

THE DECAYS $\tau \rightarrow (\pi, \pi')\nu_\tau$ IN THE NAMBU–JONA-LASINIO MODEL

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In the Nambu–Jona-Lasinio model the decay widths $\tau \rightarrow (\pi, \pi'(1300))\nu_\tau$ were calculated. For the calculation of the decay width $\tau \rightarrow \pi\nu_\tau$ the standard NJL model was used taking into account the intermediate axial-vector $a_1(1260)$ meson and the extended Nambu–Jona-Lasinio model with the intermediate $a_1(1260)$ and $a'_1(1640)$ mesons. These results are in good agreement with each other. For the calculation of the decay width $\tau \rightarrow \pi'\nu_\tau$ the extended Nambu–Jona-Lasinio model was used. The results are in satisfactory agreement with the experimental data.

Numerical estimates for the constants F_π and $F_{\pi'}$ are also in satisfactory agreement with both the experimental values for F_π and the results obtained by calculations on the lattice.

В модели Намбу–Йона-Лазинио (НИЛ) вычислены ширины распадов $\tau \rightarrow (\pi, \pi'(1300))\nu_\tau$. При вычислении ширины распада $\tau \rightarrow \pi\nu_\tau$ использована как стандартная НИЛ-модель с учетом промежуточного аксиально-векторного $a_1(1260)$ -мезона, так и расширенная модель Намбу–Йона-Лазинио с промежуточными $a_1(1260)$ - и $a'_1(1640)$ -мезонами. Эти результаты находятся в хорошем согласии друг с другом. Для вычисления ширины распада $\tau \rightarrow \pi'\nu_\tau$ использована расширенная модель Намбу–Йона-Лазинио. Полученные результаты находятся в удовлетворительном согласии с известными экспериментальными данными.

Численные оценки констант F_π и $F_{\pi'}$ также находятся в удовлетворительном согласии как с экспериментальными значениями для F_π , так и с результатами, полученными при вычислениях на решетке.

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INTRODUCTION

In the recently fulfilled works [1–4] a number of decays of the τ leptons were described. The intermediate processes with the participation of vector mesons in both ground and radial excited states were taken into account. For this, the extended NJL model was used [5–8]. This model allows us to describe all these processes without introducing any additional arbitrary parameters unlike the methods of other authors [9–12]. The decay widths $\tau \rightarrow \pi^-\pi^0\nu_\tau$, $\tau \rightarrow \pi\eta(\eta')\nu_\tau$, $\tau \rightarrow 2\pi\eta(\eta')\nu_\tau$, $\tau \rightarrow \pi\omega\nu_\tau$ were calculated.

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In this work, we will describe one of the main τ lepton decays $\tau \rightarrow \pi\nu_\tau$ and give the estimate for the decay width $\tau \rightarrow \pi'(1300)\nu_\tau$.

The calculation of the decay width $\tau \rightarrow \pi\nu_\tau$ was conducted in both standard and the extended NJL models. These results are in good agreement with each other. Calculations of the decay width $\tau \rightarrow \pi'(1300)\nu_\tau$ were performed in the extended NJL model. The estimates obtained for the decay widths $\tau \rightarrow \pi(\pi'(1300))\nu_\tau$ are in satisfactory agreement with the experimental data [14, 16]. The value obtained for $F_{\pi'}$ = 4.68 MeV is in qualitative agreement with the estimate obtained by calculation on the lattice [17].

1. LAGRANGIANS OF QUARK–MESON INTERACTIONS

In the extended NJL model, interactions of the π , $\pi'(1300)$ and $a_1(1260)$, $a'_1(1647)$ mesons with quarks are of the form

$$\begin{aligned} \Delta\mathcal{L}_E^{\text{int}}(q, \bar{q}, a_1, a'_1, \pi, \pi') &= \bar{q}(k') \left[\left(A_1 a_1^{(-)\mu} - A_2 a'_1^{(-)\mu} \right) \tau^- \gamma^\mu \gamma^5 + \right. \\ &\quad \left. + (P_1 \pi^- - P_2 \pi'^-) \tau^- \gamma^5 \right] q(k), \quad (1) \\ \tau^- &= \sqrt{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \end{aligned}$$

where a' and π' are the axial-vector and pseudoscalar meson fields in the excited state,

$$\begin{aligned} A_1 &= \frac{g_{\rho_1}}{2} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \frac{g_{\rho_2}}{2} f(k_\perp^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)}, \\ A_2 &= \frac{g_{\rho_1}}{2} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + \frac{g_{\rho_2}}{2} f(k_\perp^2) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)}, \\ P_1 &= g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k_\perp^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)}, \\ P_2 &= g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k_\perp^2) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)}. \end{aligned} \quad (2)$$

