

## FINE STRUCTURE OF STRENGTH FUNCTION FOR $\beta^+/EC$ DECAY OF $^{160g}\text{Ho}$ (25.6 min)

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A strength function for the  $\beta^+/EC$  decay of the deformed  $^{160g}\text{Ho}$  (25.6 min) nucleus has been obtained from the experimental data. The fine structure of the strength function  $S_\beta(E)$  is analyzed. It is found to have a pronounced resonant structure for Gamow–Teller transitions. In  $S_\beta(E)$  with  $\mu_\tau = +1$  the Gamow–Teller resonance is observed to split into two components. This splitting is associated with anisotropy of isovector density oscillation in deformed nuclei. The  $\beta^+/EC$  strength function for first-forbidden transitions is obtained in the Coulomb ( $\xi$ ) approximation. It is shown that  $S_\beta(E)$  for first-forbidden transitions does not have a pronounced resonant structure.

В работе из экспериментальных данных получена силовая функция  $\beta^+/EC$ -распада деформированного ядра  $^{160g}\text{Ho}$  (25,6 мин). Проанализирована тонкая структура силовой функции  $S_\beta(E)$ . Для переходов Гамова–Теллера  $S_\beta(E)$  имеет ярко выраженную резонансную структуру. Обнаружено расщепление резонанса Гамова–Теллера в  $S_\beta(E)$  с  $\mu_\tau = +1$  на две компоненты. Данное расщепление связывается с анизотропией колебаний изовекторной плотности в деформированных ядрах. В кулоновском ( $\xi$ ) приближении получена силовая функция  $\beta^+/EC$ -переходов запрета I. Показано, что  $S_\beta(E)$  для переходов запрета I не имеет выраженной резонансной структуры.

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### INTRODUCTION

The strength function for  $\beta$  transitions  $S_\beta(E)$  is one of the most important characteristics of an atomic nucleus [1–3]. It is a distribution of moduli squared of  $\beta$ -decay type matrix elements in nuclear excitation energies  $E$ . At energies  $E$  up to  $Q_\beta$  (total  $\beta$ -decay energy),  $S_\beta(E)$  defines the character of the  $\beta$  decay and the half-life  $T_{1/2}$  of a radioactive nucleus against the  $\beta$  decay. At high energies unachievable in the  $\beta$  decay,  $S_\beta(E)$  defines cross sections for various nuclear reactions that depend on the  $\beta$ -decay type matrix elements.

The  $\beta$ -decay probability is proportional to a product of the lepton part described by the Fermi function  $f(Q_\beta - E)$  and the nucleon part described by  $S_\beta(E)$ . Since the Fermi function rapidly decreases with increasing  $E$ , the probability of  $\beta$  transitions at excitation energies  $E$  larger than 2–3 MeV in medium and heavy nuclei is small. However, from the point of view of the nuclear structure and description of the  $\beta$  decay, it is the character of  $S_\beta(E)$  at excitation energies larger than 2–3 MeV that is most interesting. When  $E > 2–3$  MeV, resonances caused by the nuclear structure and residual spin–isospin interaction arise in  $S_\beta(E)$  [1–5].

Until recently the  $S_\beta(E)$  structure was experimentally studied by using total absorption gamma-ray spectrometers and total absorption spectroscopy (TAS) methods [1–3, 6, 7], where

$\gamma$ -rays accompanying the  $\beta$  decay were detected by large NaI crystals in the  $4\pi$  geometry. If the total absorption efficiency for  $\gamma$ -rays is large enough, total absorption peaks, whose intensity is governed solely by the probability of  $\beta$ -decay population of levels, can be identified in the spectra. This method allowed the resonant structure of  $S_\beta(E)$  for Gamow–Teller  $\beta$  transitions to be experimentally demonstrated [1–4]. However, TAS methods have some disadvantages arising from a low energy resolution of NaI-based spectrometers. Only one or two total absorption peaks can be identified in TAS spectra, isobaric impurities in the analyzed source often give rise to uncertainties, Gamow–Teller and first-forbidden  $\beta$  transitions cannot be separated, the fine structure of  $S_\beta(E)$  cannot be measured, and problems often arise in processing of the spectra.

Therefore, it appears important to measure  $S_\beta(E)$  by using methods of high-resolution  $\gamma$  spectroscopy. This is an arduous problem which has been solved so far only for the  $\beta^+$ /EC decay of the spherical nucleus  $^{147g}\text{Tb}$  ( $T_{1/2} = 1.6$  h,  $Q_{EC} = 4.6$  MeV) [3–5, 8].

