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## SCALAR $\sigma$ MESON AT FINITE TEMPERATURE IN NONLOCAL QUARK MODEL<sup>1</sup>

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Properties and temperature behavior of  $\pi$  and  $\sigma$  bound states are studied in the framework of the nonlocal model with a separable interaction kernel based on the quark Dyson–Schwinger and the meson Bethe–Salpeter equations.  $M_\pi(T)$ ,  $f_\pi(T)$ ,  $M_\sigma(T)$  and  $\Gamma_{\sigma \rightarrow \pi\pi}(T)$  are considered above and below the deconfinement and chiral restoration transitions.

На основе уравнений Дайсона–Швингера для кварков и Бете–Солпитера для мезонов изучаются свойства и температурное поведение связанных состояний  $\pi$  и  $\sigma$  в рамках нелокальной модели с сепарабельным ядром взаимодействия. Выше и ниже переходов деконфайнмента и кирального восстановления рассмотрены  $M_\pi(T)$ ,  $f_\pi(T)$ ,  $M_\sigma(T)$  и  $\Gamma_{\sigma \rightarrow \pi\pi}(T)$ .

### INTRODUCTION

Understanding the behavior of matter under extreme conditions is nowadays a challenge in the physics of strong interactions. Different regions of the QCD phase diagram are an object of interest, and major theoretical and experimental efforts have been dedicated to the physics of relativistic heavy-ion collisions looking for signatures of the quark–gluon plasma [QGP] [1–3].

Restoration of symmetries and deconfinement are expected to occur at high density and/or temperature. In this regard, the study of observables of pseudoscalar and scalar mesons is particularly important. Since the origin of these mesons is associated with the phenomena of spontaneous and explicit chiral symmetry breaking, its temperature behavior is expected to carry relevant signs of a possible restoration of symmetries. Usually, the restoration of chiral symmetry at high temperature is connected with the transition of hadron matter into quark–gluon plasma.

Effective quark models are useful tools to explore the behavior of matter at temperatures. Nambu–Jona-Lasinio [NJL] [4] type models have been extensively used over the past years to describe low-energy features of hadrons and also to investigate restoration of chiral symmetry with temperature [5–8].

This paper is devoted to investigation of the phase transition in hot matter and the temperature behavior of pseudoscalar as well as scalar mesons in the framework of the effective nonlocal model. This work is the continuation of [9]. In [9], a special separable form of

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effective gluon propagator is used in construction of the quark Dyson–Schwinger equation (DSE) and the Bethe–Salpeter equation (BSE) for bound states. Only pseudoscalar and vector mesons are considered in that paper. Here we concentrate on the properties of the scalar  $\sigma$  meson at finite temperature.

## 1. DYSON–SCHWINGER EQUATION WITH SEPARABLE INTERACTION

The dressed quark propagator  $S(p)$  and meson Bethe–Salpeter (BS) amplitude  $\Gamma(p, P)$  are solutions of the DSE [12–14]

$$S(p)^{-1} = i\hat{p} + m_0 + \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}^{\text{eff}}(p-q) \gamma_\mu S(q) \gamma_\nu \quad (1)$$

and the BSE equation

$$\Gamma(p, P) = \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}^{\text{eff}}(p-q) \gamma_\mu S(q_+) \Gamma(q, P) S(q_-) \gamma_\nu, \quad (2)$$

where<sup>1</sup>  $D_{\mu\nu}^{\text{eff}}(p-q)$  is an «effective gluon propagator»;  $m_0$  is the current quark mass;  $P$  is the total momentum, and  $q_\pm = q \pm P/2$ . The form of equations (1) and (2) corresponds to the rainbow-ladder truncations of DSE and BSE.

