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КОМПЬЮТЕРНЫЕ ТЕХНОЛОГИИ В ФИЗИКЕ

SYMPLECTIC STRUCTURE OF QUANTUM PHASE AND STOCHASTIC SIMULATION OF QUBITS

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Starting from the projective interpretation of the Hilbert space, a special stochastic representation of the wave function in Quantum Mechanics (QM), based on soliton realization of extended particles, is considered with the aim to model quantum states via classical computer. Entangled solitons construction having been earlier introduced in the nonlinear spinor field model for the calculation of the Einstein–Podolsky–Rosen (EPR) spin correlation for the spin-$1/2$ particles in the singlet state, another example is now studied. The latter concerns the entangled envelope solitons in Kerr dielectric with cubic nonlinearity, where we use two-soliton configurations for modeling the entangled states of photons. Finally, the concept of stochastic qubits is used for quantum computing modeling.

Исходя из проективной интерпретации гильбертова пространства, с целью моделировать квантовые состояния на классическом компьютере рассматривается специальное стохастическое представление волновой функции в квантовой механике, основанное на солитонном описании протяженных частиц. Если ранее в рамках нелинейной спинорной модели рассматривалось запутанное синглетное состояние двух частиц-солитонов со спином $1/2$, то теперь изучается другой пример. Он относится к запутанным солитонам огибающей в керровском диэлектрике с кубической нелинейностью. Эти двухсолитонные конфигурации используются для моделирования запутанных состояний фотонов. Для моделирования квантовых вычислений используется понятие стохастических кубитов.

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1. INTRODUCTION. STOCHASTIC REPRESENTATION OF QM

In recent years a very fascinating idea to put QM into geometric language attracts the attention of many physicists [1]. The starting point for such an approach is the projective interpretation of the Hilbert space $\mathcal{H}$ as the space of rays. To illustrate the main idea, it is convenient to decompose the Hermitian inner product $\langle\cdot|\cdot\rangle$ in $\mathcal{H}$ into real and imaginary parts by putting for the two L_2 vectors $|\psi_1\rangle = u_1 + \imath v_1$ and $|\psi_2\rangle = u_2 + \imath v_2$

$$\langle\psi_1|\psi_2\rangle = G\,(\psi_1, \psi_2) - \imath\Omega\,(\psi_1, \psi_2), \tag{1}$$

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where G is a Riemannian inner product on $\mathcal{H}$ and Ω is a symplectic form; that is,

$$G(\psi_1, \psi_2) = (u_1, u_2) + (v_1, v_2); \quad \Omega(\psi_1, \psi_2) = (v_1, u_2) - (u_1, v_2),$$

with $(\cdot, \cdot)$ denoting standard L_2 inner product. The symplectic form Ω revealed in (1) can acquire its dynamical content if one uses the special stochastic representation of QM suggested in [2–4].

In this representation the one-particle wave function appears to be the sum of soliton solutions with random parameters, such as their velocities, center positions and phases. The soliton solution is considered as the image of the extended particle, and the wave function proves to be the vector of the random Hilbert space, the standard QM rules being restored in the point-like limit, when the size of the extended particle-soliton tends to zero. Earlier the case of spin-$1/2$ particles-solitons was considered within the framework of the Heisenberg-like nonlinear spinor field model [5]. In the present paper another example is considered, concerning the photons as the envelope solitons in Kerr dielectric with cubic nonlinearity. For simplicity the 1D soliton solutions are used.

