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ФИЗИКА И ТЕХНИКА УСКОРИТЕЛЕЙ

LONGITUDINAL BEAM EMITTANCE BLOW-UP CAUSED BY INJECTION ERRORS IN SYNCHROTRONS WITH A LONGITUDINAL FEEDBACK SYSTEM

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The problem of longitudinal beam emittance blow-up caused by injection errors in synchrotrons with a longitudinal feedback system is studied. The obtained results are used to estimate parameters of a longitudinal feedback system.

Рассматривается проблема роста продольного эмиттанса сгустков, возникающего из-за ошибок инжекции, в синхротронах с системой демпфирования когерентных продольных колебаний. Приведен пример использования полученных результатов для оценки параметров данной системы демпфирования.

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INTRODUCTION

Emittance preservation is an important issue during injection of an ion beam into a synchrotron. An error in the position of the injected bunch relative to the synchronous phase or energy mismatch of the injected particles and ions circulated in the synchrotron can lead to an increase in the longitudinal bunch size because of decoherence or filamentation. It is well known [1, 2] that the emittance blow-up caused by an injection error is

$$\varepsilon = \left(1 + \frac{\bar{a}_\varepsilon^2}{2\sigma_0^2}\right)\varepsilon_0, \qquad (1)$$

where $\varepsilon_0 = 4\pi E_s \sigma_E \sigma_t$ is an initial longitudinal emittance of a bunch; E_s is the ion energy per nucleon; $\sigma_t = \Delta t/2$ is an initial RMS temporal length of the bunch (full length is $2\Delta t$); and $2\Delta E/E = 4\sigma_E$ is a full energy spread [3, 4]. It will be assumed further that the longitudinal RMS size σ_0 of a bunch corresponds to the Gaussian distribution of ions in the bunch. Calculations of the amplitude $\bar{a}_\varepsilon$ in (1) that leads to the emittance blow-up and its dependence on the injection error $\Delta\bar{a}_0$ (see Fig. 1) are subjects of the article.

It is assumed in (1) that all particles of the injected bunch with emittance ε_0 are redistributed on the phase space $(\xi, \dot{\xi}/\Omega_s)$, where $\xi = \delta\phi$ is rf-phase deviation of a particle from synchronous phase and Ω_s is a synchrotron angular frequency. After a long time the particles fill the larger phase space, which corresponds to emittance $\varepsilon = \varepsilon_{\rm dec}$ because of the

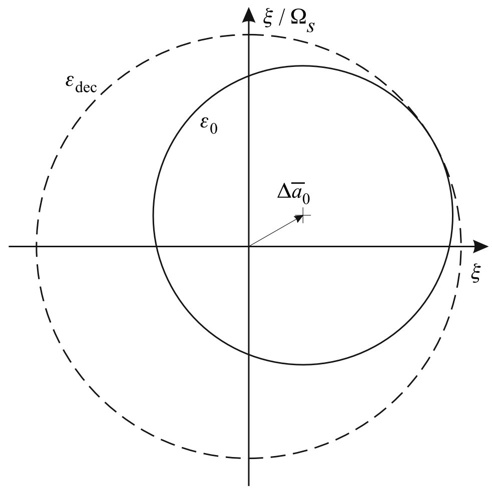

Fig. 1. Injected emittance ε_0 dilution to $\varepsilon_{\rm dec}$ because of error $\Delta\bar{a}_0$

decoherence only (see Fig. 1). Other effects such as active damping of coherent longitudinal oscillations or longitudinal instability of a beam are not taken into account in Eq. (1).

A longitudinal feedback system (LFS) in synchrotrons [5,6] is used for damping the coherent longitudinal oscillations of a bunch. The longitudinal momentum of the bunch is corrected by the rf cavity in proportion to the bunch displacement from the synchronous phase at the location of the longitudinal pick-up near the accelerating section. In this case the feedback system can lead to a steady decrease of the coherent amplitude, and the emittance blow-up does not happen without the decoherence. However, in the presence of the decoherence, the coherent amplitude decreases in time, and the displacement of the bunch centre of charge which is measured by the pick-up at every turn has a smaller magnitude than without the decoherence. Therefore, the effect of the decoherence can produce the emittance blow-up despite the active damping of the coherent oscillations by the longitudinal damper.

