в диапазоне от электрослабых взаимодействий до точки объединения. В то время как предсказания модели находятся в соответствии с наблюдаемыми в нейтринной феноменологии, модель приводит к порогу нарушения четности и массам тяжелых бозонов выше 10^7 ТэВ, т. е. энергиям, которые недостижимы на действующих в настоящее время экспериментальных установках. Ниже данного порога допустимо существование только чисто правовинтового нейтрино с массой $M(\nu_R) \approx 100$ ТэВ.

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An interesting aspect in the phenomenology of neutrinos is the emergence of important elements of new physics beyond the Standard Model (SM) since there is no doubt that the experimental results can only be understood if the neutrinos are assumed to have nonvanishing masses and mixings. Massive neutrinos require the existence of a right-handed neutrino, which makes the B–L generator free of any triangle anomaly, and the related symmetry gaugeable. Thus, the most natural extension of the SM gauge group is the left–right (LR) symmetric group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, which breaks to the SM group at some high scale [1–4]. The LR models have been discussed as embedded in larger Grand Unification (GU) models, like $SO(10)$, and their symmetry-breaking path has been discussed by several authors [5–8]. There has recently [8–11] been a renewed interest in the minimal LR symmetric extension of the SM [12, 13], a model with two scalar doublets and no bidoublets, which predicts the low-energy phenomenology of the SM at a very modest cost in terms of new particles that are required to be detected at very high energy.

Despite its simple particle content, the minimal model retains most of the interesting properties of the more complex LR models; B–L is a gauge symmetry with a triangle-anomaly-free generator; parity is spontaneously broken; massive neutrinos can be accommodated by seesaw mechanisms; right-handed neutrinos could in part account for the dark matter.

Even if the minimal LR model without bidoublets fails to predict a stable broken-symmetry vacuum [14] at tree-level, a consistent path for symmetry breaking has been predicted by the inclusion of higher order corrections that become relevant in the Higgs–Top sector [15]. Other paths towards symmetry breaking have been recently discussed [9, 11], and the minimal model seems to be a first viable step towards new physics beyond the SM.

However, while some results have been obtained on the «standard» LR model [16–19], the more recent minimal LR model has not been sufficiently studied. Like other non-supersymmetric (non-SUSY) models, there is evidence that an intermediate high scale is required prior to unification [20]. Thus, it would be interesting to investigate the issue of high-energy unification in the framework of the minimal LR model.

In this paper, we present a detailed quantitative analysis of the most simple symmetry-breaking path for the minimal LR model up to unification in order to pinpoint its predictions for the breaking scales and neutrino masses. While a similar analysis has been presented for the more complex SUSY LR models [21, 22], we notice that within the framework of the detailed quantitative analysis the behaviour of gauge couplings depends on a detailed particle content of the model. It is interesting to study the same problem in a truly minimal LR model with a minimum particle content.

We show that the simple hypothesis that a single intermediate scale exists between GU and the weak scale is enough for predicting this intermediate scale and the masses of the heavy gauge bosons Z' and W_R . The prediction of a breaking scale of order 10^{10} GeV — halfway between the electroweak scale and the GU scale — is encouraging even if that scale seems to be too large to be detected by present-day experiments. The results are compatible with a micro-milli-eV mass scenario for neutrinos and show that the non-SUSY minimal LR model is a valid and natural option as a first step toward understanding new physics beyond the SM.

An interesting point is that the present analysis does not make any use of the details of the model above GU. It requires neither full knowledge of the symmetry-breaking mechanism nor the detailed descriptions of the minimal set of Higgs representations; the beta functions depend only on the actual particle content of the model below unification. This generality makes the analysis valid for quite a wide range of mechanisms and different unifying groups. On the other hand, this choice of generality can be regarded as a shortcoming of the present study merely because no significant details can be provided on the emergence of the low-energy Lagrangian, the flow of the merged couplings above GU, the proton lifetime prediction, and the unifying group. Nevertheless, the analysis is very simple, and its generality makes it worth to be discussed together with its possible effects on the physics of neutrinos.