In this study  $S_\beta(E)$  was obtained from investigation of the  $\beta^+$ /EC decay of the deformed  $^{160g}\text{Ho}$  (25.6 min) nucleus by the high-resolution nuclear spectroscopy methods. The fine structure of  $S_\beta(E)$  was analyzed and specific features of  $S_\beta(E)$  for deformed nuclei were revealed.

### 1. ON THE $^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$ DECAY SCHEME

The most complete scheme of  $^{160}\text{Dy}$  levels excited in the decays of  $^{160m}\text{Ho}$  ( $2^-$ ; 5.02 h) and  $^{160g}\text{Ho}$  ( $5^+$ ; 25.6 min) was proposed in [9]. It comprises about 750  $\gamma$  transitions out of  $\sim 800$  observed by the authors. They measured  $^{m,g}\text{Ho}$   $\gamma$ -ray spectra in the «chain» decay

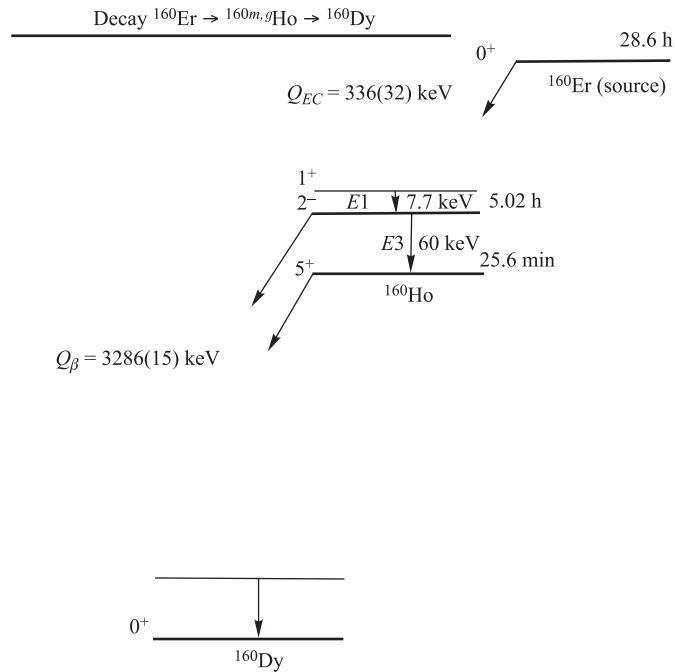


Fig. 1.  $^{160}\text{Er} \rightarrow ^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$  decay

$^{160}\text{Er}$  (28.6 h)  $\rightarrow$   $^{160m,g}\text{Ho} \rightarrow$   $^{160}\text{Dy}$  (Fig. 1) at practically equilibrium decay of isotopes. The parent isotope  $^{160}\text{Er}$  has a low decay energy  $Q_{EC} = (336 \pm 32)$  keV, and therefore its presence in the chain made it difficult to identify only soft-energy  $\gamma$  transitions.

A wealth of experimental data [9] on  $\gamma$  transitions in  $^{160}\text{Ho}$  was further analyzed in [10], which yielded, as a result, a refined scheme of excited  $^{160}\text{Dy}$  levels. The table presents a refined list of  $^{160}\text{Dy}$  levels populated by the decay of the  $^{160g}\text{Ho}$  ground state.

Levels of  $^{160}\text{Dy}$  populated by the  $\beta^+/EC$  decay of  $^{160g}\text{Ho}$  (25.6 min)