The simplest separable Ansatz  $g^2 D_{\mu\nu}^{\text{eff}}(p-q) \rightarrow \delta_{\mu\nu} D(p^2, q^2, p \cdot q)$  in a Feynman-like gauge is employed:

$$D(p^2, q^2, p \cdot q) = D_0 f_0(p^2) f_0(q^2) + D_1 f_1(p^2) (p \cdot q) f_1(q^2). \quad (3)$$

This is a rank-2 interaction with two strength parameters  $D_0$ ,  $D_1$  and the corresponding form factors  $f_i(p^2)$ . The choice for these quantities is constrained to the solution of the DSE for the quark propagator (1)

$$S(p)^{-1} = i\hat{p}A(p^2) + B(p^2) \quad (4)$$

with  $B(p^2) = m_0 + b f_0(p^2)$  and  $A(p^2) = 1 + a f_1(p^2)$ , where  $a$  and  $b$  are some constants. If there are no poles in the quark propagator  $S(p)$  for real timelike  $p^2$ , then there is no physical quark mass shell and a meson cannot decay into quarks. The propagator is confining<sup>2</sup> if  $m^2(p^2) \neq -p^2$  for real  $p^2$  where the quark mass function is  $m(p^2) = B(p^2)/A(p^2)$ . It is found that the simple choice  $f_i(p^2) = \exp(-p^2/\Lambda_i^2)$  leads to a reasonable description of  $\pi$  and  $\sigma$  meson properties. At the same time, the produced quark propagator is found to be confining and the infrared strength and shape of the quark amplitudes  $A(p^2)$  and  $B(p^2)$  are in qualitative agreement with the results of typical DSE studies. We use the exponential form factors as a minimal way to preserve these properties while realizing that the ultraviolet suppression is much greater than the power law fall-off (with logarithmic corrections) known from asymptotic QCD. The total number of model parameters is five.

<sup>1</sup>We use the Euclidean metric.

<sup>2</sup>A similar confining propagator without poles is used in a nonlocal model of NJL type [10, 11].

The extension of the separable model studies to the finite temperature case,  $T \neq 0$ , is systematically accomplished by a transcription of the Euclidean quark 4-momentum via  $q \rightarrow q_n = (\omega_n, \mathbf{q})$ , where  $\omega_n = (2n + 1)\pi T$  are the discrete Matsubara frequencies. The effective  $\bar{q}q$  interaction will automatically decrease with increasing  $T$  without the introduction of an explicit  $T$ -dependence which would require new parameters.

The result of the DSE solution for the dressed quark propagator now becomes

$$S^{-1}(p_n, T) = i\boldsymbol{\gamma} \cdot \mathbf{p} A(p_n^2, T) + i\gamma_4 \omega_n C(p_n^2, T) + B(p_n^2, T), \quad (5)$$

where  $p_n^2 = \omega_n^2 + \mathbf{p}^2$ . The solutions have the form  $B = m_0 + b(T)f_0(p_n^2)$ ,  $A = 1 + a(T)f_1(p_n^2)$ , and  $C = 1 + c(T)f_1(p_n^2)$ , and the DSE becomes a set of three nonlinear equations for  $b(T)$ ,  $a(T)$ , and  $c(T)$ . The explicit form is

$$a(T) = \frac{8D_1}{9} T \sum_n \int \frac{d^3 p}{(2\pi)^3} f_1(p_n^2) \mathbf{p}^2 [1 + a(T)f_1(p_n^2)] d^{-1}(p_n^2, T), \quad (6)$$

$$c(T) = \frac{8D_1}{3} T \sum_n \int \frac{d^3 p}{(2\pi)^3} f_1(p_n^2) \omega_n^2 [1 + c(T)f_1(p_n^2)] d^{-1}(p_n^2, T), \quad (7)$$

$$b(T) = \frac{16D_0}{3} T \sum_n \int \frac{d^3 p}{(2\pi)^3} f_0(p_n^2) [m_0 + b(T)f_0(p_n^2)] d^{-1}(p_n^2, T), \quad (8)$$

where  $d(p_n^2, T)$  is given by

$$d(p_n^2, T) = \mathbf{p}^2 A^2(p_n^2, T) + \omega_n^2 C^2(p_n^2, T) + B^2(p_n^2, T). \quad (9)$$

