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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

SELF-SIMILARITY OF PROTON SPIN AND ASYMMETRY OF JET PRODUCTION

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Self-similarity of jet production in polarized $p+p$ collisions is studied. The concept of z-scaling is applied for description of inclusive spectra obtained with different orientations of proton spin. New data on the double longitudinal spin asymmetry, A_{LL}, of jets produced in proton–proton collisions at $\sqrt{s}=200$ GeV measured by the STAR Collaboration at RHIC are analyzed in the z-scaling approach. A hypothesis of self-similarity and fractality of internal spin structure is formulated. A possibility to extract information on spin-dependent fractal dimensions of proton from the asymmetry of jet production is justified. The spin-dependent fractal dimensions for the process $\vec{p}+\vec{p}\to \mathrm{jet}+X$ are estimated.

Изучается самоподобие рождения струй в столкновениях поляризованных протонов. Для описания процесса используется концепция z-скейлинга. В рамках развиваемого подхода анализируются новые данные о двойной продольной асимметрии A_{LL} рождения струй в столкновениях $p+p$ при энергии $\sqrt{s}=200$ ГэВ, полученные коллаборацией STAR на RHIC. Сформулирована гипотеза о самоподобии и фрактальности спина протона. Обосновывается возможность извлечения из экспериментальных данных по асимметрии рождения струй информации о спин-зависимых фрактальных размерностях протона. Получена оценка о величине спин-зависимой фрактальной размерности протона для процесса $\vec{p}+\vec{p}\to$ струя $+X$.

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INTRODUCTION

Spin is one of the fundamental properties of elementary particles. Origin of spin lies out of classical analogues. The quantum field concepts are used to understand the spin of hadrons in terms of the underlying fundamental degrees of freedom such as the spins of quarks, gluons and their orbital motion. The importance of physics with polarized particles can be understood in two ways. One is elucidation of the spin structure of particles and the other one is testing symmetries in reactions of the particles utilizing information about their internal compositeness. The spin structure of nucleon has been studied for a long time in processes with longitudinally and transversally polarized leptons and protons [1–8]. The goal is to obtain a complete picture of the nucleon spin in terms of fundamental fields.

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The compositeness of hadron constituents includes interactions of the quantum fields considered within known theories. In the framework of QCD, the basic elements of hadron structure are quarks and gluons. The nonlinear Yang–Mills equations taking into account gauge invariance and Lorentz covariance regulate dynamics of constituent interactions in hard and soft regimes. Both regions contribute to proton spin at corresponding scales and, accordingly, should be taken into consideration.

Self-similarity is another property which regulates dynamics of constituent interactions at different scales. There are indications that the structure of unpolarized proton reveals self-similarity over a wide scale range [9, 10]. Motivated by the previous studies, we assume that spin content of proton has self-similar distribution in terms of polarized quarks and gluons as well. Both should reflect existence of a subtle structure of geometrical carrier of proton properties in the momentum space. An anisotropy of the momentum space may occur due to spontaneous symmetry breaking at small scales [11]. This property can be connected with scale invariance of the proton spin compositeness. Fractal character of the scale invariance and conservation of spins of the elementary quanta of particle fields imply existence of fractal topological invariants on their geometrical carrier.

The idea of self-similarity of hadron interactions is a fruitful concept to study collective phenomena in hadron and nuclear matter [12–18]. Important manifestation of such a concept is the existence of scaling itself [19–22]. The scaling in general means self-similarity at different scales. The physical content meant by behind it can be of different origin. Some of the scaling features constitute pillars of modern critical phenomena. Another category of scaling laws (self-similarity in point explosion, laminar and turbulent fluid flow, etc.) reflects features not related to phase transitions. The z-scaling established for hadron production in inclusive reactions has relevance to both the mentioned groups. It is treated as a manifestation of self-similarity of the structure of colliding objects (hadrons, nuclei), interaction mechanism of their constituents, and processes of fragmentation of produced particles into real hadrons. Properties of the z-scaling were confirmed in the regions which are far from boundaries of phase transitions. It is assumed that parameters of the scaling can be sensitive to the vicinity of a phase transition [14]. A distinctive change of the parameters c, δ_A and ε_F which enter into the scaling variable would indicate presence of a phase space boundary. The parameters have physical interpretation of "heat capacity" of the produced matter, fractal dimension of the structure of hadrons or nuclei and fractal dimension of the fragmentation process, respectively.

