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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

SELF-SIMILARITY OF HARD CUMULATIVE PROCESSES IN FIXED TARGET EXPERIMENT FOR BES-II AT STAR

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Search for signatures of phase transition in $\mathrm{Au}+\mathrm{Au}$ collisions is in the heart of the heavy ion program at RHIC. Systematic study of particle production over a wide range of collision energy revealed new phenomena such as the nuclear suppression effect expressed by nuclear modification factor, the constituent quark number scaling for elliptic flow, the “ridge effect” in $\Delta\phi-\Delta\eta$ fluctuations, etc. To determine the phase boundaries and location of the critical point of nuclear matter, the Beam Energy Scan (BES-I) program at RHIC has been suggested and performed by STAR and PHENIX Collaborations. The obtained results have shown that the program (BES-II) should be continued. In this paper a proposal to use hard cumulative processes in BES Phase-II program is outlined. Selection of the cumulative events is assumed to enrich data sample by a new type of collisions characterized by higher energy density and more compressed matter. This would allow finding clearer signatures of phase transition, location of a critical point and studying extreme conditions in heavy-ion collisions.

Поиск сигнатур фазовых переходов в столкновениях $\mathrm{Au}+\mathrm{Au}$ является центральной задачей программы по тяжелым ионам на RHIC. Систематическое изучение рождения частиц в широком диапазоне энергий столкновения ионов установило новые физические явления, такие как эффект ядерного подавления, кварковый скейлинг для эллиптического потока, “ridge”-эффект в $\Delta\phi-\Delta\eta$ флуктуациях и др. Для определения фазовой диаграммы ядерной материи и положения критической точки коллаборациями STAR и PHENIX была предложена и выполнена программа энергетического сканирования (BES-I) на RHIC. Полученные результаты показали, что необходимо ее продолжение (BES-II). В данной работе сделано предложение по изучению жестких кумулятивных процессов в рамках программы BES-II. Предполагается, что отбор кумулятивных событий в эксперименте с фиксированной мишенью позволит получить при их анализе новую информацию о высокоплотной и сжатой ядерной материи, исследовать экстремальные условия ее образования, установить четкие сигнатуры фазовых переходов и положение критической точки.

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INTRODUCTION

Experiments performed by the STAR, PHENIX, PHOBOS and BRAHMS Collaborations at RHIC have shown that the nuclear matter with new properties has been produced in the high-energy range $\sqrt{s_{NN}} = 62-200$ GeV [1–4]. Among the properties there is opacity characterized by the suppression of particle yields at high p_T and viscosity which is found to be so small that the matter looks like ideal liquid rather than an ideal gas of quarks and gluons. The new state of matter was named the strongly interacting quark–gluon plasma (sQGP). The QCD phase diagram of nuclear matter is depicted in Fig. 1. Theory predicts how transitions to sQGP depend on baryon chemical potential μ_B and temperature T. At low μ_B and high T a crossover transition occurs. At high μ_B and low T the transition is of the first order. Hence, at intermediate values, a critical point should exist. One can vary these conditions experimentally by altering the beam energy. The beam energy scan (BES) allows study of the QCD phase diagram close to the QGP-hadron gas boundary.

Search for signatures of the phase transition and location of the critical point in heavy ion collisions is the main goal of the BES Phase-I and Phase-II programs at RHIC [5–11]. Establishing the existence of the critical point would be a seminal step forward for QCD physics in the regime of strong coupling. The first phase of the BES program started after the STAR proposal [7] with data taking in the year 2010 at the energies below $\sqrt{s_{NN}} = 39$ GeV. Now the STAR Collaboration proposes a second phase of the program at RHIC (BES Phase-II) to refine understanding of the phase structure of QCD matter [10]. A similar program is suggested by the PHENIX Collaboration as well [11]. The proposal [10] is for two years (2018 and 2019) of dedicated low energy running at RHIC to make high-precision

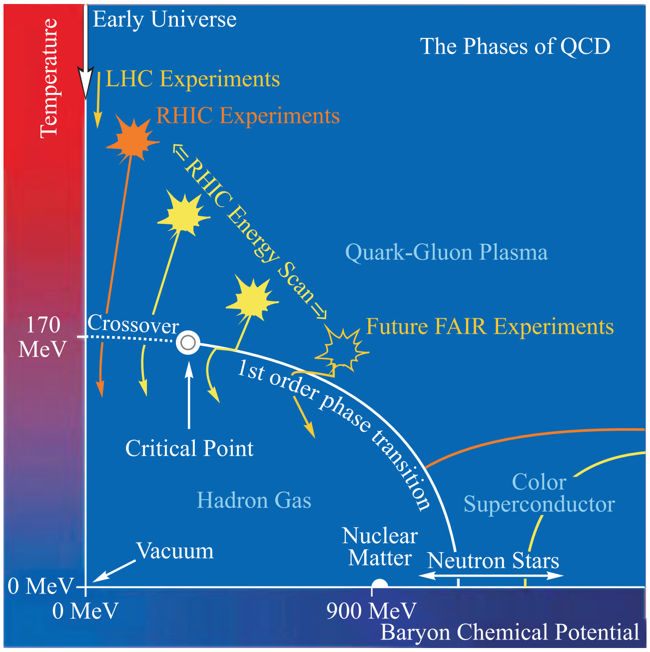

Fig. 1. Phase diagram of nuclear matter in QCD framework [10]

measurements of the observables that have been found to be sensitive to the phase structure of QCD matter in the first phase of the program.

The parameters which characterize different states of the produced matter are energy and centrality of the collisions and type of the colliding nuclei. They can regulate the density and temperature of the produced system and size of the interaction region. To investigate such states, various measurable characteristics such as the momentum and mass spectra and/or the correlation and fluctuation functions are utilized. Special interest is dedicated to observables related to the fluctuations and correlations. Significant changes of these quantities are theoretically related to vicinity of critical phenomena. In this respect, significant growth of the correlations and fluctuations at low energy is expected where a phase transition should occur.