## 2. PSEUDOSCALAR AND SCALAR CORRELATIONS

With the separable interaction the allowed form of the solution of (2) for the pion BS amplitude  $\tau \Gamma_\pi(q; P)$  is

$$\Gamma_\pi(q; P) = \gamma_5 \left( iE_\pi(P^2) + \hat{P}F_\pi(P^2) \right) f_0(q^2). \quad (10)$$

The  $q$  dependence is described only by the first form factor  $f_0(q^2)$ . The second term  $f_1$  of the interaction can contribute to BS amplitude only indirectly via the quark propagators. The pion BSE, (2), becomes a  $2 \times 2$  matrix eigenvalue problem  $\mathcal{K}(P^2)f = \lambda(P^2)f$  where the eigenvector is  $f = (E_\pi, F_\pi)$ . The kernel is

$$\mathcal{K}_{ij}(P^2) = -\frac{4D_0}{3} \text{tr}_s \int \frac{d^4 q}{(2\pi)^4} f_0^2(q^2) [\hat{t}_i S(q_+) t_j S(q_-)], \quad (11)$$

where the  $\pi$  covariants are  $t = (i\gamma_5, \gamma_5 \hat{P})$  with  $\hat{t} = (i\gamma_5, -\gamma_5 \hat{P}/2P^2)$ . We note that the separable model produces the same  $q^2$  shape for both amplitudes  $F_\pi$  and  $E_\pi$ ; the shape is that of the quark amplitude  $B(q^2)$ . Goldstone's theorem is preserved by the present separable model; in the chiral limit, whenever a nontrivial DSE solution for  $B(p^2)$  exists, there will be a massless  $\pi$  solution to (11).

The normalization condition for the pion BS amplitude can be expressed as

$$2P_\mu = 2N_c \frac{\partial}{\partial P_\mu} \int \frac{d^4q}{(2\pi)^4} \text{tr}_s \left( \bar{\Gamma}_\pi(q; -K) S(q_+) \Gamma_\pi(q; K) S(q_-) \right) \Big|_{P^2=K^2=-M_\pi^2}. \quad (12)$$

Here  $\bar{\Gamma}(q; K)$  is the charge conjugate amplitude  $[\mathcal{C}^{-1}\Gamma(-q, K)\mathcal{C}]^t$ , where  $\mathcal{C} = \gamma_2\gamma_4$  and the index  $t$  denotes a matrix transpose. The pion decay constant  $f_\pi$  can be expressed as the loop integral

$$f_\pi P_\mu \delta_{ij} = \left\langle 0 \left| \bar{q} \frac{\tau_i}{2} \gamma_\mu \gamma_5 q \right| \pi_j(P) \right\rangle = \delta_{ij} N_c \text{tr}_s \int \frac{d^4q}{(2\pi)^4} \gamma_5 \gamma_\mu S(q_+) \Gamma_\pi(q; P) S(q_-). \quad (13)$$

For a chiral partner of the pion,  $\sigma$  meson, we take the BS amplitude with only one covariant

$$\Gamma_\sigma(q; P) = E_\sigma(P^2) f_0(q^2). \quad (14)$$

At  $T = 0$  the mass-shell condition for a meson as a  $\bar{q}q$  bound state of the BSE is equivalent to the appearance of a pole in the  $\bar{q}q$  scattering amplitude as a function of  $P^2$ . At  $T \neq 0$  in the Matsubara formalism, the  $O(4)$  symmetry is broken by the heat bath and we have  $P \rightarrow (\Omega_m, \mathbf{P})$  where  $\Omega_m = 2m\pi T$ . Bound states and the poles they generate in propagators may be investigated through polarization tensors, correlators or Bethe–Salpeter eigenvalues. This pole structure is characterized by information at discrete points  $\Omega_m$  on the imaginary energy axis and at a continuum of 3-momenta. One may search for poles as a function of  $\mathbf{P}^2$ , thus identifying the so-called spatial or screening masses for each Matsubara mode. These serve as one particular characterization of the propagator and the  $T > 0$  bound states.