The emittance blow-up in case of a classical transverse feedback system was discussed in [7]. A more general approach that includes effects of transverse coherent instabilities and nonlinear damping was described in [8]. Basic ideas of these articles are used below.

BASIC NOTIONS

Longitudinal feedback systems are necessary in synchrotrons to reach the accuracy and stability required for reproducible beam performance [5, 9]. A digital bunch-by-bunch feedback [6, 10] individually steers each bunch by applying electromagnetic kicks every time the bunch passes through the rf cavity (RF cav., Fig. 2). The kick value is in proportion to the bunch deviation from the synchronous phase at the pick-up (PU) location. A stable beam rejection module removes useless stable beam components from the signal, which is eventually digitized (ADC), processed (DSP), and re-converted (DAC) to an analogue correction signal. A modulator translates the correction signal to the rf cavity operation frequency. The delay line adjusts the timing of the signal to match the bunch arrival time. It is shown in [6] that

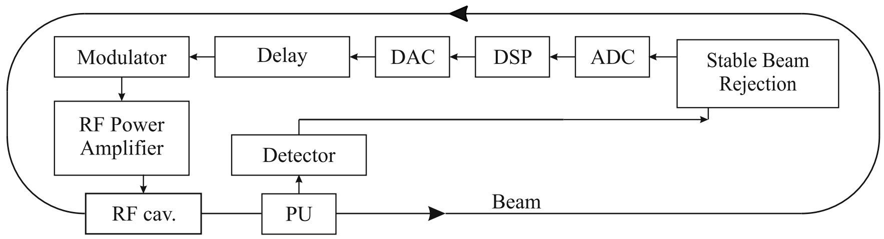

Fig. 2. Longitudinal feedback system layout

the damping effect for coherent longitudinal oscillations can be achieved because of the rf phase modulation of accelerating field in proportion to the PU signal.

Assume that $\phi(t)$ represents the phase of the rf voltage when the particle crosses the middle of the rf cavity (the origin of time for rf phase is taken at zero crossing of the rf voltage with a positive slope) and $\phi_s(t)$ is the synchronous phase [3, 5, 6]. The longitudinal phase space corresponds to coordinates (ϕ, $\dot{\phi}/\Omega_s$), where Ω_s is a synchrotron angular frequency [3,4]. An orthogonal coordinate system (x, y, $c\beta_s\, t$) that follows the ideal beam path in the synchrotron is assumed; (x, y) are coordinates in the horizontal and vertical planes, $c\beta_s = v_s$ is the velocity of a particle moving along the reference orbit (the ideal beam path).

At perturbations ΔB_s in the magnetic guide field, $\Delta V_{\rm rf}$ in the rf accelerating field and $\Delta\omega_{\rm rf}$ in the rf frequency, the differential equation of synchrotron oscillations is [3]

$$\ddot{\phi} + \Omega_s^2\, \delta\phi = -\frac{2\pi\, h_{\rm rf}\, \alpha_s}{T_{\rm rev}} \frac{d}{dt}\left(\frac{\Delta B_s}{B_s}\right) + \frac{d}{dt}\,(\Delta\omega_{\rm rf}) - \left(\frac{\Delta V_{\rm rf}}{V_{\rm rf}}\right) \Omega_s^2 \tan\phi_s, \tag{2}$$

where $h_{\rm rf}$ is the rf harmonic number; α_s is the momentum compaction factor; $T_{\rm rev}$ is the revolution period of particles in the synchrotron; and $\delta\phi \equiv \phi - \phi_s \ll 1$ is assumed. Therefore, the damping effect for injection errors can be obtained on the plateau of the magnetic field ($\dot{B}_s = 0$ and $\Delta B_s = 0$) in the case of $\Delta\omega_{\rm rf} \propto \phi$ that is the rf phase modulation. The amplitude modulation of accelerating voltage by $\Delta V_{\rm rf}$ can be used for damping the coherent longitudinal oscillations during acceleration only when $\phi_s \neq 0$.