Level energy, keV	Quantum characteristics ( $I^\pi K$ )	Population by $\beta^+/EC$ decay, %	$\log ft$
1386.46(2)	4 <sup>-</sup>	0.27(3)	7.32(5)
1438.57(3)	6 <sup>+</sup>	0.2(1)	7.4(2)
1535.14(2)	4 <sup>-</sup>	0.11(2)	7.63(8)
1603.77(8)	4 <sup>+</sup>	0.07(2)	7.8(1)
1606.9(1)	6 <sup>+</sup>	0.10(1)	7.63(5)
1607.9(1)	4 <sup>+</sup> S	0.08(2)	7.7(1)
1650.87(4)	5 <sup>-</sup>	0.02(1)	8.3(2)
1652.1(1)	(4, 5, 6)	0.16(2)	7.41(6)
1694.36(2)	4 <sup>+</sup> 4	74(5)	4.72(3)
1802.24(2)	5 <sup>+</sup> 4	10.8(9)	5.49(4)
1860.1(1)	5 <sup>-</sup> 4	0.11(1)	7.44(4)
1929.19(2)	6 <sup>+</sup> 4	0.62(7)	6.65(5)
2096.87(2)	4 <sup>+</sup> 4	2.9(2)	5.86(3)
2143.7(1)	(4 <sup>-</sup> )	0.13(2)	7.17(7)
2155.3(2)	(4 <sup>-</sup> )	0.035(3)	7.73(4)
2187.0(1)	(5, 6)	0.09(1)	7.30(5)
2194.43(3)	5 <sup>+</sup> 4	0.43(3)	6.61(4)
2208.4(1)	4 <sup>+</sup>	0.20(3)	6.93(7)
2374.5(1)	(4 <sup>-</sup> )	0.036(5)	7.52(7)
2556.8(1)	5 <sup>-</sup>	0.14(2)	6.73(7)
2572.4(1)	(4)	0.032(3)	7.35(5)
2681.9(1)	5 <sup>+</sup>	0.8(1)	5.80(6)
2727.2(1)	(4)	0.037(5)	7.06(7)
2755.0(2)	(4 <sup>-</sup> )	0.027(4)	7.15(7)
2757.1(1)	(4, 5)	0.040(7)	6.97(8)
2763.0(1)	(4, 5)	0.07(1)	6.72(7)
2777.6(1)	(4) <sup>+</sup>	0.29(2)	6.07(5)
2853.6(1)	5 <sup>-</sup>	0.055(6)	6.64(6)
2941.7(1)	(4, 5, 6)	0.023(2)	6.79(6)
2969.9(1)	(4, 5)	0.06(2)	6.3(2)
2977.5(1)	(4, 5)	0.021(5)	6.7(1)
3033.7(2)	(4, 5) <sup>-</sup>	0.0031(9)	7.3(2)
3081.4(2)	(4, 5, 6)	0.0024(7)	7.2(2)

To calculate reduced probabilities ( $\log ft$ ) of  $\beta$  transitions in  $^{160g}\text{Ho}$ , one should know population of each  $^{160}\text{Dy}$  level in per cent per  $^{160g}\text{Ho}$  decay. Unfortunately, in [9] the authors give only populations of  $^{160}\text{Dy}$  levels in % per  $^{160}\text{Er}$  decay found from the balance of  $\gamma$ -transition intensities, that does not allow calculation of  $\log ft$ . They assumed that the

sum of total intensities ( $(5960 \pm 130)$  rel. un. of intensity in Table 1 in [9]) of  $\gamma$  transitions to the ground state of  $^{160}\text{Dy}$  is 100% of  $^{160}\text{Er}$  decays. The contribution from the decay via the  $^{160m}\text{Ho}(2^-)$  isomer should be taken into account. In [9] the isomer decay branching  $b = \text{IT/total}$  was determined from the comparison of the isomeric  $E_\gamma = 59.98$  keV  $E3$  transition ( $I_{\text{tot}} = (4390 \pm 310)$  rel. un.) with the above-mentioned total intensity. It turned out that the intensity of the isomeric 59.98-keV transition was 73.6(5.2)% of decays and the intensity of the  $\beta^+/\text{EC}$  decay of  $^{160m}\text{Ho}$  ( $2^-$ ) was 26.4(5.2)%. Later, soft  $\gamma$  radiation of a «chain» of  $A = 160$  isotopes was measured by a high-resolution X-ray spectrometer, which yielded [11] a more accurate branching coefficient for the isomeric state  $^{160m}\text{Ho}$  ( $2^-$ ):  $b = 0.733(30)$ .

It is noteworthy, however, that the branching coefficients obtained above depend on how well the  $^{160m,g}\text{Ho}$  decay schemes are established and how correctly time intervals are taken into account in measurements of the  $^{160}\text{Er} \rightarrow ^{160}\text{Ho} \rightarrow ^{160}\text{Dy}$  chain decay spectra. To reduce the effect of the above factors, the authors of [12] found the branching coefficient by measuring the intensities of the 59.98-keV  $\gamma$  transition and the Kx(Ho) radiation in the decay of the parent  $^{160}\text{Er}$  nucleus:  $b = \text{IT/total} = 0.779 \pm 0.020$ . It is obvious that all three results agree within the error bars. The weighted mean for the three values is 76(2)%.