In the present context, the eigenvalues of the BSE become  $\lambda(P^2) \rightarrow \tilde{\lambda}(\Omega_m^2, \mathbf{P}^2; T)$ . The temporal meson masses identified by zeros of  $1 - \tilde{\lambda}(\Omega^2, 0; T)$  will differ in general from the spatial masses identified by zeros of  $1 - \tilde{\lambda}(0, \mathbf{P}^2; T)$ . They are however identical at  $T = 0$  and an approximate degeneracy can be expected to extend over the finite  $T$  domain, where the  $O(4)$  symmetry is not strongly broken.

The general form of the finite  $T$  pion BS amplitude allowed by the separable model is

$$\Gamma_\pi(q_n; P_m) = \gamma_5 \left( iE_\pi(P_m^2) + \gamma_4 \Omega_m \tilde{F}_\pi(P_m^2) + \boldsymbol{\gamma} \cdot \mathbf{P} F_\pi(P_m^2) \right) f_0(q_n^2). \quad (15)$$

The separable BSE becomes a  $3 \times 3$  matrix eigenvalue problem with a kernel that is a generalization of Eq. (11). In the limit  $\Omega_m \rightarrow 0$ , as is required for the spatial mode of interest here, the amplitude  $\hat{F}_\pi = \Omega_m \tilde{F}_\pi$  is trivially zero.

The BS amplitude for the  $\sigma$  at finite temperature has only one covariant.

The impulse approximation for the  $\sigma\pi\pi$  vertex, after the extension to  $T > 0$  for spatial modes characterized by  $Q = (0, \mathbf{Q})$  for the  $\sigma$  and  $P = (0, \mathbf{P})$  for the relative  $\pi\pi$  momentum, takes the following form:

$$g_{\sigma\pi\pi}(T) = -2N_c T \sum_n \text{tr}_s \int \frac{d^3q}{(2\pi)^3} \Gamma_\pi(k_{n+}; -\mathbf{P}_+) S(q_{n+-}) \Gamma_\sigma(q_{n+}; \mathbf{Q}) \times \\ \times S(q_{n++}) \Gamma_\pi(k_{n-}; \mathbf{P}_-) S(q_{n-}). \quad (16)$$

The corresponding width of the  $\sigma$  meson is equal to

$$\Gamma_{\sigma \rightarrow \pi\pi}(T) = \frac{3}{2} \frac{g_{\sigma\pi\pi}^2(T)}{16\pi M_\sigma} \sqrt{1 - \frac{4M_\pi^2(T)}{M_\sigma^2(T)}}. \quad (17)$$

### 3. NUMERICAL ANALYSIS AND CONCLUSIONS

The results for  $\pi$ ,  $\sigma$  and  $\rho$  mesons, as well as related quantities, are calculated with the parameters  $m_0 = 6.3$  MeV,  $b = 730$  MeV,  $a = 0.52$ ,  $\Lambda_0 = 698$  MeV,  $\Lambda_1 = 1.78$  GeV,  $D_0\Lambda_0^2 = 223$ ,  $D_1\Lambda_1^4 = 137$ . This set allows one to reproduce the experimental data for  $M_\pi = 140$  MeV,  $f_\pi = 93$  MeV and  $M_\rho = 770$  MeV and obtain the reasonable result for the  $\rho \rightarrow \pi\pi$  decay width  $\Gamma_{\rho \rightarrow \pi\pi} = 228$  MeV ( $\Gamma_{\rho \rightarrow \pi\pi}^{\text{exp}} = 151$  MeV). For the quark condensate we have  $\langle \bar{q}q \rangle^0 = (0.212 \text{ GeV})^3$  and the quark mass function at zero momentum equals  $m(0) = 484$  MeV.

We use this set of parameters to calculate physical observables of the scalar meson. As a result, we obtain  $M_\sigma = 762$  MeV and  $\Gamma_{\sigma \rightarrow \pi\pi} = 456$  MeV.

In this work, we consider the  $T$ -dependence of the quark mass function  $m(p)$ ,  $f_\pi$  and the spatial masses in the  $\pi$  and  $\sigma$  channels.