The z-scaling approach shows itself as an effective tool for sophisticated data analysis in searching for new phenomena, verification of theoretical models, etc. (see [9–18] and references therein). Extension of the method for analysis of processes with polarized particles and verification of the self-similarity of their interactions using spin-dependent inclusive cross sections for particle production in $\vec{p}+\vec{p}$ collisions is an interesting problem, which could give new insight into the origin of proton spin at small scales. New parameters, the spin-dependent fractal dimensions, are introduced to account for the internal spin structure of proton and particle fragmentation process within the z-scaling approach. We assume that polarization phenomena, as observed by asymmetry of inclusive cross sections in collisions of polarized protons, can be described by universal scaling functions which are bound together by the same scaling law. The arguments z of these functions are distinguished by different spin-dependent fractal dimensions for respective spin orientations of given polarizations. All these arguments are, however, of the same form as for the unpolarized processes [23].

In the paper we present results of a combined analysis of the double longitudinal asymmetry A_{LL} [3] and the cross section [24] of the inclusive jet production in proton–proton collisions measured by the STAR Collaboration at RHIC at $\sqrt{s} = 200$ GeV in the central rapidity range. The analysis is performed within the z-scaling approach and based on a hypothesis of self-similarity of the proton spin structure at different scales. The hypothesis is formulated in terms of the scaling functions and verified in the considered kinematic region. The spin-dependent fractal dimensions of proton spin structure in the process $\vec{p}+\vec{p} \to \mathrm{jet}+X$ are estimated.

1. z-SCALING AS A GENERAL CONCEPT

Here we remind some basic ideas of the z-scaling. The scaling reflects principles of locality, self-similarity and fractality of hadron interactions at a constituent level. It means that structure of colliding objects, interactions of their constituents and fragmentation process are considered to have similar properties at different scales. Fractality is a specific feature connected with the substructure of the constituents at small scales.

We assume that inclusive spectra of various types of particles are described with a universal scaling function. The function $\Psi(z)$ depends on single variable z in a wide range of the transverse momentum, registration angles, collision energies and centralities. The scaling variable is a self-similar parameter. It is expressed in the following form:

$$z = z_0 \Omega^{-1}. \tag{1}$$

Here z_0 and Ω are functions of kinematic variables:

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{\mathrm{ch}}/d\eta\,|_0)^c m_N}, \tag{2}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_a}(1-y_b)^{\varepsilon_b}. \tag{3}$$

The part z_0 is proportional to the transverse kinetic energy of a specially selected binary constituent subprocess responsible for the production of the inclusive particle and its partner (antiparticle). The multiplicity density $dN_{\mathrm{ch}}/d\eta\,|_0$ of charged particles in the central region $\eta = 0$, the nucleon mass m_N and the parameter c completely determine the functional form of the dimensionless quantity z_0. The function Ω is proportional to a relative number of the configurations at the constituent level which include the binary subprocesses corresponding to the momentum fractions x_1 and x_2 of the colliding hadrons (nuclei) and to the momentum fractions y_a and y_b of the secondary objects produced in these subprocesses. The parameters δ_1 and δ_2 are fractal dimensions of the colliding objects, whereas ε_a and ε_b stand for the fractal dimensions of the fragmentation process in the scattered and recoil direction, respectively.

Next we introduce the following notation for (3):

$$\Omega \equiv \Omega_{0000} =: \{\delta_1,\ \delta_2,\ \varepsilon_a,\ \varepsilon_b\}. \tag{3$'$}$$

The lower index (0000) corresponds to unpolarized particles in the initial and final states. For the unpolarized processes, we assume the fragmentation dimensions to have the same value $\varepsilon_a = \varepsilon_b = \varepsilon_F$ which depends on the type of the inclusive particle. The selected binary subprocess, responsible for production of the inclusive particle with mass m and its recoil partner (antiparticle with the same mass), is defined by the maximum of $\Omega\ (x_1, x_2, y_a, y_b)$ with the kinematic constraint

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2. \tag{4}$$

Here $M_X = x_1 M_1 + x_2 M_2 + m/y_b$ is the mass of the recoil system in the subprocess. The 4-momenta of the colliding objects and the inclusive particle are $P_1,\ P_2$ and p, respectively. Equation (4) reflects locality of the interaction at the constituent level and sets a restriction on the momentum fractions $x_1,\ x_2,\ y_a,\ y_b$ of particles via the kinematics of the constituent interactions. The microscopic scenario of constituent interactions developed in the framework of z-scaling is based on dependences of the momentum fractions on the collision energy, transverse momentum and collision centrality.