Different types of probes (high-p_T hadrons, direct photons, jets, lepton pairs, strange and heavy flavor particles) play an important role in determining features of the produced matter. Ratios of particle and antiparticle yields have been exploited to extract information on temperature and baryon chemical potential in the framework of the thermodynamical and statistical models. The hydrodynamic model was used for analysis of the elliptic flow and study of particle collectivity (strongly multiple particle interactions) at the quark level. Results of analyses of numerous experimental data obtained at RHIC have shown that various measurable quantities demonstrate smooth behavior as function of the energy and centrality of collisions over a wide range of transverse momentum (see [5–11] and references therein).

It is generally considered that all physical systems should reveal discontinuity in some characteristics describing their behavior nearby a phase boundary or a critical point. Therefore, the concepts of scaling and universality have been widely developed to explain the critical phenomena [12–15]. Scaling implies that systems near critical points exhibit self-similarity and are invariant with respect to scale transformations. The universality of their behavior lies in the fact that vastly different systems behave in a similar way (they are described by the same power law) near the respective critical point. The critical exponents in the power laws are determined by the interaction symmetry and space dimension only.

The scaling behavior related to the ideas of self-similarity of hadron interactions at a constituent level is manifested by z-scaling which is a fruitful concept to study collective phenomena in hadron and nuclear matter [16–18]. The concept of z-scaling [19–21] was used for analysis of inclusive spectra obtained at U70, $\mathrm{S\bar{p}pS}$, SPS, ISR, Tevatron and RHIC [22–28]. The spectra have revealed striking similarity over a wide range of energies when expressed by the variable z. The scaling is treated as manifestation of the self-similarity of the structure of the colliding objects (hadrons, nuclei), the interaction mechanism of their constituents, and the process of constituent fragmentation into real hadrons. The validity of z-scaling is confirmed in the region which is far from the boundary of a phase transition or the region where a critical point can be located. Nevertheless, the z-scaling approach can be a suitable tool to search for phase transitions and the critical point in hadron and nuclear matter. The parameters of the scaling, c, δ and ε_F, have physical interpretation as the "heat capacity" of the produced matter, the fractal dimension of the structure of hadrons or nuclei, and the fractal dimension of the fragmentation process, respectively. Signatures of new phenomena in strong interacting matter are assumed to be discontinuities of these parameters and enhancement of c–δ correlation.

Analyses of the presently available RHIC data on particle spectra in $p+p$ and $\mathrm{Au}+\mathrm{Au}$ collisions performed in the framework of z-scaling approach gave us no direct information on existence of a phase transition or a critical point [22–29]. No distinct change of the scaling parameters which would indicate vicinity of the critical phenomena was observed. We consider therefore that study of energy dependence of the parameters for the new class of events with cumulative (strongly compressed) states of nuclei could give important information on phase changes in the nuclear matter.

1. KINEMATICS OF CUMULATIVE PRODUCTION IN THE FIXED TARGET MODE

The cumulative particles are particles produced in the kinematic region forbidden for free nucleon–nucleon interactions (see [30–32] and references therein). Such particles are only produced in the processes with participation of nuclei. The cumulative effect has been traditionally studied at low transverse momentum p_T [33–35]. This corresponds to particle production in the backward hemisphere in laboratory frame of reference. Another possibility to study the cumulative processes is investigation of particles with high p_T [36–39]. Study of the cumulative processes is of great interest to search for signatures of phase transitions in highly compressed nuclear matter.

Figure 2 shows diagrams for inclusive particle production in the central rapidity range as well as in the forward and backward hemispheres in the $P_1 + P_2 \to p + X$ process.

The momenta of the colliding and produced inclusive particles are denoted by P_1, P_2 and p, respectively. The diagrams correspond to the particle production in the collider (a) and fixed target (b, c) modes. In the collider mode, the particles detected in the central detector barrel are produced mostly from the central interaction region. In the fixed target mode, the detected particles are mostly from the beam (b) and the target (c) fragmentation regions, respectively. Figure 3 shows the kinematic boundaries of pion production in the process $P_1 + P_2 \to \pi + X$ at $\sqrt{s_{NN}} = 7.7$, 9.2 and 20 GeV in the fixed target setup.

The kinematic region forbidden for pion production in $p + p$ processes is the cumulative region. The area between pp and pd lines corresponds to the single, between pd and dd lines to the double and outside of dd line to the triple cumulation. The momentum of the produced

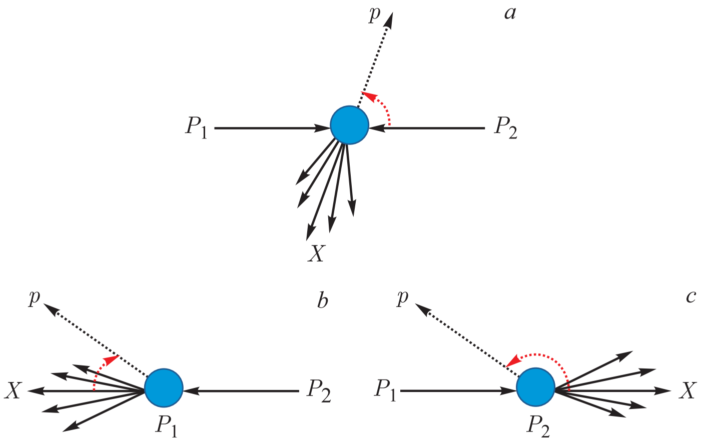

Fig. 2. Diagrams of particle production in the central rapidity range $\vartheta_{\rm cms} \approx 90^\circ$ (a), forward $\vartheta_{\rm lab} < 90^\circ$ (b) and backward $\vartheta_{\rm lab} \leqslant 180^\circ$ (c) hemisphere