Now recall that the n particles wave function Ψ_N in the stochastic representation of QM is given by the following sum of random fields:

$$\Psi_N(t, \mathbf{r}_1, \ldots, \mathbf{r}_n) = (\hbar^n N)^{-1/2} \sum_{j=1}^{N} \prod_{k=1}^{n} \varphi_j^{(k)}(t, \mathbf{r}_k), \tag{2}$$

where $N \gg 1$ stands for the number of trials (observations) and $\varphi_j^{(k)}$ is the one-particle auxiliary field function for the jth trial:

$$\varphi^{(k)}(t, \mathbf{r}) = \frac{1}{\sqrt{2}} \left(\nu_k \phi^{(k)} + \frac{\imath \pi^{(k)}}{\nu_k} \right), \tag{3}$$

with the constants ν_k satisfying the normalization condition

$$\hbar = \int d^3x \, |\varphi^{(k)}|^2,$$

where $\pi^{(k)}(t, \mathbf{r})$ stands for the conjugate momentum corresponding to the real field $\phi^{(k)}$ and $\hbar$ is the Planck constant. It is necessary to underline that the internal scalar product in $\mathcal{H}$ includes the averaging over random parameters of particles-solitons.

2. ENTANGLED OPTICAL SOLITONS IN KERR DIELECTRIC

The interest to the optical envelope solitons in cubic media is widely known [6, 7]. We intend to show that in the stochastic representation of QM one can use these soliton solutions to the nonlinear Maxwell equations for modeling entangled states of photons. Consider the Kerr nonlinear dielectric, with the permeability ϵ being the quadratic function of the electric strength $\mathbf{E}$:

$$\epsilon = \epsilon_0 + \epsilon_1 |\mathbf{E}|^2, \tag{4}$$

where ϵ_0 and ϵ_1 stand for some positive constants. The Maxwell equations read

$$\mathrm{rot}\,\mathbf{E} = -\partial_t \mathbf{B}, \qquad \mathrm{div}\,(\epsilon\mathbf{E}) = 0, \tag{5}$$

$$\mathrm{rot}\,\mathbf{B} = \partial_t\left[\epsilon\mathbf{E}\right], \qquad \mathrm{div}\,\mathbf{B} = 0, \tag{6}$$

with the unit vacuum velocity of light. From (5) and (6) one immediately derives the nonlinear wave equation for $\mathbf{E}$:

$$\mathrm{rot}^2\,\mathbf{E} = -\partial_t^2\left[\epsilon(\mathbf{E})\mathbf{E}\right]. \tag{7}$$

Substituting into (7) the following vector field:

$$\mathbf{E}_R = \mathbf{e}_R A\,\mathrm{sech}\,(k\xi), \quad \xi = z - Vt, \tag{8}$$

where $\mathbf{e}_R$ stands for the unit vector corresponding to the right circular polarization:

$$\mathbf{e}_R = \mathbf{e}_x\cos\phi + \mathbf{e}_y\sin\phi, \quad \phi = \omega t - k_0 z, \tag{9}$$

one gets three algebraic equations for the constant parameters A, V, k, k_0, ω:

$$\epsilon_0\left(\omega^2 - k^2V^2\right) = k_0^2 - k^2, \tag{10}$$

$$\epsilon_1 A^2\left(9k^2V^2 - \omega^2\right) = 2k^2\left(\epsilon_0 V^2 - 1\right), \tag{11}$$

$$k_0 = \omega V\left(\epsilon_0 + 3\epsilon_1 A^2\right), \tag{12}$$

where the natural supposition concerning the envelope pulse was made:

$$k_0 \gg k. \tag{13}$$

Introducing the independent parameters $X = k^2/\epsilon_0\omega^2$, $Z = 3A^2\epsilon_1/\epsilon_0$, one easily finds from (10), (11) and (12) the minimal value of X: $X_{\min} \equiv X_0 \approx 0.049$ and estimates the following useful parameters:

$$\lambda^2 \equiv \frac{k^2V^2}{\omega^2} \in \left[\frac{1}{27}, X_0\right]; \quad \frac{k^2}{k_0^2} \in [X_0, 1].$$

