Письма в ЭЧАЯ. 2015. Т. 12, № 2(193). С. 372–388

ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

FLUCTUATION OF FLUCTUATIONS IN PIONIZATION: TARGET EXCITATION DEPENDENCE

D. Ghosh [a], *A. Deb* [a, 1], *M. Mondal* [b], *R. Saha* [a], *A. Mondal* [c]

[a] Nuclear and Particle Physics Research Centre, Department of Physics, Jadavpur University, Kolkata, India
[b] S. A. Jaipuria College, Kolkata, India
[c] RCC Institute of Information Technology, Beliaghata, Kolkata, India

This paper presents some interesting and informative results of experimental investigation on target excitation dependence of fluctuation of fluctuations studies on produced pions emitted in ^{16}O–AgBr interactions at $60A$ GeV and ^{32}S–AgBr interactions at $200A$ GeV. A search for the possible signature of chaotic behavior of multiparticle production in nucleus–nucleus collision has been performed with the help of entropy index, μ_q. To study the target excitation dependence of the chaotic behavior, the whole dataset is divided into subsamples having different number of recoil target protons, which may be considered as a measure of target excitation.

В работе представлены некоторые результаты экспериментального исследования зависимости флуктуации флуктуаций пионов, рожденных во взаимодействиях ^{16}O–AgBr при $60A$ ГэВ и ^{32}S–AgBr при $200A$ ГэВ, от возбуждения мишени. Обсуждаются результаты поиска возможных признаков хаотического поведения в процессах множественного рождения в столкновении ядро–ядро на примере индекса энтропии μ_q. С целью изучения зависимости хаотического поведения от возбуждения мишени полный набор разделяется на поднаборы с разным числом протонов отдачи мишени, которое может считаться мерой возбуждения мишени.

PACS: 25.75-q; 24.60.ky; 29.40.Rg

INTRODUCTION

The technique of multiparticle data analysis underwent a paradigm shift when Bialas and Peschanski suggested that the particle density function should be examined locally within narrow regions of phase space [1, 2]. They applied the technique of scaled factorial moments (SFM) to the JACEE events induced by ultrahigh-energy cosmic-ray nuclei [3]. The SFMs of integer orders capable of detecting and characterizing nonstatistical density fluctuations, are found to depend on the phase-space resolution obeying power law. This power-law scaling behavior of SFMs is known as “intermittency”, the term which was coined from the hydrodynamics of turbulence.

However, the method of factorial moments does not extract the complete fluctuations of the system. In case of vertically averaged horizontal moments, only the spatial fluctuations

[1] E-mail: argha_deb@yahoo.com

are taken into account neglecting event-to-event fluctuations. On the other hand, horizontally averaged vertical moments measure event-to-event fluctuations, while information regarding spatial fluctuations is lost. Thus, the common methods lead to the loss of information on the chaotic nature of the multiparticle production process.

Cao and Hwa [4] invented a novel method, which turned out to be quite successful in capturing both fluctuations. They introduced new moments, $C_{p,q}$, which are the moments of factorial-moment distributions and take into account the spatial as well as the event-to-event fluctuations (p is the order of event-to-event fluctuations and q is the order of spatial fluctuations). This erraticity approach is a useful technique for investigating the chaotic behavior of multiparticle production in high-energy collisions. The erraticity moments ($C_{p,q}$ s) like F_q moments, also scale with phase-space partition size obeying a generalized power law. From these moments, an effective parameter, the entropy index, μ_q, is defined by the authors [4] to characterize the chaotic behavior of the multiparticle production process.

To extract the entropy index, one should study the phase space and the event space simultaneously. For each event, one can calculate the factorial moment in pseudorapidity (η) space. Event by event, the value of factorial moments fluctuates greatly. The entropy index, μ_q, describes the degree of such fluctuations from event to event. The small μ_q implies no chaotic behavior, while the large one can be considered as a signature of chaotic dynamics in multiparticle production process. The entropy indices also provide an efficient way to distinguish whether the dominant initiator of the perturbative QCD branching process is a quark or a gluon [5].

Any system involving repeated samplings, whose outcome can fluctuate from event to event, can be investigated in the framework of erraticity analysis. Many problems of nature involve spatial patterns; they can range from phase transition in condensed matter to galactic clustering in astrophysics. Erraticity analysis has, therefore, a wide scope of applicability. It is a general measure of event-to-event fluctuation.

The erraticity analysis has been applied to multiparticle production in branching processes [4, 6], classical chaos [7], phase transition study [8], hadron–hadron collisions [9–12], hadron–nucleus collisions [13–15], and in nucleus–nucleus collisions [16–25]. A similar type of analysis has also been performed to analyze the heartbeat wave form [26].

It is to mention that Fu et al. [27] and Fuming et al. [28] have demonstrated that in a low-multiplicity sample, erraticity analysis is dominated by the statistical fluctuations. So, the dominance of dynamical fluctuation over statistical fluctuation should be checked. In order to make a faithful comparison between the results from experimental data and the pure statistical-fluctuation case, the authors of [28] have demonstrated a method of constructing a model of pure statistical fluctuations.

Target protons, which are also known as grey tracks in nuclear emulsion, are the low-energy part of intra-nuclear cascade formed in high-energy interactions. The number of grey particles gives an indirect measure of the impact parameter or the collision centrality. Centrality increases with the number of grey particles. It is to be mentioned that n_g may be considered as a measure of violence of target fragmentation [29]. Therefore, it would be no doubt interesting to study the behavior of pions as a function of n_g, which has been considered as the number of collisions, more generally as a measure of target excitation [30]. In order to get better insight into the inner dynamics of the particle production in high-energy nuclear collision, the target excitation dependence of fluctuation of fluctuations in pionization has been analyzed thoroughly.

The possible signature of chaotic pion emission and its target excitation dependence have been investigated in this paper using ^{16}O–AgBr and ^{32}S–AgBr interactions at 60 and $200A$ GeV, respectively. Entropy index, μ_q, in each case has been calculated to see whether the particle production is chaotic in nature for these heavy-ion interactions.

1. EXPERIMENTAL DETAILS

The data were obtained by exposing $200A$ GeV sulphur beam and $60A$ GeV oxygen beam on Illford G5 emulsion stacks at CERN SPS [31]. A Leitz Metalloplan microscope with a 10X objective and 10X ocular lens provided with a semi-automatic scanning stage is used to scan the plates. Each plate is scanned by two independent observers to increase the scanning efficiency. For measurement, 100X oil-immersion objective is used. The measuring system fitted with it has 1 μm resolution along the X- and Y-axes and 0.5 μm resolution along the Z-axis. Details of events selection criteria and classification of tracks can be found in our earlier communication [16].

The emission angle (θ) is measured for each shower track by taking the readings of the coordinate of the interaction point (X_0, Y_0, Z_0), coordinate (X_i, Y_i, Z_i) at a point on each secondary track and coordinate (X_1, Y_1, Z_1) of a point on the incident beam. In case of shower particles, the variable used is pseudorapidity (η), which is defined as $\eta = -\ln\tan(\theta/2)$. The accuracy in pseudorapidity in the region of interest is of the order of 0.1 pseudorapidity units.

Nuclear emulsion covers 4π geometry and provides very good accuracy in the measurement of emission angles of target protons due to high spatial resolution and thus, is suitable as a detector for the study of fluctuations in the fine resolution of the phase space considered.

2. METHOD OF ANALYSIS

The single-particle density distribution in pseudorapidity space is nonflat. The nonuniformity of particle spectra influences the scaling behavior of factorial moments. Bialas and Gazdzicki [32] proposed a method to construct a set of variables, which drastically reduces the distortion of intermittency due to nonuniformity of single-particle density distribution. According to them, the new scaled variable X_z is related to the single-particle density distribution $\rho(Z)$ as

$$X_Z = \int\limits_{Z_1}^{Z} \rho(Z)\,\partial Z \Bigg/ \int\limits_{Z_1}^{Z_2} \rho(Z)\,\partial(Z), \tag{1}$$

where Z_1 and Z_2 are the two extreme points of the distribution. The variable X_Z varies between 0.0 and 1.0 keeping $\rho(Z)$ almost constant.