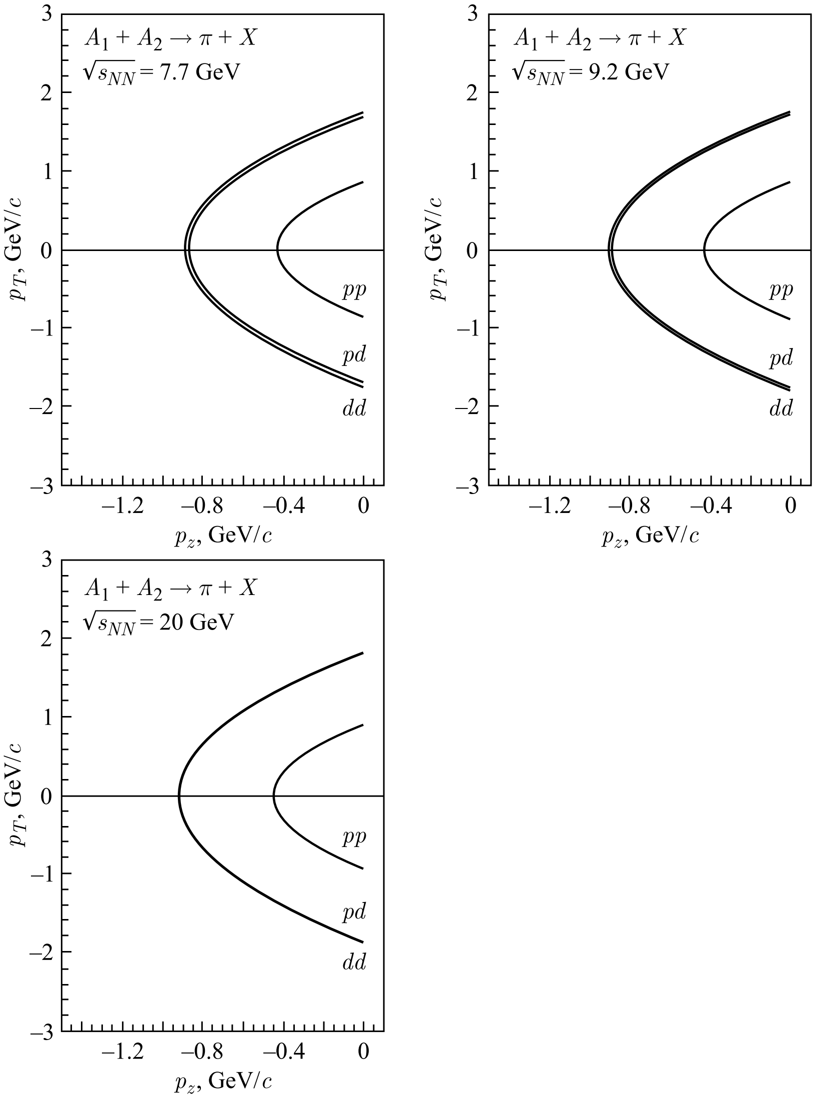

Fig. 3. Kinematic boundaries for the pion production in the backward hemisphere in $p+p$, $p+d$, $d+d$ collisions at $\sqrt{s_{NN}} = 7.7, 9.2$ and 20 GeV in $\{p_z,\, p_T\}$ plane

pion grows with the collision energy but is asymptotically restricted. For $d+d$ processes at $\vartheta_{\rm lab} = 180°$, the maximal backward momentum is $p_{\max} = (M_d^2 - m_\pi^2)/2M_d$. The cumulative region for heavier nuclei is wider. It increases with the atomic weight as $p_{\max} \approx Am_N/2$. Therefore, interactions of compressed nuclei can be studied over a wide kinematic range. For heavy-ion interactions, however, the region near the kinematic boundary is practically not reachable. The collisions of light and medium nuclei are preferable in this respect.

The main physical motivation to investigate the cumulative processes is related to the assumption that cumulative regions correspond to the extreme regime of particle production in which the nuclear matter is strongly compressed. The conclusion is based on the validity of the momentum conservation law and the Heisenberg uncertainty principle. Both conditions strongly select the state from which the cumulative particles are produced. Such processes are rare events. The probability of production of cumulative states is usually small enough though not zero. The cumulative effect (particle production in the cumulative region) is a quantum phenomenon. The uncertainty of the momentum of the particle produced in the nuclear matter is related to the uncertainty of the size of the region in which the particle was created.

In the most extreme case of a deep-cumulative region (the region near the kinematic boundary of the reaction), the nucleus size is assumed to be compressed to the size of a single nucleon. In such very rare processes the momentum of the inclusive particle should be fully balanced by the momentum of a compressed recoil system consisting of mutually slow moving constituents. The system in this state is expected to demonstrate the property of collectivity and transition regime from single to multiple constituent interactions. In this regime the limiting fragmentation of the highly compressed nuclear matter should be observed.

2. z-SCALING

One of the approaches that can be useful to search for signatures of phase transition in the cumulative region of particle production in the suggested BES Phase-II program at RHIC is z-scaling [19–21]. The scaling has been suggested to describe regularities found in inclusive hadron production in high energy proton–(anti)proton and nucleus–nucleus collisions [22–28, 40–43]. It manifests itself in the fact that the inclusive spectra of various types of particles are described with a universal scaling function $\Psi(z)$. The function depends on a single variable z in a wide range of the transverse momentum, registration angles, collision energies and centralities. The scaling variable is expressed by the formula

$$z = z_0 \Omega^{-1}, \tag{1}$$

where z_0 and Ω are functions of some kinematic and dynamical variables:

$$z_0 = \frac{\sqrt{s_\perp}}{(dN_{\rm ch}/d\eta|_0)^c m_N}, \tag{2}$$

$$\Omega = (1-x_1)^{\delta_1}(1-x_2)^{\delta_2}(1-y_a)^{\varepsilon_F}(1-y_b)^{\varepsilon_F}. \tag{3}$$

The quantity z_0 is proportional to the transverse kinetic energy of the selected binary constituent subprocess required for the production of the inclusive particle m and its partner (antiparticle). The multiplicity density $dN_{\rm ch}/d\eta|_0$ of charged particles in the central interaction region $\eta = 0$, the nucleon mass m_N and the parameter c completely determine the functional relationship of the dimensionless variable z_0. The parameter c has meaning of the "specific heat capacity" of the medium produced in the collisions.

The quantity Ω is proportional to the relative number of the configurations at the constituent level which include the binary subprocesses corresponding to the momentum fractions x_1 and x_2 of colliding hadrons (nuclei) and to the momentum fractions y_a and y_b of the secondary objects just produced in these subprocesses. The parameters δ_1 and δ_2 are fractal dimensions of the colliding objects, and ε_F stands for the fractal dimension of the fragmentation process.

