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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

# ON GRAVITATIONAL COUPLING FOR MASSIVE HIGHER-SPIN FIELDS IN $d = 3$ SPACE

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In this paper we study the gravitational interaction for massive higher integer spin fields in three-dimensional AdS space. Within of gauge-invariant description we have constructed cubic vertex $s-s-2$ and discuss the possible going beyond the first-order approximation.

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## INTRODUCTION

Besides the basic interest of three-dimensional ($d = 3$) models related to various dualities, it turns out that a theory in three dimensions looks a lot easier than in four dimensions and higher. From this point of view, it is useful to consider such a technically complicated theory as a higher-spin one [1, 2]. In particular, one can try to study the possible structure of the massive higher-spin theory which unlike the massless one we know is not much.

The paper is organized as follows. In the rest part of Introduction we formulate a general scheme for constructing interactions based on the gauge-invariant description. Then using this approach, in Sec. 1 we will consider free theory and gravitational coupling for massless higher-spin fields in $d = 3$. In this section a number of known results are reproduced. Then in Sec. 2 the same aspects are investigated for massive higher-spin fields in $d = 3$.

Both for massless and for massive higher-spin fields, we will use the gauge-invariant description. Within such a consideration the general scheme of constructing interactions looks as follows. The Lagrangian and gauge transformations are represented as a series in powers of some coupling constant $\kappa$:

$$\mathcal{L} = \mathcal{L}_0 + \kappa\mathcal{L}_1 + \kappa^2\mathcal{L}_2 + \ldots, \quad \delta = \delta_0 + \kappa\delta_1 + \kappa^2\delta_2 + \ldots,$$

where $\mathcal{L}_0$ is quadratic in the fields Lagrangian corresponding to the free theory; $\mathcal{L}_1$ is cubic in the fields Lagrangian corresponding to first-order interactions and so on, and we have the

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same for gauge transformations $\delta_0, \delta_1, \ldots$ Then the condition of gauge invariance $\delta\mathcal{L} = 0$ is also represented as a series in powers of coupling constant $\kappa$:

$$\begin{aligned}
&\kappa^0 \quad \delta_0\mathcal{L}_0 = 0,\\
&\kappa^1 \quad \delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0,\\
&\kappa^2 \quad \delta_0\mathcal{L}_2 + \delta_1\mathcal{L}_1 + \delta_2\mathcal{L}_0 = 0,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

In zero order the gauge invariance corresponds to a free theory, in the first order we have linear interaction and so on. The goal of this paper is to formulate free theory and construct first-order gravitational interaction for massless and massive higher-spin fields [1], i.e., to find $\mathcal{L}_0, \delta_0$ and $\mathcal{L}_1, \delta_1$. The problem of going beyond the first-order approximation is also discussed.

## 1. MASSLESS HIGHER-SPIN FIELDS IN $d = 3$

**1.1. Free Theory.** Let us firstly consider massless higher-spin fields. We will work in three-dimensional AdS space and use frame-like formulation [3]. The main objects are generalized vielbein $\Phi_\mu{}^{a_1\ldots a_{s-1}}$ and dual field to the generalized connection $\Omega_\mu{}^{a_1\ldots a_{s-1}} = \varepsilon^{bc(a_1}\Omega_\mu{}^{a_2\ldots a_{s-1})b,c}$. For these fields, one can determine the new field variables

$$\hat{\Omega} = \Omega + \lambda\Phi, \quad \tilde{\Omega} = \Omega - \lambda\Phi.$$

Here $\lambda$ is the dimensional parameter related to cosmological constant $\Lambda$ as $\lambda^2 = -\Lambda$. In terms of new fields the free Lagrangian is rewritten in form of two independent parts [5]:

$$\mathcal{L}_0 \sim \mathcal{L}_0(\hat{\Omega}) - \mathcal{L}_0(\tilde{\Omega}).$$

Each part depends on its own one field $A$ and represents the linearized Chern–Simons action with Abelian gauge transformations:

$$\mathcal{L}_0(A) \sim ADA + \lambda AA, \quad \delta_0 A = D\eta + \lambda\eta.$$