One can consider a two-dimensional space, like pseudorapidity (η) space, as the horizontal axis, which is divided into M bins, and the vertical axis has N sites corresponding to N events in the event space. For each event, the normalized factorial moment is calculated according to the formula

$$F_q^e(M) = \frac{M^{q-1} \sum\limits_{m=1}^{M} n_m(n_m-1)\cdots(n_m-q+1)}{\left(\dfrac{1}{M}\sum\limits_{n=1}^{M} n_m\right)^q}, \tag{2}$$

where M is the partition number in phase space; n_m is the number of particles in the mth bin for eth event, and $q = 2, 3, 4 \ldots$ are the order of moments. The factorial moment $F_q^e(M)$ describes the pattern of the distribution of produced pions of the eth event. As the pattern changes from event to event, $F_q^e(M)$ also changes resulting in a distribution $P(F_q^e)$ of the whole sample. Let $P(F_q)$ be normalized as

$$\int_0^{\propto} P(F_q)\, dF_q = 1. \tag{3}$$

Let the average of $F_q^e(M)$ determined from $P(F_q^e)$ be denoted by $\langle F_q^e(M) \rangle$. In order to capture the spatial fluctuation and the event-to-event fluctuation simultaneously, a new normalized moment is defined as [4]:

$$C_{p,q}(M) = \frac{\langle F_q^p(M) \rangle}{\langle F_q(M) \rangle^p}, \tag{4}$$

where $\langle F_q \rangle = \dfrac{1}{N} \sum\limits_{e=1}^{N} F_q^e = \int\limits_0^{\propto} F_q^e P(F_q)\, dF_q$, N being the total number of events. The value of order p can be any positive real number. For $p > 1$, $C_{p,q}$ reflects the large F_q behavior of $P(F_q)$, which is sensitive to the spikes in phase space. For $p < 1$, $C_{p,q}$ probes the low F_q behavior of $P(F_q)$, which is influenced mainly by bins with low multiplicities, including empty bins. Thus, knowledge of $C_{p,q}$ for $0 < p < 2$ can reveal a great deal about the properties of $P(F_q)$, all of which are not probed by intermittency analysis.

If $C_{p,q}(M)$ has a power-law behavior as the division number of bins, M goes to infinity, i.e.,

$$C_{p,q}(M) \propto M^{\psi_q(p)}, \quad M \rightarrow \propto, \tag{5}$$

then the phenomenon is referred to as erraticity [33] of the multiparticle system and $\Psi_q(p)$ is known as erraticity exponent. Since $C_{p,q}$ are the moments of F_q^e, they describe the deviation of event factorial moment F_q^e from the mean $\langle F_q \rangle$, i.e., to quantify event-to-event fluctuations of factorial moments, we calculate the moment of factorial moments $C_{p,q}$. Those fluctuations depend on the bin size, because F_q^e itself is a description of spatial pattern that varies according to resolution. Thus, if those fluctuations scale with bin size, then the erraticity exponent $\Psi_q(p)$ is an economical way of characterizing the aspect of the self-similar dynamics that has some order in its erratic fluctuations [33].

Erraticity is characterized by the slope μ_q of $\Psi_q(p)$ at $p = 1$, which is called entropy index defined by [4]:

$$\mu_q = \frac{d}{dp} \Psi_q(p) \bigg|_{p=1}, \tag{6}$$

and it describes the width of fluctuation [4–6], i.e., μ_q describes the event-to-event fluctuation of factorial moments, which measure the spatial fluctuation of the multiplicity distribution. A positive value of μ_q ($\mu_q > 0$) would correspond to a broad $P(F_q)$ distribution, which, in turn, would mean large fluctuations of the spatial pattern from event to event.

The entropy index μ_q is related to the entropy (S_q) of the event space as [4, 6]:

$$S_q = \ln (N M^{-\mu_q}). \tag{7}$$

It is evident from Eq. (7) that a small μ_q corresponds to large entropy, which means less chaotic behavior of particle production in branching processes. As μ_q increases, i.e., event-to-event fluctuation of the factorial moment increases, the system becomes more chaotic reducing the entropy, S_q.

To emphasize that S_q is defined in event space, one may call it "eventropy" [6]. One can think of the event space as of a one-dimensional space with N sites. At each site a number F_q^e can be registered. The meaning of Eq. (7) can better be understood following a very simple example cited by Cao and Hwa [4]. Two extreme cases can be considered here: a) If F_q^e is the same for every event, then $P(F_q^e) = 1/N$ and $S_q = \ln N$. b) If only one event has $F_q^e \neq 0$, and $F_q^e = 0$ in all others, then $S_q = 0$. We should think of case (a) as being highly disordered to spread out an observable (F_q^e in this case) over all events than to confine it to a few events having nonzero values (analogous to the increase of entropy of an expanding gas). The larger the number of events, the larger is the "eventropy". However, a branching dynamics that results in the same F_q^e for every event does not fluctuate in the branching processes. It corresponds to nearby trajectories staying nearby throughout. In short, the dynamics is not chaotic. In other words, $S_q = \ln N$ implies that μ_q must vanish following Eq. (7). Thus, small μ_q corresponds to large "eventropy", which, in turn, implies no chaotic behavior.

On the other hand, if all $F_q^e = 0$ except only one event e', then $P(F_q^e) = \delta_{ee'}$ and $S_q = 0$. This is highly ordered in the event space, but the fluctuation of F_q^e from zero to a nonzero value is large. Thus, if the fluctuation is large, initially nearby trajectories become widely separated in the final states of different events, and the dynamics is chaotic. In order for the "eventropy" to be small, μ_q must be large. Thus, large entropy index implies chaotic behavior.

At large M, only large spikes in small bins contribute to $F_q^e(M)$, specially when q is large. Events with large spikes are rare. As a result, the fluctuation in $F_q^e(M)$ from event to event becomes more pronounced with increasing q. That behavior is quantified by μ_q. Thus, μ_q can be used to characterize the "spatial" properties of the chaotic behavior of multiparticle production.

3. RESULTS AND DISCUSSIONS

The whole dataset for each type of interactions has been divided into two sets ($0 \leqslant n_g \leqslant 4$ and $n_g > 4$ for ^{16}O–AgBr interactions and $0 \leqslant n_g \leqslant 5$ and $n_g > 5$ for ^{32}S–AgBr interactions in such a way that the two sets for each type of interactions contain almost equal number

Table 1. **Values of number of events and average multiplicity of shower tracks ($\langle n_s \rangle$) for different sets of n_g**

Type of interaction	Value of n_g	No. of events	Average multiplicity ($\langle n_s \rangle$)
^{16}O–AgBr interactions at $60A$ GeV	$0 \leqslant n_g \leqslant 4$	125	56.81 ± 2.59
	$n_g > 4$	125	69.60 ± 2.65
^{32}S–AgBr interactions at $200A$ GeV	$0 \leqslant n_g \leqslant 5$	71	90.13 ± 5.56
	$n_g > 5$	69	102.85 ± 6.22

of events) depending upon the number of grey tracks (n_g) to study the target excitation dependence of erratic behavior of pions. The details are provided in Table 1.

In the present analysis, "cumulative" variable $X(\eta)$ as mentioned before, has been used. The $X(\eta)$ region is divided into $M = 4, 5, 6, \ldots, 20$ bins for all the subsets. Here q, the order of spatial fluctuation, is varied from 2 to 4 in step of 1.