The values of the angles $\beta = 79.85^\circ$ and $\beta_0 = 61.44^\circ$ are taken from [6], and the angles $\alpha = 59.38^\circ$ and $\alpha_0 = 59.06^\circ$ correspond to the mixing angle taken from the vector meson sector [18].

The form factors $f(k_\perp^2)$ used in the extended NJL model have the form $f(k_\perp^2) = (1 + dk_\perp^2)$, where $k_\perp = k - \frac{(kp)p}{p^2}$ is the transverse quark momentum, for light quarks $d = -1.784 \text{ GeV}^{-2}$ [6]. Here k and p are the quark and meson momenta, respectively. In the meson rest frame, this formula takes the simple form $k_\perp = (0, \mathbf{k})$. π and π' are the pion fields in the ground and first radial excited states. a_1 and a'_1 are the axial-vector meson fields in the ground and first radial excited states. q is the field of the u and d quarks and $\bar{q} = (\bar{u}, \bar{d})$.

The quark–meson coupling constants are

$$\begin{aligned} g_{\rho_2} &= \left(\frac{2}{3} I_2^{f^2}(m_u) \right)^{-1/2}, & g_{\pi_2} &= \left(4 I_2^{f^2}(m_u) \right)^{-1/2}, \\ g_{\rho_1} &= \left(\frac{2}{3} I_2^{(0)}(m_u, m_d) \right)^{-1/2} = 6.14, & g_{\pi_1} &= g_\pi = \frac{m_u}{F_\pi}, \end{aligned} \quad (3)$$

where the integrals $I_m^{f^n}$ read

$$I_m^{f^n}(m_q) = -i \frac{N_c}{(2\pi)^4} \int d^4k \frac{(f_q(k_\perp^2))^n}{(m_q^2 - k^2)^m} \Theta(\Lambda_3^2 - k_\perp^2), \quad (4)$$

and the cut-off parameter $\Lambda_3 = 1.03$ GeV [2, 6].

Add here the Lagrangian describing a weak interaction of the lepton and quark current

$$\mathcal{L}^{\text{weak}} = \bar{\tau} \gamma_\mu \gamma_5 \nu \frac{G_F}{\sqrt{2}} |V_{ud}| \bar{d} \gamma_5 \gamma_\mu u. \quad (5)$$

These Lagrangians allow us to calculate the decay widths $\tau \rightarrow (\pi\pi'(1300))\nu_\tau$ without introducing any extra parameters.

2. THE DECAY $\tau \rightarrow \pi\nu_\tau$

First, let us calculate the decay width $\tau \rightarrow \pi\nu_\tau$ in the standard NJL model. Here the Lagrangian of the quark–meson interaction is

$$\mathcal{L} = \bar{q} [\tau^- \gamma_5 (i g_\pi \pi^- + \gamma_\mu a_{1\mu}^-)] q. \quad (6)$$

In the standard NJL model the main contribution to the decay width $\tau \rightarrow \pi\nu_\tau$ is given by one of the quark-loop diagrams shown in Fig. 1.

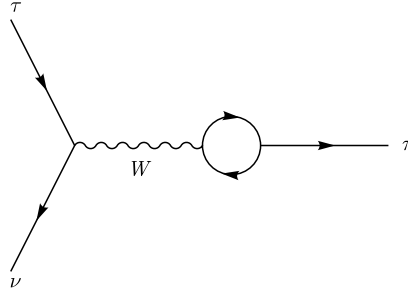


Fig. 1. Feynman diagram describing the $\tau \rightarrow \pi\nu_\tau$ decay

Using the Lagrangian (6) for the amplitude shown in Fig. 1, we obtain the corresponding expression:

$$A_{\tau \rightarrow \pi\nu_\tau} = \frac{G_F}{\sqrt{2}} \bar{u}_\nu \gamma_\mu \gamma_5 u_\tau g_{\mu\alpha} |V_{ud}| \int \frac{d^4k}{(2\pi)^4} \frac{\text{tr} (i\gamma_5)(\hat{k} + m_u) \gamma_\alpha \gamma_5 (\hat{k} + \hat{p} + m_u)}{(m_u^2 - k^2)(m_u^2 - (k+p)^2)}. \quad (7)$$

