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NEW FORMULAE FOR THE RADIATION INTENSITY  
IN TAMM'S PROBLEM

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# 1 Introduction.

The aim of this consideration is to analyze frequency and angular distributions of the radiation in the so-called Tamm problem. The latter treats the point charge which is at rest in medium at the space point  $z = -z_0$  up to a moment  $t = -t_0$ . In the time interval  $-t_0 < t < t_0$ , the charge moves with the velocity  $v$  that can be smaller or greater than the light velocity in medium  $c_n$ . After the moment  $t = t_0$ , the charge is again at rest at the point  $z = z_0$ . This problem was first considered by Tamm [1] in 1939. Later, it was qualitatively analyzed by Lawson [2,3] and numerically by Zrelov and Ruzicka [4,5]. In 1996, the exact solution of Tamm's problem was found for nondispersive medium [6]. A careful analysis of this solution was given in [7]. It was shown there that Tamm's formulae do not always describe the Cherenkov radiation properly.

The plan of our exposition is as follows.

In Section 2, we reproduce Tamm's derivation of frequency and angular distributions of the radiation intensity produced by a point charge moving uniformly on a finite space interval. Criteria for the validity of Tamm's formula are given in Sect. 3. Exact electromagnetic fields of Tamm's problem and radiation intensity are explicitly written out in Section 4. Suitable approximations made in Sect. 5 permit us to find analytical expression for the radiation intensity which has a greater range of applicability than the original Tamm's formula. The analytical formula taking into account possible deceleration of a moving charge is presented in Sect. 6. It generalizes the formula found earlier in Ref.[8]. Short resume of the results obtained is given in Sect. 8.

## 2 Tamm's original approach

Tamm considered the following problem. A point charge is at rest at the point  $z = -z_0$  of the  $z$  axis up to a moment  $t = -t_0$ . In the time interval  $-t_0 < t < t_0$  it uniformly moves along the  $z$  axis with the velocity  $v$  greater than the light velocity in medium  $c_n$ . For  $t > t_0$ , the charge is again at rest at the point  $z = z_0$ . The nonvanishing Fourier component  $z$  of the vector potential is given by

$$A_\omega = \frac{1}{c} \int_{-z_0}^{z_0} \frac{1}{R} j_\omega(x', y', z') \exp(-i\omega R/c_n) dx' dy' dz',$$

where  $R = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{1/2}$ ,  $j_\omega = 0$  for  $z' < -z_0$  and  $z' > z_0$  and  $j_\omega = e\delta(x')\delta(y') \exp(-i\omega z'/v)/2\pi$  for  $-z_0 < z' < z_0$ . Inserting all this into  $A_\omega$  and integrating over  $x'$  and  $y'$ , one gets

$$A_\omega(x, y, z) = \frac{e}{2\pi c} \int_{-z_0}^{z_0} \frac{dz'}{R} \exp[-i\omega(\frac{z'}{v} + \frac{R}{c_n})],$$

$$R = [\rho^2 + (z - z')^2]^{1/2}, \quad \rho^2 = x^2 + y^2. \quad (2.1)$$

At large distances from the charge ( $R \gg z_0$ ), one has  $R = R_0 - z' \cos \theta$ ,  $\cos \theta = z/R_0$ ,  $R_0 = (x^2 + y^2 + z^2)^{1/2}$ . Inserting this into (2.1) and integrating over  $z'$ , one gets

$$A_\omega(\rho, z) = \frac{e\beta q(\omega)}{\pi R_0 \omega} \exp(-i\omega R_0/c_n),$$

$$q(\omega) = \frac{\sin[\omega t_0(1 - \beta_n \cos \theta)]}{1 - \beta_n \cos \theta}, \quad \beta = \frac{v}{c}, \quad \beta_n = \frac{v}{c_n}. \quad (2.2)$$

Now we evaluate the field strengths. In the wave zone, where  $R_0 \gg c/n\omega$ , the nonvanishing spherical components are

$$E_\theta = H_\phi = -\frac{2e\beta}{\pi c R_0} \sin \theta \int_0^\infty nq(\omega) \sin[\omega(t - R_0/c_n)] d\omega. \quad (2.3)$$