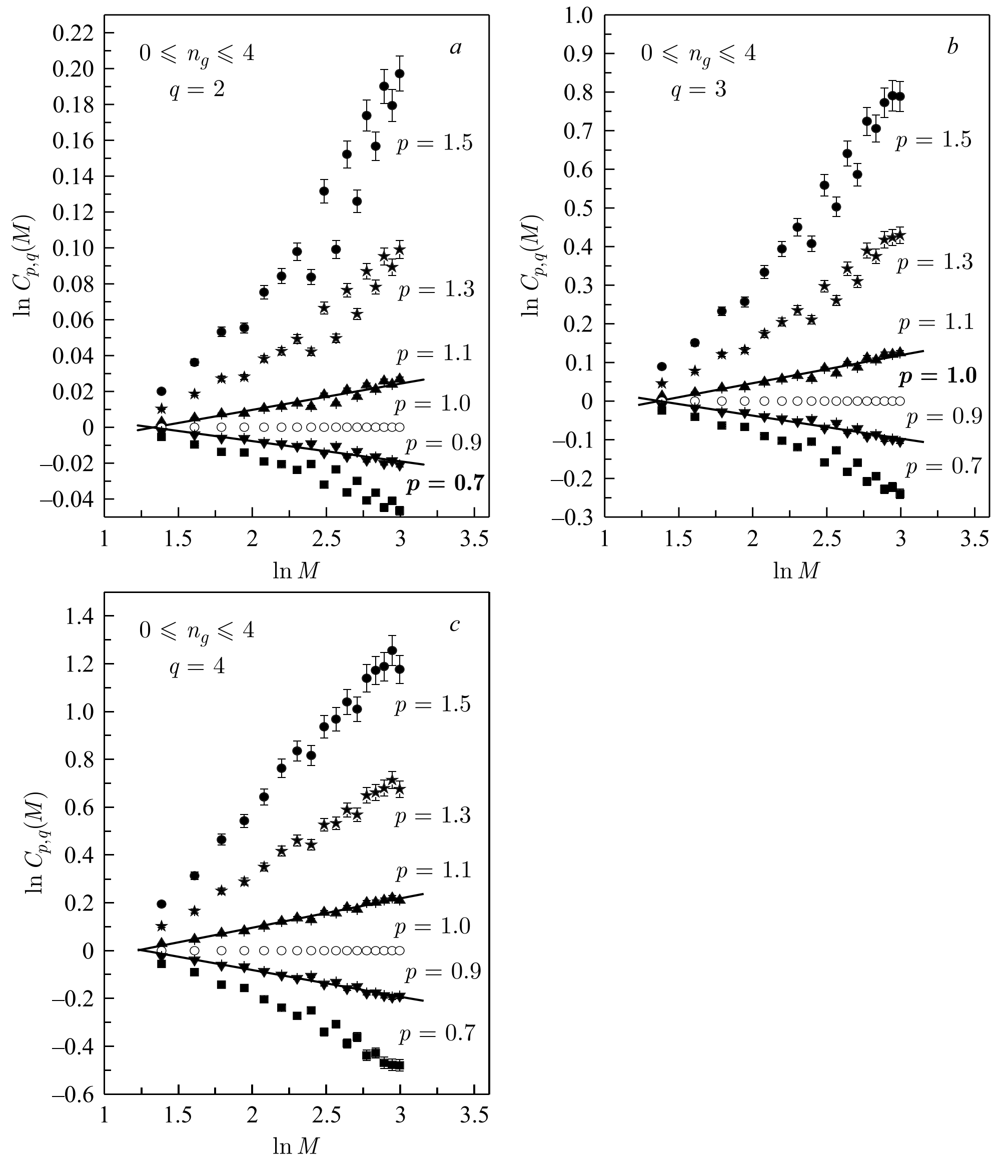

Fig. 1. Plot of $\ln C_{p,q}(M)$ vs. $\ln M$ for $p = 0.7$, 0.9, 1.0, 1.1, 1.3 and 1.5 in the range $0 \leqslant n_g \leqslant 4$ for ^{16}O–AgBr interactions at $60A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

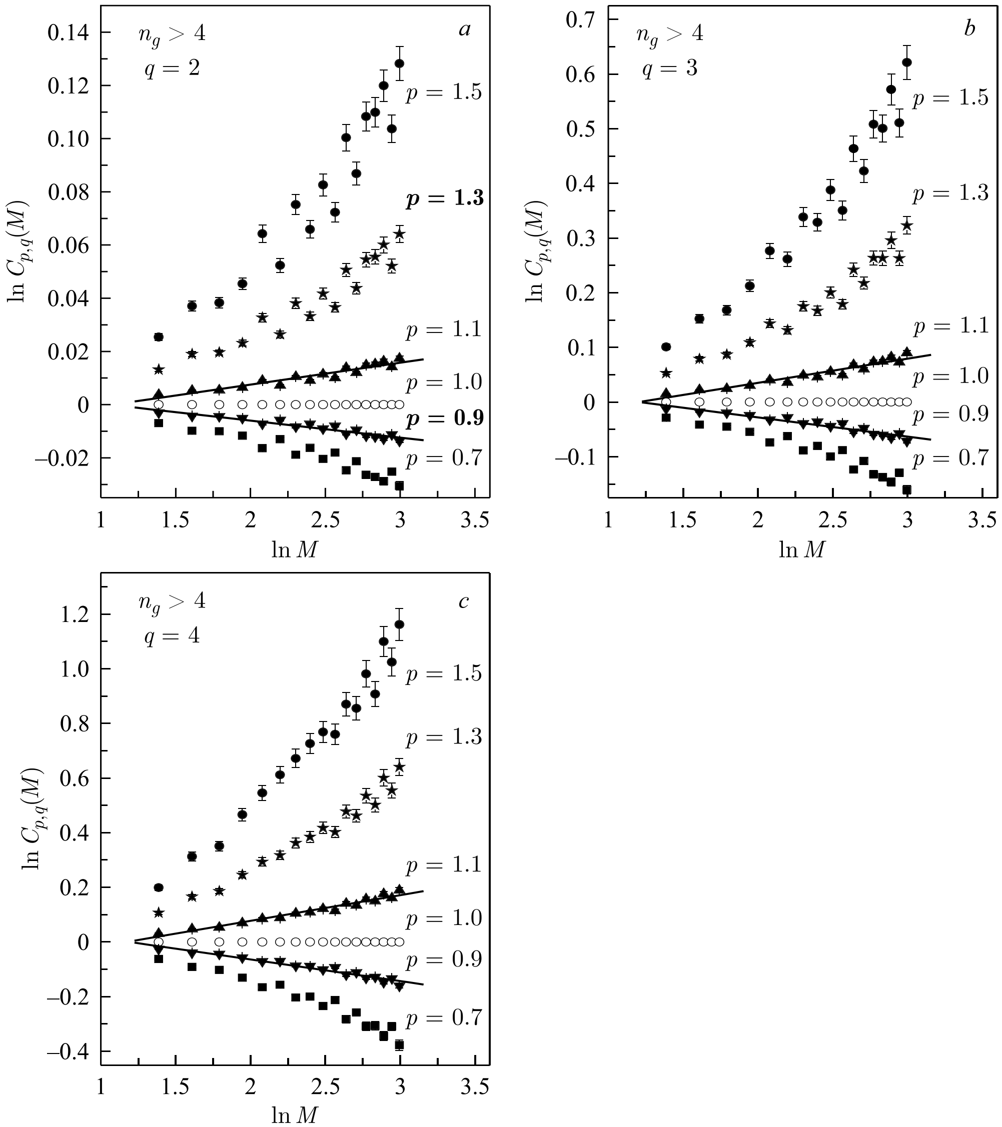

Fig. 2. Plot of $\ln C_{p,q}(M)$ vs. $\ln M$ for $p = 0.7$, 0.9, 1.0, 1.1, 1.3 and 1.5 in the range $n_g > 4$ for ^{16}O–AgBr interactions at $60A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

$C_{p,q}(M)$, the moment of factorial moments, has been calculated using Eq. (4) to probe the event-to-event fluctuation of $F_q^e(M)$. For each q, the values of $C_{p,q}(M)$ have been calculated for $p = 0.7$, 0.9, 1.0, 1.1, 1.3 and 1.5. The variation of $\ln C_{p,q}(M)$ with $\ln M$ is depicted in Figs. 1, *a–c* and 2, *a–c* for the sets $0 \leqslant n_g \leqslant 4$, and $n_g > 4$, respectively, for ^{16}O–AgBr interactions. Figures *a*, *b* and *c* correspond to $q = 2$, 3 and 4 for a particular n_g interval.