The scaling variable z has property of a fractal measure. It grows in a power-like manner with the increasing resolution Ω^{-1} defined with respect to the constituent subprocesses satisfying (4). The scaling function $\Psi(z)$ is expressed in terms of the inclusive cross section $Ed^3\sigma/dp^3$, multiplicity density $dN/d\eta$, and total inelastic cross section σ_{in}. All the quantities are measurable for the inclusive reaction $P_1 + P_2 \to p + X$. The function is determined as follows:

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\sigma_{\mathrm{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{5}$$

Here J is Jacobian for the transition from the variables $\{p_T^2, y\}$ to $\{z, \eta\}$. The function $\Psi(z)$ satisfies the normalization condition:

$$\int\limits_0^\infty \Psi(z)\, dz = 1. \tag{6}$$

Equation (6) allows us to interpret $\Psi(z)$ as probability density of the production of the inclusive particle with the corresponding value of the variable z.

2. SCALING OF JET PRODUCTION IN UNPOLARIZED $p + p$ COLLISIONS

Self-similarity of jet production in $p+p$ and $\bar{p}+p$ collisions has been studied in the framework of z-scaling in [17, 18]. The inclusive jet transverse momentum distributions measured by the STAR Collaboration at RHIC, by the CDF and D∅ Collaborations at Tevatron and by the CMS and ATLAS Collaborations at the LHC have been analyzed. The experimental spectra were compared with next-to-leading order pQCD calculations in p_T- and z-presentations. The parameters δ, ε_F and c used in the definition of the variable z were determined from analyses of many different sets of experimental data (see [17] and references therein). They are found to be constant ($\delta = 1,\ \varepsilon_F \approx 0,\ c = 1$) and independent of the multiplicity density and of the kinematic quantities such as collision energy, detection angle and transverse momentum of the inclusive particle. Possible change of the parameters was suggested as a signature of new phenomena in the kinematic regions not experimentally explored yet.

Results of the analyses showed the energy and angular independence of the scaling function $\Psi(z)$ and indicated its asymptotic behavior in a large interval of collision energies. A power law of $\Psi(z)$ over a wide range of z was found. It was shown that self-similar features of jet cross sections manifested by the z-scaling give strong restriction on the behavior of the scaling function at high z. The obtained results are considered as confirmation of the self-similarity of jet production, locality of constituent interactions and fractality of hadron structure at small scales.

The inclusive cross sections of jet production in $p+p$ collisions at $\sqrt{s} = 200$ GeV obtained by the STAR Collaboration at RHIC [24] are shown in Fig. 1, *a*. The data cover the kinematic range of the pseudorapidity $|\eta| < 0.8$ and the transverse momentum $p_T = 13-57$ GeV/c. They are in good agreement with NLO pQCD calculations with the CTEQ6M parton distribution functions (PDFs) at the factorization and renormalization scales equal to the transverse momentum ($\mu_F = \mu_R = p_T$). The same data in z-presentation are shown in Fig. 1, *b*. As seen from Fig. 1, *b*, the STAR data are in good agreement with the ISR data obtained at $\sqrt{s} = 38-63$ GeV and Sp$\bar{\rm p}$S data at $\sqrt{s} = 200$ GeV (see [17, 18] and references therein). A power behavior of the scaling function is demonstrated for $z > 80$. Experimental verification of this regime at high values of z is of interest.