The  $T$ -dependence of  $m(0)$ ,  $m_0(0)$  and  $f_\pi$  is displayed in Fig. 1;  $m_0(0)$  corresponds to the solution of DSE in the chiral limit  $m_0 = 0$ . The temperature DSE equation contains

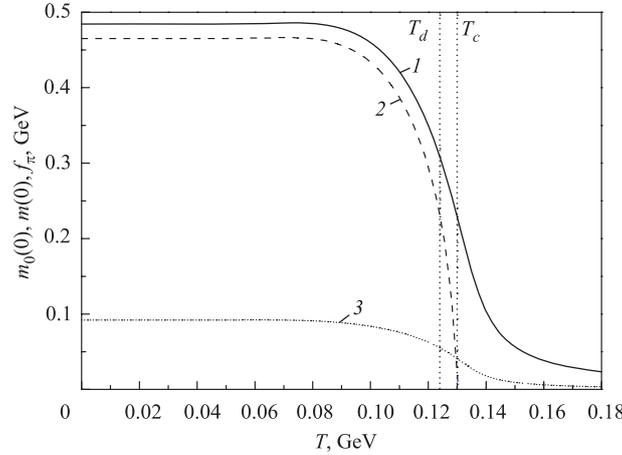


Fig. 1. Temperature dependence of the quark mass function  $m(0)$  (1). Curve 2 corresponds to the quark mass function in chiral limit,  $m_0(0)$ . Curve 3 shows the temperature dependence of the decay constant  $f_\pi$

three functions  $A(p_n^2, T)$ ,  $C(p_n^2, T)$  and  $B(p_n^2, T)$ . The numerical calculations show that the solutions  $A(p_n^2, T)$  and  $C(p_n^2, T)$  as functions of temperature practically coincide in the region  $0 < T < 200$  MeV. The temperature  $T_d = 123$  MeV (quark deconfinement temperature) presented in Fig.1 is determined by the properties of Green quark function in the DSE

equation. It corresponds to the appearance of the poles at real momentum in the lowest Matsubara quark mode.

The critical temperature  $T_c$  is that when the quark condensate disappears and chiral symmetry is restored.

In the chiral limit the pion is massless below  $T_c$  and its mass increases above  $T_c$ ; the  $\sigma$ -meson mass drops to zero at  $T_c$  and above this temperature it is degenerated with  $M_\pi$ . When  $m_0 \neq 0$   $M_\pi(T)$  is seen to be only weakly  $T$ -dependent up to near  $T_c$ , where a sharp rise begins, as displayed in Fig. 2. These qualitative features of the response of the pion mode with  $T$  agree with the results deduced from the DSE in [12–14].

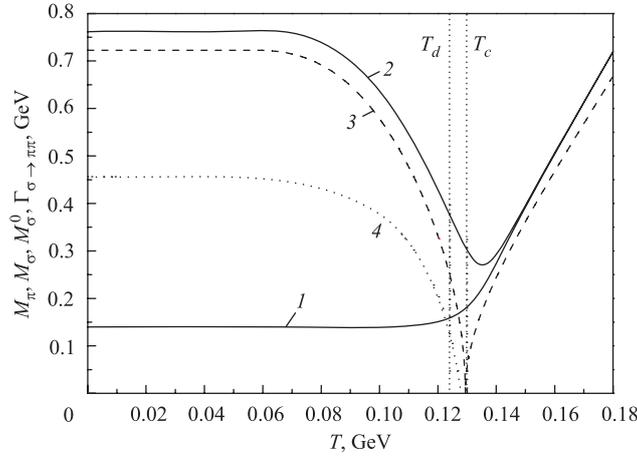


Fig. 2. Temperature dependence of  $M_\pi$  (1),  $M_\sigma$  (2),  $M_\sigma^0$  (3) and  $\Gamma_\sigma$  (4)

The temperature dependence of the width  $\Gamma_{\sigma \rightarrow \pi\pi}$  is also shown in Fig. 2.

In conclusion, we have studied the  $\pi$ - and  $\sigma$ -meson properties and finite temperature in the framework of the nonlocal quark model which was used in [9]. The temperature dependence of these quantities shows that the restoration of chiral symmetry,  $M_\sigma \sim M_\pi$ , occurs at the critical temperature  $T_c = 130$  MeV.

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