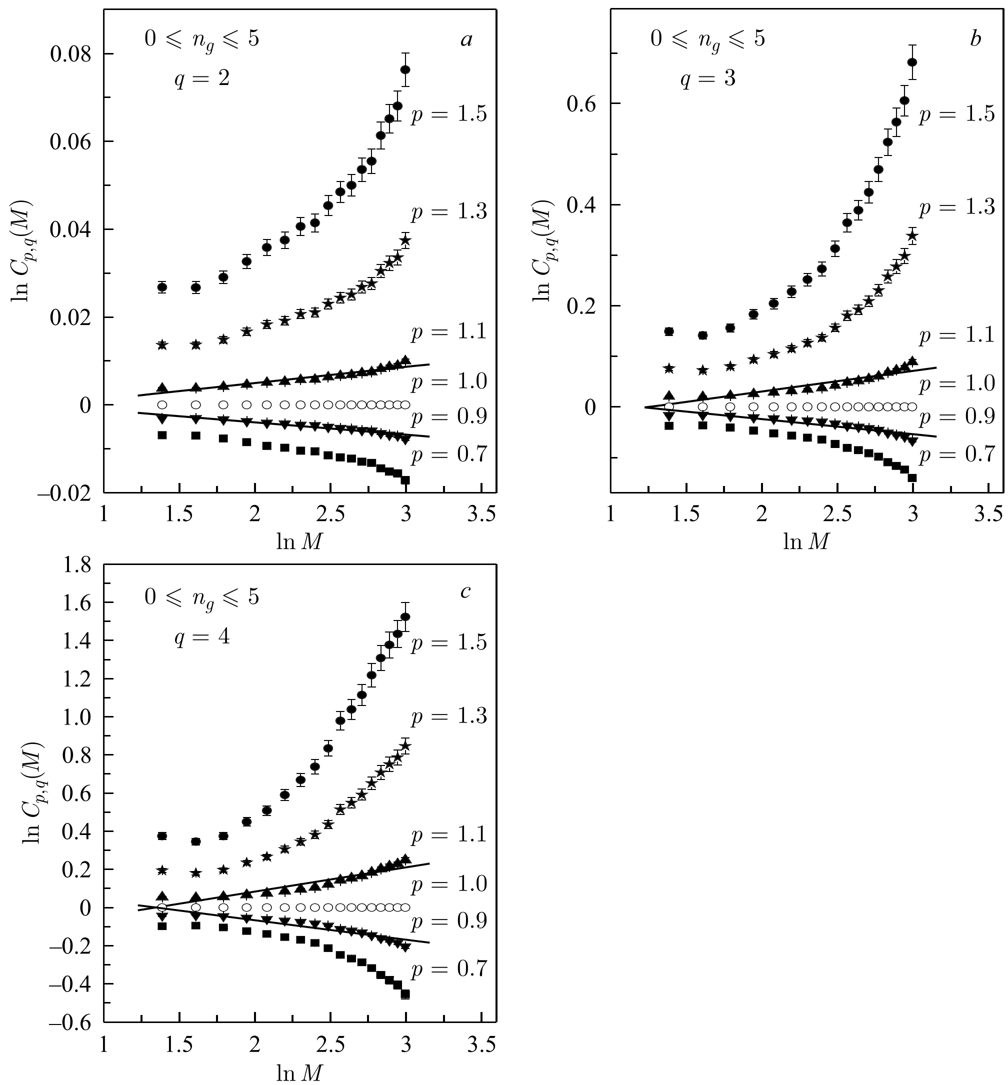

Fig. 3. Plot of $\ln C_{p,q}(M)$ vs. $\ln M$ for $p = 0.7$, 0.9, 1.0, 1.1, 1.3 and 1.5 in the range $0 \leqslant n_g \leqslant 5$ for ^{32}S–AgBr interactions at $200A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

Similarly, Figs. 3, *a–c* and 4, *a–c* display the variation of the same quantity for ^{32}S–AgBr interactions.

$C_{p,q}(M)$ shows power-law behavior with M in the neighbourhood of $p = 1$ for the entire range of M for all the cases. This indicates that pion fluctuation patterns are erratic in all the cases. The best linear fits of $\ln C_{p,q}(M)$ versus $\ln M$ plots corresponding to $p = 0.9$ and 1.1 have been performed. According to Eq. (5), the slopes of the plots give $\Psi_q(p)$. Using these slopes and following Eq. (6), the values of entropy index μ_q, which is instrumental in

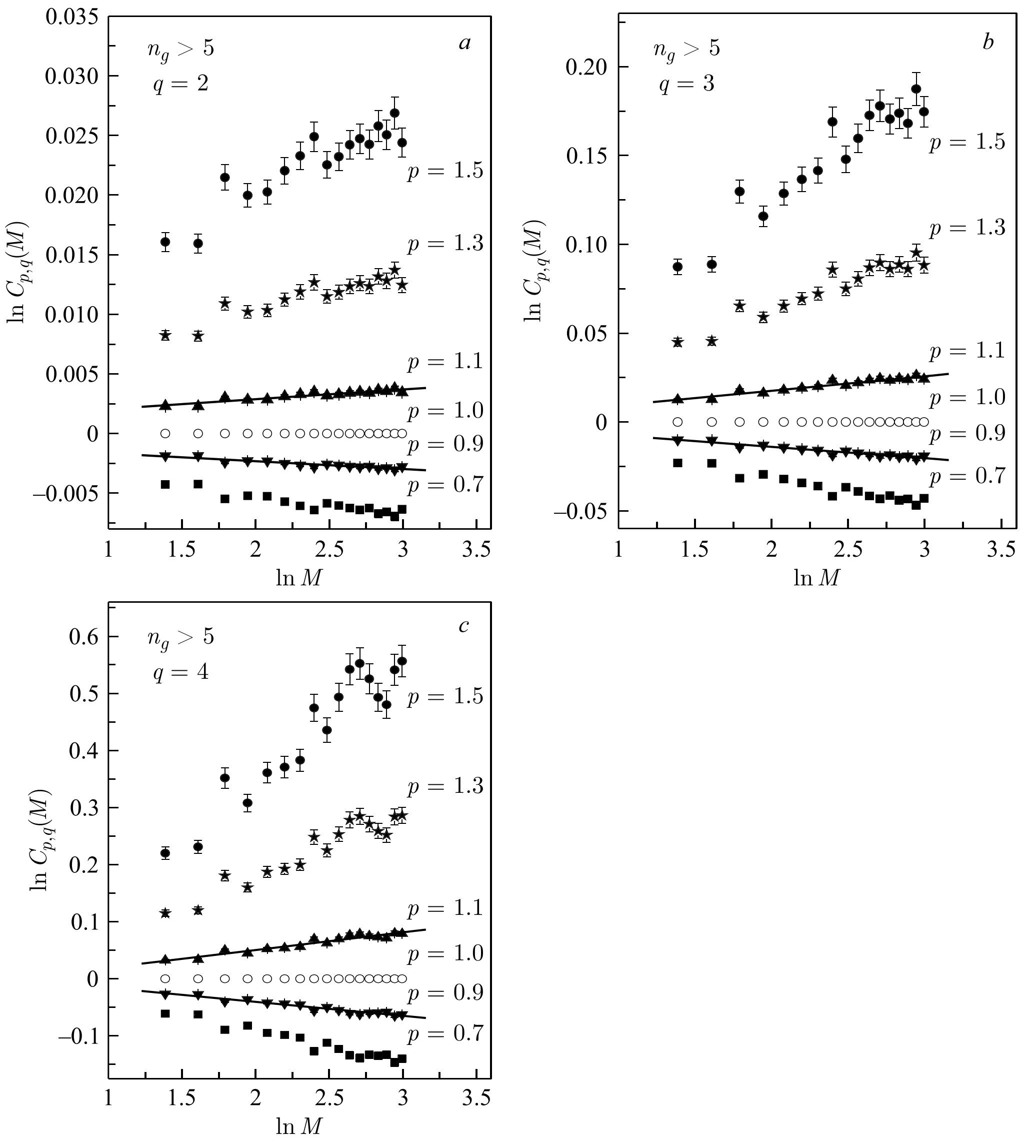

Fig. 4. Plot of $\ln C_{p,q}(M)$ vs. $\ln M$ for $p = 0.7$, 0.9, 1.0, 1.1, 1.3 and 1.5 in the range $n_g > 5$ for ^{32}S–AgBr interactions at $200A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

quantifying the degree of fluctuation of $F_q^e(M)$ from event to event, have been calculated. The values of slope and entropy indices are listed in Tables 2 and 3 for ^{16}O–AgBr and ^{32}S–AgBr interactions, respectively, considering two n_g intervals. It is observed from the tables that entropy indices have nonzero value, from which it can be inferred that pion production process for both types of interactions is chaotic irrespective of degrees of target excitation. It is also