The energy flux through the sphere of the radius  $R_0$  for the whole time of observation

$$\mathcal{E} = R_0^2 \int S_r d\Omega dt, \quad d\Omega = \sin \theta d\theta d\phi, \quad S_r = \frac{c}{4\pi} E_\theta H_\phi.$$

can be presented in the form

$$\mathcal{E} = \int \frac{d^2 \mathcal{E}}{d\Omega d\omega} d\Omega d\omega, \quad (2.4)$$

where

$$\frac{d^2 \mathcal{E}}{d\Omega d\omega} = \frac{e^2}{\pi^2 c n} \left[ \sin \theta \frac{\sin \omega t_0 (1 - \beta_n \cos \theta)}{\cos \theta - 1/\beta_n} \right]^2. \quad (2.5)$$

is the energy emitted during the whole charge motion into the solid angle  $d\Omega$ , in the frequency interval  $d\omega$ . This famous formula obtained by Tamm is frequently used by experimentalists (see, e.g., [9-12]) for the identification of the Cherenkov radiation.

### 3 Criteria for the validity of Tamm's approximation

We elucidate here the approximations made during the transition from the exact vector potential (2.1) to Tamm's formula (2.5):

1) Changing  $R$  by  $R_0$  outside the exponent means that observation is made on the sphere with the radius  $R_0$  much larger than the motion interval  $z_0$ , i.e.:

$$R_0 \gg z_0. \quad (3.1)$$

2) Tamm's field strengths (2.3) are valid only in the wave zone where

$$k_n R_0 \gg 1, \quad k_n = \omega/c_n, \quad c_n = c/n. \quad (3.2)$$

3) When changing  $R$  under the sign of exponent in (2.1) by  $R_0 - z' \cos \theta$ , it is implicitly assumed that the quadratic term in the expansion of  $R$  is small as compared to the linear one. Consider this more carefully. We expand  $R$  up to the second order

$$R \approx R_0 - z' \cos \theta + \frac{z'^2}{2R_0} \sin^2 \theta.$$

Under the sign of exponent in (2.1), the following terms

$$\frac{z'}{v} + \frac{1}{c_n} (R_0 - z' \cos \theta + \frac{z'^2}{2R_0} \sin^2 \theta).$$

appear. We collect terms involving  $z'$

$$\frac{z'}{c_n} \left[ \left( \frac{1}{\beta_n} - \cos \theta \right) + \frac{z'}{2R_0} \sin^2 \theta \right].$$

Taking for  $z'$  its maximal value  $z_0$ , we present the condition for the second term in the expansion of  $R$  to be small in the form

$$z_0 \ll 2R_0 \left( \frac{1}{\beta_n} - \cos \theta \right) / \sin^2 \theta. \quad (3.3)$$

It is seen that the right-hand side of this equation vanishes for  $\cos \theta = 1/\beta_n$ , i.e., at the angle where the Cherenkov radiation exists. Therefore, in this angular region the second-order terms may be important.

4) Under the sign of exponent in (2.1) the second-order term should be small compared to  $\pi$ , i.e., the inequality

$$\frac{z'^2 \omega \sin^2 \theta}{2R_0 c_n} \ll \pi \quad (3.4)$$

should hold. Or, taking for  $\theta$  and  $z'$  their maximal values ( $\theta = \pi/2$ ,  $z' = z_0$ ), one gets [9]

$$\frac{z_0^2 \omega}{2R_0 c_n} \ll \pi. \quad (3.5)$$

From (3.2) and (3.5) one finds the following restriction on  $\omega$ :

$$\frac{c_n}{R_0} \ll \omega \ll \frac{2\pi R_0 c_n}{z_0^2}. \quad (3.6)$$

In the  $\lambda$  language ( $\omega = 2\pi c/\lambda$ ), this condition looks like

$$\frac{n z_0^2}{R_0} \ll \lambda \ll 2\pi n R_0. \quad (3.7)$$

Let  $\lambda = 4 \cdot 10^{-5} cm$ , (the middle of the optical region),  $n = 1.5$  (glass). For the typical value  $R = 100 cm$ , the r.h.s. of inequality (3.7) is fulfilled with a great accuracy. Then, the l.h.s. of (3.7) gives  $z_0 \ll 5 \cdot 10^{-2} cm$ . On the other hand,  $z_0$  should not be too small. In fact, for  $k_n z_0 \ll 1$ , Tamm's formula (2.8) is reduced to

$$\frac{d^2 \mathcal{E}}{d\omega d\Omega} \sim \frac{e^2 \sin^2 \theta \omega^2 t_0^2}{\pi^2 c n \beta_n^2} \quad t_0 = \frac{z_0}{v},$$

i.e., the Cherenkov diffraction picture disappears. Therefore, the width interval  $10^{-4} cm < z_0 < 10^{-2} cm$  turns out to be optimal for the validity of Tamm's formula and existence of the pronounced Cherenkov maximum in the treated case.