As seen from Fig. 1, the scaling function $\Psi(z)$ exhibits a power behavior $\Psi(z) \sim z^{-\beta}$ over a wide z-region with a constant value of the slope parameter β. The observed power shape of the scaling function is considered to reflect self-similarity of hadron interactions and

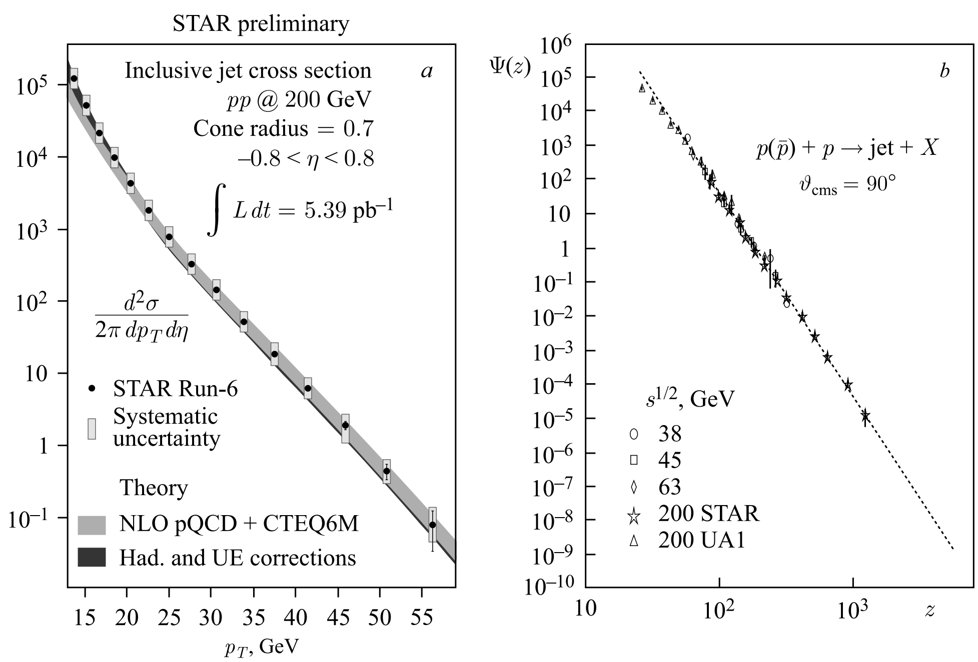

Fig. 1. *a*) Inclusive spectra of jet production in $p+p$ collisions at $\sqrt{s} = 200$ GeV and $|\eta| < 0.8$ measured by the STAR Collaboration [24] at RHIC. *b*) The same data in z-presentation. The ISR and Sp$\bar{\rm p}$S data from the analysis [17, 18] are shown for comparison

fractal structure of their constituents at small scales. The asymptotic form of $\Psi(z)$ imposes restrictions on the cross sections at high p_T. It can be used to perform the global QCD fit for construction of quark and gluon distribution functions in the regions where the experimental data are still missing.

3. ASYMMETRY OF JET PRODUCTION IN POLARIZED $p+p$ COLLISIONS

Recently, the STAR Collaboration has presented new high-precision data [3] on the mid-rapidity inclusive jet longitudinal double spin asymmetry, A_{LL}, in polarized $p+p$ collisions at the center-of-mass energy $\sqrt{s} = 200$ GeV. The data was obtained from an integrated luminosity of 20 pb^{-1} recorded in the year 2009. As noted in [3], the measurements place stringent constraints on polarized PDFs extracted at next-to-leading order from global analyses of inclusive deep inelastic scattering (DIS), semi-inclusive DIS, and the RHIC $p+p$ data. Moreover, the reported asymmetries provide evidence for positive gluon polarization in the region $x_{\text{Bj}} > 0.05$. Figure 2 shows the STAR data on A_{LL} as a function of p_T. The