Now we intend to search for the magnetic field $\mathbf{B} = \mathrm{rot}\,\mathbf{A}$. To this end, it is necessary to find the transversal vector potential:

$$\mathbf{A} = -\int^t \mathbf{E}\,dt. \tag{14}$$

The integral in (14) can be found via the integration by parts that gives the asymptotic series. Thus, inserting (8) into (14), one gets

$$\mathbf{A}_R = \frac{A}{\omega}\mathrm{sech}\,(k\xi)\left[\mathbf{e}_L - \mathbf{e}_R\lambda\tanh(\mathrm{k}\xi) + \mathcal{O}(\lambda^2)\right]. \tag{15}$$

From (15) one easily finds the magnetic field

$$\mathbf{B}_R = \frac{A}{\omega}\mathrm{sech}\,(k\xi)\left[\mathbf{e}_L\left(k_0 - k\lambda + 2k\lambda\tanh^2(k\xi)\right) + \mathbf{e}_R(k - \lambda k_0)\tanh(k\xi) + \mathcal{O}(\lambda^2)\right]. \tag{16}$$

It is worthwhile to underline that the soliton solution corresponding to the left circular polarization can be obtained from (8) and (15) via the transposition $\mathbf{e}_R \Longrightarrow \mathbf{e}'_L$; $\mathbf{e}_L \Longrightarrow \mathbf{e}'_R$, where the denotation is used:

$$\mathbf{e}'_R = \mathbf{e}_x \cos\phi \, - \mathbf{e}_y \sin\phi, \quad \mathbf{e}'_L = \mathbf{e}_x \sin\phi \, + \mathbf{e}_y \cos\phi.$$

Using the solution found, it is possible to calculate the integrals of motion describing the soliton configuration, that is, the physical observables: the energy W, the spin $\mathbf{S}$ and the momentum $\mathbf{P}$. These observables can be constructed via the Lagrangian density

$$\mathcal{L} = \frac{1}{4}\left(2\epsilon_0\mathbf{E}^2 + \epsilon_1\mathbf{E}^4 - 2\mathbf{B}^2\right)$$

if one follows the standard variational procedure:

$$W = \frac{1}{4}\int dz \, \left(2\epsilon_0\mathbf{E}^2 + 3\epsilon_1\mathbf{E}^4 + 2\mathbf{B}^2\right), \tag{17}$$

$$\mathbf{S} = \int dz \, \epsilon[\mathbf{E}\mathbf{A}], \tag{18}$$

$$P_z = \int dz \, \epsilon \left(\mathbf{E}\partial_z\mathbf{A}\right). \tag{19}$$

Inserting (8) and (16) into (17), (18) and (19), one gets

$$W \approx \frac{A^2}{k}\left[\epsilon_0 + \epsilon_1 A^2 + \frac{1}{3\omega^2}\left(3k_0^2 - 4k\lambda + k^2\right)\right], \tag{20}$$

$$\mathbf{S}_R = -\mathbf{S}_L = \mathbf{e}_z S, \quad S \approx \frac{2A^2}{3k\omega}\left(3\epsilon_0 + 2\epsilon_1 A^2\right), \tag{21}$$

$$\mathbf{P} = \mathbf{e}_z P, \quad P = k_0 S. \tag{22}$$

Now we define the subsidiary complex vector function analogous to that in (3):

$$\boldsymbol{\varphi} = \frac{1}{\sqrt{2}}\left(\nu\mathbf{A} + \frac{\imath}{\nu}\boldsymbol{\pi}\right), \quad \boldsymbol{\pi} = -\epsilon\mathbf{E}, \tag{23}$$

where the constant ν is defined by the normalization condition

$$\hbar = \int dz \, |\boldsymbol{\varphi}|^2. \tag{24}$$

Stochastic representation of the one-particle wave function $\boldsymbol{\Psi}_N$ can be defined as the linear combination of the functions (23) determined in N independent trials:

$$\boldsymbol{\Psi}_N(t,z) = (\hbar N)^{-1/2}\sum_{j=1}^{N}\boldsymbol{\varphi}_j(t,z). \tag{25}$$