The minimal LR symmetric model has been described in several papers [12–15]. The LR symmetric Lagrangian is a sum of a fermionic term \mathcal{L}_f , a standard Yang–Mills term \mathcal{L}_{YM} for the gauge bosons, a Higgs term \mathcal{L}_H , and eventually the Higgs–fermion interaction term \mathcal{L}_{int} . A special feature of the minimal model is its limited particle content. The Higgs sector contains two scalar doublets but no bidoublets, and is described by the simple Lagrangian

$$\mathcal{L}_H = -\frac{1}{2}|D_L^\mu \chi_L|^2 - \frac{1}{2}|D_R^\mu \chi_R|^2 + V(\chi_L, \chi_R), \quad (1)$$

where the covariant derivative D_a^μ is defined according to

$$D_a^\mu = \left(\partial^\mu - ig_a \mathbf{A}_a^\mu \mathbf{T}_a + i\tilde{g} B^\mu \frac{Y}{2} \right), \quad a = L, R. \quad (2)$$

$\mathbf{T}_L, \mathbf{T}_R$ and Y are the generators of $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$, respectively, with couplings $g_L = g_R = g$ and \tilde{g} . The electric charge is given by $Q = T_{L3} + T_{R3} + Y/2$. The Higgs fields χ_a are the scalar doublets

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^0 \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \quad (3)$$

with the transformation properties

$$\chi_L \equiv (2, 1, 1), \quad \chi_R \equiv (1, 2, 1). \quad (4)$$

A standard \mathcal{L}_{YM} is considered for the seven gauge fields $\mathbf{A}_L^\mu, \mathbf{A}_R^\mu$, and B^μ .

Fermions are described by doublets of spinors ψ_L and ψ_R with the transformation properties

$$\psi_L \equiv (2, 1, B - L), \quad \psi_R \equiv (1, 2, B - L). \quad (5)$$

Their Lagrangian term \mathcal{L}_f is

$$\mathcal{L}_f = -\bar{\psi}_L \gamma_\mu D_L^\mu \psi_L - \bar{\psi}_R \gamma_\mu D_R^\mu \psi_R. \quad (6)$$

The Lagrangian $\mathcal{L} = \mathcal{L}_f + \mathcal{L}_{\text{YM}} + \mathcal{L}_H$ is fully symmetric for the LR exchange, if the Higgs potential $V(\chi_L, \chi_R)$ is assumed to be symmetric for the exchange of χ_L and χ_R .

The simplest path for symmetry breaking requires two energy scales [15]; parity is assumed to be broken at a large energy scale $\mu = \Lambda_R$, where the scalar R-doublet χ_R takes a broken-symmetry vacuum expectation value (VEV), $\langle \chi_R \rangle = w$, while the L-doublet χ_L still retains a vanishing VEV. Below this energy scale, the gauge group is broken down to the SM gauge group $SU(2)_L \otimes U(1)$. At the electroweak scale, the L-doublet χ_L takes a broken-symmetry VEV $\langle \chi_L \rangle = v$, breaking the SM gauge group into a simple $U(1)_{\text{em}}$ group of electromagnetism. Provided that $w \gg v = 246 \text{ GeV}$, the model predicts the same phenomenology as of the SM. In unitarity gauge, we set $\chi_a^+ = 0$ and take χ_a^0 real with a finite VEV $\langle \chi_L^0 \rangle = v$, $\langle \chi_R^0 \rangle = w$. Assuming that $v \ll w$, the mass matrix for the gauge bosons has two charged eigenvectors [12], W_L^\pm and W_R^\pm , which are decoupled with the masses

$$M_{W(L)} = \frac{gv}{2}, \quad M_{W(R)} = \frac{gw}{2}, \quad (7)$$

a vanishing eigenvalue for the electromagnetic unbroken $U(1)_{\text{em}}$ eigenvector, and two massive neutral eigenvectors with a small mass