The differential equation (2) can be rewritten as a system of linear difference equations to describe a motion of the bunch centre of charge. Let $\xi[n] \equiv \langle\delta\phi[n]\rangle = \langle\phi[n]\rangle - \phi_s[n]$ be the first moment (centre of charge) of $\phi[n]$ at the nth turn for a stationary distribution of particles in the bunch with respect to synchronous phase $\phi_s[n]$ and energy $E_s[n]$. For the phase deviation $\langle\delta\phi[n]\rangle$ and the energy deviation $\langle\delta E[n]\rangle \equiv \langle E[n]\rangle - E_s[n]$, one can write [6]

$$\begin{gathered}\langle\delta\phi[n+1]\rangle = \langle\delta\phi[n]\rangle - \frac{4\pi^2\nu_s^2}{qV_{\rm rf}\cos\phi_s}\langle\delta E[n+1]\rangle + \langle\Delta\phi_{\rm rf}[n+1]\rangle - 2\pi h_{\rm rf}\left\langle\frac{\Delta B_s[n+1]}{B_s[n+1]}\right\rangle, \\ \langle\delta E[n+1]\rangle = \langle\delta E[n]\rangle + \langle\delta\phi[n]\rangle\, qV_{\rm rf}\cos\phi_s + q\,\langle\Delta V_{\rm rf}[n]\rangle\sin\phi_s + q\,\langle\Delta V_{\rm rf}[n]\,\delta\phi[n]\rangle\cos\phi_s,\end{gathered} \tag{3}$$

where q is the charge of the particle; $qV_{\rm rf}$ is the maximal energy gain per turn; and $\langle\Delta\phi_{\rm rf}[n+1]\rangle$ is the phase shift because of the rf phase perturbation that the bunch «sees» passing the rf cavity in comparison with the phase shift without the rf phase perturbation [6]:

$$\langle \Delta\phi_{\rm rf}[n+1]\rangle \equiv \left\langle \int\limits_{t[n]}^{t[n+1]} \Delta\omega_{\rm rf}(t)dt \right\rangle \approx \Delta\omega_{\rm rf} T_{\rm rev}.$$

The phase trajectory on $(\xi,\ \dot{\xi}/\Omega_s)$ phase plane in the case of free synchrotron oscillations is a circle with a radius that equals the amplitude $\bar{a}(t)$ of synchrotron oscillations. Consequently, $\Delta\bar{a}_0$ is the total injection error of the bunch centre of charge relative to the synchronous particle because of the phase and energy deviations at $t = 0$.

The phase shift $\langle\Delta\phi_{\rm rf}[n+1]\rangle$ can be produced by a small modulation of the rf frequency during the gap before the point of time when the damped bunch passes the rf cavity. Let the phase shift $\langle\Delta\phi_{\rm rf}[n+1]\rangle$ be chosen in proportion to $\langle\delta\phi[n]\rangle$ but deviations $\Delta V_{\rm rf} = \Delta B_s = 0$:

$$\langle\Delta\phi_{\rm rf}[n+1]\rangle = -g_\phi\langle\delta\phi[n]\rangle, \tag{4}$$

where g_ϕ is the gain of the feedback loop. Consequently, Eqs. (3) can be written in the following matrix form:

$$\begin{pmatrix}\delta\phi[n+1]\\ \delta E[n+1]\end{pmatrix} = \widehat{M} \times \begin{pmatrix}\delta\phi[n]\\ \delta E[n]\end{pmatrix}, \qquad \widehat{M} \equiv \begin{pmatrix} 1-4\pi^2\nu_s^2 - g_\phi & -\dfrac{4\pi^2\nu_s^2}{qV_{\rm rf}\cos\phi_s} \\ qV_{\rm rf}\cos\phi_s & 1\end{pmatrix}, \tag{5}$$

so that the particle dynamics is determined by roots z_k of the characteristic equation:

$$z_k^2 - (2 - g_\phi - 4\pi^2\nu_s^2)z_k + 1 - g_\phi = 0.$$

Eigenvalues of the characteristic equation for small g_ϕ are

$$z_\pm = \exp\left(-\frac{g_\phi}{2}\right)\exp\left(\pm j2\pi\nu\right), \qquad \sin\pi\nu = \pi\nu_s\exp\left(\frac{g_\phi}{4}\right). \tag{6}$$