Thus, formula (25) gives the stochastic realization of the one-photon wave function via the infinite dimensional objects-solitons. For calculating the mean values of observables one can use the standard quantum mechanical rule:

$$\langle A\rangle = \int dz\, \mathbb{E}\left[\Psi_{Nl}^{*} A_{lm} \Psi_{Nm}\right], \tag{26}$$

with A_{lm} standing for some Hermitian operator (matrix) and $\mathbb{E}$ signifying the averaging over random soliton parameters. For example, the spin operator has the standard form $(S_k)_{lm} = -\imath\hbar\epsilon_{klm}$.

Now we consider two-soliton (photon) singlet states, that is, construct the entangled solitons configuration with the zero spin and momentum:

$$\varphi^{(12)}(t, z_1, z_2) = \frac{1}{\sqrt{2}}\Big[\boldsymbol{\varphi}_L(t, -z_1) \otimes \boldsymbol{\varphi}_R(t, z_2) - \boldsymbol{\varphi}_R(t, -z_1) \otimes \boldsymbol{\varphi}_L(t, z_2)\Big]. \tag{27}$$

Now it is not difficult to find the stochastic wave function for the singlet two-soliton entangled state:

$$\Psi_N\left(t, z_1, z_2\right) = \left(\hbar^2 N\right)^{-1/2} \sum_{j=1}^{N} \varphi_j^{(12)}, \tag{28}$$

where $\varphi_j^{(12)}$ corresponds to the entangled solitons configuration in the jth trial. We hope that the formalism of entangled solitons can also be applied to 3D optical solitons for modeling the real photons.

3. CONCLUSION. SIMULATION OF STOCHASTIC QUBITS BY PROBABILISTIC BITS

Now we intend to explain how the stochastic qubits introduced previously could be simulated by standard probabilistic bits. To this end, one should define the random phase Φ_j for the jth trial in our system of n solitons-particles. Let $\varphi^{(k)}(\mathbf{r})$ denote the standard (etalon) profile for the kth soliton. The most probable position $\mathbf{d}_j^{(k)}(t)$ of the kth soliton's center for jth trial can be found from the following variational problem:

$$\left|\int d^3x\, \varphi_j^{*(k)}(t, \mathbf{r})\varphi^{(k)}\left(\mathbf{r} - \mathbf{d}_j^{(k)}\right)\right| \to \max,$$

thus giving the random phase structure

$$\Phi_j = \sum_{k=1}^{n} \arg \int d^3x\, \varphi_j^{*(k)}(t, \mathbf{r})\varphi^{(k)}\left(\mathbf{r} - \mathbf{d}_j^{(k)}\right). \tag{29}$$

The random phase (29) can be used for simulating quantum computing via generating the following K random dichotomic functions:

$$f_s(\theta_s) = \operatorname{sign}\left[\cos\left(\Phi_j + \theta_s\right)\right], \quad s = \overline{1, K}, \tag{30}$$

with θ_s being arbitrary fixed phases. It is worthwhile to compare the standard EPR correlation $P(\mathbf{a},\mathbf{b}) = -\cos\theta$ with the random phases one for the case of $n = 2$ particles:

$$\mathbb{E}\,(f_1\,f_2) = 1 - \frac{2}{\pi}|\triangle\theta|,$$

where $\triangle\theta = \theta_1 - \theta_2$. The similarity of these two functions of the angular variable θ seems to be a good motivation for the K qubits simulation by the dichotomic random functions (30) popularized in paper [8]. This very simple model of stochastic qubits simulation can be employed for simulating Bi-photons, EPR states and other entanglement states. We hope that this model will be useful for Shor's and Grover's Quantum Algorithms realization. All elementary qubits operations can be realized via classical computer through simulating the phase structure of realistic solitons by the generator of random numbers connected to the model solitons' generator, e.g., Kerr dielectric with the optical excitations or magnetic with the excitations of localized spin inversion domains.

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