$$M_Z^2 = \frac{g^2 v^2 (g^2 + 2\tilde{g}^2)}{4(g^2 + \tilde{g}^2)} + \mathcal{O}\left(\frac{v^2}{w^2}\right) \quad (8)$$

for the «light» Z boson and a large mass

$$M_{Z'}^2 = (M_{W(L)}^2 + M_{W(R)}^2) \left(1 + \frac{\tilde{g}^2}{g^2}\right) - M_Z^2 \quad (9)$$

for the «heavy» boson Z' . All the effects of the heavy Z' and W_R^\pm are certainly suppressed at low energy [12].

In an intermediate energy range, above the electroweak scale up to the parity-breaking scale Λ_R , the minimal LR model becomes identical to the SM with a $SU(2)_L$ gauge coupling $g_2 = g$ and a $U(1)$ coupling g_1 , which according to Eq. (8), must satisfy the matching condition

$$\frac{1}{g_1^2} = \frac{1}{\tilde{g}^2} + \frac{1}{g_2^2} \quad (10)$$

at the scale $\mu = \Lambda_R$ in order to recover the known SM result $2M_Z^2/v^2 = g_2^2 + g_1^2$.

The GU hypothesis of a single unified gauge symmetry describing all forces and matter at very short distances is very attractive. According to it, the couplings are expected to merge at a very high energy scale $\mu = \Lambda_{\text{GUT}}$. However, as in other non-SUSY models, an intermediate high scale is required prior to unification [20]. In this paper, we explore the simplest hypothesis that the intermediate scale is the parity-breaking scale $\mu = \Lambda_R$, and that above that scale the gauge couplings of the full gauge group $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ run up to the unification scale $\mu = \Lambda_{\text{GUT}}$, where they merge. We show that this simple hypothesis is sufficient for determining the scales Λ_R and Λ_{GUT} by simply using the known beta functions of the model.

We prefer to use simple one-loop beta functions that are decoupled and allow for a thorough analytical discussion of the problem. Two-loop beta functions are known for the Standard Model [23] and would be required for a thorough quantitative discussion, but their use would not change the qualitative result in any way.

At one-loop, the gauge couplings satisfy the renormalization group (RG) equation

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_i) = -b_i \frac{g_i^3}{16\pi^2}, \quad (11)$$

where the coefficient b_i is known to be [24]

$$b_i = \frac{11}{3}C_A - \frac{4}{3}(2n_g T_F) - \frac{1}{3}T_s n_s, \quad (12)$$

n_g is the number of fermion generations, and n_s is the number of complex scalars.

For the gauge groups $SU(2)$ and $SU(3)$, the running of the couplings is not affected by symmetry breaking at the scale $\mu = \Lambda_R$; the coefficients are $b_2 = 19/6$ for $SU(2)$ ($C_A = 2$, $T_F = 1/2$, $n_g = 3$, $T_s = 1/2$ and $n_s = 1$) and $b_3 = 7$ for $SU(3)$ ($C_A = 3$, $T_F = 1/2$, $n_g = 3$ and $n_s = 0$). In fact, the trace of the square of a generator T for both groups reads

$$\text{Tr}(g_i T \cdot g_i T) = 4g_i^2 n_g T_F = 2g_i^2 n_g. \quad (13)$$

For the $U(1)_{B-L}$ gauge group, the running of the coupling depends on the particle content that differs below and above the symmetry-breaking scale. For $\mu > \Lambda_R$, the LR symmetry is unbroken, and for each generation there are six left-handed quarks with $Y = 1/6$, six right-handed quarks with $Y = 1/6$, two left-handed leptons with $Y = 1/2$, and two right-handed leptons with $Y = 1/2$. Thus,

$$\text{Tr}(\tilde{g}T \cdot \tilde{g}T) = \sum_{\text{fermions}} (\tilde{g}^2 Y^2) = \frac{4}{3}\tilde{g}^2 n_g, \quad (14)$$