As an illustration, we turn to Ref. [13] in which the angular distribution of the radiation ( $\lambda \approx 4 \cdot 10^{-5} cm$ ) arising from the passage of *Au* heavy ions ( $\beta \approx 0.87$ ) through the *LiF* slab ( $n \approx 1.39$ ) of width  $L = 0.5 cm$  was studied.

Substituting the parameters of Ref. [13] into (3.5) defining the validity of Tamm's formula (2.5), we find that the l.h.s. of (3.5) coincides with  $\pi$  for the observation sphere radius  $R_0 \approx 20 m$ . Obviously, this value is unrealistic. Since the realistic  $R_0$  is about  $1 - 2 m$ ,

the strong violation of (3.5) takes place. In this case Tamm's formula does not describe properly the experimental situation.

It should be noted that for gases, these conditions are less restrictive than for solids and liquids. In fact, since for them  $\beta_n \approx 1$ , one gets

$$\left(\frac{1}{\beta_n} - \cos \theta\right) / \sin^2 \theta \approx \frac{1 - \cos \theta}{\sin^2 \theta} = 1/2 \cos^2(\theta/2).$$

Since for gases the angular spectrum is confined to the  $\theta \approx 0$  region, Eq.(3.3) is reduced to (3.1). The same is true for Eq. (3.4). As a result, for gases, Tamm's expression (2.8) for the radiated power works when Eqs. (3.1) and (3.2) are fulfilled.

Conditions (3.1)-(3.7) ensuring the validity of Tamm's expressions are spread over different sources. We collected them together to make the interpretation of numerical results given below easier.

## 4 Exact electromagnetic field strengths and angular-frequency distribution of the radiated energy

The energy flux through the unit solid angle of the sphere of the radius  $R_0$  for the whole time of a charge motion is given by

$$\frac{dW}{d\Omega} = \frac{c}{4\pi} R_0^2 \int_{-\infty}^{\infty} dt (\vec{E} \times \vec{H})_r. \quad (4.1)$$

Expressing  $\vec{E}$  and  $\vec{H}$  through their Fourier transforms

$$\vec{E} = \int \exp(i\omega t) \vec{E}_\omega d\omega, \quad \vec{H} = \int \exp(i\omega t) \vec{H}_\omega d\omega$$

and integrating over  $t$ , one gets

$$\frac{dW}{d\Omega} = \frac{cR_0^2}{2} \int_{-\infty}^{\infty} (\vec{E}(\omega) \times \vec{H}(-\omega))_r d\omega = \int_0^{\infty} S(\omega) d\omega, \quad (4.2)$$

where

$$S(\omega) = \frac{d^2W}{d\omega d\Omega} = cR_0^2 [\vec{E}_\theta^{(r)}(\omega) \vec{H}_\phi^{(r)}(\omega) + \vec{E}_\theta^{(i)}(\omega) \vec{H}_\phi^{(i)}(\omega)]. \quad (4.3)$$

This quantity shows how the Fourier component of the energy radiated for the whole time of a charge motion is distributed over the sphere  $S$ . It does not depend on time. The superscripts  $(r)$  and  $(i)$  mean the real and imaginary parts of  $E_\theta$  and  $H_\phi$ . The exact field strengths obtained by differentiation of the exact vector potential (2.1) are given by

$$H_\phi^{(r)}(\omega) = \frac{ek_n z_0}{2\pi c R_0} \sin \theta \int \frac{G}{R^2} dz', \quad H_\phi^{(i)}(\omega) = \frac{ek_n z_0}{2\pi c R_0} \sin \theta \int \frac{F}{R^2} dz',$$