Damping of coherent longitudinal oscillations is obtained at $g_\phi > 0$ so that the time constant of damping is

$$\tau_d = 2\,T_{\rm rev}/g_\phi. \tag{7}$$

Graphs of the signal from the pick-up and the phase trajectory of the bunch centre of charge calculated in accordance with Eqs. (5) are shown in Fig. 3. The following parameters

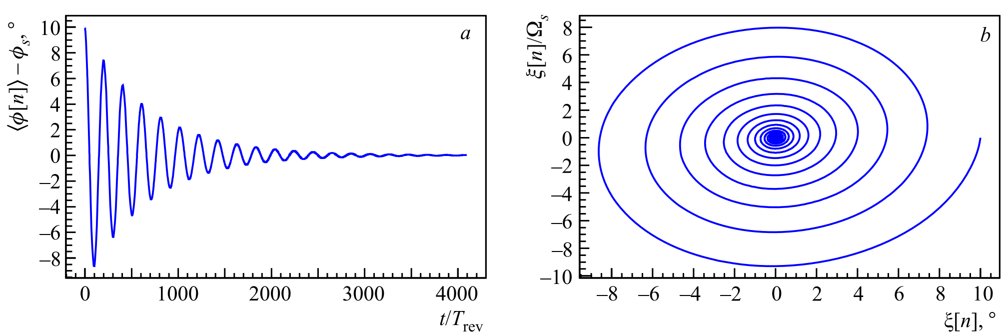

Fig. 3. Signal from pick-up (a) and phase trajectory (b)

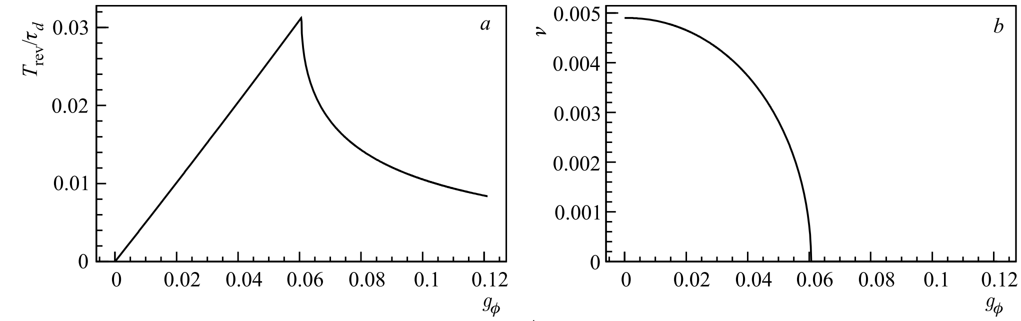

Fig. 4. Dependences of $T_{\rm rev}/\tau_d$ (a) and ν (b) on gain g_ϕ

for the proton bunch injected on the plateau of the magnetic field with errors of $\langle \delta\phi[0]\rangle = 10^\circ$ and $\langle \delta E[0]\rangle = 0$ were used in calculations: $\nu_s = 4.899 \cdot 10^{-3}$, $V_{\rm rf} = 6$ MV, $g_\phi = 0.003$.

It should be noted that coherent longitudinal frequency $\nu/T_{\rm rev}$ of the damped bunch does not coincide with synchrotron frequency $\nu_s/T_{\rm rev}$ of free oscillations. Dependences of decrement $T_{\rm rev}/\tau_d$ and number ν of coherent synchrotron oscillations per turn on gain g_ϕ are shown in Fig. 4 in the case of $\nu_s/T_{\rm rev} = 55$ Hz. The damped bunch returns to the synchronous phase as quickly as possible without coherent oscillations ($\nu = 0 \ \Rightarrow$ *critically damped* oscillator) for

$$(g_\phi)_{\rm opt} \equiv g_\phi^* = 4\pi\nu_s - 4\pi^2\nu_s^2. \tag{8}$$

For example, $g_\phi^* = 0.0606$ in the case of $\nu_s = 4.899 \cdot 10^{-3}$.

