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THREE-FLUID RELATIVISTIC HYDRODYNAMICS FOR HEAVY-ION COLLISIONS

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A relativistic three-fluid 3D hydrodynamic model has been developed for describing heavy-ion collisions at high energies. In addition to two baryon-rich fluids, the new model incorporates evolution of a third, retarded baryon-free fluid created by this decelerated baryonic matter. Different equations of state, including those with the deconfinement phase transition, are treated. A reasonable agreement with experiment is illustrated by proton rapidity spectra, their dependence on collision centrality and beam energy.

Разработана трехструйная трехмерная гидродинамическая модель для описания столкновений тяжелых ионов при больших энергиях. В дополнение к двум барионно-насыщенным струям новая модель включает в себя эволюцию третьей, запаздывающей безбарионной струи, порожденной замедленной барионной материей. Представлены различные уравнения состояния, в том числе с учетом фазового перехода деконфайнмента. Приемлемое согласие с экспериментом продемонстрировано на примерах спектров быстрот протонов, их зависимости от центровки столкновения и энергии пучка.

1. INTRODUCTORY REMARKS

During past twenty years, hydrodynamics proved to be quite a reasonable tool for studying collective properties of hot and dense nuclear matter and its equation of state (EoS) (see, for example, references in [1]). The very recent fascinating finding [2] of an anomalous dependence of strange particles abundance and of the inverse slope of transversal momentum distributions on the heavy-ion collision energy can be considered as a new experimental signal of the deconfinement transition and possible formation of a mixed quark–hadron phase. The analysis of these effects calls for advanced dynamical models of hydrodynamical nature allowing one to use various EoS. In the present paper we demonstrate the first results of a relativistic 3-fluid 3-dimensional hydrodynamic code developed for describing highly relativistic nucleus–nucleus collisions, i.e., just at energies where a partial deconfinement of hadrons may occur. Below, basic features of the hydrodynamic model will be presented, and a numerical solution of 3-fluid hydrodynamics will be compared with global observables.

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2. THREE-FLUID HYDRODYNAMIC MODEL

Supported by experiment, a specific feature of high-energy heavy-ion collisions is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. We simulate this counter-streaming behavior by introducing the multi-fluid concept. The basic idea of a 3-fluid approximation to heavy-ion collisions [3,4] is that at each space–time point $x = (t, \mathbf{x})$ the distribution function of baryon-rich matter, $f_b(x,p)$, can be represented by a sum of two distinct contributions

$$f_b(x,p) = f_p(x,p) + f_t(x,p),$$

initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, we shall associate newly produced particles, populating the midrapidity region with a fireball (f) fluid described by the distribution function $f_f(x,p)$.

Using the standard procedure for deriving hydrodynamic equations from the coupled set of relativistic Boltzmann equations with the above-introduced distribution functions f_α $(\alpha = p, t, f)$, we arrive at equations for the baryon charge conservation

$$\partial_\mu J^\mu_\alpha(x) \quad = \quad 0 \tag{1}$$

(for $\alpha = p$ and t) and the energy-momentum conservation of the fluids

$$\partial_\mu T^{\mu\nu}_p(x) \quad = \quad -F^\nu_p(x) + F^\nu_{fp}(x), \tag{2}$$

$$\partial_\mu T^{\mu\nu}_t(x) \quad = \quad -F^\nu_t(x) + F^\nu_{ft}(x), \tag{3}$$

$$\partial_\mu T^{\mu\nu}_f(x) \quad = \quad F^\nu_p(x) + F^\nu_t(x) - F^\nu_{fp}(x) - F^\nu_{ft}(x). \tag{4}$$

Here the baryon current $J^\mu_\alpha = n_\alpha u^\mu_\alpha$ is defined in terms of proper baryon density n_α and hydrodynamic 4-velocity u^μ_α. Equation (1) implies that there is no baryon-charge exchange between p and t fluids, as well as that the baryon current of the fireball fluid is identically zero, $J^\mu_f = 0$. The energy-momentum tensors

$$T^{\mu\nu}_\alpha = (\varepsilon_\alpha + P_\alpha)\, u^\mu_\alpha\, u^\nu_\alpha - g^{\mu\nu} P_\alpha \tag{5}$$

in Eqs. (2), (3) are affected by friction forces F^ν_p, F^ν_t, F^ν_{fp}, and F^ν_{ft}. Friction forces between baryon-rich fluids, F^ν_p and F^ν_t, partially transform the collective incident energy into thermal excitation of these fluids $(|F^\nu_p - F^\nu_t|)$ and give rise to particle production into the fireball fluid $(F^\nu_p + F^\nu_t)$, see Eq. (4). F^ν_{fp} and F^ν_{ft} correspond to friction of the fireball fluid with the p and t fluids, respectively.