$$\begin{aligned}
E_{\theta}^{(r)}(\omega) &= \frac{ek_n^2 z_0}{2\pi\omega\epsilon R_0} \sin\theta \left( \int \frac{1-z'\epsilon_0\cos\theta}{R^3} F_1 dz' - \frac{2}{k_n R_0} \int \frac{F}{R^2} dz' \right), \\
E_{\theta}^{(i)}(\omega) &= \frac{ek_n^2 z_0}{2\pi\omega\epsilon R_0} \sin\theta \left( \int \frac{1-z'\epsilon_0\cos\theta}{R^3} G_1 dz' + \frac{2}{k_n R_0} \int \frac{G}{R^2} dz' \right),
\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}
F &= \cos\psi - \frac{1}{k_n R_0 R} \sin\psi, & G &= \sin\psi + \frac{1}{k_n R_0 R} \cos\psi, \\
F_1 &= \sin\psi + 3\frac{\cos\psi}{k_n R_0 R} - 3\frac{\sin\psi}{k_n^2 R_0^2 R^2}, & G_1 &= \cos\psi - 3\frac{\sin\psi}{k_n R_0 R} - 3\frac{\cos\psi}{k_n^2 R_0^2 R^2}, \\
\psi &= k_n R_0 \left( \frac{z'\epsilon_0}{\beta_n} + R \right), & R &= (1 - 2z'\epsilon_0\cos\theta + \epsilon_0^2 z'^2)^{1/2}, \quad \epsilon_0 = z_0/R_0.
\end{aligned} \tag{4.5}$$

The  $z'$  integration in (4.3) is performed over the interval  $(-1, 1)$ . When Eqs. (3.1),(3.2) and (3.5) are satisfied,  $S(\omega)$  given by (4.3) transforms into the Tamm formula (2.5). Unfortunately, Eqs. (4.4) are not suitable for large frequencies. In fact, for the visible light  $k = \omega/c$  is of the order  $10^5 \text{ cm}^{-1}$ . For the observation distance  $R_0 \sim 1 \text{ m}$ , one gets  $kR_0 \sim 10^7$ . A great number of integration steps is needed to obtain the required accuracy. Therefore, some approximations are needed.

## 5 Approximations

In the wave zone where  $k_n R_0 \gg 1$ , we omit the terms of the order  $(k_n R_0)^{-1}$  and higher outside  $\psi$

$$\begin{aligned}
S(\omega, \theta) &= \frac{e^2 k^2 z_0^2 n}{4\pi^2 c} \sin^2\theta \left[ \int \frac{\sin\psi_1}{R^2} dz' \cdot \int \frac{\sin\psi_1}{R^3} (1 - z'\epsilon_0\cos\theta) dz' + \right. \\
&\quad \left. + \int \frac{\cos\psi_1}{R^2} dz' \cdot \int \frac{\cos\psi_1}{R^3} (1 - z'\epsilon_0\cos\theta) dz' \right],
\end{aligned} \tag{5.1}$$

where

$$\psi_1 = \omega t_0 z' + k_n R_0 (R - 1), \quad t_0 = z_0/v. \tag{5.2}$$

Since the condition  $k_n R_0 \gg 1$  in real experiments is satisfied with a great accuracy (we have seen,  $kR_0$  is of the order  $10^7$ ), Eq.(5.1) is almost exact. It is important that a maximal value of  $\psi_1$  in (5.2) is of the order  $k_n z_0$ , not  $k_n R_0$  as in Eq. (4.5). This makes integration easier.

If  $z_0 \ll R_0$ , one may disregard  $\epsilon_0$  outside  $\psi_1$ . Then,

$$S(\omega, \theta) = \frac{e^2 k^2 z_0^2 n}{4\pi^2 c} \sin^2\theta \left[ \left( \int \sin\psi_1 dz' \right)^2 + \left( \int \cos\psi_1 dz' \right)^2 \right]. \tag{5.3}$$

The expansion of  $\psi_1$  up to the first order of  $\epsilon_0$  gives Tamm's formula (2.5) which does not always describe properly the real experimental situation. Therefore, we expand  $\psi_1$  up to the second order of  $\epsilon_0$

$$\psi_1 = \frac{\omega z' z_0}{c_n} \left( \frac{1}{\beta_n} - \cos\theta + \frac{z' z_0}{2R_0} \sin^2\theta \right). \tag{5.4}$$