Here p is the momentum of the pion, $G_F = 1.16637 \cdot 10^{-11} \text{ MeV}^{-2}$ is the Fermi constant, k is the momentum of the quark in the loop k , m_u is the u quark mass, and $|V_{ud}| = 0.97428$ is the cosine of the Cabibbo angle.

We show that without the π - a_1 transitions this diagram completely determines the decay width $\tau \rightarrow \pi\nu_\tau$. Really, the amplitude (7) corresponding to this diagram is

$$A_{\tau \rightarrow \pi\nu_\tau} = L_\mu \frac{G_F}{\sqrt{2}} |V_{ud}| \left[\sqrt{2}g4m_u I_2 = \sqrt{2} \frac{m_u}{g} = \sqrt{2}F_\pi \right] p^\mu, \quad (8)$$

where $L_\mu = \bar{u}_\nu \gamma_\mu \gamma_5 u_\tau$ is the lepton current.

Here we used the following ratio:

$$4I_2 = \frac{1}{g^2},$$

where

$$F_\pi = 93 \text{ MeV}, \quad m_u = m_d = 280 \text{ MeV}.$$

As a result, we get the well-known formula which can be found, for example, in [13].

Taking into account the π - a_1 transitions, the coupling constant of the pion with the quarks g_π has the form

$$g_\pi = \sqrt{Z}g = \frac{m_u}{F_\pi}, \quad \text{where } Z = \left(1 - \frac{6m_u^2}{m_{a_1}^2} \right)^{-1}, \quad (9)$$

$m_{a_1} = 1230 \text{ MeV}$ is the mass of the a_1 meson.

2.1. The Calculation of the Decay Width $\tau \rightarrow \pi\nu_\tau$ in the Standard NJL Model. The decay amplitude of $\tau \rightarrow \pi(\pi')\nu_\tau$ has the form

$$F_{\tau \rightarrow \pi\nu_\tau} = L_\mu \frac{G_F}{\sqrt{2}} |V_{ud}| \left| F_1^q(\pi, \pi') + F_2^{a_1}(\pi, \pi') + F_3^{a'_1}(\pi, \pi') \right| p^\mu \pi(\pi'), \quad (10)$$

where F_1^q corresponds to the contribution of the amplitude from the one-loop quark diagram (Fig. 1). $F_2^{a_1}$ corresponds to the contribution of the amplitude from the diagram with intermediate $a_1(1260)$ mesons (Fig. 2). $F_3^{a'_1}$ corresponds to the contribution of the amplitude from the diagram with intermediate $a'_1(1640)$ mesons (Fig. 2).

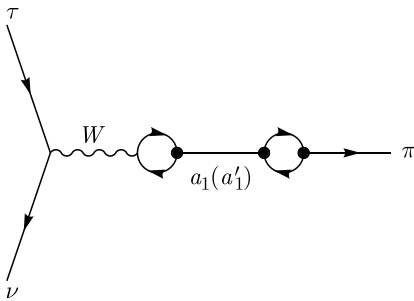


Fig. 2. Diagrams describing the $\tau \rightarrow \pi\nu_\tau$ decay with the intermediate axial-vector mesons

The model parameters F_π and $F_{\pi'}$ are determined by the equation

$$F = |F_1^q + F_2^{a_1} + F_3^{a'_1}|. \quad (11)$$

In the standard NJL model for the quantities F_i we obtain the following values:

$$F_1^q = ZF_\pi, \quad F_2^{a_1} = (1 - Z)F_\pi, \quad F_3^{a'_1} = 0. \quad (12)$$

As a result, we get $F = F_\pi$.

It means that taking into account the π - a_1 transitions, we once again have the standard expression for the decay amplitude $\tau \rightarrow \pi\nu_\tau$.