The selected binary subprocess, which results in production of the inclusive particle and its recoil partner (antiparticle), is defined by the maximum of $\Omega(x_1, x_2, y_a, y_b)$ with the kinematic constraint:

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2. \tag{4}$$

Here $M_X = x_1 M_1 + x_2 M_2 + m/y_b$ is the mass of the recoil system in the subprocess. The 4-momenta of the colliding objects and the inclusive particle are P_1, P_2 and p, respectively. Equation (4) accounts for the locality of the interaction at the constituent level and sets a restriction on the momentum fractions x_1, x_2, y_a, y_b of particles via the kinematics of the constituent interactions. A microscopic scenario of constituent interactions developed within the scaling approach is based on dependences of the momentum fractions on the collision energy, transverse momentum and centrality.

The scaling variable z has a property of the fractal measure. It grows in the power manner with the increasing resolution Ω^{-1} with respect to the constituent subprocesses. The scaling function $\Psi(z)$ is expressed in terms of the experimentally measurable quantities — the inclusive cross section $Ed^3\sigma/dp^3$, the multiplicity density $dN/d\eta$, and the total inelastic cross section σ_{in} for the inclusive reaction $P_1 + P_2 \to p + X$. It is determined by the following expression:

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\,\sigma_{\text{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}. \tag{5}$$

Here J is Jacobian for the transition from the variables $\{p_T^2, y\}$ to $\{z, \eta\}$. The function $\Psi(z)$ satisfies the normalization condition:

$$\int\limits_0^\infty \Psi(z)\,dz = 1. \tag{6}$$

Equation (6) allows us to interpret $\Psi(z)$ as the probability density to produce the inclusive particle with the corresponding value of the variable z.

3. SELF-SIMILARITY IN $p + p$ COLLISIONS

Proton–proton collisions provide the basis for analyzing more complicated proton–nucleus and nucleus–nucleus interactions. Figure 4 shows spectra of light hadrons produced in proton–proton interactions in z-presentation. The kinematic region covers a wide range of the collision energies, registration angles, and transverse momenta. The scale factors are introduced to split the data into different groups. We see a collapse of the data points onto a single curve. The solid line is a fitting curve for these data. The derived representation demonstrates the universality of the shape of the scaling curve for different types of hadrons. Found regularity (universality of the shape of the function $\Psi(z)$ and its scaling behavior in the wide kinematic range at constant values of the parameters δ, ε_F and c) is treated as a manifestation of self-similarity of the structure of the colliding objects, interaction mechanism of their constituents, and processes of fragmentation into real particles. The fractal dimension ε_F of the fragmentation process varies for different types of hadrons. The compatibility of the corresponding scaling curves for single hadrons in the plane $\{z, \Psi\}$ was obtained by the scale

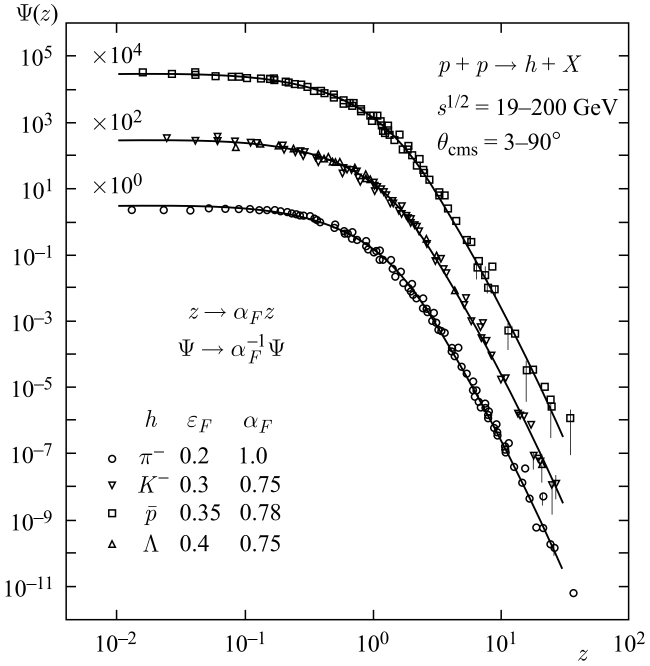

Fig. 4. Inclusive spectra of hadrons produced in proton–proton collisions in the z-presentation. The symbols denote the experimental data obtained in the experiments performed at CERN, FNAL and BNL (see [16, 20] and references therein)

transformation $z \to \alpha_F\, z,\ \Psi \to \alpha_F^{-1}\,\Psi$. The scale parameter α_F depends on a flavor only. The normalization condition (6) is conserved under the transformation.

The scaling function $\Psi(z)$ exhibits two regimes of scaling behavior: one in the low-z and the other in the high-z region. The low-z region corresponds to saturation of the scaling function with the typical flattening-out. The behavior of $\Psi(z)$ at low z depends mainly on the parameter c. The parameter is determined from the multiplicity dependence of the inclusive spectra. The region of low z (transverse momentum $<$ 100 MeV) and of high multiplicity density is preferable to study collective effects and possible phase transition in hadron matter. The region of high z (high transverse momenta) is characterized by the power behavior of $\Psi(z) \sim z^{-\beta}$ with the constant value of the slope parameter β. At high z, the observed power character of the scaling function points to self-similarity in constituent interactions at small scales. The asymptotic behavior of $\Psi(z)$ imposes restrictions on the behavior of the cross sections at high p_T. Note that these restrictions can be used to perform the global QCD fit and construct quark and gluon distribution functions in the regions where the experimental data are missing [41, 43].

The parameters δ, ε_F and c introduced to construct the variable z are determined from analyses of many different sets of experimental data (see [19, 20, 22–27, 40–43] and references therein). They are found to be constant and independent of the kinematic quantities — the collision energy, angle, transverse momentum of the inclusive particle and multiplicity density at high energies. A possible change of these parameters is assumed to be used as a signature of new phenomena in the kinematic regions not yet explored experimentally.

4. SELF-SIMILARITY IN $A+A$ COLLISIONS

The phase transitions and other collective effects should show up more prominently in a larger space volume in the collisions of heavy nuclei than in proton–proton interactions. It is expected that they have influence on the production mechanisms of particles, i.e., interaction of nuclear constituents and fragmentation process in the final state.