With this  $\psi_1$ ,  $S(\theta)$  can be obtained in the closed form

$$S(\theta) = \frac{e^2 k R_0}{4\pi c} \{ [S(z_+) - S(z_-)]^2 + [C(z_+) - C(z_-)]^2 \}, \quad (5.5)$$

where

$$S(x) = \sqrt{\frac{2}{\pi}} \int_0^x dt \sin t^2 \quad \text{and} \quad C(x) = \sqrt{\frac{2}{\pi}} \int_0^x dt \cos t^2$$

are Fresnel's integrals and

$$z_{\pm} = \sqrt{\frac{\epsilon_0 k_n z_0}{2}} \sin \theta \left( \frac{1 - \beta_n \cos \theta}{\epsilon_0 \beta_n \sin^2 \theta} \pm 1 \right).$$

Equation (5.5) is valid if the third-order terms are small compared with  $\pi$ :

$$\frac{1}{2} k_n R_0 \epsilon_0^3 z^3 \cos \theta \sin^2 \theta \ll \pi. \quad (5.6)$$

If we take for  $z'$  and  $\cos \theta \sin^2 \theta$  their maximal values, one gets

$$\frac{n z_0^3}{\lambda R_0^2} \ll 1. \quad (5.7)$$

We collect all approximations involved in derivation of (5.5)

$$k_n R_0 \gg 1, \quad z_0 \ll R_0, \quad \frac{n z_0^3}{\lambda R_0^2} \ll 1. \quad (5.8)$$

## 6 Accelerated charge motion

Consider the following problem. Let a point charge be at rest at the point  $z = -z_0$  up to a moment  $t = -t_0$ . At the moment  $t = t_0$ , the charge acquires the velocity  $v_1$ . In the time interval  $(-t_0 < t < t_0)$  the charge decelerates according to the law

$$\frac{z}{z_0} = \frac{t}{t_0} + \frac{at_0^2}{z_0} \left( 1 - \frac{t^2}{t_0^2} \right), \quad \frac{dz}{dt} = \frac{z_0}{t_0} - 2at.$$

After the moment  $t = t_0$ , the charge is again at rest at the point  $z = z_0$ . The initial and final velocities of charge are equal to

$$v_{i,f} = v \pm 2at_0.$$

Here

$$v = \frac{v_i + v_f}{2} = \frac{z_0}{t_0}$$

is the charge velocity at the moment  $t = 0$  and  $at_0 = (v_i - v_f)/4$ . It turns out that the same equations (4.3), (4.4) are valid for the treated decelerated charge motion with the exception that the function  $\psi$  should be changed by

$$\psi = \omega t_0 T + k_n R_0 R,$$

where

$$T = \frac{1}{\delta} [1 - (1 + \delta^2 - 2\delta z')^{1/2}], \quad \delta = \frac{at_0}{v} = \frac{1}{2} \frac{v_i - v_f}{v_i + v_f}, \quad t_0 = z_0/v.$$

In the wave zone, the same equation (5.1) is valid if one puts

$$\psi_1 = \omega t_0 T + k_n R_0 (R - 1). \quad (6.1)$$

Dropping  $\epsilon_0$  outside the sin and cos, one arrives at (5.3) with  $\psi_1$  given by (6.1). Expanding square roots entering into  $T$  and  $R$ , we get

$$R - 1 = -z' \epsilon_0 \cos \theta + \frac{1}{2} \epsilon_0^2 z'^2 \sin^2 \theta, \quad T = z' - \frac{1}{2} \delta (1 - z'^2).$$

Then,

$$\psi_1 \approx \frac{1}{2} \omega t_0 [z'^2 (\delta + \epsilon_0 \beta_n \sin^2 \theta) + 2z' (1 - \beta_n \cos \theta) - \delta]. \quad (6.2)$$

With such  $\psi_1$ , integrals entering into (5.3) can be taken analytically and one gets for  $S(\theta)$

$$S(\theta) = \frac{e^2 k R_0}{4\pi c} \frac{\epsilon_0 \beta_n \sin^2 \theta}{\epsilon_0 \beta_n \sin^2 \theta + \delta} \{ [S(z_+) - S(z_-)]^2 + [C(z_+) - C(z_-)]^2 \}, \quad (6.3)$$

where

$$z_{\pm} = \left[ \frac{\omega t_0}{2} (\delta + \beta_n \epsilon_0 \sin^2 \theta) \right]^{1/2} \left[ \frac{1 - \beta_n \cos \theta}{\delta + \beta_n \epsilon_0 \sin^2 \theta} \pm 1 \right].$$