**1.2. Gravitational Coupling.** It is assumed that after switching on an interaction such separation of field variables is also preserved, then we can say that every massless higher-spin field is described by its one field variable $A$. Let $A_2$ and $A_s$ correspond to massless spin-2 and spin-$s$ fields, respectively. The gravitational coupling in the first order corresponds to the cubic vertex $s - s - 2$ with suitable first-order corrections to gauge transformations

$$\mathcal{L}_1 \sim A_s A_s A_2, \quad \delta_1 A_s \sim A_s\eta_2 + A_2\eta_s, \quad \delta_1 A_2 \sim A_s\eta_s.$$

Their explicit form is uniquely determined by the gauge invariance in first-order approximation $\delta_0\mathcal{L}_1 + \delta_1\mathcal{L}_0 = 0$. It is natural to try to close the theory by requiring that $\delta_1\mathcal{L}_1 = 0$. For example, for spin-3 it is achieved by adding to the constructed vertex $3 - 3 - 2$ the vertex of the self-interaction of gravitational field $2 - 2 - 2$ with corresponding correction to gauge transformation. The resulting model is the Chern–Simons theory with the symmetry group $SL(3, R)$ [4,5]. Thus, unlike four dimensions, in three dimensions a system of interacting massless spin-2 and spin-3 fields is allowed without involving any new dynamical fields.

[1] We consider only integer higher spins.

## 2. MASSIVE HIGHER-SPIN FIELDS IN $d = 3$

**2.1. Free Theory.** Now we study the theory of massive higher-spin fields in three dimensions. For the gauge-invariant frame-like description of massive spin-$s$ field, we need a set of field variables $(\Phi_k, \Omega_k)$, $k = 0, 1, \dots, s$. Here $k$ runs from 0 to s and characterizes the spin of corresponding massless field being auxiliary Stueckelberg fields up to $k \leqslant s - 1$. For example, for spin-2 we should introduce three pairs of fields $(\Phi_\mu{}^a, \Omega_\mu{}^a), (\Phi_\mu, \Omega^a), (\Phi, \pi^a)$. The last two pairs play the role of auxiliary Stueckelberg spin-1 and spin-0 fields. Doing as in the massless case, we wish to separate field variables. But we see that in the last pair the fields do not have a symmetrical structure, one of them being a scalar and another being a vector. To avoid this problem, we impose the gauge $\Phi = 0$ and eliminate Stueckelberg spin-0 field. Thereafter the new field variables $(\hat{\Phi}_k, \tilde{\Omega}_k)$ can be introduced as follows:

$$\hat{\Omega}_k = \Omega_k + m_k \Phi_k, \quad \tilde{\Omega}_k = \Omega_k - m_k \Phi_k, \quad k = 1, \dots, s.$$

Here $m_k$ is the dimensional parameter related to mass $m$ and $\lambda$ parameters $m_k \sim m_k(m, \lambda)$. In terms of new fields, the free Lagrangian takes the form [6]

$$\mathcal{L}_0 \sim \sum_{k=1}^{s} \left[ \mathcal{L}_0(\hat{\Omega}_k) - \mathcal{L}_0(\tilde{\Omega}_k) \right],$$

where each half has the same structure and depends on its own set of fields $A_k$:

$$\mathcal{L}_0(A_k) \sim A_k D A_k + m A_k A_{k-1} + m A_k A_k,$$

with gauge transformations

$$\delta_0 A_k = D\hat{\eta}_k + m(\hat{\eta}_{k-1} + \hat{\eta}_k + \hat{\eta}_{k+1}).$$

In some sense this Lagrangian looks like an Abelian Chern–Simons action. Turning to the construction of gravitational interaction, it will be natural to write the vertex of first-order one $s - s - 2$ as follows:

$$\mathcal{L}_1(A_k) \sim A_{sk} A_{sk} A_2 + A_{sk} A_{sk-1} A_2.$$

Unfortunately, the analysis of this naive vertex shows that, when we return to the initial fields, the $\Phi_k, \Omega_k$ part of the auxiliary fields (actually only one field) becomes dynamical. The reason is that the gauge $\Phi = 0$ is inconsistent with this type of interaction.