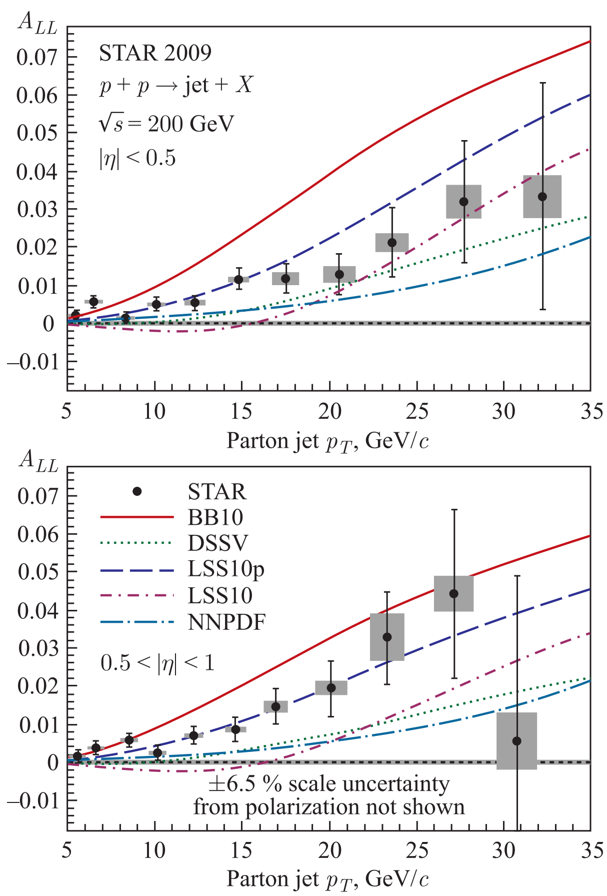

Fig. 2. Double longitudinal spin asymmetry of jet production in $\vec{p}+\vec{p}$ collisions at $\sqrt{s} = 200$ GeV, $|\eta| < 0.5$ and $0.5 < |\eta| < 1$ measured by the STAR Collaboration at RHIC [3]

data on the double spin asymmetry is small but nonzero in the measured pseudorapidity regions. The asymmetry increases with the transverse momentum up to the value of 3–4% at $p_T = 30$ GeV/c.

The curves in Fig. 2 represent expectations with different polarized PDFs. The cross section is well described by the NLO pQCD calculations over the transverse momentum range $5 < p_T < 50$ GeV/c [3]. The calculations indicate that midrapidity jet production at RHIC is dominated by quark–gluon (qg) and gluon–gluon (gg) scattering. It was emphasized that qg and gg scattering cross sections are sensitive to the longitudinal helicities of the participating partons, so the inclusive jet longitudinal double spin asymmetry provides direct sensitivity to the gluon polarization in the proton.

The information on the asymmetries extracted from the experiments with polarized proton beams is complementary to the measurements with unpolarized particles. It allowed us to formulate new hypothesis of self-similarity of spin-dependent interactions of protons (generally hadrons) at high energies [23]. The scaling hypothesis assumes fractal structure of spin compositeness of the protons characterized by spin-dependent fractal dimensions.

4. SCALING HYPOTHESIS FOR POLARIZED $p + p$ COLLISIONS

The reaction $\vec{p} + \vec{p} \to \mathrm{jet} + X$ with two longitudinally polarized protons in the initial state is described by spin-dependent cross sections $\sigma_{++}, \sigma_{--}, \sigma_{+-}, \sigma_{-+}$. The symbols $+$ and $-$ denote the positive and negative helicities of the protons, respectively. The double spin asymmetry, A_{LL}, of the process is expressed via combination of the spin-dependent cross sections in the following form:

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}. \tag{7}$$

The data on the cross sections measured at a given angle ϑ_{cms} for different polarizations of protons allow us to express the double spin asymmetry A_{LL} and the unpolarized cross section σ_0 as a function of the transverse momentum p_T.

Exploiting the information on the asymmetry and the unpolarized cross section, the spin-dependent functions $\Psi_{++}, \Psi_{--}, \Psi_{+-}, \Psi_{-+}, \Psi_{00}$ can be constructed. These functions have different arguments which we denote as z_{++}, z_{--}, z_{+-}, z_{-+}, z_{00}, respectively. It means that both dimensionless quantities, Ψ and z, can depend on spin orientations of the colliding protons. The corresponding notation for the functions Ω is written as follows:

$$\Omega_{++00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}, \tag{8}$$

$$\Omega_{+-00} =: \{\delta, \delta + \Delta\delta, \varepsilon_F, \varepsilon_F\}, \tag{9}$$

$$\Omega_{--00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\}, \tag{10}$$

$$\Omega_{-+00} =: \{\delta + \Delta\delta, \delta, \varepsilon_F, \varepsilon_F\}. \tag{11}$$

All the functions have the same form as in (3) and are expressed via spin-dependent fractal dimensions as indicated.