Equation (6.3) works if, in addition to (5.8), the following inequality is satisfied:

$$\frac{\omega t_0}{2} \delta^2 \ll \pi. \quad (6.4)$$

In the limit  $\delta \rightarrow 0$  (zero acceleration), Eq.(6.3) is reduced to (5.5). For  $\epsilon_0 \rightarrow 0$  (large radius of the observation sphere), one gets

$$S(\theta) = \frac{e^2 k z_0 \beta_n \sin^2 \theta}{4\pi c \delta} \{ [S(x_+) - S(x_-)]^2 + [C(x_+) - C(x_-)]^2 \}, \quad (6.5)$$

where

$$x_{\pm} = \sqrt{\frac{k z_0 \delta}{2\beta}} \left( \frac{1 - \beta_n \cos \theta}{\delta} \pm 1 \right).$$

Equation (6.5) was obtained earlier in Ref. [8].

## 7 Numerical results

For the values  $n, L, \lambda$  the same as in Ref.[13] (see Sect. 3), one finds that for  $R_0 = 100cm$ , the inequality (5.7) ensuring the validity of (5.5) is reduced to  $0.02 \ll 1$ . This means that Eq. (5.5) should describe properly the experimental situation of Ref. [13].

With these parameters we evaluated the exact (5.1) and approximate (5.5) angular distributions of the radiated energy on the spheres of the radii  $R_0 = 10cm$  (Fig. 1),  $R_0 = 1m$  (Fig. 2) and  $R_0 = 10m$  (Fig. 3). These figures demonstrate good agreement between the exact (5.1) and Fresnel (5.5) intensities. Even in the  $R_0 = 10cm$  case, for which



(5.7) is strongly violated (it looks like  $11 \ll 1$ ), the agreement of (5.1) and (5.5) is quite satisfactory. On the other hand, both of them sharply disagree with Tamm's intensity (2.5). This proves Fig. 4, where the exact (5.1) intensity on the sphere of the radius  $R_0 = 10m$  is compared with Tamm's intensity (2.5) (which does not depend on  $R_0$ ).

Formula (6.5) was used earlier in Ref. [8] to evaluate the angular distribution arising from the decelerated motion of heavy ions through the  $LiF$  slab with the same parameters as in Ref. [13]. The initial and final velocities were  $\beta_i = 0.875$ ,  $\beta_f = 0.861$ . This gives  $\beta = 0.868$  and  $\delta \approx 4.1 \cdot 10^{-3}$ . Inequality (6.4) takes the form  $0.25 \ll 1$ . The angular dependencies (6.3) for  $R_0 = 10cm$  and  $R_0 = 100cm$  are presented in Fig. 5. In Fig. 6, the intensity (6.5) obtained in [8] is compared with the intensity (6.3) for  $R_0 = 1m$ . These intensities almost coincide. The reason for this is that  $\delta$  appears in Eq.(6.3) through the combination  $\delta + \epsilon_0 \beta_n \sin^2 \theta$ . For the given experimental conditions, the second term is 1/4 of the first one in the neighbourhood of the radiation maximum.

The moral of this section is that one should be very careful when applying Tamm's formula (2.5) to analyse experimental data. The validity of the conditions (3.1)-(3.7) ensuring the validity of (2.5) should be verified. The exact energy flux (5.1) or approximate expressions (5.5) or (6.3) should be used if these conditions are violated.

## 8 Conclusion

Let us briefly summarize the main results obtained:

We found analytical formulae describing the frequency-angular distribution of the radiated energy in the so-called Tamm's problem. They generalize the famous Tamm's formula to the cases when the intensity measurements are performed on the finite distances or when the moving charge exhibits deceleration due to the energy losses. The formulae obtained have a much greater range of applicability than Tamm's one. We hope, they will be useful to experimentalists.

## Acknowledgement

The authors are indebted to Prof. V.P. Zrelow for many stimulating and interesting discussions and to Prof. J. Ruzicka for attracting our attention to Ref. [8].