LONGITUDINAL EMITTANCE BLOW-UP CAUSED BY INJECTION ERRORS IN SYNCHROTRONS WITH LFS

The amplitude $\bar{a}(t)$ of coherent longitudinal oscillations of a bunch decreases in time because of decoherence with time constant $\tau_{\rm dec}$ and satisfies the following differential equation:

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm dec}} \tag{9}$$

with the starting condition $\bar{a}(t=0) = \Delta a_0$. The term $\bar{a}(t)$ describes the dependence of the amplitude of the oscillations of the bunch centre of charge on time because of the filamentation that leads to redistribution of particles on the phase space. At the longitudinal pick-up it looks like a damped coherent oscillation. Hence, at the decoherence effect only, the impact of the injection error $\Delta\bar{a}_0$ on the emittance growth in time can be described by function $\bar{a}_\varepsilon(t) = \Delta\bar{a}_0 - \bar{a}(t)$. Consequently, the part $\bar{a}_\varepsilon(t)$ of the amplitude of coherent longitudinal oscillations $\bar{a}(t)$ leading to the emittance blow-up because of decoherence satisfies the differential equation

$$\frac{d\bar{a}_\varepsilon(t)}{dt} = \frac{\bar{a}(t)}{\tau_{\rm dec}} \tag{10}$$

with the starting condition $\bar{a}_\varepsilon(t=0) = 0$. The differential equation (10) can be used to obtain a new dependence of $\bar{a}_\varepsilon(t)$ on time after including the active damping and instability effects in dependence of $\bar{a}(t)$ on time in the differential equation (9).

An action of a longitudinal feedback system can be taken into account in (9) by including additional term $d\bar{a}_d(t)/dt$, which corresponds to the decrease in the amplitude of oscillation of the bunch centre of charge. The longitudinal instability with the time constant of growth $\tau_{\rm inst}$ leads to additional positive term $\bar{a}(t)/\tau_{\rm inst}$. Therefore, the differential equation for amplitude $\bar{a}(t)$ is given by

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm dec}} + \frac{d\bar{a}_d(t)}{dt} + \frac{\bar{a}(t)}{\tau_{\rm inst}}. \tag{11}$$

Let us assume that dependences $\bar{a}(t)$ and $\bar{a}_\varepsilon(t)$ have been obtained from Eqs. (11) and (10). The total amplitude not corrected by the active feedback at the longitudinal instability is as follows:

$$\lim_{t\to\infty} \bar{a}_\varepsilon(t) = F_\varepsilon \cdot \Delta\bar{a}_0, \tag{12}$$

where F_ε is the form factor. Its value determines the part of initial error Δa_0 that leads to the emittance blow-up. So, $F_\varepsilon = 1$ at the decoherence effect only and $F_\varepsilon < 1$ in case of active damping. Therefore, the relative emittance blow-up can be expressed by the formula

$$\frac{\Delta\varepsilon}{\varepsilon_0} = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} = \frac{(\Delta\bar{a}_0)^2}{2\sigma_0^2}\, F_\varepsilon^2\,. \tag{13}$$

The term $d\bar{a}_d(t)/dt$ in (11) depends on the type of a feedback transfer function, and in accordance with Eq. (6) for the linear feedback system it is given by

$$\frac{d\bar{a}_d(t)}{dt} = -\frac{\bar{a}(t)}{\tau_d}, \tag{14}$$

where the time constant of damping τ_d in the case of the feedback with the rf phase modulation in accordance with (7) is $\tau_d = 2\,T_{\rm rev}/g_\phi$. By substituting (14) in (11), the differential equation for amplitude $\bar{a}(t)$ can be written as follows:

$$\frac{d\bar{a}(t)}{dt} = -\frac{\bar{a}(t)}{\tau_{\rm dec}} - \frac{\bar{a}(t)}{\tau_d} + \frac{\bar{a}(t)}{\tau_{\rm inst}} = -\frac{\bar{a}(t)}{\tau}, \tag{15}$$