In this paper we analyzed all excited  $^{160}\text{Dy}$  states known from [9] and singled out those which, according to angular momentum and parity selection rules, can be populated only by the  $\beta^+/\text{EC}$  decay of the ground state  $^{160}\text{Ho}$  (25.6 min) with the quantum characteristics  $I^\pi = 5^+$ . Then we found population of each of these levels by the  $^{160g}\text{Ho}$   $\beta^+/\text{EC}$  decay and calculated the reduced probabilities  $\log ft$  using the weighted mean branching 76(2)%. The results are presented in the table.

## 2. STRUCTURE OF THE STRENGTH FUNCTION FOR THE $\beta^+/\text{EC}$ DECAY OF $^{160g}\text{Ho}$

For the Gamow–Teller and first-forbidden  $\beta$  transitions in the  $\xi$  approximation and unique first-forbidden  $\beta$  transitions the reduced probabilities  $B(\text{GT})$ , [ $B(\lambda^\pi = 0^-) + B(\lambda^\pi = 1^-)$ ], [ $B(\lambda^\pi = 2^-)$ ], half-life  $T_{1/2}$ , level populations  $I(E)$ , strength function  $S_\beta(E)$ , and  $ft$  are related as follows [1, 2, 4, 5, 13]:

$$\frac{d(I(E))}{dE} = S_\beta(E)T_{1/2}f(Q - E), \quad (1)$$

$$(T_{1/2})^{-1} = \int S_\beta(E)f(Q - E)dE, \quad (2)$$

$$\int_{\Delta E} S_\beta(E)dE = \sum_{\Delta E} \frac{1}{(ft)}. \quad (3)$$

$$B(\text{GT}, E) = \frac{D(g_V^2/4\pi)}{ft}, \quad (4)$$

$$B(\text{GT}, E) = \frac{g_A^2}{4\pi} \frac{|\langle I_f || \sum t_\pm(k) \sigma_\mu(k) || I_i \rangle|^2}{2I_i + 1}, \quad (5)$$

$$[B(\lambda^\pi = 2^-)] = \frac{3}{4} \frac{[D g_V^2 / 4\pi]}{ft}, \quad (6)$$

$$[B(\lambda^\pi = 0^-) + B(\lambda^\pi = 1^-)] = \frac{[D g_V^2 / 4\pi]}{ft}, \quad (7)$$

where  $E$  is the excitation energy of the daughter nucleus;  $Q$  is the total  $\beta$ -decay energy;  $f(Q - E)$  is the Fermi function;  $|\langle I_f || \sum t_\pm(k) \sigma_\mu(k) || I_i \rangle|$  is the reduced nuclear matrix element for the Gamow-Teller transition;  $I_i$  is the spin of the parent nucleus;  $I_f$  is the level spin of the daughter nucleus;  $D = (6147 \pm 7)$  s. Measuring populations of levels by the  $\beta$  decay, one can find the reduced probabilities and the strength function of the  $\beta$  decay.

The scheme of states essential for analysis of Gamow-Teller transition strength functions is shown in Fig. 1. As the  $\beta^+/EC$  decay of nuclei with  $N > Z$  takes place, there is only one value of the isospin  $T_0 + 1$  in the coupling of the isospin ( $\tau = 1, \mu_\tau = +1$ ) of configurations like proton hole-neutron particle  $[\nu p \times \pi h]_{1+}$  with the neutron excess isospin  $T_0$ . The most collective state arising from excitations like  $[\nu p \times \pi h]_{1+}$  with the isospin  $\tau = 1$  and isospin projection  $\mu_\tau = +1$  is also called [2-4] the Gamow-Teller resonance with  $\mu_\tau = +1$ . While for  $\beta^-$  decays of nuclei with  $N > Z$  the Gamow-Teller resonance ( $\tau = 1, \mu_\tau = -1$ ) is (Fig. 2) at the excitation energy higher than  $Q_\beta$  and is inaccessible in terms of energy for population by the  $\beta^-$  decay, the Gamow-Teller resonance with  $\mu_\tau = +1$  may be populated by the  $\beta^+/EC$  decay [2-4]. Now there is no a theory that adequately describes strength functions for the  $\beta$  decay of deformed nuclei. The theory allows rather correct calculations of positions and relative intensities of peaks in the Gamow-Teller transition strength functions for spherical and slightly deformed nuclei [3, 14, 15]. The difference