The decay width of the $\tau \rightarrow \pi\nu_\tau$ process is equal to

$$\Gamma(\tau \rightarrow \pi\nu_\tau) = \frac{|F|^2}{2 \cdot 2m_\tau} \Phi, \quad (13)$$

where Φ is the phase volume

$$\Phi = \frac{E_\nu}{4\pi m_\tau}, \quad (14)$$

where E_ν is determined in the following form:

$$E_\nu = \frac{m_\tau^2 - m_\pi^2}{2m_\tau}. \quad (15)$$

We also used $(p_\nu p_\tau) = m_\tau E_\nu$.

For the square of the amplitude, we obtain the following expression:

$$|F|^2 = 8G_F^2 |V_{ud}|^2 m_\tau^3 E_\nu F_\pi^2. \quad (16)$$

This expression is in satisfactory agreement with the experimental data, $\Gamma_{\tau \rightarrow \pi\nu_\tau}^{\text{theor}} = 2.5 \cdot 10^{-10}$ MeV, $\Gamma_{\tau \rightarrow \pi\nu_\tau}^{\text{exp}} = 2.1 \cdot 10^{-10}$ MeV [14].

To describe the process $\tau \rightarrow \pi'\nu_\tau$, the extended NJL model should be used, that will be done below. To describe the process $\tau \rightarrow \pi\nu_\tau$, it is enough to use the standard NJL model.

2.2. The Decay $\tau \rightarrow \pi\nu_\tau$ in the Extended NJL Model. Using the extended model, we obtain only minor corrections to formulas from the standard NJL model. Indeed, for example, we give the expression obtained in the extended model for the amplitude of the one-loop approximation (Fig. 1).

$$F_1^q = Z \left[F_\pi \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + \Gamma \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] p^\mu, \quad (17)$$

$$F_2^{a_1} = D_1 + D_2 + D_3 + D_4, \quad F_3^{a_1} = C_1 + C_2 + C_3 + C_4,$$

where

$$\Gamma = \frac{I_2^f}{\sqrt{I_2 I_2^f}} = 0.54. \quad (18)$$

Expressions D_1, D_2, D_3, D_4 and C_1, C_2, C_3, C_4 are shown in (19) and (21), respectively. Hence, it is easy to see that the contribution to the amplitude of the term which contains the form factor is negligibly small.

Therefore, for the calculation of the decay width $\tau \rightarrow \pi\nu_\tau$ one can use only the standard NJL model. Since the angles α and α_0 are close to each other, it is easy to see that part of the amplitude form factor provides a very small contribution, as compared to the first term, and can be neglected.

Now we consider the $\tau \rightarrow \pi\nu_\tau$ decay which contains two quark-loop diagrams. The corresponding quark-loop diagram for this decay is shown in Fig. 2. First, we consider the

diagram with the intermediate a_1 meson. As a result, we obtain

$$\begin{aligned}
D_1 &= -\frac{6m_u^2}{m_{a_1}^2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] D_{F0}, \\
D_2 &= -\frac{6m_u^2}{m_{a_1}^2} \frac{1}{g_\rho/2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] D_{F1}, \\
D_3 &= -\frac{6m_u^2}{m_{a_1}^2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] D_{F11}, \\
D_4 &= -\frac{6m_u^2}{m_{a_1}^2} \frac{1}{g_\rho/2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] D_{F2},
\end{aligned} \tag{19}$$

where $D_{F0}, D_{F1}, D_{F11}, D_{F2}$ correspond to the contribution from the second loop in Fig. 2. They have the following form:

$$\begin{aligned}
D_{F0} &= Z F_\pi \left[\frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \right] p^\alpha, \\
D_{F1} &= \sqrt{\frac{3}{2}} \sqrt{Z} \Gamma F_\pi \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] p^\alpha, \\
D_{F11} &= \sqrt{Z} \Gamma F_\pi \left[\frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} \right] p^\alpha, \\
D_{F2} &= \sqrt{\frac{3}{2}} F_\pi \left[\frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] p^\alpha.
\end{aligned} \tag{20}$$

We proceed to consider the contribution from the diagram with the intermediate $a'_1(1640)$ meson. The amplitude comprising the $a'_1(1640)$ meson has the form

$$\begin{aligned}
C_1 &= -\frac{6m_u^2}{m_{a'_1}^2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] C_{F0}, \\
C_2 &= -\frac{6m_u^2}{m_{a'_1}^2} \frac{1}{g_\rho/2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] C_{F1}, \\
C_3 &= -\frac{6m_u^2}{m_{a'_1}^2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] C_{F11}, \\
C_4 &= -\frac{6m_u^2}{m_{a'_1}^2} \frac{1}{g_\rho/2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] C_{F2},
\end{aligned} \tag{21}$$

where $C_{F0}, C_{F1}, C_{F11}, C_{F2}$ correspond to the contribution from the second loop in Fig. 2.