As to the created fireball, the conventional hydrodynamic form (5) in terms of the proper energy density, ε_α, and pressure, P_α, is valid only for the thermalized part of the energy-momentum tensor

$$T^{(\mathrm{eq})\mu\nu}_f = (\varepsilon_f + P_f)\, u^\mu_f\, u^\nu_f - g^{\mu\nu} P_f. \tag{6}$$

Its evolution is defined by the Euler equation with a retarded source term

$$\partial_\mu T^{(\mathrm{eq})\mu\nu}_f(x) = \int d^4x' \delta^4 \left(x - x' - U_F(x')\tau\right) \left[F^\nu_p(x') + F^\nu_t(x')\right] - \\ - F^\nu_{fp}(x) - F^\nu_{ft}(x), \tag{7}$$

where τ is the formation time, and $U_F^\nu(x')$ is a free-streaming 4-velocity of the produced fireball matter [1]. In fact, this is the velocity at the moment of production of the fireball matter. According to Eq. (7), the energy and momentum of this matter appear as a source in the Euler equation only later, at the time $U_F^0\tau$ after production, and in different space point $\mathbf{x}' - \mathbf{U}_F(x')\tau$, as compared to the production point $\mathbf{x}'$.

The residual part of $T_f^{\mu\nu}$ (the free-streaming one) is defined as

$$T_f^{(\mathrm{fs})\mu\nu} = T_f^{\mu\nu} - T_f^{(\mathrm{eq})\mu\nu}. \tag{8}$$

The equation for $T_f^{(\mathrm{fs})\mu\nu}$ can be easily obtained by taking the difference between Eqs. (4) and (7). If all the fireball matter turns out to be formed before freeze-out (which is the case, in fact), then this equation is not needed. Thus, the 3-fluid model introduced here contains both the original 2-fluid model with pion radiation [3,5,6] and the (2+1)-fluid model [7,8] as limiting cases for $\tau \to \infty$ and $\tau = 0$, respectively.

Friction forces F_α^ν are parameterized using nucleon–nucleon cross sections for α fluids and assuming that pion–nucleon absorption is a dominant channel in friction $F_{f\alpha}^\nu$ between the fireball and baryonic matter [1].

Equations (1)–(3) and (7), supplemented by a certain equation of state (EoS) and expressions for friction forces F^ν, form a full set of equations of the relativistic 3-fluid hydrodynamic model. These equations are solved numerically by the particle-in-cell method.

3. COLLISION DYNAMICS AND PROTON RAPIDITY SPECTRA

The relativistic 3D code for the above described 3-fluid model was constructed by means of modifying the existing 2-fluid 3D code of Refs. [3,5,6]. In actual calculations we used the mixed-phase EoS developed in [10,11]. This phenomenological EoS takes into account a possible deconfinement phase transition of nuclear matter. The underlying assumption of this EoS is that unbound quarks and gluons may coexist with hadrons in the nuclear environment. In accordance with lattice QCD data, the statistical mixed-phase model describes the first-order deconfinement phase transition for pure gluon matter and crossover for that with quarks [10,11]. For details concerning the used EoS's, please, refer to [12].

In Fig. 1, global dynamics of heavy-ion collisions is illustrated by the energy-density evolution of the baryon-rich fluids ($\varepsilon_b = \varepsilon_p + \varepsilon_t$) in the reaction plane of the Pb + Pb collision at $E_{\mathrm{lab}} = 158$ $A\cdot$GeV. Different stages of interaction at relativistic energies are clearly seen in this example: Two Lorentz-contracted nuclei (note the different scales along the x and z axes in Fig. 1) start to interpenetrate through each other, reach a maximal energy density by the time ~ 1.1 fm/c and then expand predominantly in longitudinal direction forming a «sausage-like» freeze-out system. At this and lower incident energies the baryon-rich dynamics is not really disturbed by the fireball fluid and hence the cases $\tau = 0$ and 1 fm/c turned to be indistinguishable in terms of ε_b.

Time evolution of ε_b in Fig. 1 is calculated for the mixed phase model. Topologically results for EoS of pure hadronic and that of two-phase models look very similar. Due to essential softening of the equation of state near the deconfinement phase transition, in the last case, the system evolves noticeably slower what may have observable consequences [10,11].