Table 2. **The slopes of linear fits and entropy indices for the experimental and randomized data of ^{16}O–AgBr interactions at $60A$ GeV**

Value of n_g	q	p	$\Psi_q(p)$ (exp.)	μ_q (exp.)	$\Psi_q(p)$ (random.)	μ_q (random.)
$0 \leqslant n_g \leqslant 4$	2	0.9	–0.011 ± 0.001	0.130 ± 0.010	–0.00072 ± 0.00009	0.0079 ± 0.0010
		1.1	0.015 ± 0.001		0.00085 ± 0.00010	
	3	0.9	–0.060 ± 0.004	0.665 ± 0.040	–0.00519 ± 0.00043	0.0562 ± 0.0047
		1.1	0.073 ± 0.004		0.00604 ± 0.00051	
	4	0.9	–0.111 ± 0.004	1.170 ± 0.040	–0.01772 ± 0.00143	0.1927 ± 0.0159
		1.1	0.123 ± 0.004		0.02082 ± 0.00175	
$n_g > 4$	2	0.9	–0.006 ± 0.001	0.070 ± 0.010	–0.00023 ± 0.00002	0.0026 ± 0.0002
		1.1	0.008 ± 0.001		0.00028 ± 0.00002	
	3	0.9	–0.035 ± 0.003	0.395 ± 0.030	–0.00195 ± 0.00015	0.0216 ± 0.0017
		1.1	0.044 ± 0.003		0.00237 ± 0.00018	
	4	0.9	–0.079 ± 0.005	0.865 ± 0.050	–0.00779 ± 0.00061	0.0859 ± 0.0068
		1.1	0.094 ± 0.005		0.00938 ± 0.00075	

Table 3. **The slopes of linear fits and entropy indices for the experimental and randomized data of ^{32}S–AgBr interactions at $200A$ GeV**

Value of n_g	q	p	$\Psi_q(p)$ (exp.)	μ_q (exp.)	$\Psi_q(p)$ (random.)	μ_q (random.)
$0 \leqslant n_g \leqslant 5$	2	0.9	–0.0028 ± 0.0002	0.033 ± 0.003	–0.00076 ± 0.00004	0.0086 ± 0.0005
		1.1	0.0037 ± 0.0003		0.00095 ± 0.00005	
	3	0.9	–0.0299 ± 0.0031	0.354 ± 0.038	–0.00731 ± 0.00046	0.0831 ± 0.0054
		1.1	0.0408 ± 0.0044		0.00930 ± 0.00062	
	4	0.9	–0.1019 ± 0.0105	1.145 ± 0.115	–0.02758 ± 0.00214	0.3032 ± 0.0252
		1.1	0.1271 ± 0.0124		0.03305 ± 0.00290	
$n_g > 5$	2	0.9	–0.0007 ± 0.0001	0.008 ± 0.001	–0.00012 ± 0.00003	0.0021 ± 0.0004
		1.1	0.0008 ± 0.0001		0.00030 ± 0.00004	
	3	0.9	–0.0064 ± 0.0005	0.073 ± 0.006	–0.00121 ± 0.00048	0.0163 ± 0.0058
		1.1	0.0082 ± 0.0006		0.00205 ± 0.00068	
	4	0.9	–0.0245 ± 0.0023	0.277 ± 0.020	–0.00520 ± 0.00280	0.0631 ± 0.0335
		1.1	0.0309 ± 0.0017		0.00742 ± 0.00389	

observed (considering Tables 1, 2 and 3) that for both interactions the value of μ_q decreases as the degree of target excitation increases. This implies that the multiparticle production process becomes less chaotic with the increase of degree of target excitation. Furthermore, the values of μ_q suggest that chaoticity decreases as the colliding system becomes more complex.

The q dependence of μ_q is shown in Fig. 5, *a, b* for ^{16}O–AgBr and in Fig. 6, *a, b* for ^{32}S–AgBr interactions for the two n_g intervals. It is evident from the figures that the value of entropy index μ_q increases with the order q for all the n_g intervals suggesting increase in

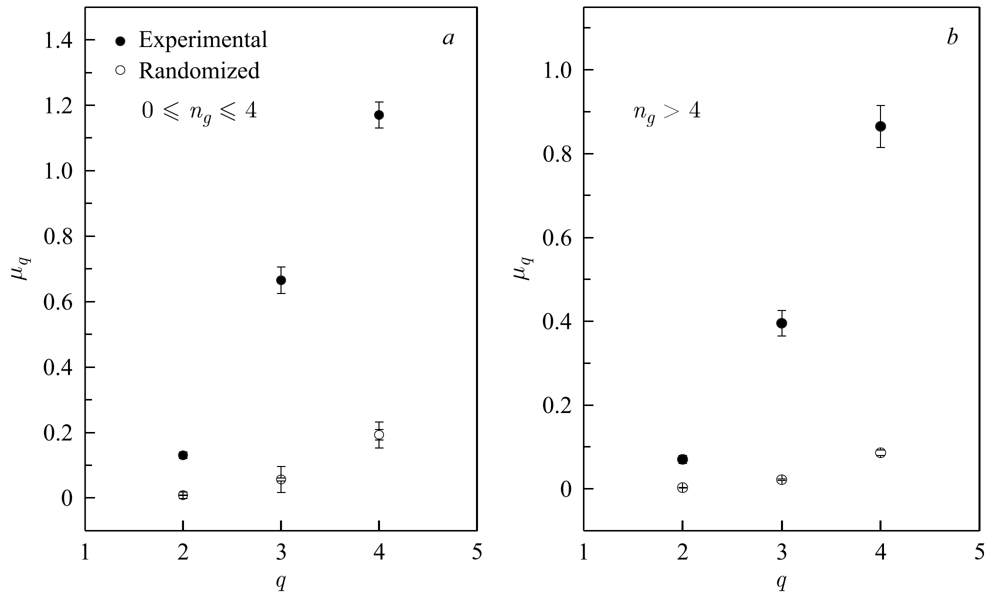

Fig. 5. The dependence of μ_q on q for experimental and randomized data for ^{16}O–AgBr interactions at $60A$ GeV in the range $0 \leqslant n_g \leqslant 4$ (*a*) and $n_g > 4$ (*b*)

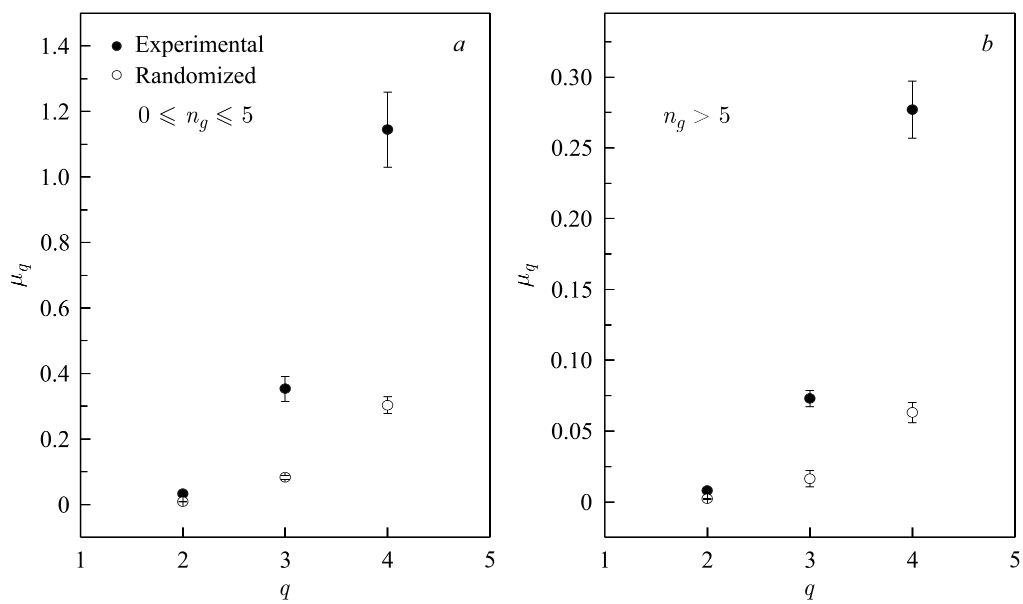

Fig. 6. The dependence of μ_q on q for experimental and randomized data for ^{32}S–AgBr interactions at $200A$ GeV in the range $0 \leqslant n_g \leqslant 5$ (*a*) and $n_g > 5$ (*b*)

chaoticity with order of spatial fluctuation for all degrees of target excitations. In other words, the event-to-event fluctuations become more erratic with the increase of q implying decrease in event-space entropy S_q.