where the overall damping time τ

$$\frac{1}{\tau} = \frac{1}{\tau_{\rm dec}} + \frac{1}{\tau_d} - \frac{1}{\tau_{\rm inst}} \tag{16}$$

corresponds to the damped oscillation if $\tau_d < \tau_{\rm inst} < \tau_{\rm dec}$. The solution of (15) is given by

$$\bar{a}(t) = \Delta\bar{a}_0 \exp\left(-\frac{t}{\tau}\right), \tag{17}$$

and the solution of Eq. (10) with $\bar{a}(t)$ from (17) is

$$\bar{a}_\varepsilon(t) = \frac{\tau}{\tau_{\rm dec}}\left(1 - \exp\left(-\frac{t}{\tau}\right)\right)\Delta a_0. \tag{18}$$

Therefore, in accordance with (12), the form factor F_ε for the total amplitude not corrected by the active linear feedback at the longitudinal instability is

$$F_\varepsilon = \frac{1}{\Delta a_0}\lim_{t\to\infty}\bar{a}_\varepsilon(t) = \frac{\tau}{\tau_{\rm dec}} = \left(1 + \frac{\tau_{\rm dec}}{\tau_d} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-1}. \tag{19}$$

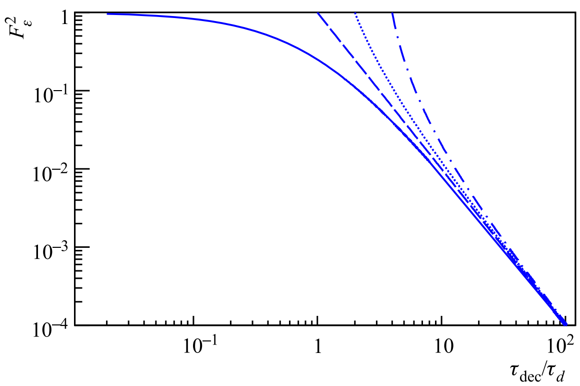

Fig. 5. Dependence of F_ε^2 on $\tau_{\rm dec}/\tau_d$ without instability (solid curve), $\tau_{\rm inst} = \tau_{\rm dec}$ (dashed), $\tau_{\rm inst} = \tau_{\rm dec}/2$ (dotted), $\tau_{\rm inst} = \tau_{\rm dec}/4$ (dash-dotted)

Consequently, the emittance blow-up is given by

$$\frac{\Delta\varepsilon}{\varepsilon_0} = \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} = \frac{(\Delta\bar{a}_0)^2}{2\sigma_0^2} F_\varepsilon^2, \qquad F_\varepsilon = \left(1 + \frac{\tau_{\rm dec}}{\tau_d} - \frac{\tau_{\rm dec}}{\tau_{\rm inst}}\right)^{-1}. \tag{20}$$

If $\tau_{\rm inst} = \tau_d$ or $\tau_{\rm inst} > \tau_d \to \infty$, then (20) coincides with (1), that is $\bar{a}_\varepsilon = \Delta\bar{a}_0$. If $\tau_{\rm inst} \to \infty$, then

$$\varepsilon = \left(1 + \frac{(\Delta\bar{a}_0)^2}{2\sigma_0^2}\left(1 + \frac{\tau_{\rm dec}}{\tau_d}\right)^{-2}\right)\varepsilon_0. \tag{21}$$

It is clear from (20) and (21) that faster decoherence (a smaller magnitude of $\tau_{\rm dec}$) for the fixed parameters τ_d and $\tau_{\rm inst}$ leads to a larger emittance blow-up (Fig. 5).

LFS PARAMETERS

It is clear from (20) that the parameters of a longitudinal feedback system can be calculated in the context of longitudinal emittance blow-up taking into account synchrotron project settings of $\Delta\bar{a}_0/\sigma_0$, $\Delta\varepsilon/\varepsilon_0$ and $\tau_{\rm dec}/\tau_d$. Let bunches be injected into synchrotron as a set of groups separated in time. This scheme is used in the LHC (CERN), where $T_{\rm rev} = 88.92\ \mu$s at injection, $h_{\rm rf} = 35640$, $V_{\rm rf} = 6$ MV, $\alpha_s = 0.000322$, $E_s = 450$ GeV for protons and the gap between groups of bunches injected from the SPS to the LHC is about 1 μs [11]. It was observed that $\tau_{\rm dec} = 100$ ms $\approx 1100\, T_{\rm rev}$ [12]. The injected bunch should be adjusted to the rf bucket on the true slope of rf sinusoidal voltage in agreement with the transition energy.