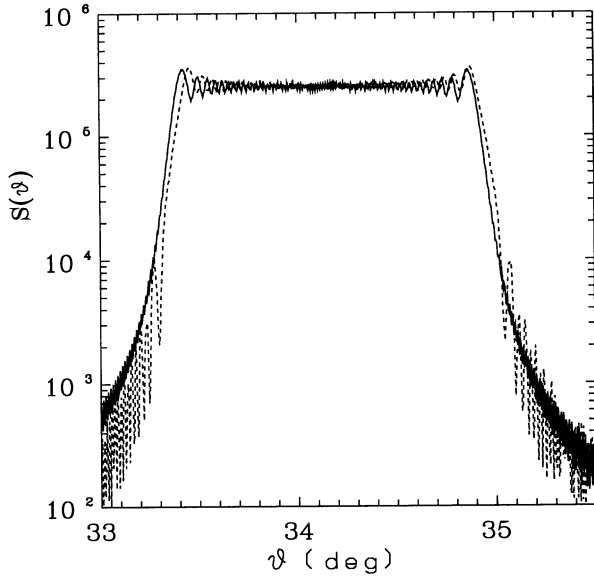


Figure 1: Exact (solid line) and Fresnel (broken line) intensities (in units  $e^2/c$ ) on the observation sphere of radius  $R_0 = 10\text{cm}$ . Parameters of Tamm's problem: charge's path and velocity  $L = 0,5\text{cm}$  and  $\beta = 0.868$ , resp.; wavelength  $\lambda = 4 \cdot 10^{-5}\text{cm}$ ; refractive index  $n = 1.392$ .

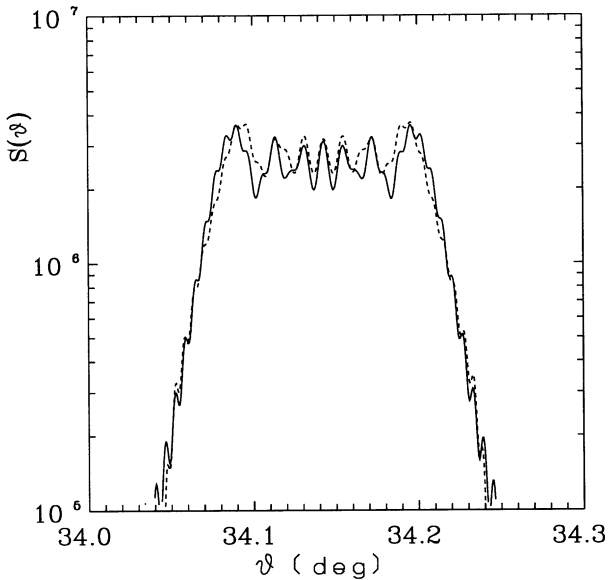


Figure 2: The same as in Fig.1, but for  $R_0 = 1\text{m}$ .

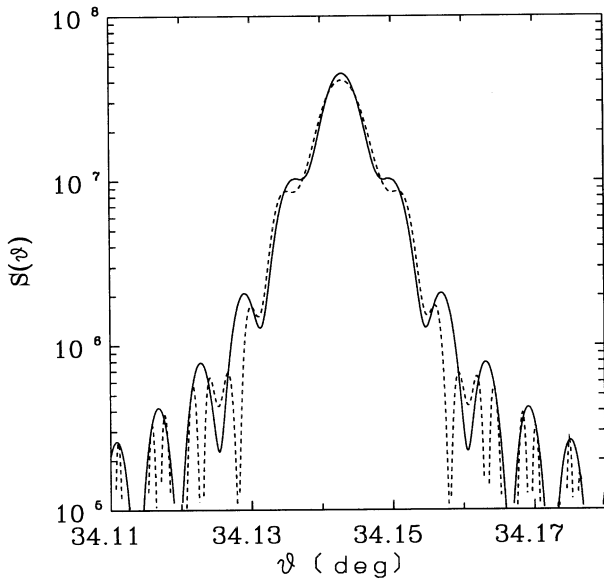


Figure 3: The same as in Fig.1, but for  $R_0 = 10m$  .