Following Fuming et al. [28], $C^{\mathrm{st}}_{p,q}(M)$s have been calculated using Eq. (4) for assessing the statistical contribution of fluctuation. For the calculation of $C^{\mathrm{st}}_{p,q}(M)$, the data corresponding to each n_g interval are randomly shuffled so that particles of an event become

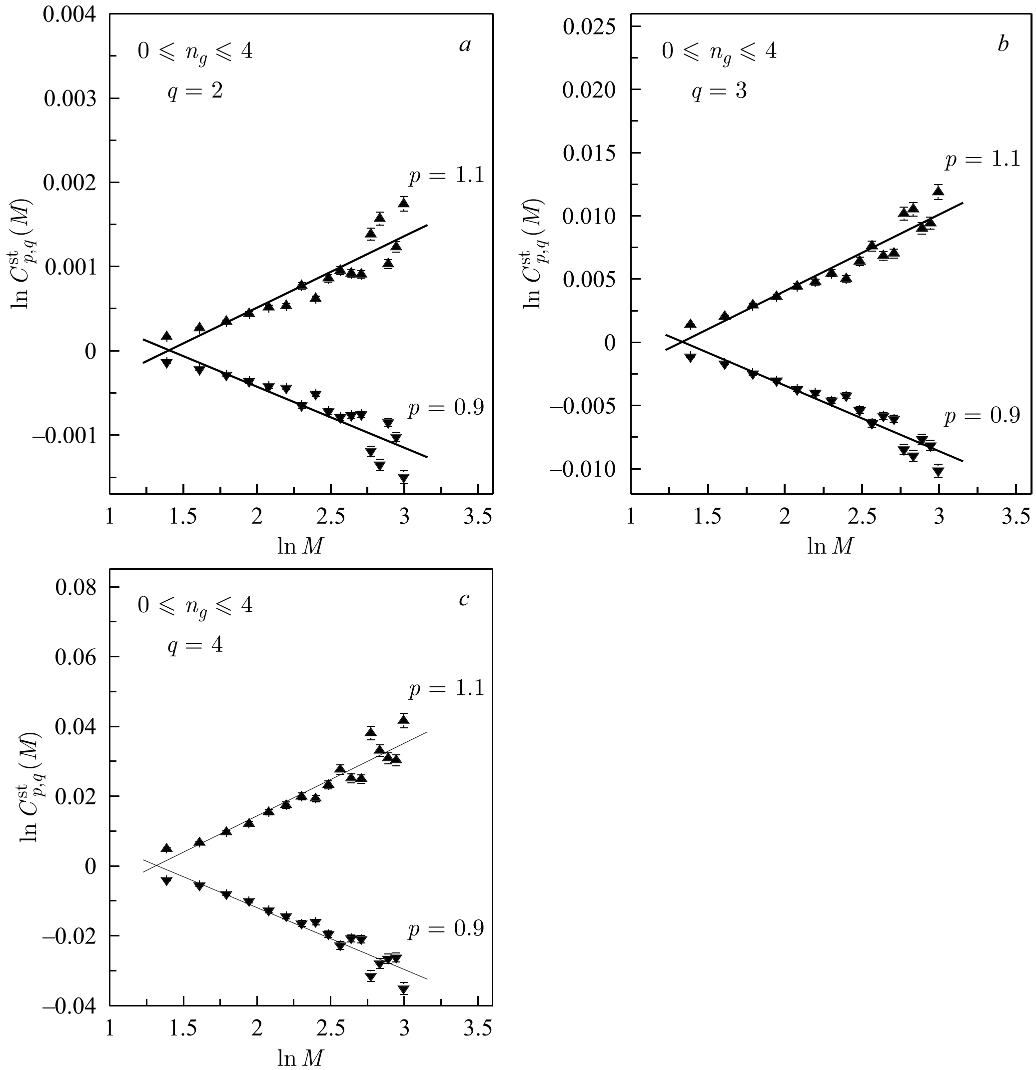

Fig. 7. Plot of $\ln C^{\mathrm{st}}_{p,q}(M)$ vs. $\ln M$ for $p = 0.9$ and 1.1 for randomized dataset in the range $0 \leqslant n_g \leqslant 4$ for ^{16}O–AgBr interactions at $60A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

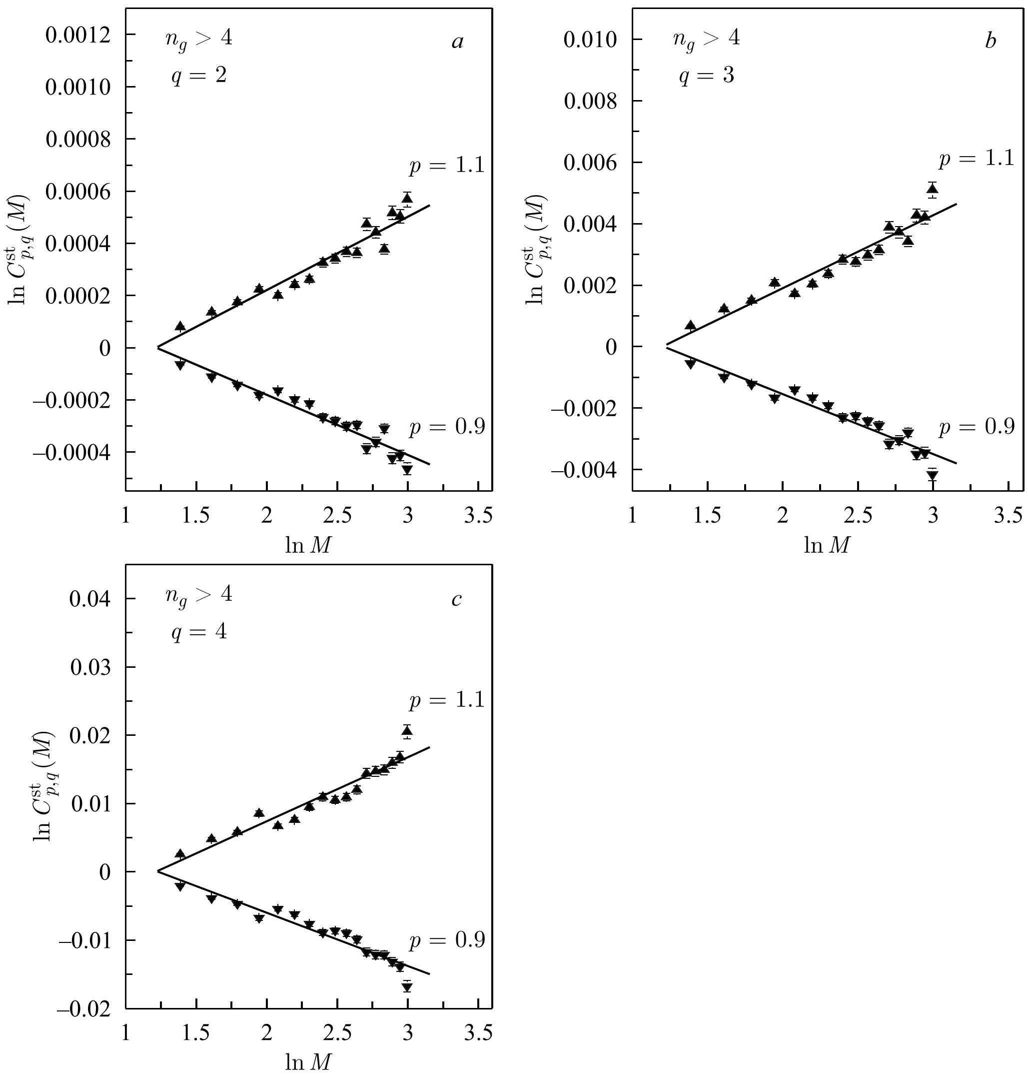

Fig. 8. Plot of $\ln C^{\rm st}_{p,q}(M)$ vs. $\ln M$ for $p = 0.9$ and 1.1 for randomized dataset in the range $n_g > 4$ for ^{16}O–AgBr interactions at $60A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

uncorrelated. For each q, the values of $C^{\rm st}_{p,q}(M)$ have been calculated for $p = 0.9$ and 1.1. The variation of $\ln C^{\rm st}_{p,q}(M)$ with $\ln M$ has been shown in Figs. 7, *a*–*c* and 8, *a*–*c* for ^{16}O–AgBr interactions and in Figs. 9, *a*–*c* and 10, *a*–*c* for ^{32}S–AgBr interactions. The best linear fits of $\ln C^{\rm st}_{p,q}(M)$ versus $\ln M$ plots corresponding to $p = 0.9$ and 1.1 have been performed. The slopes $\Psi_q(p)$ are listed in Tables 2 and 3 for ^{16}O–AgBr and ^{32}S–AgBr interactions, respectively.