**2.2. Gravitational Coupling.** Let us refuse the condition $\Phi = 0$ and study gravitational coupling in terms of initial fields $(\Phi_k, \Omega_k)$. The Lagrangian and gauge transformations for free massive spin-$s$ field will have the following form:

$$\mathcal{L}_0 \sim \sum_{k=0}^{s} \left[ \Omega_k \Omega_k + \Omega_k D \Phi_k + m(\Omega_k \Phi_{k-1} + \Omega_{k-1} \Phi_k) + m^2 \Phi_k \Phi_k \right], \qquad (1)$$

$$\delta_0 \Omega_k \sim D\eta_k + m(\eta_{k+1} + \eta_{k-1}) + m^2 \xi_k, \quad \delta_0 \Phi_k \sim D\xi_k + \eta_k + m(\xi_{k+1} + \xi_{k-1}). \qquad (2)$$

Initial field variables for the gravity are the vielbein $h_\mu{}^a$ and dual Lorentz connection $\omega_\mu{}^a = \varepsilon^{abc} \omega_\mu{}^{bc}$. In the first-order approximation, from gauge invariance $\delta_0 \mathcal{L}_1 + \delta_1 \mathcal{L}_0 = 0$ it follows that the most general solution for gravitational interaction up to a field redefinition corresponds to the vertex $s-s-2$, which is obtained from the free theory (1), (2) by replacing the background vielbein ${e_\mu}^a$ and derivative $D_\mu$ on vielbein and connection of gravitational field, respectively, [7, 8]

$$\mathcal{L}_1 = \mathcal{L}_0({e_\mu}^a \to {h_\mu}^a, D_\mu \to {\omega_\mu}^a)$$

and the same replacing for gauge transformations. This coupling represents the minimal one. The minimal coupling is possible because we work in three dimensions. In four dimensions and higher minimal coupling is not consistent, we must also consider the nonminimal interaction with higher derivatives. What about the closing of the theory? A common question looks very complicated. We only present some arguments for the spin-3 case. To close the theory, we should require $\delta_1\mathcal{L}_1 = 0$. Note that when spin-3 field is massless this variation vanishes by adding the vertex of gravitational self-interaction $2-2-2$ with corresponding correction to gauge transformation. In the massive spin-3 case, such adding is not sufficient. Hence, we need more contributions by which this variation would be canceled. Since in three dimensions vertices higher than cubic one cannot be built, it is required to introduce new dynamical fields and interactions in the system. It seems that the natural candidate for the role of such a field is a massive spin-4 field. It will give the vertex $4-3-2$ with the suitable contribution to the variation. Thus, there is an important difference as compared with massless case where the system of massless spin-2 and spin-3 fields is closed without adding any new dynamical fields.

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## REFERENCES

1. *Blencowe M. P.* A Consistent Interacting Massless Higher-Spin Field Theory in $D = 2+1$ // Class. Quant. Grav. 1989. V. 6. P. 443–452.
2. *Vasiliev M. A.* Higher Spin Gauge Theories in Various Dimensions // Fortsch. Phys. 2004. V. 52. P. 702–717.
3. *Vasiliev M. A.* “Gauge” Form of Description of Massless Fields with Arbitrary Spin // Yad. Fiz. 1980. V. 32. P. 855–861.
4. *Witten E.* $(2+1)$-Dimensional Gravity as an Exactly Soluble System // Nucl. Phys. B. 1988. V. 311. P. 46–78.
5. *Campoleoni A. et al.* Asymptotic Symmetries of Three-Dimensional Gravity Coupled to Higher-Spin Fields // JHEP. 2010. V. 11. P. 007.
6. *Buchbinder I. L., Snegirev T. V., Zinoviev Yu. M.* Gauge Invariant Lagrangian Formulation of Massive Higher Spin Fields in $AdS_3$ Space // Phys. Lett. B. 2012. V. 716. P. 243–248.
7. *Zinoviev Yu. M.* On Massive Gravity and Bigravity in Three Dimensions // Class. Quant. Grav. 2013. V. 30. P. 055005.
8. *Buchbinder I. L., Snegirev T. V., Zinoviev Yu. M.* On Gravitational Interactions for Massive Higher Spins in $AdS_3$ // J. Phys. A: Math. Theor. 2013. V. 46, No. 21. P. 214015.