They have the following form:

$$\begin{aligned}
 C_{F0} &= ZF_\pi \left[\frac{\cos(\beta + \beta_0) \sin(\alpha + \alpha_0)}{\sin(2\beta_0) \sin(2\alpha_0)} \right] p^\alpha, \\
 C_{F1} &= \sqrt{\frac{3}{2}} \sqrt{Z} \Gamma F_\pi \left[\frac{\cos(\beta + \beta_0) \sin(\alpha - \alpha_0)}{\sin(2\beta_0) \sin(2\alpha_0)} \right] p^\alpha, \\
 C_{F11} &= \sqrt{Z} \Gamma F_\pi \left[\frac{\cos(\beta - \beta_0) \sin(\alpha + \alpha_0)}{\sin(2\beta_0) \sin(2\alpha_0)} \right] p^\alpha, \\
 C_{F2} &= \sqrt{\frac{3}{2}} F_\pi \left[\frac{\cos(\beta - \beta_0) \sin(\alpha - \alpha_0)}{\sin(2\beta_0) \sin(2\alpha_0)} \right] p^\alpha.
 \end{aligned} \tag{22}$$

As a result, after the calculation we get the following values:

$$F_1^q = 133.5 \text{ MeV}, \quad F_2^{a_1} = -46.76 \text{ MeV}, \quad F_3^{a'_1} = 5.9 \text{ MeV}.$$

Finally, for F_π we obtain $F_\pi = 92.8 \text{ MeV}$. It is easy to see that these results are in good agreement with each other.

3. THE DECAY $\tau \rightarrow \pi'\nu_\tau$ IN THE EXTENDED NJL MODEL

Now let us consider the decay of $\tau \rightarrow \pi'\nu_\tau$. The corresponding quark diagram for this decay is shown in Fig. 3.

Note that in the vertex with π' there are two cases: in the first case, this vertex does not contain the vertex form factor, and in the second case, this vertex does. As a result, this amplitude takes the following form:

$$\begin{aligned}
 A_0 &= L_\mu \frac{G_F}{\sqrt{2}} |V_{ud}| \sqrt{2} \sqrt{Z} F_\pi \left[\sqrt{Z} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + \Gamma \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] p^\mu = \\
 &= L_\mu \frac{G_F}{\sqrt{2}} |V_{ud}| (-5.64) p^\mu \text{ MeV}. \tag{23}
 \end{aligned}$$

Besides the one-loop contribution to the decay $\tau \rightarrow \pi'\nu_\tau$, an important role is played by the contributions of the intermediate $a_1(1260)$ and $a'_1(1640)$ axial-vector mesons. The corresponding two-loop diagrams for this contribution are shown in Fig. 4. Here the first loop

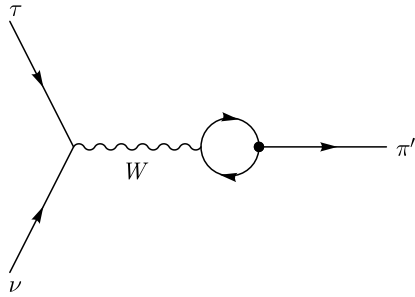


Fig. 3. Feynman diagram for the $\tau \rightarrow \pi'\nu_\tau$ decay

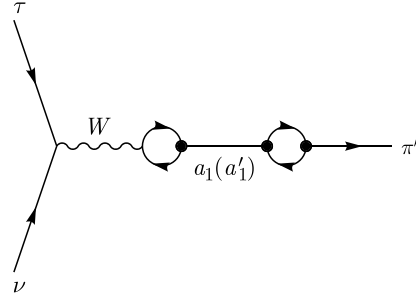


Fig. 4. Feynman diagrams for the $\tau \rightarrow \pi'\nu_\tau$ decay with the axial-vector transitions

consists of two parts, one of which does not contain a form factor at the vertices with the a_1 (a'_1) mesons. The second loop of each of the vertices consists of two parts, one of which comprises a form factor and the other does not.