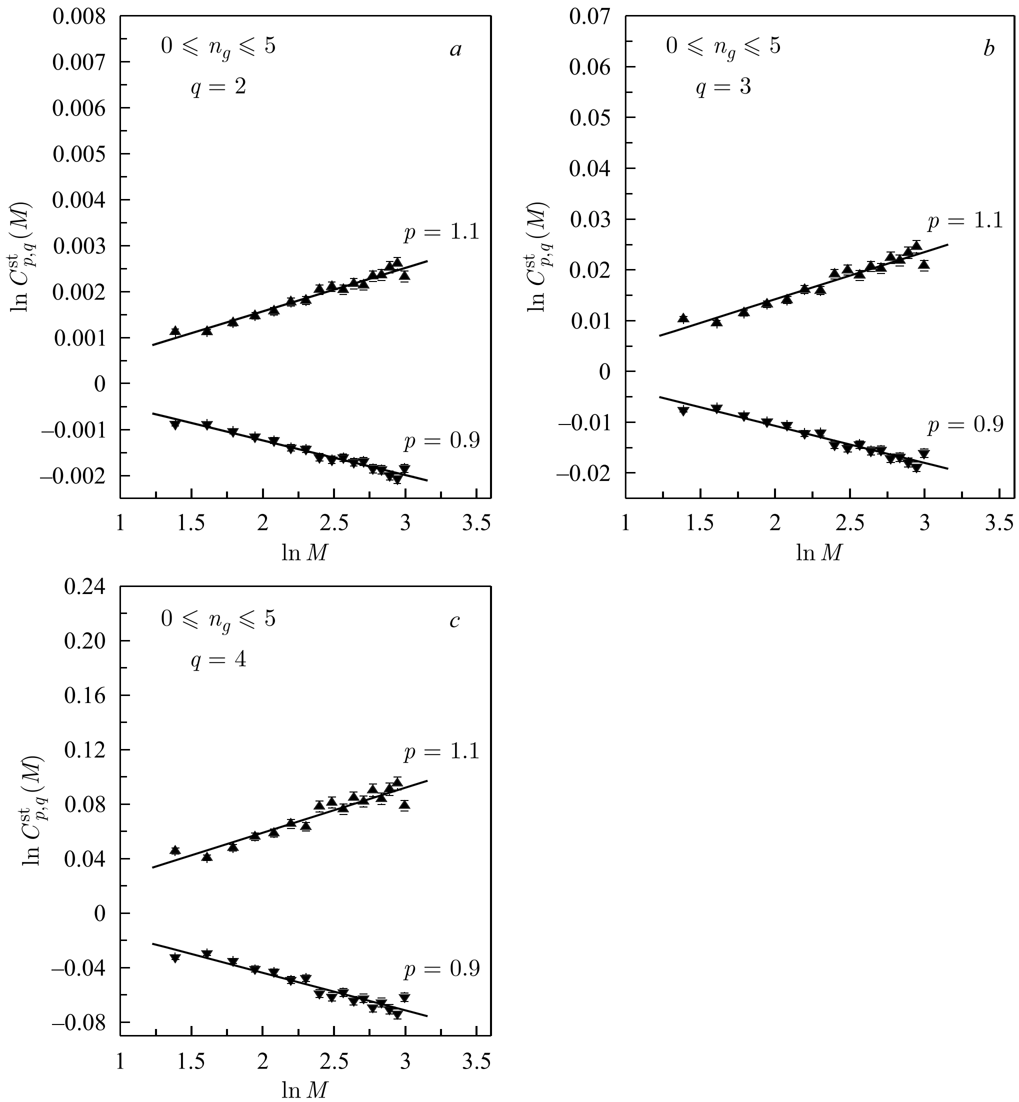

Fig. 9. Plot of $\ln C^{\rm st}_{p,q}(M)$ vs. $\ln M$ for $p = 0.9$ and 1.1 for randomized dataset in the range $0 \leqslant n_g \leqslant 5$ for ^{32}S–AgBr interactions at $200A$ GeV with $q = 2$ (a), $q = 3$ (b) and $q = 4$ (c)

To extract the values of entropy index, $\mu_q^{\rm st}$, same procedure as before has been followed. The values are listed in Tables 2 and 3. The variation of $\mu_q^{\rm st}$ with q is depicted in Figs. 5, *a, b* and 6, *a, b* for ^{16}O–AgBr and ^{32}S–AgBr interactions, respectively, for the two different degrees of target excitations along with the experimental μ_qs. It can clearly be seen that the values of entropy index for randomly generated data are very small compared to the exper-

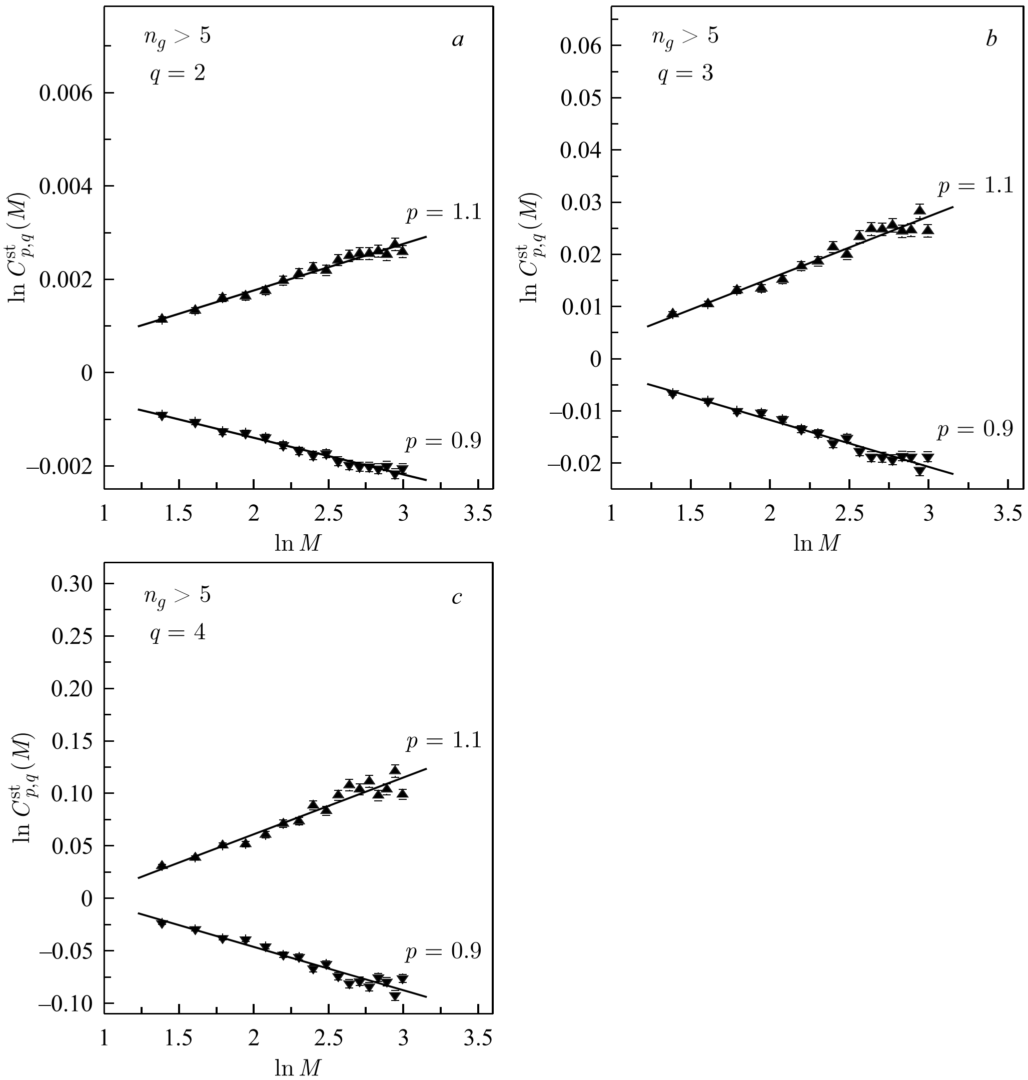

Fig. 10. Plot of $\ln C^{\rm st}_{p,q}(M)$ vs. $\ln M$ for $p = 0.9$ and 1.1 for randomized dataset in the range $n_g > 5$ for ^{32}S–AgBr interactions at $200A$ GeV with $q = 2$ (*a*), $q = 3$ (*b*) and $q = 4$ (*c*)

