Письма в ЭЧАЯ. 2015. Т. 12, № 3(194). С. 614–618

HYDRODYNAMIC PRESSURE COMPUTATION UNDER REAL SEA SURFACE ON BASIS OF AUTOREGRESSIVE MODEL OF IRREGULAR WAVES

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Determining the impact of external excitations on a dynamic marine object such as ship hull in a seaway is the main goal of simulations. Now such simulations are most often based on approximate mathematical models that use results of the theory of small-amplitude waves. The most complicated software for marine objects behavior simulation LAMP IV (Large Amplitude Motion Program) uses numerical solution of traditional hydrodynamic problem without often-used approximations but on the basis of theory of small-amplitude waves. For efficiency reasons these simulations can be based on autoregressive model to generate real wave surface. Such a surface possesses all the hydrodynamic characteristics of sea waves, preserves dispersion relation and also shows superior performance compared to other wind wave models. Naturally, the known surface can be used to compute velocity field and in turn to determine pressures in any point under sea surface. The resulting computational algorithm can be used to determine pressures without the use of theory of small-amplitude waves.

Определение степени воздействия взволнованной морской поверхности на корпус судна является главной задачей испытаний, которые к настоящему времени основаны на результатах теории волн малой амплитуды: эту теорию, исключая некоторые общепринятые аппроксимации, использует пакет прикладного программного обеспечения LAMP IV (Large Amplitude Motion Program) для исследования поведения морских объектов. Для повышения эффективности такие испытания можно проводить на основе авторегрессионной модели, используя ее для генерации взволнованной морской поверхности. Такая поверхность обладает гидродинамическими характеристиками морских волн, сохраняет дисперсионное соотношение, а также имеет более быстрый численный алгоритм по сравнению с другими моделями ветрового волнения. Полученную таким образом морскую поверхность можно использовать для расчета поля скоростей, а его — для расчета поля давлений в любой точке под поверхностью. Окончательный алгоритм позволяет определять давления, не используя предположения теории волн малой амплитуды.

PACS: 92.10.hd; 92.10.Hm

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INTRODUCTION

For many years marine object behavior in a seaway was investigated through experiments conducted in a towing tank and, although, in some cases, this approach proved to be useful, it has some disadvantages compared to modern techniques. First of all, conducting a single experiment in a towing tank and collecting desired data takes as long as one month to complete. Second, a towing tank provides machinery to generate only plane waves, which propagate at most in one direction and the process of propagation is disturbed by walls of a pool so that real three-dimensional sea waves cannot be generated in experiment. Finally, all the simulations in a towing tank are carried out not for real-sized ship but for its model and using fitting criteria to generalize experiment results for real ship is not always feasible; so, not every aspect of real behavior can be captured in a towing tank. As a result of these deficiencies and also as a consequence of development of high-performance computer machines, more and more experiments conducted in a towing tank are replaced by computer-based simulations conducted in a virtual testbed.

Virtual testbed being a computer program to simulate physical and anthropogenic phenomena can be seen as evolution and virtual analog of a towing tank and it not only lacks disadvantages of a towing tank mentioned above, but also offers much broader set of simulation options. For example, in a computer program with the help of a proper sea wave generator it is possible to combine climatic and wind wave model [5] and to use assimilated wind velocity field data to simulate wind waves and swell, which occur in a particular region of ocean, and also to simulate evolution of wave climate between normal and storm weather. Another option is to simulate water streams, ice cover, wave deflection and wave diffraction. However, none of these options were implemented in software to a full extent and often-used wind wave models are capable of generating only linear sea. So, virtual testbed approach takes marine object behavior simulations one level higher than the level offered by a towing tank, however, not all potential of this approach is realized.

Not only different weather scenarios are not implemented in a virtual testbed, but wind wave models, such as the Longuet–Higgins model, are capable of generating only linear sea and more effective models can be developed. An alternative autoregressive model is a wind wave model proposed by Rozhkov, Gurgenidze and Trapeznikov [2] and it is advantageous in many ways over the Longuet–Higgins model when conducting simulations in a virtual testbed. First, it allows generating realizations of arbitrary amplitude ocean waves, whereas the Longuet–Higgins model formulas are derived using assumptions of small-amplitude wave theory and are not suitable to generate surfaces of large-amplitude waves [4]. Second, it lacks disadvantages of the Longuet–Higgins model: it has high convergence rate, its period is limited only by the period of implemented pseudorandom number generator and it can model certain nonlinearities of wave motion such as asymmetric distribution of wave surface elevation [3]. Finally, autoregressive model has efficient and fast numerical algorithm compared to the Longuet–Higgins model, which reduces simulation time [6]. However, autoregressive model formulas are not derived from partial differential equations of wave motion, but instead represent nonphysical approach to wavy surface generation and, to prove adequacy of such an approach, series of experiments were conducted to show that wavy surface generated by this model possesses integral characteristics as well as dispersion relation of a real ocean waves and an ability to reproduce storm weather [4].

Theory of small-amplitude waves is also used to determine pressures under sea surface and methods for determining pressures should also be modified to match autoregressive model.