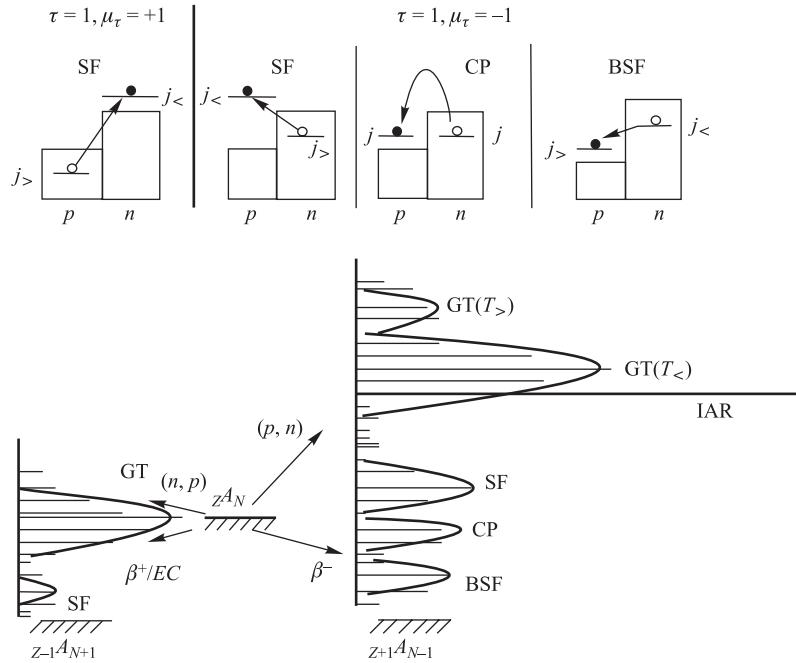


Fig. 2. Schematic view of the strength function for the Gamow-Teller  $\beta$  transitions and configurations responsible for formation of resonances in  $S_\beta(E)$

between the experimental and theoretical absolute intensities of peaks in strength functions for spherical nuclei varies from tens to hundreds of per cent. The theory predicts more intense peaks in strength functions than experimentally observed [3, 16, 17]. Macroscopically, Gamow–Teller-like collective excitations are oscillations of spin–isospin density without a change in the shape of the nucleus. Therefore, the position of the peak in the strength function in the spherical limit should approximately correspond to the center of gravity of the strength function for the deformed nucleus [2]. The resonant structure of the strength functions for Gamow–Teller-like  $\beta$  transitions is caused by the residual spin–isospin interaction and partial  $SU(4)$  spin–isospin symmetry in nuclei [2, 4].

Experimental data on population of  $^{160}\text{Dy}$  levels by the  $\beta^+/EC$  decay of  $^{160g}\text{Ho}$  are presented in the table. Based on the experimental data (see the table), we found reduced probabilities of the  $\beta^+/EC$  decay of  $^{160g}\text{Ho}$  for Gamow–Teller and first-forbidden transitions (Fig. 3). The reduced probabilities of the  $\beta$  decay are proportional to squares of nuclear matrix elements and reflect the fine structure of  $\beta$ -decay strength functions.

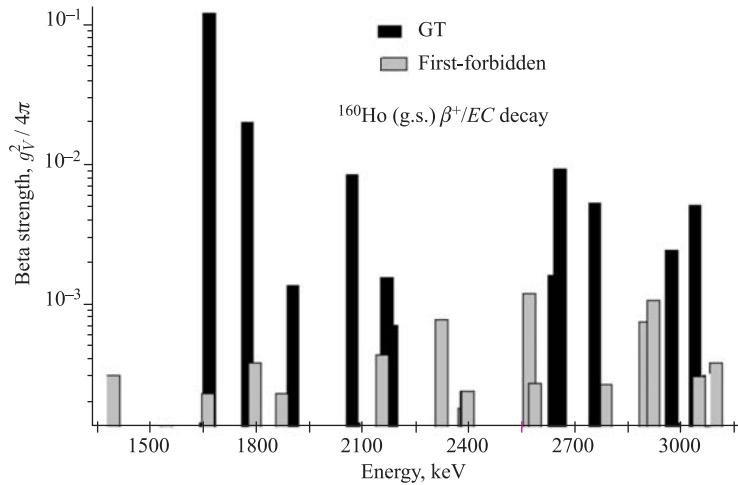


Fig. 3. Structure of the strength function for the  $\beta^+/EC$  decay of  $^{160g}\text{Ho}$

The strength function for the Gamow–Teller  $\beta^+/EC$  transitions shows a pronounced resonant structure (Fig. 3). The strongest peak in the region of 2–3 MeV is identified with the  $\mu_\tau = +1$  Gamow–Teller resonance because evaluation by the model described in [2] predicts such a resonance in the region of 2–4 MeV and the value of  $ft$  for the 1694-keV level (see the table) is typical of the  $\mu_\tau = +1$  Gamow–Teller resonance [2]. In Fig. 3 the peak for the Gamow–Teller transitions is seen to split into two components, one in the region of 1700–2200 keV and the other in the region of 2680–3100 keV.