Here we give a short review of the obtained results of $A+A$ data in z-scaling approach. Figure 5 shows z-presentation of the spectra of π mesons produced in $\mathrm{Au}+\mathrm{Au}$ collisions at RHIC energies $\sqrt{s_{NN}} = 9.2$, 62.4 and 200 GeV in the central rapidity region $|\eta| < 0.5$ for different centralities [18, 25]. A consistent description of the data presentation has been obtained by the condition that the fractal dimension of the nucleus δ_A is expressed in terms of the nucleon fractal dimension δ and the atomic number A as $\delta_A = A\delta$ [44]. It has been found that the "specific heat" (parameter c) is independent of the energy and centrality of the collision and decreases with the increase of the atomic number of the nucleus. A strong suppression of the function $\Psi(z)$ with the increasing centrality in nuclear collisions has been found for the centrality-independent value of ε_{AA}. The suppression is enhanced with increasing transverse momentum p_T. It has been found that the universal shape of $\Psi(z)$ for $A+A$ collisions can be restored if a dependence of the fractal dimension ε_{AA} of the fragmentation process on the event centrality (multiplicity density) is assumed. The dependence is taken in the following form:

$$\varepsilon_{AA} = \varepsilon_0 \left(\frac{dN_{\mathrm{ch}}}{d\eta}\right) + \varepsilon_{pp}. \tag{7}$$

The value of ε_{pp} is the same as for proton–proton collisions. The coefficient ε_0 depends on the collision energy. The same type of behavior has been observed for interactions of Cu, Au

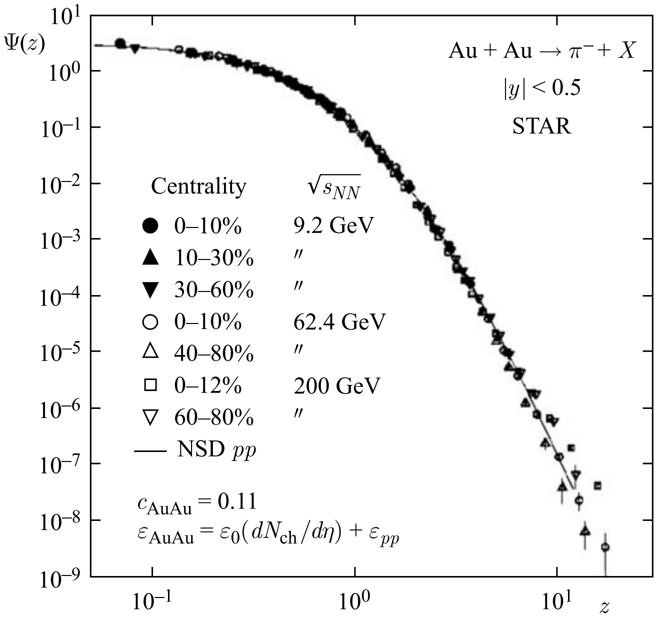

Fig. 5. Inclusive spectra of pions produced in $\mathrm{Au}+\mathrm{Au}$ collisions in z-presentation. The data obtained by the STAR Collaboration have been used in the analysis (see [16] and references therein)

and Pb nuclei at the energies $\sqrt{s_{NN}} = 17.3, 62.4$, 130 GeV [18, 25]. We thus assume that in $A + A$ interactions, similarly as in $p + p$ collisions, a discontinuity of the model parameters (specific heat and fractal dimensions) should indicate new critical phenomena.

5. DISCUSSION

Here we discuss a possibility of using z-scaling approach to search for critical phenomena in relativistic collisions of heavy nuclei in more detail. The endeavor for a unique description of the spectra of hadrons produced in $A+A$ interactions by the universal scaling function $\Psi(z)$ gives a strong restriction on the parameters of z-scaling. A sharp change (or discontinuity) of the fractal dimensions δ_A and ε_{AA} and/or the "heat capacity" c is proposed as a signature of new effects, in particular, of the phase transition. Such effects can be, however, smeared by the large constituent energy losses especially in the central collisions of heavy nuclei. The growth of ε_{AA} with the collision centrality (multiplicity) corresponds to the increased energy losses of the secondary particles in the produced nuclear medium during their fragmentation. This adds to the difficulties in determination of the region where the phase transition or a critical point could exist.

The problem can be partially evaded in the cumulative region (x_1A_1, $x_2A_2 > 1$). This is the case of the hard cumulative processes corresponding to the region of particle production with high transverse momenta. Such processes have not been investigated up to now. The transition into the cumulative region at fixed centrality is considered as an essential condition for this type of searching for the phase transition and localization of the critical point. The z-scaling predicts the dependence of the energy losses on the collision energy and centrality, transverse momentum, type of the inclusive particle, and order of cumulativity [16–18]. The cumulative region $x_1A_1,\ x_2A_2 > 1$ is practically achievable at relatively low energies only (see Fig. 3). The decrease of energy losses with the increasing p_T is significant especially at lower energies and high transverse momenta, which corresponds to the cumulative and central rapidity region $x_1A_1 \approx x_2A_2 > 1$. Cumulative kinematics is also available in the beam or target fragmentation region (Fig. 2) in which case $x_1A_1 \ll 1,\ x_2A_2 > 1$ or $x_1A_1 > 1,\ x_2A_2 \ll 1$, respectively. The corresponding values of momenta of the produced particles are essentially different. For production in the backward hemisphere in the fixed target mode, the momenta are substantially less than for the forward direction. In this case, the background from the beam fragmentation decreases with increasing collision energy. This is the main advantage to study the rare processes — cumulative production in the backward hemisphere. The kinematic boundary and hence the size of the cumulative range depend on the collision energy. Therefore, one can study different cumulative regions in the RHIC BES program [10, 11].