1. PRESSURE DETERMINATION

The problem of pressure determination under real sea surface in case of inviscid incompressible fluid is reduced to solving the Laplace equation with dynamic and kinematic boundary conditions [7] and, in two-dimensional case, an analytical solution can be obtained. In two-dimensional case, the corresponding system of equations

$$\begin{aligned} &\phi_{xx} + \phi_{zz} = 0, \\ &\phi_t + \frac{1}{2}\left({\phi_x}^2 + {\phi_z}^2\right) + g\zeta = \frac{p}{\rho}, \quad \text{at} \quad z = \zeta(x,t), \\ &\zeta_t + \zeta_x \phi_x = \phi_z, \qquad\qquad\qquad \text{at} \quad z = \zeta(x,t) \end{aligned} \tag{1}$$

can be solved in three steps. The first step is to solve the Laplace equation using the Fourier method and to obtain solution of the form of the Fourier integral

$$\phi(x,z,t) = \int\limits_{-\infty}^{\infty} E(\lambda)\, \mathrm{e}^{\lambda(z+\iota x)}\, d\lambda. \tag{2}$$

The second step is to determine coefficients $E(\lambda)$ by substituting this integral into the second (kinematic) boundary condition. The boundary condition is held on the free wavy surface $z = \zeta(x,t)$, so that velocity potential derivative $\phi_z(x,t)$ can be evaluated using the chain rule. After performing these steps, the equation

$$\frac{\zeta_t}{\zeta_x + \zeta_t - \zeta_x\left(\zeta_x + i\right)} = \int\limits_{-\infty}^{\infty} \lambda E(\lambda)\, \mathrm{e}^{\lambda(\zeta+\iota x)} d\lambda,$$

which represents the Laplace transform formula, can be obtained and inversed to obtain formula for coefficients $E(\lambda)$:

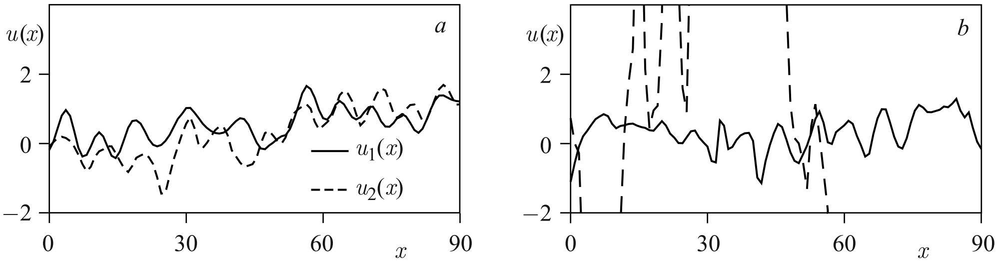
$$E(\lambda) = \frac{1}{2\pi\iota}\frac{1}{\lambda} \int\limits_{-\infty}^{\infty} \frac{\zeta_t}{\zeta_x + \zeta_t - \zeta_x\left(\zeta_x + i\right)}\, \mathrm{e}^{-\lambda(\zeta+\iota x)}\, dx. \tag{3}$$

The final step is to substitute (3) into (2), which yields the equation

$$\phi(x,t) = \frac{1}{2\pi\iota} \int\limits_{-\infty}^{\infty} \frac{1}{\lambda}\, \mathrm{e}^{\lambda(\zeta+\iota x)} d\lambda \int\limits_{-\infty}^{\infty} \frac{\zeta_t}{\zeta_x + \zeta_t - \zeta_x\left(\zeta_x + i\right)}\, \mathrm{e}^{-\lambda(\zeta+\iota x)}\, dx. \tag{4}$$

Using this equation, an explicit formula for pressure determination can be obtained directly from the first boundary condition:

$$p(x_0, z_0) = -\rho\phi_t - \frac{\rho}{2}\left({\phi_x}^2 + {\phi_z}^2\right) - \rho g z_0.$$



Comparison of velocity fields produced by general solution (u_1) and by solution for small-amplitude waves (u_2). Velocity fields for small-amplitude (*a*) and large-amplitude (*b*) cases

Analytical solution (4) was compared to the solution

$$\left.\frac{\partial\phi}{\partial x}\right|_{x,t} = -\frac{1}{\sqrt{1+\alpha^2}}\,\mathrm{e}^{-I(x)} \int\limits_0^x \frac{\partial\dot{z}/\partial z + \alpha\dot{\alpha}}{\sqrt{1+\alpha^2}}\,\mathrm{e}^{I(x)}dx, \quad I(x) = \int\limits_0^x \frac{\partial\alpha/\partial z}{1+\alpha^2}dx \qquad (5)$$

obtained for small-amplitude waves [1], and numerical experiments showed good correspondence rate between resulting velocity potential fields. In order to obtain velocity potential fields, realizations of the wavy sea surface were generated by autoregressive model differing only in wave amplitude. In numerical implementation, infinite outer and inner integral limits of (4) were replaced by the corresponding wavy surface size (x_0, x_1) and wave number interval (λ_0, λ_1), so that inner integral of (4) converges. Experiments were conducted for waves of both small and large amplitudes and, in case of small-amplitude waves, both solutions produced similar results, whereas, in case of large-amplitude waves, only general solution (4) produced stable velocity field (Figure). Therefore, general solution works for different wavy sea surfaces and does not impose restrictions on wave amplitude.

2. CONCLUSION AND FUTURE WORK

Resulting solution (4) can be used to compute impact of hydrodynamic forces on a ship's hull and is advantageous in several ways. First, it can be used for wavy surfaces of arbitrary amplitudes to support simulations for small-sized ships or storm weather in a virtual testbed. Second, the formula is analytical and explicit, so that no numerical scheme is needed to implement solution of initial system of partial differential equations (1) on a computer. Moreover, resulting algorithm is fast and easily scalable on a multiprocessor computer, because it is based on an explicit formula. The future work is to extend this solution to three-dimensional case.

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