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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

EXACT SUPERPROPAGATORS IN $\mathcal{N}=2$ THREE-DIMENSIONAL SUPERSYMMETRIC ELECTRODYNAMICS

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Using the heat kernel approach, we obtain exact propagators of matter fields in three-dimensional $\mathcal{N}=2$ supersymmetric electrodynamics in case of covariantly constant gauge background.

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INTRODUCTION

Three-dimensional gauge theories with extended supersymmetry have many useful properties such as the mirror symmetry and Seiberg-like dualities (see, e.g., [1–4]). Due to the achievements in study of the BLG [5–7] and ABJM [8] models, three-dimensional field theories with extended supersymmetry attract modern interest.

In the present paper we consider certain aspects of the three-dimensional $\mathcal{N}=2$ supersymmetric electrodynamics without Chern–Simons kinetic form. We calculate exact Green's function associated with the model under consideration using the heat kernel approach in $\mathcal{N}=2$ three-dimensional superspace. We base our consideration on the recent work [9].

The classical action of the $\mathcal{N}=2$, $d=3$ supersymmetric electrodynamics reads

$$S_{\mathcal{N}=2} = \frac{1}{e^2}\int d^7z\, G^2 - \int d^7z\,(\bar{Q}_+\, e^{2V} Q_+ + \bar{Q}_-\, e^{-2V} Q_-) - \left(m\int d^5z\, Q_+Q_- + \text{c.c.}\right), \tag{1}$$

where $Q_\pm$ are chiral superfields with opposite charges with respect to the gauge superfield V. G is the superfield strength for the gauge superfield V,

$$G = \frac{i}{2}\bar{D}^\alpha D_\alpha V. \tag{2}$$

The action (1) can be obtained by dimensional reduction from the $\mathcal{N}=1$, $d=4$ electrodynamics [10, 11]. We are interested in the propagators of chiral superfields which depend

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on the gauge background superfield. For this purpose the background field method in the $\mathcal{N}=2$, $d=3$ superspace [12] appears to be useful. We split the gauge superfield V into the background V and quantum v parts:

$$V \to V + ev. \tag{3}$$

For our case it is enough to consider only quadratic part of quantum action

$$S_2[V] = -\int d^7z\left(v\Box v + \bar{\mathcal{Q}}_+\mathcal{Q}_+ + \bar{\mathcal{Q}}_-\mathcal{Q}_-\right) - \left(m\int d^5z\,\mathcal{Q}_+\mathcal{Q}_- + \text{c.c.}\right), \tag{4}$$

where $\mathcal{Q}_\pm$ and $\bar{\mathcal{Q}}_\pm$ are covariantly (anti)chiral superfields with respect to the background gauge superfield,

$$\bar{\mathcal{Q}}_+ = \bar{Q}_+\,\mathrm{e}^{2V}, \quad \mathcal{Q}_+ = Q_+, \quad \bar{\mathcal{Q}}_- = \bar{Q}_-\,\mathrm{e}^{-2V}, \quad \mathcal{Q}_- = Q_-. \tag{5}$$

Also, we assume that the background gauge field strengths are constant in space-time $\partial_m G = \partial_m W_\alpha = 0$.

1. EXACT SUPERPROPAGATORS

The quadratic action $S_2[V]$ (4) obtained above is responsible for the propagators,

$$\begin{aligned} &i\langle \mathcal{Q}_+(z)\mathcal{Q}_-(z')\rangle = -mG_+(z,z'),\\ &i\langle \mathcal{Q}_+(z)\bar{\mathcal{Q}}_+(z')\rangle = G_{+-}(z,z') = G_{-+}(z',z), \end{aligned} \tag{6}$$

where Green's functions $G_+(z,z')$ and $G_{-+}(z,z')$ obey the equations

$$(\Box_+ + m^2)G_+(z,z') = -\delta_+(z,z'), \tag{7}$$

$$\frac{1}{4}\bar{\nabla}^2 G_{-+}(z,z') + m^2 G_+(z,z') = -\delta_+(z,z'). \tag{8}$$

Here $\delta_+(z,z')$ are chiral delta function and $\Box_+$ are d'Alembertians acting in the space of chiral superfields,

$$\Box_+ = \nabla^m\nabla_m + G^2 + \frac{i}{2}(\nabla^\alpha W_\alpha) + iW^\alpha\nabla_\alpha. \tag{9}$$

Green's functions (7), (8) can be expressed in terms of corresponding heat kernels,

$$G_+(z,z') = i\int_0^\infty ds\,K_+(z,z'|s)\,\mathrm{e}^{is(m^2+i\epsilon)}, \tag{10}$$

$$G_{+-}(z,z') = i\int_0^\infty ds\,K_{+-}(z,z'|s)\,\mathrm{e}^{is(m^2+i\epsilon)}, \tag{11}$$

where the standard $\epsilon \to +0$ prescription is assumed. These heat kernels were computed exactly for the on-shell gauge superfield background $D^\alpha W_\alpha = 0$ subject to $\partial_m W_\alpha = 0$ [9]:

$$K_+(z,z'|s) = \mathbf{U}(s)\,\exp\left[\frac{i}{4}(F\coth(sF))_{mn}\rho^m(s)\rho^n(s) - \frac{1}{2}\bar{\zeta}^\beta(s)\rho_{\beta\gamma}(s)W^\gamma(s) + \right.$$
$$\left. + \int_0^s dt\,\Sigma(t)\right]\zeta^2(s)I(z,z'), \quad (12)$$

$$K_{+-}(z,z'|s) = -\mathbf{U}(s)\,\exp\left[\frac{i}{4}(F\coth(sF))_{mn}\tilde{\rho}^m(s)\tilde{\rho}^n(s) + R(z,z') + \right.$$
$$\left. + \int_0^s dt(R'(t)+\Sigma(t))\right]I(z,z'), \quad (13)$$