The constituent energy losses increase with energy and centrality of the collision and decrease as the transverse momentum of inclusive particle increases [24]. The essential problem is that the energy losses smear characteristic behavior of experimental observables near a phase boundary and a critical point [16]. The cumulative processes with high p_T (hard cumulative processes) are, therefore, the most preferable to search for signature of these critical phenomena. In this region we expect discontinuity and strong correlation of the parameters δ_A, ε_{AA} and c. The fractal dimension δ_A can be sensitive, in particular, to particle-like fluctuations of the nuclear matter. The fragmentation properties of the particles produced in the collisions of such fluctuations in nuclei (flucton collisions) could influence the value of the fragmentation dimension ε_{AA}. We expect that properties of the compressed matter in cumulated nuclei could change the observed value of the "specific heat" c as well. We also assume that the fractal dimension δ_A will grow as the nucleus cumulation increases. It should be greater for flucton (local cumulations of the nuclear matter in the nucleus) than for the ordinary nuclei. The additive property $\delta_A = A\delta$ found for interactions of uncompressed nuclei can be violated as well. Due to fluctons, the change of nuclear structure with $\delta_A = A^d\delta,\ d > 1$ is expected in the cumulative region.

Different scenarios of energy dependence of the fractal dimension δ_A and the "specific heat" c have been analyzed in [18]. Here we use values of the z-scaling parameters for the scenario with energy-independent parameter c found in the central $\mathrm{Au}+\mathrm{Au}$ collisions at $\sqrt{s_{NN}} = 9.2$ GeV to illustrate the p_T-dependence of the momentum fractions $x_1,\ x_2$ and y_a

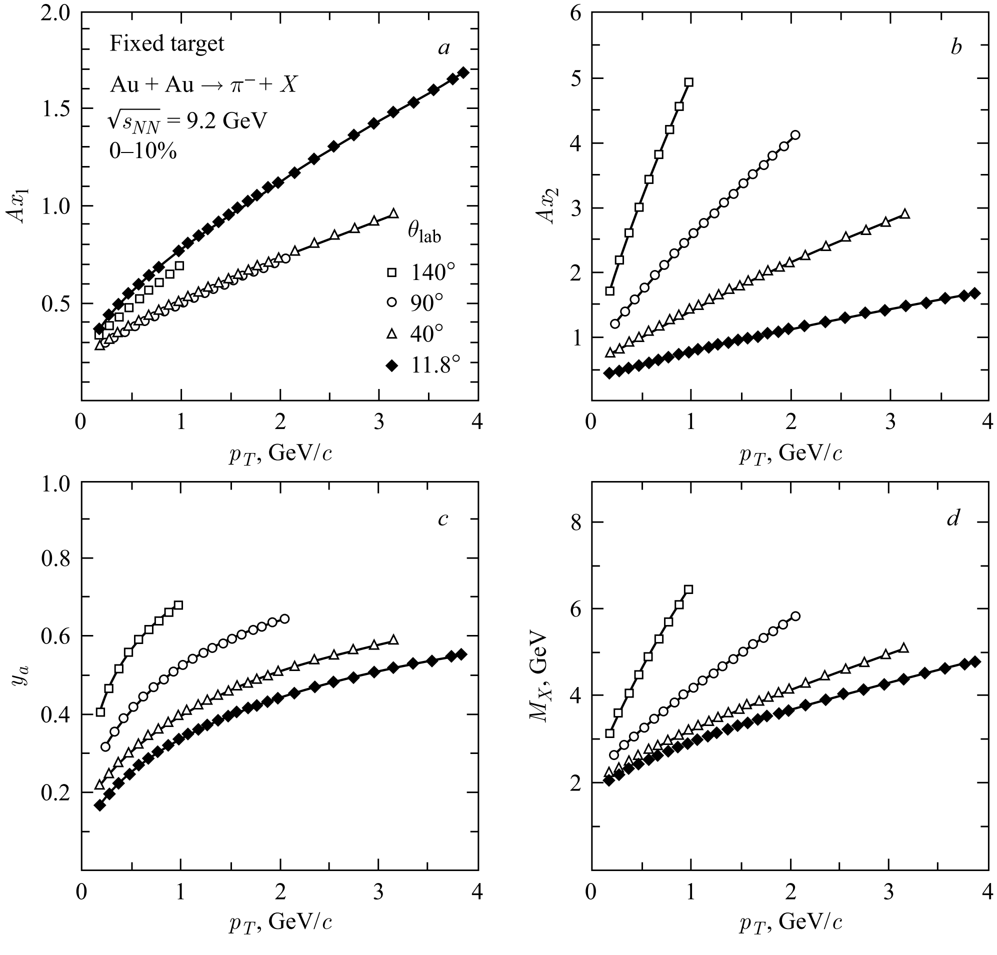

Fig. 6. The dependence of the momentum fractions $x_1,\ x_2,\ y_a$ and missing mass M_X on transverse momentum p_T for pion production in the central (0–10%) $\mathrm{Au}+\mathrm{Au}$ collisions at $\sqrt{s_{NN}} = 9.2$ GeV and $\vartheta_{\mathrm{lab}} = 11.8{-}140°$

and the missing mass M_X for pions produced at $\vartheta_{\rm lab} = 11.8{-}140^\circ$. The laboratory angle $\vartheta_{\rm lab} = 11.8^\circ$ corresponds approximately to the angle $\vartheta_{\rm cms} \approx 90^\circ$ in the nucleon–nucleon center-of-mass system at this energy.

As seen from Figs. 6,*a* and 6,*b*, the fractions x_1 and x_2 increase with p_T. The double cumulative region (cumulation of both colliding nuclei) corresponds to $x_1 A_1 > 1$ and $x_2 A_2 > 1$. This region reachable at $p_T \approx 2$ GeV/*c* for $\vartheta_{\rm lab} = 11.8^\circ$ is suitable for study in the collider mode. The cumulative numbers $x_1 A = x_2 A \approx 1$ are equal to each other at this momentum. We would like to emphasize that cumulation of the target (particle 2) increases with $\vartheta_{\rm lab}$. From Figs. 6,*a* and 6,*b* one can see that the fixed target mode is best suitable for study of the large cumulations of the target nucleus, especially in the backward hemisphere. The cumulative region $Ax_1 \to 0,\ Ax_2 \gg 1$ overlaps with the limiting fragmentation of the target. We found the cumulative number $Ax_2 \approx 5$ at the angle $\vartheta_{\rm lab} = 140^\circ$ and $p_T \approx 1$ GeV/*c*.