which is tantamount to setting $T_F = 1/3$ in Eq. (12). Since there are two scalars, $n_s = 2$, we obtain the coefficient $\tilde{b} = -3$ for the coupling \tilde{g} of $U(1)_{B-L}$. For $\mu < \Lambda_R$, the LR symmetry

is broken, and the coupling g_1 is determined by the beta function of the SM $U(1)$ gauge group; for each generation, there are six left-handed quarks with $Y = 1/6$, three right-handed quarks with $Y = 2/3$, three right-handed quarks with $Y = -1/3$, two left-handed leptons with $Y = -1/2$, and one right-handed lepton with $Y = -1$. Thus,

$$\text{Tr}(g_1 T \cdot g_1 T) = \sum_{\text{fermions}} (g_1^2 Y^2) = \frac{10}{3} g_1^2 n_g, \quad (15)$$

which is tantamount to setting $T_F = 5/6$ in Eq. (12). Since there is one scalar, $n_s = 1$ (heavy fields are integrated out), we obtain the SM coefficient $b_1 = -41/6$ for the coupling g_1 .

It is useful to rescale the couplings in order to make the equivalence of the trace in Eqs. (13)–(15) explicit. Let us define a new set of couplings:

$$\alpha_1 = \frac{5}{3} \frac{g_1^2}{4\pi}; \quad \tilde{\alpha} = \frac{2}{3} \frac{\tilde{g}^2}{4\pi}, \quad (16)$$

$$\alpha_2 = \frac{g_2^2}{4\pi}; \quad \alpha_3 = \frac{g_3^2}{4\pi}. \quad (17)$$

In fact, the GU hypothesis requires the trace in Eqs. (13) and (15) to be the same at the GU scale $\mu = \Lambda_{\text{GUT}}$, where $SU(3)$, $SU(2)_L$, $SU(2)_R$, and $U(1)_{B-L}$ are restored as subgroups of the same larger group. In terms of the new set of rescaled couplings, the equivalence of the trace is satisfied whenever the couplings are equal, and the condition for GU is simply stated as $\tilde{\alpha} = \alpha_2 = \alpha_3$.

The new set of couplings satisfies the RG equation

$$\mu \frac{d\alpha_i}{d\mu} = \beta_i(\alpha_i) = -2c_i \frac{\alpha_i^2}{4\pi}, \quad (18)$$

where $c_2 = b_2$, $c_3 = b_3$, $c_1 = 3b_1/5$, and $\tilde{c} = 3\tilde{b}/2$.

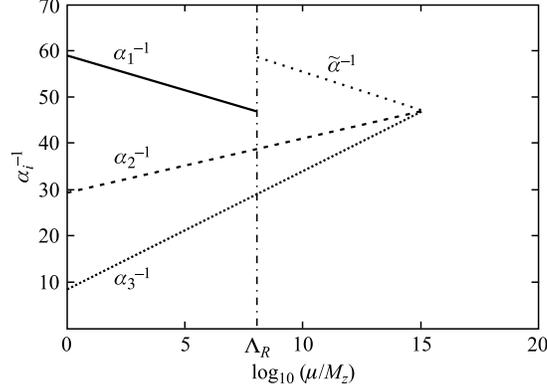
Equation (18) can be easily integrated, yielding the linear equations

$$\alpha_i^{-1}(\mu) = \alpha_i^{-1}(\mu_0) + \frac{c_i}{2\pi} \ln \left(\frac{\mu}{\mu_0} \right) \quad (19)$$

that are reported in the figure. In this scenario, the scale Λ_{GUT} is determined by the crossing of α_2 and α_3 . The inclusion of two-loop corrections would decrease the value of $\ln \Lambda_{\text{GUT}}$ by less than 3% [23] and would not affect the order of magnitude of Λ_{GUT} . Two-loop corrections are even smaller at the intermediate scale Λ_R and are completely negligible at the electroweak scale.