Below we discuss hypothesis of self-similarity of processes with polarized protons [23] using z-presentation of the spin-dependent scaling functions $\Psi(z)$. Based on the existence of

z-scaling for unpolarized collisions, we assume self-similarity of the polarization processes at a constituent level expressed in the following unified form:

$$\Psi_{00} = \Psi(z_{00}), \quad \Psi_{++} = \Psi(z_{++}), \quad \Psi_{+-} = \Psi(z_{+-}). \qquad (12)$$

The self-similarity hypothesis is considered here for the reaction $\vec{p} + \vec{p} \to \text{jet} + X$ with the production of inclusive jet. The last relations include the correction factor $\Delta\delta$ to the fractal dimension δ established in the analyses [18] of jet production in the reactions with unpolarized protons. The correction can be determined using data on the cross section and asymmetry of the processes under consideration. Information on both, the polarized and unpolarized

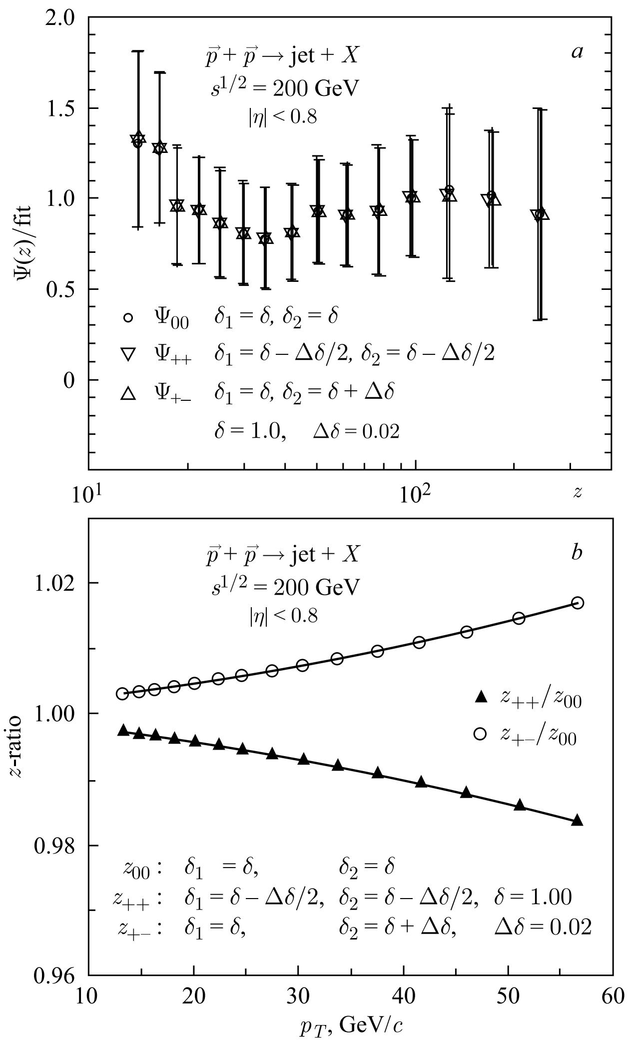

Fig. 3. *a*) The scaled spin-dependent Ψ_{++}, Ψ_{+-} and spin-independent Ψ_{00} functions for the process $\vec{p}+\vec{p} \to \text{jet}+X$ at $\sqrt{s} = 200$ GeV and $|\eta| < 0.8$ in z-presentation. *b*) The ratios of the spin-dependent self-similarity parameters

cross sections, are necessary to extract the spin-dependent fractal dimensions and verify the hypothesis (12) for the given process. Such data allow us to obtain restrictions on the model parameter $\Delta\delta$. For jet production we found that the fragmentation dimension ε_F is small and therefore we set $\varepsilon_F = 0$. In such a case, the kinematic conditions of the constituent interactions are determined in terms of x_1 and x_2 only. The momentum fractions depend on the measured momenta and the values of δ and $\Delta\delta$.