The increase of the fraction y_a with p_T and $\vartheta_{\rm lab}$ is shown in Fig. 6,*c*. One can see that the constituent energy loss $\Delta E/E \approx 1 - y_a$ decreases with the increase of both p_T and $\vartheta_{\rm lab}$. At $p_T = 1$ GeV/*c*, the relative energy loss for $\vartheta_{\rm lab} = 11.8^\circ$ and 140° is equal to 70% and 30%, respectively. Therefore, large cumulation ($Ax_2 \gg 1$) with small energy losses ($1 - y_a$) in the hard cumulative region ($p_T \geqslant 0.5$ GeV/*c*) in the backward hemisphere is considered to be preferable to search for signatures of phase transitions. The corresponding dependences of the recoil mass M_X on p_T are shown in Fig. 6,*d*. The higher momentum of the inclusive particle the stronger cumulation and the larger recoil mass. For the momentum $p_T = 1$ GeV/*c*, the mass $M_X \approx 2.5$ and 6.5 GeV at $\vartheta_{\rm lab} = 11.8^\circ$ and 140°, respectively.

We would like to note that the most stringent condition in the cumulative region is the multiplicity which can also be used to select events to control the properties of the medium in which the flucton interactions take place. It is expected that the transition into the cumulative region for events with high multiplicity can involve additional selection of events with higher density of the nuclear matter. Small energy losses with additional compression of the nuclear matter can allow us to find more accurate localization of the critical point, detection of the phase transition and determination of their boundaries on the phase diagram.

CONCLUSIONS

A proposal to use the hard cumulative processes in BES Phase-II program at RHIC was suggested. Selection of cumulative events is assumed to enrich data sample by a new type of events characterized by higher energy density and more compressed matter. It allows us to find clearer signatures of new physics phenomena. The transition into the cumulative region is considered as an essential condition to search for the phase transition and localization of the critical point. The hard cumulative production in the backward hemisphere in the fixed target mode has great advantages. The main one is that the background decreases with increasing collision energy. The method of data analysis known as z-scaling was suggested to search for self-similarity behavior of the pion production in this region. It was noted that the requirement of the universal description of the hadron spectra in nuclear collisions at different energies and centralities gives restrictions on the values of the model parameters of z-scaling and their dependences on energy and multiplicity density. The parameters δ_A, ε_{AA}, and c interpreted as the fractal dimension of the nucleus, the fractal dimension of the fragmentation process, and the "heat capacity" of the produced medium are important ingredients of the physical scenario of nucleus interactions. A discontinuity of the parameters and strong correlation of δ_A and c is assumed to play an important role in hard cumulative region in the backward hemisphere and in the search of phase transition and location of the critical point of nuclear matter.

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REFERENCES

1. *Arsene I. et al. (BRAHMS Collab.).* Quark–Gluon Plasma and the Color Glass Condensate at RHIC? The Perspective from the BRAHMS Experiment // Nucl. Phys. A. 2005. V. 757. P. 1.
2. *Back B. B. et al. (PHOBOS Collab.).* The PHOBOS Perspective on Discoveries at RHIC // Ibid. P. 28.
3. *Adams J. et al. (STAR Collab.).* Experimental and Theoretical Challenges in the Search for the Quark–Gluon Plasma: The STAR Collaboration's Critical Assessment of the Evidence from RHIC Collisions // Ibid. P. 102.
4. *Adcox K. et al. (PHENIX Collab.).* Formation of Dense Partonic Matter in Relativistic Nucleus–Nucleus Collisions at RHIC: Experimental Evaluation by the PHENIX Collaboration // Ibid. P. 184.
5. *Caines H. (for the STAR Collab.).* The RHIC Beam Energy Scan — STAR'S Perspective // Proc. for the Rencontres de Moriond 2009, QCD Session; arXiv: 0906.0305v1 [nucl-ex]. June 1, 2009.
6. *Abelev B. I. et al. (STAR Collab.).* Experimental Study of the QCD Phase Diagram & Search for the Critical Point: Selected Arguments for the Run-10 Beam Energy Scan; http://drupal.star.bnl.gov/STAR/starnotes/public/sn0493. June 4, 2009.
7. *Aggarwal M. M. et al. (STAR Collab.).* An Experimental Exploration of the QCD Phase Diagram: The Search for the Critical Point and the Onset of Deconfinement. arXiv:1007.2613 [nucl-ex]. July 15, 2010.
8. *STAR Collab.* STAR Collaboration Decadal Plan. Brookhaven National Laboratory, Relativistic Heavy Ion Collider. Dec. 2010; http://www.bnl.gov/npp/docs/STAR_Decadal_Plan_Final%5B1%5D.pdf.
9. Hot and Dense QCD Matter, A Community White Paper on the Future of Relativistic Heavy-Ion Physics in the US. Unraveling the Mysteries of the Strongly Interacting Quark–Gluon Plasma; http://www.bnl.gov/npp/.
10. *STAR Collab.* Studying the Phase Diagram of QCD Matter at RHIC: A STAR White Paper Summarizing the Current Understanding and Describing Future Plans. SN0598. June 1, 2014.
11. *PHENIX Collab.* The PHENIX Experiment at RHIC, Decadal Plan 2011–2020. Brookhaven National Laboratory.
12. *Stanley H. E.* Introduction to Phase Transitions and Critical Phenomena. London: Oxford Univ. Press, 1971.
13. *Stanley H. E.* Scaling, Universality, and Renormalization: Three Pillars of Modern Critical Phenomena // Rev. Mod. Phys. 1999. V. 71. P. S358.
14. *Hankey A., Stanley H. E.* Systematic Application of Generalized Homogeneous Functions to Static Scaling, Dynamic Scaling, and Universality // Phys. Rev. B. 1972. V. 6. P. 3515.
15. *Lübeck S.* Universal Scaling Behavior of Non-Equilibrium Phase Transitions // Intern. J. Mod. Phys. B. 2004. V. 18. P. 3977.