imental data. Therefore, we can safely conclude that our erraticity results are not reflections of mere statistics only.

CONCLUSIONS

The following important results are obtained from the above analysis:

• For ^{32}S–AgBr and ^{16}O–AgBr interactions the pion density fluctuation is erratic in nature and, hence, the pionization process for the considered $A{-}A$ interactions is chaotic.

• The values of entropy index suggest that the particle production process becomes less chaotic with the increase of degree of target excitation.

• Chaoticity increases with the order of spatial fluctuation for all degrees of target excitation.

• For ^{16}O–AgBr interactions at $60A$ GeV, the pionization process is more chaotic compared to ^{32}S–AgBr interactions at $200A$ GeV. This may suggest that chaoticity decreases as the complexity of the interaction increases.

REFERENCES

1. *Bialas A., Peschanski R.* Moments of Rapidity Distributions as a Measure of Short-Range Fluctuations in High-Energy Collisions // Nucl. Phys. B. 1986. V. 273. P. 703.
2. *Bialas A., Peschanski R.* Intermittency in Multiparticle Production at High Energy // Nucl. Phys. B. 1988. V. 308. P. 857.
3. *Burnett T. H. et al.* Multiplicities in High-Energy Nucleus–Nucleus Collisions // Phys. Rev. Lett. 1983. V. 50. P. 2062.
4. *Cao Z., Hwa R. C.* In Search for Signs of Chaos in Branching Processes // Phys. Rev. Lett. 1995. V. 75. P. 1268.
5. *Cao Z., Hwa R. C.* Fluctuations and Entropy Indices of QCD Parton Showers // Phys. Rev. D. 1996. V. 54. P. 6674.
6. *Cao Z., Hwa R. C.* Chaotic Behavior of Particle Production in Branching Processes // Ibid. V. 53. P. 6608.
7. *Cao Z., Hwa R. C.* Fluctuations of Spatial Patterns as a Measure of Classical Chaos // Phys. Rev. E. 1997. V. 56. P. 326.
8. *Hwa R. C., Wu Y.* Critical Behavior of Hadronic Fluctuations and the Effect of Final-State Randomization // Phys. Rev. C. 1999. V. 60. P. 054904.
9. *Shaoshun W., Zhaomin W.* The Measurement of Entropy Indices in *pp* Collisions at 400 GeV/*c* // Phys. Lett. B. 1998. V. 416. P. 216.
10. *Shaoshun W., Zhaomin W.* Chaotic Behavior of Multiparticle Production in *pp* Collisions at 400 GeV/*c* // Phys. Rev. D. 1998. V. 57. P. 3036.
11. *Shaoshun W. Chong, Zhaomin W.* Erraticity Analysis of the NA27 Data // Phys. Lett. B. 1999. V. 458. P. 505.
12. *Atayan M. R. et al.* Erraticity Analysis of Multiparticle Production in $\pi + p$ and $K + p$ Collisions at 250 GeV/*c* // Phys. Lett. B. 2003. V. 558. P. 22.
13. *Ghosh D. et al.* Target Excitation Dependence of the Entropy Index in 350 GeV/*c* π^-–AgBr Collisions // J. Phys. G: Nucl. Part. Phys. 2008. V. 35. P. 125005.
14. *Ghosh D. et al.* Chaos in Compound Hadrons in High-Energy Hadronic Interactions // Phys. Scripta. 2010. V. 81. P. 055101.
15. *Ghosh D. et al.* Quantitative Assessment of Target Dependence of Pion Fluctuation in Hadronic Interactions — Estimation through Erraticity // Pramana. 2012. V. 79. P. 1395.
16. *Ghosh D. et al.* Analysis of Fluctuation of Fluctuations in ^{32}S–AgBr Interactions at $200A$ GeV // Phys. Lett. B. 2002. V. 540. P. 52.
17. *Ghosh D. et al.* Erraticity Analysis of Multipion Data in ^{32}S–AgBr Interactions at $200A$ GeV // Phys. Rev. C. 2003. V. 68. P. 024908.
18. *Ghosh D. et al.* Multiplicity-Dependent Chaotic Pionization in Ultra-Relativistic Nuclear Interactions // J. Phys. G: Nucl. Part. Phys. 2003. V. 29. P. 2087.

19. *Ghosh D. et al.* Fluctuations of Rapidity Gaps in Nucleus–Nucleus Collisions: Evidence of Erraticity // Phys. Rev. C. 2003. V. 68. P. 027901.
20. *Ghosh D. et al.* Multidimensional Erraticity Analysis in Ultra-Relativistic Nuclear Collisions // J. Phys. G: Nucl. Part. Phys. 2005. V. 31. P. 1083.
21. *Ghosh D. et al.* Chaos Analysis of Nuclear Fragments in High-Energy Collisions // J. Phys. G: Nucl. Part. Phys. 2007. V. 34. P. 2165.
22. *Ghosh M. K., Mukhopadhyay A.* Erraticity Analysis of Particle Production in ^{32}S–Ag/Br Interaction at 200A GeV/c // Phys. Rev. C. 2003. V. 68. P. 034907.
23. *Ahmad S. et al.* Erraticity Behaviour in Relativistic Nucleus–Nucleus Collisions // J. Phys. G: Nucl. Part. Phys. 2004. V. 30. P. 1145.
24. *Ahmad S. et al.* Erratic Fluctuations in Rapidity Gaps in Relativistic Nucleus–Nucleus Collisions // Acta Phys. Polon. B. 2004. V. 35. P. 809.
25. *Ahmad N.* // Acta Phys. Hung. A. 2006. V. 25. P. 105.
26. *Hwa R. C.* Fluctuation Index as a Measure of Heartbeat Irregularity // Nonlinear Phenomena Complex Syst. 2000. V. 3. P. 93.
27. *Fu J. et al.* The Influence of Statistical Fluctuations on the Erraticity Behavior of Multiparticle System // Phys. Lett. B. 2000. V. 472. P. 161.
28. *Fuming L. et al.* A Monte Carlo Study of Erraticity Behavior in Nucleus–Nucleus Collisions at High Energies // Phys. Lett. B. 2001. V. 516. P. 293.
29. *Babecki J., Nowak G.* Characteristics of Slow Particles in Hadron–Nucleus Interactions and Their Relation to the Models of High-Energy Interactions // Acta Phys. Polon. B. 1978. V. 9. P. 401.
30. *Andersson B. et al.* On the Correlation between Fast Target Protons and the Number of Hadron–Nucleon Collisions in High-Energy Hadron–Nucleus Reactions // Phys. Lett. B. 1978. V. 73. P. 343.
31. *Jain P. L., Sengupta K., Singh G.* Production of Fast and Slow Particles in Nucleus–Nucleus Collisions at Ultra-Relativistic Energies // Phys. Rev. C. 1991. V. 44. P. 844.
32. *Bialas A., Gazdzicki M.* A New Variable to Study Intermittency // Phys. Lett. B. 1990. V. 252. P. 483.
33. *Hwa R. C.* Beyond Intermittency: Erraticity // Acta Phys. Polon. B. 1996. V. 27. P. 1789.

Received on January 13, 2014.