We first consider the diagram with the intermediate a_1 meson. As a result, we obtain

$$\begin{aligned}
A_1 &= -6m_u^2 \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \text{BW } A_{F0}, \\
A_2 &= -6m_u^2 \frac{1}{g_\rho/2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \text{BW } A_{F1}, \\
A_3 &= -6m_u^2 \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \text{BW } A_{F11}, \\
A_4 &= -6m_u^2 \frac{1}{g_\rho/2} \sqrt{2} \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \Gamma \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] \text{BW } A_{F2},
\end{aligned} \tag{24}$$

where $A_{F0}, A_{F1}, A_{F11}, A_{F2}$ correspond to the contribution from the second loop in Fig. 4. They have the following form:

$$\begin{aligned}
A_{F0} &= Z F_\pi \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \left(-\frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
A_{F1} &= \sqrt{\frac{3}{2}} \sqrt{Z} \Gamma F_\pi \left[\frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \left(-\frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
A_{F11} &= \sqrt{Z} \Gamma F_\pi \left[\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \left(-\frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
A_{F2} &= \sqrt{\frac{3}{2}} F_\pi \left[\frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \left(-\frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha.
\end{aligned} \tag{25}$$

We proceed to consider the contribution from the diagram with the intermediate $a'_1(1640)$ mesons. The amplitude containing the $a'_1(1640)$ meson has the form

$$\begin{aligned}
B_1 &= -6m_u^2 \sqrt{2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] \text{BW } B_{F0}, \\
B_2 &= -6m_u^2 \frac{1}{g_\rho/2} \sqrt{2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] \text{BW } B_{F1}, \\
B_3 &= -6m_u^2 \sqrt{2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] \text{BW } B_{F11}, \\
B_4 &= -6m_u^2 \frac{1}{g_\rho/2} \sqrt{2} \left[\left(\frac{-\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) + \Gamma \left(\frac{-\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \right] \text{BW } B_{F2},
\end{aligned} \tag{26}$$

where $B_{F0}, B_{F1}, B_{F11}, B_{F2}$ correspond to the contributions from the second loop in Fig. 4.

They have the following form:

$$\begin{aligned}
 B_{F0} &= Z F_\pi \left[\left(-\frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \right) \left(-\frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
 B_{F1} &= \sqrt{\frac{3}{2}} \sqrt{Z} \Gamma F_\pi \left[\left(-\frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \left(-\frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
 B_{F11} &= \sqrt{Z} \Gamma F_\pi \left[\left(-\frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} \right) \left(-\frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha, \\
 B_{F2} &= \sqrt{\frac{3}{2}} F_\pi \left[\left(-\frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right) \left(-\frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right) \right] p^\alpha,
 \end{aligned} \tag{27}$$

where BW is the Breit–Wigner formula for the intermediate state of the $a_1(a'_1)$ mesons and has the form

$$\text{BW} = \frac{1}{M_{a_1(a'_1)}^2 - M_{\pi'}^2 - i M_{a_1(a'_1)} \Gamma_{a_1(a'_1)}}. \tag{28}$$

For the $\tau \rightarrow \pi' \nu_\tau$ decay in the extended NJL model we obtain the following value for the quantities $F_{1(\pi')}^q$, $F_{2(\pi')}^{a_1}$, $F_{3(\pi')}^{a'_1}$:

$$F_{1(\pi')}^q = 5.64, \quad F_{2(\pi')}^{a_1} = -2.3 - i \cdot 6.39, \quad F_{3(\pi')}^{a'_1} = -4.59 + i \cdot 1.88, \tag{29}$$

$$\tilde{F}_{\pi'} = F_{1(\pi')}^q + F_{2(\pi')}^{a_1} + F_{3(\pi')}^{a'_1} = -1.25 - i \cdot 4.51, \tag{30}$$

$$F_{\pi'} = |\tilde{F}_{\pi'}| = \sqrt{\text{Re } \tilde{F}_{\pi'}^2 + \text{Im } \tilde{F}_{\pi'}^2} = 4.68 \text{ MeV}.$$

Calculations on the lattice gave the following results for $F_{\pi'} \approx 2.4 \text{ MeV}$ [17]. We can see that our results are in qualitative agreement with the estimates obtained by calculations on the lattice.