16. *Tokarev M. V., Zborovský I.* Self-Similarity of High-p_T Hadron Production in Cumulative Processes and Violation of Discrete Symmetries at Small Scales (Suggestion for Experiment) // Phys. Part. Nucl. Lett. 2010. V. 7. P. 160.
17. *Tokarev M. V. et al.* Search for Signatures of Phase Transition and Critical Point in Heavy-Ion Collisions // Phys. Part. Nucl. Lett. 2011. V. 8. P. 533.
18. *Tokarev M. V., Zborovský I.* Energy Scan in Heavy-Ion Collisions and Search for a Critical Point // Phys. At. Nucl. 2012. V. 75. P. 700.
19. *Zborovský I., Tokarev M. V.* Generalized z-Scaling in Proton–Proton Collisions at High Energies // Phys. Rev. D. 2007. V. 75. P. 094008.
20. *Zborovský I., Tokarev M. V.* New Properties of z-Scaling: Flavor Independence and Saturation at Low z // Intern. J. Mod. Phys. A. 2009. V. 24. P. 1417.
21. *Tokarev M. V., Zborovský I.* z-Scaling as Manifestation of Symmetry in Nature // Selected Papers of the Seminar (2002–2005) "Symmetries and Integrable Systems" / Ed. by A. N. Sissakian. Dubna: JINR, 2006. V. II. P. 154.
22. *Tokarev M. V.* z-Scaling at RHIC // Phys. Part. Nucl. Lett. 2006. V. 3. P. 7.
23. *Tokarev M. V.* z-Scaling in Heavy-Ion Collisions at the RHIC // Phys. Part. Nucl. Lett. 2007. V. 4. P. 676.
24. *Zborovský I., Tokarev M. V.* Energy Scan in Heavy-Ion Collisions and Search for a Critical Point // Phys. At. Nucl. 2012. V. 7. P. 700.
25. *Tokarev M. V., Zborovský I.* Self-Similarity of Pion Production in AA Collisions at RHIC // Phys. Part. Nucl. Lett. 2010. V. 7, No. 3. P. 171.
26. *Tokarev M. V. (for the STAR Collab.).* High-p_T Spectra of Charged Hadrons in Au + Au Collisions at $\sqrt{s_{NN}} = 9.2$ GeV in STAR // Phys. At. Nucl. 2011. V. 74, No. 5. P. 799.
27. *Tokarev M. V., Zborovský I.* Beam Energy Scan at RHIC and z-Scaling // Nucl. Phys. Proc. Suppl. 2013. V. 245. P. 231.
28. *Tokarev M. V., Zborovský I.* Energy Loss in Heavy Ion Collisions // Proc. of the 40th Intern. Symp. on Multiparticle Dynamics (ISMD 2010), Antwerp, Belgium, Sept. 21–25, 2010. P. 301.
29. *Tokarev M. V., Zborovský I.* Self-Similarity of Hadron Production in Heavy Ion Collisions at RHIC // Nonlin. Phenom. Complex Syst. 2009. V. 12. P. 459.
30. *Baldin A. M.* The Physics of Relativistic Nuclei // Sov. J. Part. Nucl. 1977. V. 8. P. 175.
31. *Stavinsky V. S.* Limiting Fragmentation of Nuclei — Cumulative Effect // Sov. J. Part. Nucl. 1979. V. 10. P. 949.
32. *Leksin G. A.* Nuclear Scaling. Elementary Particles // Proc. of the 3rd Phys. School. ITEP. M., 1975. No. 2. P. 5.;
    Leksin G. A. Nuclear Scaling. M., 1975. P. 90.
33. *Nikiforov N. A. et al.* Backward Production of Pions and Kaons in the Interaction of 400 GeV Protons with Nuclei // Phys. Rev. C. 1980. V. 22. P. 700.
34. *Gavrishchuk O. P. et al.* Charged Pion Backward Production in 15–65 GeV Proton–Nucleus Collisions // Nucl. Phys. A. 1991. V. 523. P. 589.
35. *Belyaev I. M. et al.* Production of Cumulative Pions and Kaons in Proton–Nucleus Interactions at Energies from 15 to 65 GeV // Phys. At. Nucl. 1993. V. 56. P. 1378.
36. *Leksin G. A.* Methods for Investigating Nuclear Matter under the Conditions Characteristic of Its Transition to Quark–Gluon Plasma // Phys. At. Nucl. 2002. V. 65, No. 11. P. 1985.
37. *Antonov N. N. et al.* Measurement of Positive Charged Particle Yields in Proton–Nucleus Interactions at $\sqrt{s_{NN}} \approx 10$ GeV and the Angle of 35 Degrees // Physics of Fundamental Interactions, Rus. Acad. of Sci., ITEP, Moscow, Nov. 21–25, 2011; http://matras.itep.ru/npd2k11/.

38. *Ammosov V. V. et al.* First Measurements of Cumulative Particle Production in Proton–Nucleus Interactions at Energy 50 GeV in the Region $p_T > 1$ GeV/c // Seminar of LHEP, JINR, Dubna, June 6, 2012; http://lhe.jinr.ru/seminar-archive.shtml/.
39. *Ammosov V. V. et al.* A Measurement of the Yield of the Positive Particles Escaping at 35° Angle from Proton Interactions with Nuclear Targets at Energy of 50 GeV // Yad. Fiz. 2013. V. 76. P. 1275.
40. *Tokarev M. V., Rogachevsky O. V., Dedovich T. G.* Scaling Features of π^0-Meson Production in High-Energy pp Collisions // J. Phys. G: Nucl. Part. Phys. 2000. V. 26. P. 1671.
41. *Tokarev M. V.* Neutral-Meson Production in pp Collisions at RHIC and QCD Test of z-Scaling // Phys. At. Nucl. 2009. V. 72. P. 541.
42. *Tokarev M. V., Zborovský I.* On Self-Similarity of Top Production at Tevatron // J. Mod. Phys. 2012. V. 3. P. 815.
43. *Tokarev M. V., Zborovský I., Dedovich T. G.* Self-Similarity of Jet Production in pp and $\bar{p}p$ Collisions at RHIC, Tevatron and LHC // Intern. J. Mod. Phys. A. 2012. V. 27. P. 1250115.
44. *Tokarev M. V. et al.* A-Dependence of z-Scaling // Intern. J. Mod. Phys. A. 2001. V. 16, No. 7. P. 1281.

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