As discussed above, we assume that all the couplings cross at the scale $\mu = \Lambda_{\text{GUT}}$, yielding $\tilde{\alpha}(\Lambda_{\text{GUT}}) = \alpha_2(\Lambda_{\text{GUT}}) = \alpha_3(\Lambda_{\text{GUT}})$. According to the RG Equation (19), we let the coupling $\tilde{\alpha}$ flow down the parity-breaking scale Λ_R , where, according to the matching condition in Eq. (10), it must satisfy the constraint

$$\alpha_1(\Lambda_R) = \frac{5\alpha_2(\Lambda_R)\tilde{\alpha}(\Lambda_R)}{2\alpha_2(\Lambda_R) + 3\tilde{\alpha}(\Lambda_R)}, \quad (20)$$



Running of the inverse couplings α_1^{-1} , α_2^{-1} , and α_3^{-1} for the SM $SU(3) \otimes SU(2)_L \otimes U(1)$ gauge group, below the parity-breaking scale ($\mu < \Lambda_R$, on the left), and of the inverse couplings $\tilde{\alpha}^{-1}$, α_2^{-1} , and α_3^{-1} for the LR symmetric $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group, above the parity-breaking scale ($\mu > \Lambda_R$, on the right)

where according to Eq. (19), $\alpha_1(\Lambda_R)$ can be determined by the SM beta function for $\mu < \Lambda_R$, starting from the known value at the electroweak scale $\alpha_1(M_Z)$ and flowing up to the matching point Λ_R . The unknown scale Λ_R is pinpointed by the matching Eq. (20) as shown in the figure.

The analytical solution is

$$\ln\left(\frac{\Lambda_{\text{GUT}}}{M_Z}\right) = 2\pi \left(\frac{\alpha_2^{-1} - \alpha_3^{-1}}{c_3 - c_2}\right) = \frac{12\pi}{23} (\alpha_2^{-1} - \alpha_3^{-1}), \quad (21)$$

$$\ln\left(\frac{\Lambda_R}{M_Z}\right) = 2\pi (A\alpha_1^{-1} + B\alpha_2^{-1} + C\alpha_3^{-1}), \quad (22)$$

where $\alpha_i = \alpha_i(M_Z)$ are the couplings evaluated at the electroweak scale $\mu = M_Z$, and

$$A = \left(\frac{3}{5}c_2 + \frac{2}{5}\tilde{c} - c_1\right)^{-1} = \frac{5}{21}, \quad (23)$$

$$B = \frac{2A}{5} \left(\frac{\tilde{c} - c_3}{c_3 - c_2} - \frac{3}{2}\right) = -\frac{3}{7}, \quad (24)$$

$$C = \frac{2A}{5} \left(\frac{c_2 - \tilde{c}}{c_3 - c_2}\right) = \frac{4}{21}. \quad (25)$$

Inserting the actual phenomenological values [25] $\alpha_1^{-1} = 59.01$, $\alpha_2^{-1} = 29.57$, and $\alpha_3^{-1} = 8.33$ in Eq. (22), we obtain a parity-breaking scale

$$\frac{\Lambda_R}{M_Z} = 1.2 \cdot 10^8 \quad (26)$$

that is halfway between the electroweak scale and the GU scale, which according to Eq. (21) is predicted to be

$$\frac{\Lambda_{\text{GUT}}}{M_Z} = 1.3 \cdot 10^{15}. \quad (27)$$

While a scale $\Lambda_R \approx 10^7$ TeV and is beyond the rich of present-day experiments, it is in agreement with the predictions of other «standard» LR models [17–19]. It is quite reasonable to believe that VEV of the R -scalar χ_R is $w \approx \Lambda_R$ [15] and $v/w \approx 10^{-8}$. At the LHC energy $\sqrt{s} = 14$ TeV, the existence of tiny corrections of order $\sqrt{s}/w \approx 10^{-6}$ would be hardly detected and once more confirm that the present scenario could only come from the physics of neutrinos.