As a result, the total decay amplitude $\tau \rightarrow \pi' \nu_\tau$ takes the form

$$T = -A_0 + A_1 + A_2 + A_3 + A_4 + B_1 + B_2 + B_3 + B_4. \tag{31}$$

Considering the decay amplitude $\tau \rightarrow \pi' \nu_\tau$ via one loop (23), for the decay width (13), we obtain the value of $\Gamma_{\tau \rightarrow \pi' \nu_\tau} = 1.04 \cdot 10^{-13} \text{ MeV}$. For the decay width via two loops with the intermediate $a_1(1260)$ mesons, we obtain the value of $\Gamma_{\tau \rightarrow (a_1) \rightarrow \pi' \nu_\tau} = 3.03 \cdot 10^{-13} \text{ MeV}$. With the intermediate $a'_1(1640)$ mesons we obtain the value of $\Gamma_{\tau \rightarrow (a'_1) \rightarrow \pi' \nu_\tau} = 1.62 \cdot 10^{-13} \text{ MeV}$. The total decay width of $\tau \rightarrow \pi' \nu_\tau$ with the interference between the amplitudes (31) is $\Gamma_{\tau \rightarrow \pi' \nu_\tau}^{\text{tot}} = 2.21 \cdot 10^{-13} \text{ MeV}$.

The resulting value for the decay width of $\tau \rightarrow \pi' \nu_\tau$ is in satisfactory agreement with the experimental evaluation obtained in [16]. There, the following restrictions on the quantity of the decay width $\tau \rightarrow \pi' \nu_\tau$ were obtained, that is, $\Gamma_{\tau \rightarrow (\pi') \nu_\tau}^{\text{exp}} = (2.27 - 4.31) \cdot 10^{-13} \text{ MeV}$.

In addition, the evaluation obtained for $F_{\pi'} = 4.68 \text{ MeV}$ is in qualitative agreement with the estimates obtained in the lattice model [17].

CONCLUSIONS

The decay mechanism $\tau \rightarrow \pi\nu_\tau$ is similar to the decay $\pi^- \rightarrow \mu^- \nu$, which was described in [15]. Those calculations were performed in the standard NJL model. It is interesting to note that in the calculation of these processes in the quark one-loop approximation with the decay constant F_π there appears a factor Z , that is, due to taking into account of π - a_1 transitions. To obtain a correct form for the decay amplitude $\tau \rightarrow \pi\nu_\tau$, it is also necessary to take into account the two-loop approximation with the intermediate $a_1(1260)$ mesons. This approach takes into account the possibility of the a_1 - π transition to the intermediate state with the a_1 meson. As a result, for the same value of $F_\pi = 93$ MeV we obtain satisfactory agreement with the experimental data in both cases, as the decay $\tau \rightarrow \pi\nu_\tau$ and the decay $\pi^- \rightarrow \mu^- \nu$ (11).

It is interesting to note that the calculation of the width $\tau \rightarrow \pi\nu_\tau$ conducted in the expanded NJL model taking into account the intermediate $a_1(1260)$ and $a'_1(1640)$ mesons leads to the same results for the width of the $\tau \rightarrow \pi\nu_\tau$ decay and of the constant F_π . The extended NJL model allows us to describe the decay $\tau \rightarrow \pi'\nu_\tau$.

Unlike the case of the decay from $\tau \rightarrow \pi\nu_\tau$, in the $\tau \rightarrow \pi'\nu_\tau$ decay amplitude the intermediate $a_1(1260)$ and $a'_1(1640)$ mesons play a decisive role.

Note that the results obtained here for the widths of the $\tau \rightarrow \pi'\nu_\tau$ decay are in satisfactory agreement with the experimental data [16]. In our calculation, we can claim only qualitative agreement with experiment, as in the extended NJL model to describe the excited state of the axial-vector mesons, we used the mixing angles taken from the vector meson sector [18].

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