The critical damped mode can be used at injection of the first bunch. Its injection errors on the phase and energy are supressed by matching of rf bucket with the longitudinal emittance of the injected bunch because of rf accelerating phase shifting. This mode is used in the LHC where $\nu_s = 4.899 \cdot 10^{-3}$ and consequently $g_\phi^* = 0.0606$ in accordance with Eq. (8). Therefore, the rf phase variation has been achieved to be 2.7° per turn for the injection error of 45° in agreement with the requirements on the rf synchronisation system [13]. Signals from the pick-up in cases of the phase or energy errors with the amplitude $\Delta\bar{a}_0 = 45°$ are shown in Fig. 6.

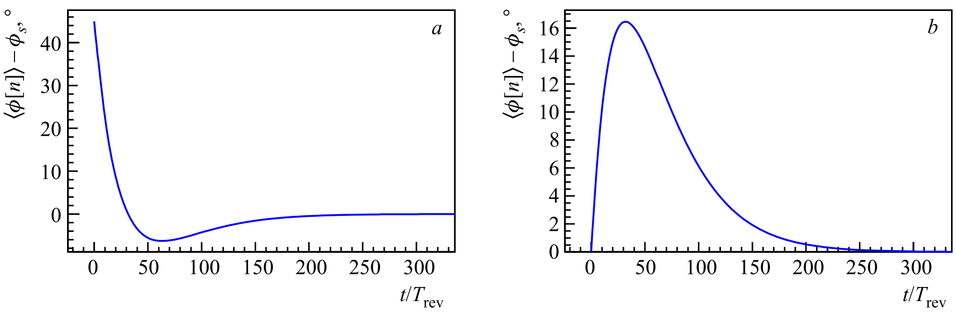

Fig. 6. Signals from pick-up (critical damped mode): phase (a) and energy (b) errors

When the next group of bunches is injected, the gap of 1 μs can be used to adjust the rf accelerating system to damp injection errors. Consequently, $g_\phi \approx 0.003$ is available for the equivalent phase shift during 1 μs and injection errors of 10°. Hence, $\tau_d = 2\,T_{\rm rev}/g_\phi \approx 700\,T_{\rm rev}$.

For the Gaussian distribution of ions in the bunch and a small amplitude approximation for synchrotron oscillations, one can write

$$\sigma_0 = \sigma_\phi = \frac{2\pi}{T_{\rm rf}}\,\sigma_t, \qquad h_{\rm rf}T_{\rm rf} = T_{\rm rev}.$$

If longitudinal emittance of a proton beam is $\varepsilon = 0.48$ eV · s, then $\sigma_\phi = 54°$. Let $\Delta\bar{a}_0 \leqslant 10°$ [12]. In this case $\Delta\bar{a}_0/\sigma_\phi \approx 0.19$. Taking into account Eq. (20) and the graph without instability in Fig. 5, one can conclude that $F_\varepsilon^2 = 0.15$. Consequently, $\Delta\varepsilon/\varepsilon_0 = 0.3\%$. The corresponding graphs of the signal from the pick-up and the phase trajectory of the bunch centre of charge are shown in Fig. 3.

SUMMARY

The longitudinal emittance blow-up caused by injection errors in synchrotrons with a longitudinal feedback system has been investigated on the basis of the original model that takes into account influence of decoherence processes on damping at the longitudinal instability. The time constant of damping has been calculated for longitudinal coherent oscillations in synchrotrons where a longitudinal feedback system with rf phase modulation is used for damping coherent oscillations. The system of linear difference equations has been deduced for computation of signals from pick-ups and phase trajectories of the bunch centre of charge in the case of damped oscillations. The obtained results have been used to estimate parameters of a longitudinal feedback system.

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