In the minimal LR model, the mass generation can be understood in terms of nonrenormalizable effective operators that are generated at low energy below the symmetry-breaking scale. Mass terms can be generated by bilinear fermionic operators that must be coupled with Higgs bidoublets or triplets for Dirac or Majorana masses, respectively, in order to preserve gauge invariance. In the minimal model, a Higgs bidoublet can be written as a product $\chi_L \chi_R^\dagger$ of a $SU(2)_L$ doublet times a $SU(2)_R$ doublet, yielding a factor vw in the low-energy limit and the Dirac mass terms $m_D \bar{\psi}_L \psi_R = \gamma_D \bar{\psi}_L \psi_R vw$. A triplet can be built up from the two $SU(2)_L$ doublets (or two $SU(2)_R$ doublets), yielding a factor v^2 (or w^2) in the low-energy limit, and the Majorana mass terms $M_L \bar{\psi}_L^C \psi_L = \gamma_M \bar{\psi}_L^C \psi_L v^2$ and $M_R \bar{\psi}_R^C \psi_R = \gamma_M \bar{\psi}_R^C \psi_R w^2$. Here, the couplings γ_D and γ_M are expected to scale like an inverse of some large energy scale Λ .

Thus, for neutrinos, the mass matrix can be written as

$$\begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix} = m_D \begin{pmatrix} y \frac{v}{w} & 1 \\ 1 & y \frac{w}{v} \end{pmatrix}, \quad (28)$$

where $y = \gamma_M/\gamma_D$ and is of order unity, and the Dirac mass m_D is expected to fall in the MeV–GeV range like for other fermions. In fact, for charged fermions, $y = 0$ and the mass matrix contains only the Dirac terms. In the present argument, neutrinos are taken into consideration in a single generation. The discussion of important aspects, like mixing among light neutrinos, would certainly require a full mass matrix, but the existence of mixing terms would not significantly change the qualitative nature of the argument. The eigenvalues of the mass matrix in Eq. (28) show the usual seesaw behaviour of a light neutrino ν_L

$$M(\nu_L) = \frac{y^2 - 1}{2} \left(\frac{v}{w}\right) m_D + \mathcal{O}\left(\frac{v^2}{w^2}\right) \quad (29)$$

and a heavy neutrino ν_R

$$M(\nu_R) = y \left(\frac{w}{v}\right) m_D + \mathcal{O}\left(\frac{v}{w}\right). \quad (30)$$

Assuming that $v/w \approx M_Z/\Lambda_R \approx 10^{-8}$, the mass of the light neutrino ν_L would be pushed below the eV scale, while the heavy neutrino ν_R would be conceivable with a mass of $M(\nu_R) = m_D \cdot 10^8 \approx 10^8$ MeV = 100 TeV. At the LHC energy, the ratio $\sqrt{s}/M(\nu_R) \approx 0.1$; however, the heavy neutrino interacts only through the heavy gauge bosons W_R and Z' with an effective weak coupling that scales like $M_{W_L}^2/M_{W_R}^2 = v^2/w^2 \approx 10^{-16}$ compared to the light neutrino. Thus, its sterile nature would prevent its detection anyway. Astrophysical effects could be considered as the large mass of the heavy neutrino would imply important gravitational effects, and sterile neutrinos could in part account for the dark matter of the Universe.

To summarize, the simplest minimal LR extension of the SM has been studied in the high energy limit. Some consequences of the GU hypothesis have been explored assuming that the parity-breaking scale Λ_R is the only relevant energy above the electroweak scale up to GU. In this scenario, which is shown to be compatible with the observed neutrino phenomenology, the parity-breaking scale and the heavy boson masses are pushed up to 10^7 TeV, which is beyond the reach of present-day experiments. Only an almost sterile right-handed neutrino with a mass $M(\nu_R)$ of ≈ 100 TeV could exist below that scale.

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