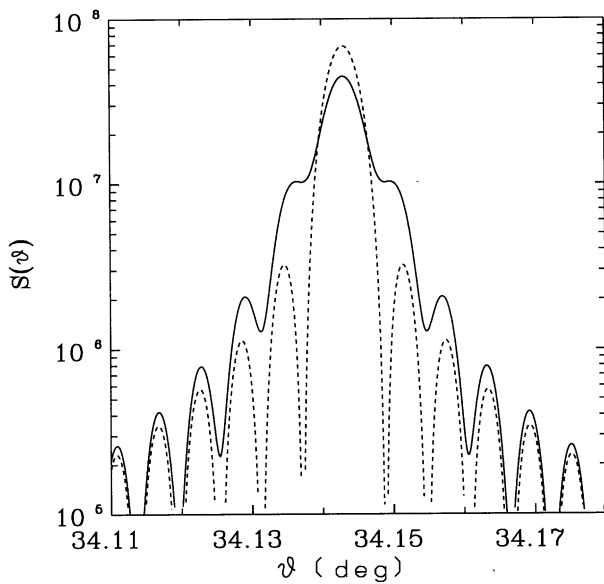


Figure 4: Exact intensity for  $R_0 = 10m$  (solid line) versus Tamm's intensity (broken line) which corresponds to  $R_0 = \infty$ .

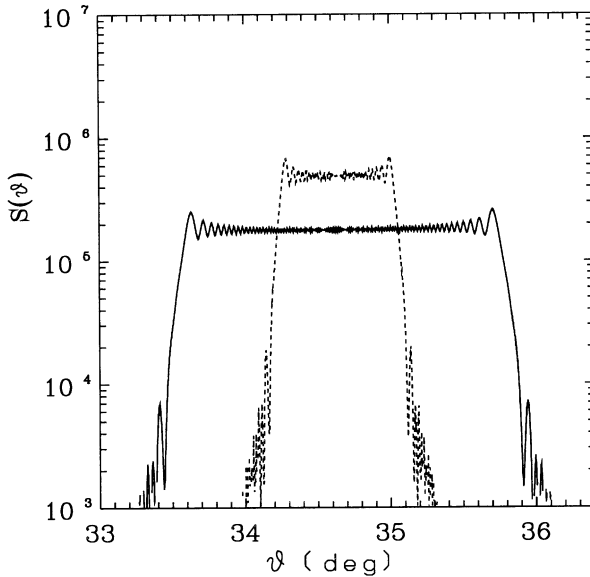


Figure 5: Fresnel intensities (in units  $e^2/c$ ) for decelerated motion of charge for  $R_0 = 10\text{cm}$  (solid line) and  $R_0 = 1\text{m}$  (broken line). Initial and final velocities are  $\beta_i = 0.875$  and  $\beta_f = 0.861$ , resp. Other parameters are the same as in Fig. 1.

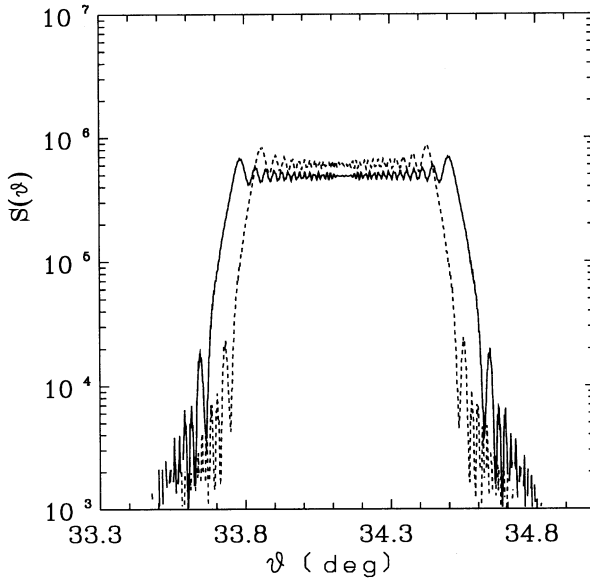


Figure 6: The same as in Fig.5, but for  $R_0 = 1\text{m}$  (solid line) and  $R_0 = \infty$  (broken line).

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Новые формулы для интенсивности излучения в задаче Тамма

Получены аналитические выражения, описывающие интенсивность излучения заряда, движущегося на конечном пространственном интервале. Они имеют большую область применимости по сравнению с известной формулой Тамма. Кроме того, учтены эффекты возможного ускорения заряда.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

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New Formulae for the Radiation Intensity in Tamm's Problem

Analytical formulae are obtained which describe the radiation intensity of a charge moving on a finite space interval. They have greater range of applicability than Tamm's original formula. In addition, the effects arising from the possible acceleration of charge are taken into account.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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