The strength function for first-forbidden  $\beta^+/EC$  transitions does not show a pronounced resonant structure (Fig. 3). This may indicate the absence of the first-forbiddenness symmetry of interaction in the nucleus, which means that configurations populated by first-forbidden transitions are not distinguished in quantum numbers among neighboring levels of the daughter nucleus, and stronger mixing of configurations occurs. Mixing of configurations populated by Gamow–Teller  $\beta^+/EC$  transitions is weaker because of partial  $SU(4)$  spin–isospin symmetry of interaction in the nucleus [2, 4, 17].

### 3. DISCUSSION

Charge-exchange particle-hole excitations (Fig. 3) populated by the  $\beta$  decay are related to the oscillation of the  $\mu_\tau = \pm 1$  components of the isovector density [18]  $\rho_{\tau=1,\mu\tau}$ :

$$\rho_{\tau=1,\mu\tau}(r) = \sum_k 2t_{\mu\tau}(k)\delta(r - r_k), \quad (8)$$

where summation is taken over all nucleons  $k$ ;  $t_{\mu\tau}$  is the spherical component of the nucleon isospin  $t$ .

$$t_{\mu\tau} = \begin{cases} (1/2)^{1/2}(t_x - it_y), & \mu_\tau = -1, \\ t_z, & \mu_\tau = 0, \\ -(1/2)^{1/2}(t_x + it_y), & \mu_\tau = +1. \end{cases} \quad (9)$$

Oscillations with  $\tau = 0$  correspond to oscillation of the isoscalar (total) density. Oscillations with  $\tau = 1$ ,  $\mu_\tau = 0$ ,  $I^\pi = 1^-$  correspond to the oscillation of the  $\rho_{\tau=1,0}$  component of the isovector density and describe oscillation of protons and neutrons moving in antiphase,  $E1$  giant resonance (GDR). The isovector density oscillation amplitudes are tensors in isospace and orbital space, which leads to splitting of the  $E1$  resonance into deformed nuclei [19, 20].

Oscillations with  $\tau = 1$ ,  $\mu_\tau = \pm 1$  describe  $\beta^+/EC$  and  $\beta^-$  decays, and the peaks in the strength functions for deformed nuclei should also be split.

In this study we experimentally observed splitting of the peak in the strength function for the Gamow–Teller  $\beta^+/EC$  decay of the deformed  $^{160g}\text{Ho}$  nucleus, which corresponds to oscillation anisotropy of the isovector density component  $\rho_{\tau,\mu=1,1}$ .

No significant manifestations of the resonance structure have been found in the strength function for first-forbidden  $\beta^+/EC$  transitions.

### CONCLUSIONS

The experimental data obtained by high-resolution nuclear spectroscopy methods for the  $\beta^+/EC$  decay of the deformed nucleus  $^{160g}\text{Ho}$  (25.6 min) allow the following conclusions:

1. The strength function for the Gamow–Teller  $\beta^+/EC$  decay of the deformed  $^{160g}\text{Ho}$  nucleus has a pronounced resonant character. Earlier [3, 5, 8], we performed similar measurements and draw a similar conclusion for the Gamow–Teller  $\beta^+/EC$  decay of the spherical  $^{147g}\text{Tb}$  nucleus.

2. No significant manifestations of the resonant structure are revealed in the strength function for the first-forbidden  $\beta^+/EC$  decay of the deformed  $^{160g}\text{Ho}$  nucleus.

3. Unlike the case in the strength function for the Gamow–Teller  $\beta^+/EC$  decay of the spherical  $^{147g}\text{Tb}$  nucleus, the peak in the strength function for the Gamow–Teller  $\beta^+/EC$  decay of the deformed  $^{160g}\text{Ho}$  nucleus is split into two components. By analogy with the splitting of the peak of the  $E1$  giant resonance in deformed nuclei, this splitting can be associated with anisotropy of oscillation of the isovector density component  $\rho_{\tau,\mu=1,1}$ .

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