The transverse momentum dependence of the unpolarized cross section (Fig. 1, *a*) and the double longitudinal spin asymmetry (Fig. 2, *a*) approximated linearly and extrapolated up to $p_T = 55$ GeV/c were used in our analysis. We have constructed the functions $\Psi_{++}(z_{++})$ and $\Psi_{+-}(z_{+-})$, exploiting the hypothesis on universality of their shapes written in the form (12). The functions were used to estimate the correction $\Delta\delta$ to the proton fractal dimension from data on A_{LL} at $\sqrt{s} = 200$ GeV and $|\eta| < 0.5$. The correction is found to be $\Delta\delta = 0.02$.

Figure 3, *a* shows the scaled spin-dependent functions $\Psi(z)/\text{fit}$ for the reaction $\vec{p} + \vec{p} \rightarrow$ jet $+ X$ and the corresponding parameters δ_1 and δ_2. The functions coincide with each other with high accuracy in the whole considered region of z. The coincidence justifies the condition (12) as an expression of self-similarity of polarization processes in the z-scaling approach.

The respective ratios of the spin-dependent self-similarity parameters $(z_{++}/z_{00},\ z_{+-}/z_{00})$ as functions of the transverse momentum are shown in Fig. 3, *b*. The effect of spin–spin interactions, as seen from the ratios, does not exceed 4%. Such a difference agrees with values of the spin asymmetry A_{LL} which have similar magnitudes in the high-p_T region (Fig. 2, *a*). The ratio decreases as a function of p_T for the interactions of protons with the same (positive or negative) helicities and increases with p_T for the opposite orientation of proton helicities. The larger values of z_{+-} mean that the spin structure of the proton can be probed with higher resolution (i.e., at smaller scales) in the collisions of protons with opposite helicities relative to the interactions where the protons have the same helicities. If the collision is viewed as a clash of two fractals, the collisions of polarized protons can be viewed as clashing of mutually spinning fractals with fractal spin connections like an "internal screw–screw connection". We expect that the features of the proton spin structure could manifest itself more prominently at high p_T. As in the case of unpolarized processes, a discontinuity of the spin-dependent fractal dimensions should indicate a spin phase transition. To test such a hypothesis, the measurements of spin asymmetries and cross sections need very good accuracy. New precise data on A_{LL} over a wide range of x_1 and x_2 could give more detailed information regarding polarized constituent interactions and provide complementary restriction on parameters of the scaling variable z. The hypothesis of self-similarity of the spin-dependent structure of proton interactions encoded in the parameters δ, $\Delta\delta$ and in the functions Ψ_{++}, Ψ_{+-} is considered to play an important role to understand origin of the proton spin.

CONCLUSIONS

New data [3] on the double longitudinal spin asymmetry, A_{LL}, of the inclusive jet production in proton–proton collisions at $\sqrt{s} = 200$ GeV measured by the STAR Collaboration at RHIC were analyzed in the framework of the z-scaling approach. The hypothesis of self-similarity of the proton spin structure was formulated. The requirement of universality of the shape of the scaling function for different proton polarizations was assumed. The spin-dependent fractal dimensions δ_1 and δ_2 for the reaction $\vec{p}+\vec{p} \to \text{jet}+X$ are expressed via the spin-independent dimension δ and its correction $\Delta\delta$ according to different spin orientations of the colliding protons. The latter are found to be $\delta = 1$ and $\Delta\,\delta = 0.02$. Study of the behavior of the respective ratios of spin-dependent self-similarity parameters $(z_{++}/z_{00},\ z_{+-}/z_{00})$ leads us to a conclusion that interaction of the protons with mutually parallel spin orientations along the collision axis allows us to probe their spin structure at smaller scales in comparison with the situation where the spins of both polarized protons are aligned in the opposite direction. Such an inference relies on the fractal structure of hadron constituents as implemented in the z-scaling formalism and, as we consider, reflects the self-similarity of hadron interactions in spin-dependent processes at a constituent level.

The measurements of nonzero asymmetry, A_{LL}, give us strong motivation to study fractal properties of proton spin in the reactions with inclusive jet production. We believe that the considered scaling property for polarization processes reflects the self-similarity of the spin structure of the colliding objects and interaction mechanism of their constituents. We hope that systematic experimental investigations of such processes will contribute to further development of theory and understanding of spin as one of the most important and basic properties of particles.

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