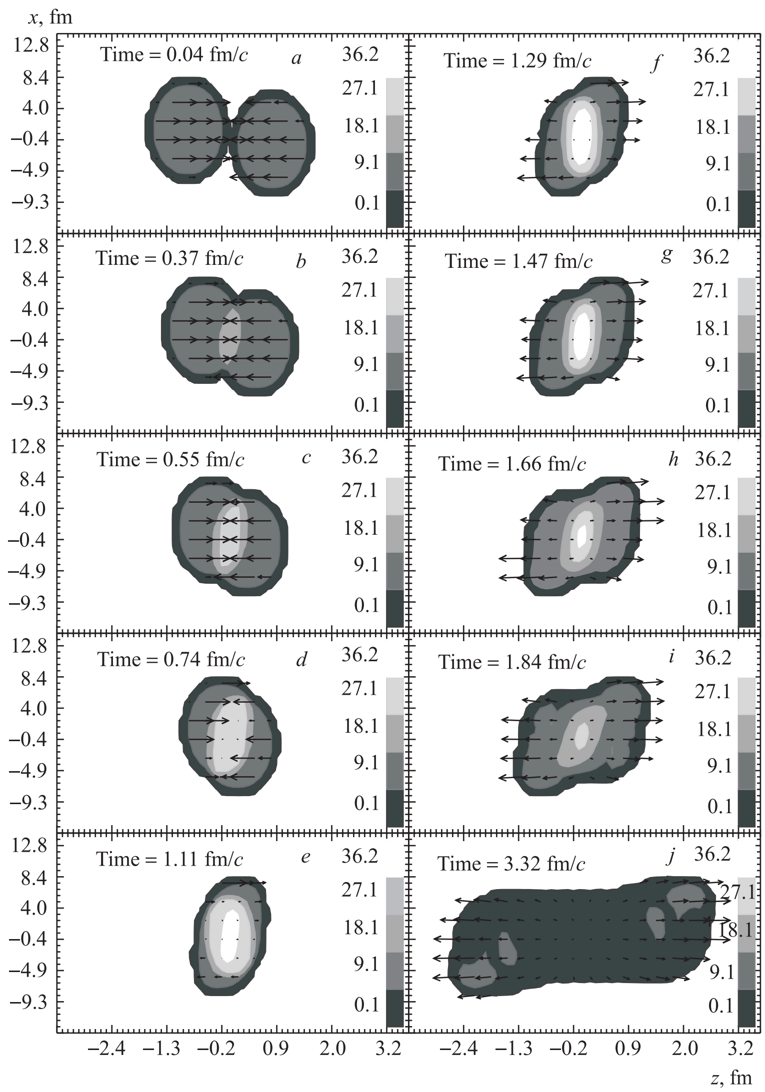

Fig. 1. Time evolution of the energy density, $\varepsilon_b = \varepsilon_p + \varepsilon_t$, for the baryon-rich fluids in the reaction plane (xz plane) for the Pb + Pb collision ($E_{\text{lab}} = 158\ A\cdot$GeV) at impact parameter $b = 2$ fm. Shades of gray represent different levels of ε_b as indicated at the right side of each panel. Numbers at this palette show the ε_b values (in GeV/fm^3) at which the shades change. Arrows indicate the hydrodynamic velocities of the fluids

The energy released in the fireball fluids is of an order of magnitude smaller than that stored in baryon-rich fluids and depends on the formation time. At realistic values of the formation times, $\tau \sim 1$ fm/c, the effect of the interaction is substantially reduced. It happens because the fireball fluid starts to interact only near the end of the interpenetration stage.