$$\mathbf{U}(s) = \frac{1}{8(i\pi s)^{3/2}}\,\frac{sB}{\sinh(sB)}\,\mathrm{e}^{isG^2},$$

where we denote the components of the supersymmetric interval $\zeta^A = \{\rho^m, \zeta^\alpha, \bar{\zeta}_\alpha\}$:

$$\zeta^\alpha = (\theta-\theta')^\alpha, \quad \bar{\zeta}^\alpha = (\bar{\theta}-\bar{\theta}')^\alpha, \quad \rho^m = (x-x')^m - i\gamma^m_{\alpha\beta}\zeta^\alpha\bar{\theta}'^\beta + i\gamma^m_{\alpha\beta}\theta'^\alpha\bar{\zeta}^\beta \quad (14)$$

and $I(z,z')$ is the parallel displacement propagator [13] in $\mathcal{N}=2$, $d=3$ superspace [9]. The heat kernel (13) also contains the supersymmetric two-point function $\tilde{\rho}^m$ which is chiral with respect to the first argument and is antichiral with respect to the second one,

$$\tilde{\rho}^m = \rho^m + i\zeta^\alpha\gamma^m_{\alpha\beta}\bar{\zeta}^\beta, \quad D'_\alpha\tilde{\rho}^m = \bar{D}_\alpha\tilde{\rho}^m = 0. \quad (15)$$

The two-point function $R(z,z')$ in (13) was found in the form

$$R(z,z') = -i\zeta\bar{\zeta}G + \frac{7i}{12}\bar{\zeta}^2\zeta W + \frac{i}{12}\zeta^2\bar{\zeta}\bar{W} - \frac{1}{2}\bar{\zeta}^\alpha\tilde{\rho}_{\alpha\beta}W^\beta - \frac{1}{2}\zeta^\alpha\tilde{\rho}_{\alpha\beta}\bar{W}^\beta +$$
$$+ \frac{1}{12}\zeta^\alpha\bar{\zeta}^\beta[\tilde{\rho}^\gamma_\beta D_\alpha W_\gamma - 7\tilde{\rho}^\gamma_\alpha D_\gamma W_\beta], \quad (16)$$

and the $\Sigma(z,z')$ reads

$$\Sigma(z,z') = -i(\bar{W}^\beta\zeta_\beta - W^\beta\bar{\zeta}_\beta)G - \frac{i}{3}\zeta^\alpha\bar{\zeta}^\beta W_\beta\bar{W}_\alpha + \frac{2i}{3}\zeta^\alpha\bar{\zeta}_\alpha W^\beta\bar{W}_\beta +$$
$$+ \frac{i}{12}\zeta^2[\bar{W}^2 - \bar{\zeta}^\alpha\bar{W}_\alpha D^\beta W_\beta] + \frac{i}{12}\bar{\zeta}^2[W^2 + \zeta^\alpha W_\alpha\bar{D}^\beta\bar{W}_\beta] +$$
$$+ \frac{1}{12}(\zeta^\alpha\bar{W}^\beta - \bar{\zeta}^\beta W^\alpha)[\rho_{\alpha\gamma}D^\gamma W_\beta + \rho_{\beta\gamma}\bar{D}^\gamma\bar{W}_\alpha]. \quad (17)$$

Also we introduced the s-dependent variables by the rule

$$\begin{aligned} W^\alpha(s) &\equiv \mathcal{O}(s) W^\alpha \mathcal{O}(-s) = W^\beta (\mathrm{e}^{-sN})_\beta{}^\alpha, \\ \zeta^\alpha(s) &\equiv \mathcal{O}(s) \zeta^\alpha \mathcal{O}(-s) = \zeta^\alpha + W^\beta ((\mathrm{e}^{-sN} - 1) N^{-1})_\beta{}^\alpha, \\ \rho^m(s) &\equiv \mathcal{O}(s) \rho^m \mathcal{O}(-s) = \rho^m - i(\gamma^m)^{\alpha\beta} \int\limits_0^s dt \left(W_\alpha(t) \bar\zeta_\beta(t) + \bar W_\alpha(t) \zeta_\beta(t) \right). \end{aligned} \tag{18}$$

The expression for $R'(t)$ can be found explicitly, $R'(t) = \mathcal{O}(t)[\bar W^\alpha \bar D_\alpha - W^\alpha D_\alpha, R]\mathcal{O}(-t)$ and then combined with (17),

$$R'(t) + \Sigma(t) = \mathcal{O}(t) \Big\{ 2i\bar\zeta W G + 2i(\zeta\bar\zeta\, W \bar W - \zeta W\, \bar\zeta \bar W) + \\ + i\bar\zeta^2 [W^2 - \zeta^\alpha W^\beta D_\alpha W_\beta] - \frac{1}{2} \bar\zeta^\beta W^\alpha [\tilde\rho_{\beta\gamma} \bar D^\gamma \bar W_\beta - \tilde\rho_{\alpha\gamma} D^\gamma W_\beta] \Big\} \mathcal{O}(-t). \tag{19}$$

We note that at coincident superspace points the function $R(z, z')$ vanishes, $R(z, z')|_{\zeta \to 0} = 0$, and does not contribute, but the function $R'(t) + \Sigma(t)$ has convenient form for further calculation.

CONCLUSIONS

The kernels (12) and (13) are useful for loop computations [9] and obtained exactly for our case of gauge superfield background. It should be emphasized that the parallel displacement propagator $I(z, z')$ in (12) and (13) provides the correct transformation properties of the heat kernel under the gauge symmetry.

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