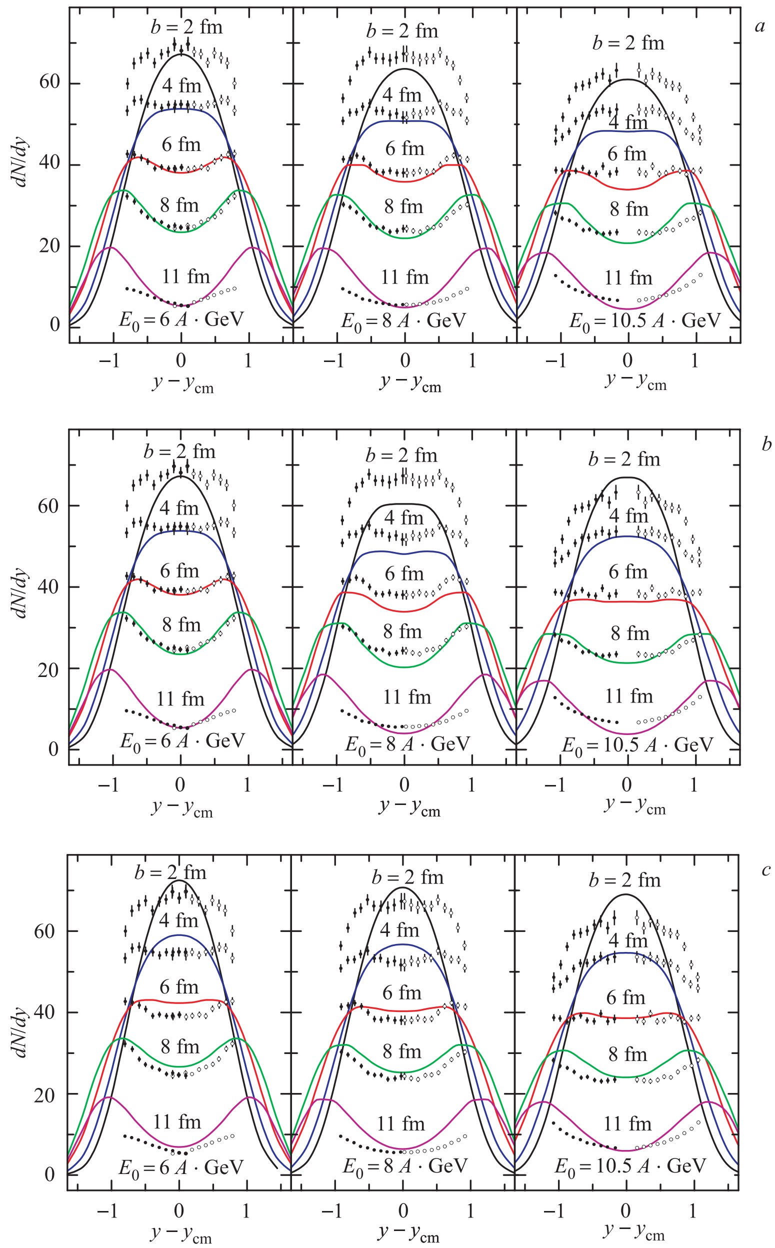

Fig. 2. Proton rapidity spectra from Au + Au collisions at three bombarding energies and different impact parameters b. Three panels correspond to different equations of state: a) hadron gas EoS; b) two-phase model EoS; c) mixed-phase model EoS. Experimental points are from [13]

As a result, by the end of the collision process it looses only 10% of its available energy at $E_{\rm lab} = 158$ $A\cdot$GeV; and 30%, at $E_{\rm lab} = 10.5$ $A\cdot$GeV. Certainly, this effect should be observable in mesonic quantities, in particular, in such fine observables as directed and elliptic flows. The global baryonic quantities stay practically unchanged at finite τ [1].

To calculate observables, hydrodynamic calculations should be stopped at some freeze-out point. In our model it is assumed that a fluid element decouples from the hydrodynamic regime, when its energy density ε and densities in the eight surrounding cells become smaller than a fixed value $\varepsilon_{\rm fr}$. A value $\varepsilon_{\rm fr} = 0.15$ GeV/fm^3 was used for this local freeze-out density which corresponds to the actual energy density of the freeze-out fluid element of ~ 0.12 GeV/fm^3. To proceed observing free hadron gas, the shock-like freeze-out [14] is assumed, conserving energy and baryon charge.

Proton rapidity spectra calculated for Au + Au collisions are presented in Fig. 2 for $E_{\rm beam} = 6, 8$, and 10.5 $A\cdot$GeV. One should note that hydrodynamics does not make difference between bounded into fragments and free nucleons. In the results presented the contribution from light fragments (d, t, ^{3}He and ^{4}He) has been substracted using a simple coalescence model [15]. This procedure allows one to reasonably reproduce evolution of the spectra shape with changing the impact parameter b for all the energies considered. Some discrepancy observed for peripheral collisions near the target and projectile rapidity are mainly due to a not-subtacted contribution of heavier fragments. As is seen, the rapidity spectra are only slightly sensitive to the EoS used.

CONCLUSIONS

In this paper the developed 3-fluid model has been presented for simulating heavy-ion collisions in the range of incident energies between few and about 200 $A\cdot$GeV. In addition to two baryon-rich fluids, which constitute the 2-fluid model, a delayed evolution of the produced baryon-free (fireball) fluid is incorporated. This delay is governed by a formation time, during which the fireball fluid neither thermalizes nor interacts with the baryon-rich fluids. After the formation, it thermalizes and comes into interaction with the baryon-rich fluids. Implementation of different EoS, including those with the deconfinement phase transition, may open great opportunities for analysis of collective-effects deconfinement phase transition in relativistic heavy-ion collisions. Unfortunately, in spite of a reasonable reproduction of observable proton rapidity spectra in the wide range of bombarding energies and centrality parameters, we are unable to favor any of considered EoS. Analysis of more delicate characteristics such as the excitation functions for strange hadrons, mentioned in introduction, is needed. This work is in progress now.

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